

# Stochastic Modeling And Analysis of A Repairable Single Unit Systems.

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## Abstract

In this paper we study the stochastic modeling and analysis of a repairable single unit system using Weibull distribution as it models a broad range of random variables, largely in the nature of a time to failure or time between events. Here we have considered three types of failures- abrupt, wear-out, and intermittent failures and concept of MOT (Maximum operation time) in which system goes under preventive maintenance. It is assumed that all types of failure and repair rates follow the Weibull distribution, and a single repairman can attend to all kinds of failures and preventive maintenance. By employing Semi-Markov, Regenerative Point Techniques and the Weibull distribution various system performance measures, cost analysis and busy period analysis also examined in this model. In addition, by using some system performance measures we provide the numerical illustration with numerically and graphically.

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**Keywords:** Availability, preventive maintenance, Weibull distribution, M.O.T.

## 1 Introduction

Reliability analysis plays a pivotal role in ensuring the performance of complex systems across various domains, from manufacturing and healthcare to transportation and telecommunications. The assessment of a system's reliability not only aids in minimizing downtime and optimizing maintenance schedules but also contributes significantly to cost-effective operations and enhanced safety. The study of reliability in single unit repairable systems has long been a subject of interest within the field of reliability engineering. Researchers have endeavoured to develop models that accurately depict the dynamic behaviour of such systems, accounting for the inherent complexities of the repair processes and various failure modes. Early works by Barlow and Hunter (1960) [1] laid the foundation for understanding the reliability of repairable systems, emphasizing the importance of incorporating failure and repair rates into models. The application of Semi-Markov and Regenerative Point Techniques in reliability modeling is a relatively recent development. This technique, as introduced by Grabski and Tondel (2010), focuses on identifying regenerative points within a system's life cycle, enabling a more detailed and accurate modeling of stochastic processes. The Weibull distribution has been widely used in reliability analysis due to its flexibility in modeling the distribution of failure times. Researchers have successfully integrated the Weibull distribution into various reliability models, enhancing their accuracy in predicting failure and repair times. This integration is particularly relevant when dealing with wear-out failures, where the Weibull distribution can capture the characteristic wear-out behavior. D Kuntal, Kumar Ashish, et al. [2] studied the analysis of the redundant system of non-identical units using the Weibull Distribution. Another study by Hemant Kumar, Pathak [3] has been carried out in Stochastic Modelling and Reliability Analysis of an R O Membrane System Used in Water Purification System with Patience – Time for Repair. Indeewar Kumar, Ashish, et al. [4] further extended their work in Stochastic modeling of non-identical redundant systems with priority, preventive maintenance, and

Weibull failure and repair distributions. Ashish Kumar, Monika S Barak, Kuntal Devi [5] approached a reliability model of a redundant system having one original and one duplicate unit developed with an immediate repair facility. Repairman conducts the preventive maintenance of the unit after a pre-specific time to enhance the performance and efficiency of the system. All random variables follow the Weibull distribution. Ashish Kumar and Monika Saini [6] carried out their work with the objective to perform RAMD analysis, and Failure Modes and Effects Analysis (FMEA) unified with the development of a novel stochastic model using Markovian approach to estimate the Steady-State Availability (SSA) of the TIUP. Nivedita Gupta, Ashish Kumar, and Monika Saini [7] investigate various reliability measures of generators used in STP through the RAMD approach at the component level. For this purpose, mathematical models using the Markovian birth-death process have been developed for all subsystems of the generator. further Hemant Kumar saw and V.K.Pathak [8] have extended their work in reliability modeling and analysis of single unit system with environmental failure and PM AT MOT.

In this Paper authors have paid attention to a single unit repairable systems which can be repaired using minimum scheduled maintenance. Three types of failures are taken abrupt, wear out and intermittent failures. The characterization of the three types of failures individually, there remains a gap in the integration of these elements with the Weibull distribution. The majority of the studies' researchers assumed that all random variables contributing to the unit's failure time distributed exponentially and repair intervals were either randomly or constantly distributed. However, the majority's performance of most of the industrial systems changes as time goes on. Thus, their repair and failure are not always constantly distributed they can also act as a arbitrary distribution. So, weibull distribution act as essential alternative to exponential distribution. This research seeks to bridge this gap by applying Semi-Markov Regenerative Point techniques and the Weibull distribution to develop a comprehensive model that accounts for abrupt, wear-out, and intermittent failures within a single unit repairable system, offering a holistic approach to reliability analysis that combines the benefits of both techniques.

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## 2 Model description

In this study system without standby unit is taken. The performance of the systems gets effected due to abrupt, wear-out, and intermittent failures but requires less maintenance. An abrupt failure refers to a sudden and unexpected breakdown, so it leads to total failure of the system, intermittent failure includes unpredictable, irregular fault which can also lead to total failure whereas wear-out failure occurs due to fatigue. So system will experience partial failure. After Maximum operation time preventive maintenance has been applied to the system to ensure the continuous operation of the system. The system restores its normal working condition after the fault is removed by the single repairman. All random variables are statistically independent. The performance of most of the systems varies with respect to passes of time. So, their repair and failure are not necessarily constantly distributed but may behave as any arbitrary distribution. So, here Weibull distribution is useful.

### Assumptions

For modeling of the system following assumptions have been taken.

1. A single repairman always available for the failure situation.
2. Abrupt and intermittent failures lead system to total failure and wearout failure leads to partial failure.
3. When the failure of the unit is detected the regular repairmen immediately attends the unit.
4. The system might fail from a degraded state, repair has been performed in failed and partially failed state.
5. Preventive maintenance has been applied to the system after Maximum operation time.
6. All random variables are statistically distributed.
7. Both the failure and repair rate follows the weibull distribution.
8. It is possible to simulate both increasing and decreasing failures rates using the Weibull failure distribution.

### System Notation:

P M	Preventive Maintenance
MTSF	Mean time to system failure
/ $*$	Symbol for Laplace -Steiltjes transform/ Laplace Transform
$\odot$	Symbol for Laplace-Stieltjes convolution /Laplace convolution
$S_0$	Initially the components are in working condition and the system works with full efficiency
$S_1$	The system gets partially failed due to wearout failure.
$S_2$	The system is failed due to abrupt failure.
$S_3$	System is failed due to intermittent failure.
$S_4$	Preventive Maintenance state.
$\alpha/\beta/\gamma/\lambda/\theta/\chi/k/h/l$	Scale Parameter.
$\eta > 0$	Common Shape Parameter.
$f(t)$	Probability density function and cumulative density function of time for P.M after maximum operation time.
$g_i(t)/G_i(t)$	pdf /cdf of repair time for

	partial failure due to wearout, Total failure due to abrupt and intermittent failure.
$f_1(t)$	PDF of failure rate of normal to partial failure due to wearout failure.
$f_2(t)$	PDF of failure rate from normal to Total failure due to abrupt failure.
$f_3(t)$	PDF of failure rate of Total failure due to intermittent failure.
$f_4(t)$	PDF of failure rate from partial failure to Total failure due to wearout failure.

$f_1(t)$	$= \lambda = \lambda \eta t^{\eta-1} e^{-(\lambda t^\eta)} dt$
$f_2(t)$	$= \gamma = \gamma \eta t^{\eta-1} e^{-(\gamma t^\eta)} dt$
$f_3(t)$	$= k \eta t^{\eta-1} e^{-(k t^\eta)} dt$
$f_4(t)$	$= \alpha \eta t^{\eta-1} e^{-(\alpha t^\eta)} dt$
$g_1(t)$	$= \theta \eta t^{\eta-1} e^{-(\theta t^\eta)} dt$
$g_2(t)$	$= l \eta t^{\eta-1} e^{-(l t^\eta)} dt$
$g_3(t)$	$= h \eta t^{\eta-1} e^{-(h t^\eta)} dt$
$g_4(t)$	

	$= \beta =$ $\beta \eta t^{\eta-1} e^{-(\beta t^\eta)}$
$f(t)$	$= \chi =$ $\chi \eta t^{\eta-1} e^{-(\chi t^\eta)}$

Table 1: Set Of Notations

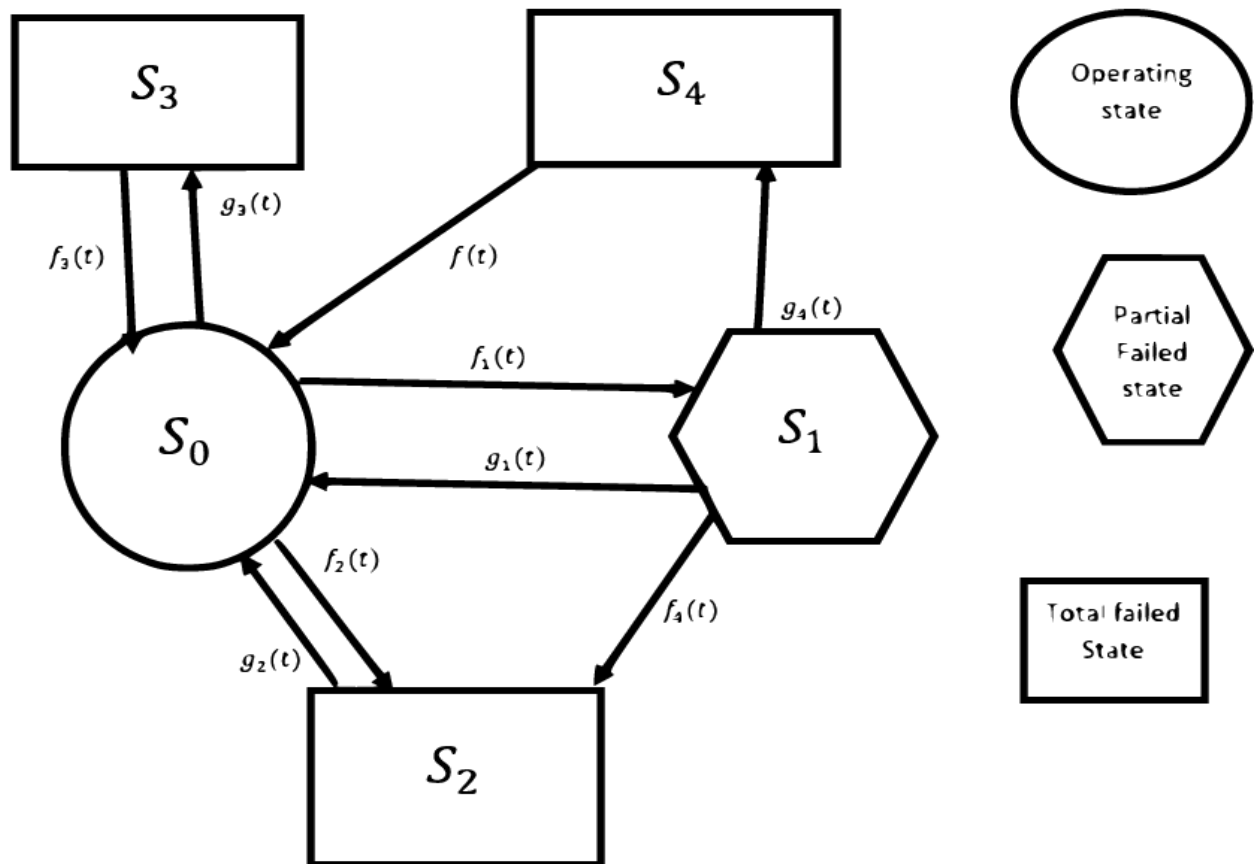


Figure 1: State Transition diagram

### 3 Reliability Analysis Of The Steystem

#### 3.1 Transition Probabilities and Mean Sojourn Time

Using simple Probailistic formula,the expressions for transition probabilities in steady state are as follows:

$$P_{ij} = Q_{ij}(\infty) = \int q_{ij}(t) dt \quad (3.1)$$

$$Q_{01}(t) = \int_0^\infty f_1(t) \overline{F(t)} G_2(t) dt = \int_0^\infty \lambda \eta t^{\eta-1} e^{-(\lambda t^\eta)} \cdot e^{-(\alpha t^\eta)} e^{-(\theta t^\eta)} dt \quad (3.2)$$

$$= \lambda \int_0^\infty \eta t^{\eta-1} \cdot e^{-(\lambda+\alpha+\theta) \cdot t^\eta} dt \quad (3.3)$$

put,

$$t^\eta = z$$

$$= \lambda \int_0^\infty e^{-(\lambda+\alpha+\theta)z} dz \quad (3.4)$$

$$= \lambda \left[ \frac{e^{-(\lambda+\alpha+\theta)z}}{-(\lambda+\alpha+\theta)} \right]_0^\infty \quad (3.5)$$

$$= \frac{\lambda}{-(\lambda+\alpha+\theta)} [0 - 1] \quad (3.6)$$

$$= \frac{\lambda}{\lambda+\alpha+\theta} \quad (3.7)$$

$$Q_{02}(t) = \int_0^\infty g_2(t) \overline{F_1(t)} F(t) dt = \int_0^\infty \theta \eta t^{n-1} e^{-(\theta t^n)} \cdot e^{-(\lambda t^n)} e^{-(\alpha t^n)} dt \quad (3.8)$$

$$Q_{03}(t) = \int_0^\infty f(t) \overline{F_1(t)} G_2(t) dt = \int_0^\infty \alpha \eta t^{n-1} e^{-(\alpha t^n)} \cdot e^{-(\lambda t^n)} e^{-(\theta t^n)} dt \quad (3.9)$$

$$P_{01} + P_{02} + P_{03} = \frac{\lambda}{\lambda+\alpha+\theta} + \frac{\theta}{\lambda+\alpha+\theta} + \frac{\alpha}{\lambda+\alpha+\theta} = 1 \quad (3.10)$$

$$Q_{10}(t) = \int_0^\infty g_r(t) \overline{F_2(t)} \cdot \overline{F_3(t)} dt = \int_0^\infty k \eta t^{n-1} e^{-(k t^n)} \cdot e^{-(\beta t^n)} \cdot e^{-(\lambda t^n)} dt \quad (3.11)$$

$$P_{10} = \frac{k}{k+\beta+\lambda} \quad (3.12)$$

$$Q_{12} = Q_{12}(t) = \int_0^\infty f_3(t) \cdot \overline{G_r(t)} \cdot \overline{F_2(t)} dt = \int_0^\infty \gamma \eta t^{n-1} e^{-(\lambda t^n)} \cdot e^{-(k t^n)} \cdot e^{-(\beta t^n)} dt \quad (3.13)$$

$$P_{12} = \frac{\gamma}{k+\beta+\lambda} \quad (3.14)$$

$$Q_{14} = Q_{14}(t) = \int_0^\infty f_2(t) \cdot \overline{G_r(t)} \cdot \overline{F_3(t)} dt = \int_0^\infty \beta \eta t^{n-1} e^{-(\beta t^n)} \cdot e^{-(k t^n)} \cdot e^{-(\gamma t^n)} dt \quad (3.15)$$

$$P_{14} = \frac{\beta}{\beta+k+\lambda} \quad (3.16)$$

$$p_{10} + p_{12} + p_{14} = 1 \quad (3.17)$$

$$Q_{20}(t) = \int_0^\infty g_{re}(t) \overline{F(t)} dt = \int_0^\infty h \eta t^{n-1} e^{-(h t^n)} \cdot e^{-(\alpha t^n)} dt \quad (3.18)$$

$$= \frac{h}{h+\alpha} = P_{20} \quad (3.19)$$

$$P_{20} = 1 \quad (3.20)$$

$$Q_{30}(t) = \int_0^\infty f(t) dt = \int_0^\infty \alpha \eta t^{n-1} e^{-(\alpha t^n)} dt \quad (3.21)$$

$$P_{30} = 1 \quad (3.22)$$

$$Q_{40}(t) = \int_0^\infty g_r(t) dt = \int_0^\infty k \eta t^{n-1} e^{-(k t^n)} dt \quad (3.23)$$

$$P_{40} = 1 \quad (3.24)$$

Let T denote the time to system failure then the mean sojourn times  $(\mu_i)$  in the state  $S_i$  are given by

$$(\mu_i) = E(t) = \int_0^\infty P[T > t] dt \quad (3.25)$$

Therefore, the mean sojourn times  $(\mu_i)$  at regenerative states  $S_i$  are as follows:-

$$\mu_0(t) = \int_0^\infty \overline{F_1(t)} G_2(t) F(t) dt \quad (3.26)$$

$$= \frac{\Gamma(1+\frac{1}{\eta})}{(\alpha+\lambda+\theta)^{\frac{1}{\eta}}} \quad (3.27)$$

$$\mu_1(t) = \int_0^\infty \overline{G_r(t)F_2(t)F_3(t)} dt = \frac{\Gamma(1+\frac{1}{\eta})}{(\gamma+k+\beta)^{\frac{1}{\eta}}} \quad (3.28)$$

$$\mu_2(t) = \int_0^\infty \overline{G_{re}(t)F(t)} dt = \frac{\Gamma(1+\frac{1}{\eta})}{(h+\alpha)^{\frac{1}{\eta}}} \quad (3.29)$$

$$\mu_3(t) = \int_0^\infty \overline{G_1(t)} dt = \frac{\Gamma(1+\frac{1}{\eta})}{(l)^{\frac{1}{\eta}}} \quad (3.30)$$

#### 4 Mean Time To System Failure:

Let  $\pi_i(t)$  be the c.d.f of first passage time from the regenerative state  $S_i$  to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for  $\pi_i(t)$  :

$$\pi_i(t) = \sum_j Q_{ij}(t) \odot \pi_j(t) + \sum_k Q_{i,k}(t) \quad (4.1)$$

$$\pi_0(t) = Q_{01}(t) \odot \pi_1(t) + Q_{02}(t) + Q_{03}(t) \quad (4.2)$$

$$\pi_1(t) = Q_{10}(t) \odot \pi_0(t) + Q_{12}(t) + Q_{14}(t) \quad (4.3)$$

We are taking Laplace Steiltjes transform of the above equation:

$$(\pi_0^{**}, \pi_1^{**}) = Q^{**(-1)}(Q_{02}^{**} + Q_{03}^{**}, Q_{12}^{**} + Q_{14}^{**}) \quad (4.4)$$

where,

$$Q^{**(-1)} = \begin{bmatrix} 1 & -Q_{01}^{**} & -Q_{02}^{**} & -Q_{03}^{**} & 0 \\ -Q_{10}^{**} & 1 & -Q_{12}^{**} & 0 & Q_{14}^{**} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_0^{**} \\ \pi_1^{**} \\ \pi_2^{**} \\ \pi_3^{**} \\ \pi_4^{**} \end{bmatrix} = \begin{bmatrix} Q_{02}^{**} + Q_{03}^{**} \\ Q_{12}^{**} + Q_{14}^{**} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (4.5)$$

Now, by solving the above equation for  $\pi_0^{**}(s)$ , we get

$$\pi_0^{**}(s) = \frac{N(s)}{D(s)} \quad (4.6)$$

where;

$$D(s) = \begin{vmatrix} -1 & Q_{01}^{**} & Q_{02}^{**} & Q_{03}^{**} & 0 \\ Q_{10}^{**} & -1 & Q_{12}^{**} & 0 & Q_{14}^{**} \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{vmatrix} \quad (4.7)$$

and

$$N(s) = \begin{vmatrix} Q_{02} + Q_{03} & Q_0^{**} & Q_{02}^{**} & Q_{03}^{**} & 0 \\ Q_{12} + Q_{14} & -1 & Q_{12}^{**} & 0 & Q_{14}^{**} \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{vmatrix} \quad (4.8)$$

Therefore,

$$D = 1 - p_{01}p_{10} \quad (4.9)$$

and,

$$N = \mu_0 + \mu_1 p_{01} \quad (4.10)$$

The MTSF when the given system starts from the state '0' is:

$$MTSF = \lim_{s \rightarrow 0} \frac{1 - \phi_0^*(s)}{s} = \frac{D'(0) - N'(0)}{D(0)} = \frac{N}{D} \quad (4.11)$$

$$E(T) = \frac{\mu_0 + \mu_1 p_{01}}{1 - p_{01} p_{10}} \quad (4.12)$$

## 5 Availability:

The availability  $A_i(t)$  of a system is defined as the probability that the system is in up-state and provide service when needed, at the instant 't' given that the system entered regenerative state  $S_i$  at  $t=0$ . The recursive relations for  $A_i(t)$  are given as

$$A_0(t) = M_0(t) + q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) + q_{03}(t) \odot A_3(t) \quad (5.1)$$

$$A_1(t) = M_1(t) + q_{10}(t) \odot A_0(t) + q_{12} + q_{14} \quad (5.2)$$

$$A_2(t) = M_2(t) + q_{20}(t) \odot A_0(t) + q_{23} \quad (5.3)$$

$$A_3(t) = q_{30}(t) \odot A_0(t) \quad (5.4)$$

$$A_4(t) = q_{40}(t) \quad (5.5)$$

Applying Laplace transformation of the above relations we get,

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)} \quad (5.6)$$

Writing in matrix form:

$$(A_0^*, A_1^*, A_2^*, A_3^*, A_4^*) = q^{-1} (M_0^*, M_1^*, M_2^*, 0, 0) \quad (5.7)$$

Where,

$$q^{-1} = \begin{bmatrix} 1 & -q_{01}^* & -q_{02}^* & -q_{03}^* & 0 \\ -q_{10}^* & 1 & -q_{12} & 0 & -q_{14} \\ -q_{20}^* & 0 & 1 & 0 & 0 \\ -q_{30}^* & 0 & 0 & 1 & 0 \\ -q_{40}^* & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_0^* \\ A_1^* \\ A_2^* \\ A_3^* \\ A_4^* \end{bmatrix} = \begin{bmatrix} M_0^* \\ M_1^* \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (5.8)$$

Therefore,

$$N_1(S) = \begin{vmatrix} M_0 & q_{01}^* & q_{02}^* & q_{03}^* & 0 \\ M_1 & -1 & q_{12} & 0 & q_{14} \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{vmatrix} \quad (5.9)$$

$$N_1 = \mu_0 + \mu_1 p_{01} \quad (5.10)$$

## 6 Busy Period Of Regular Repair Person:

Let  $B_i(t)$  be the probability that the server is busy repairing the unit at an instant 't' given that the system entered state  $S_i$  at  $t=0$ .

The steady state, the function of time for which the regular repair facility is busy in repair is given by:-

$$B_0 = \lim_{s \rightarrow 0} s B_0^*(s) \quad (6.1)$$

$$= \frac{N_3(s)}{D_1(s)} \quad (6.2)$$

$$B_0(t) = q_{01}(t) \odot B_1(t) + q_{02}(t) \odot B_2(t) + q_{03}(t) \odot B_3(t) \quad (6.3)$$



$$B_1(t) = u_1(t) + q_{10}(t) \odot B_0(t) + q_{12}(t) \odot B_2(t) + q_{14}(t) \odot B_4(t) \quad (6.4)$$

$$B_2(t) = u_2(t) + q_{20}(t) \odot B_0(t) \quad (6.5)$$

$$B_3(t) = u_3(t) + q_{30}(t) \odot B_0(t) \quad (6.6)$$

$$B_4(t) = q_{40}(t) \odot B_0(t) \quad (6.7)$$

Therefore,

$$N_3(S) = \begin{vmatrix} 0 & q_{01}^* & q_{02}^* & q_{03}^* & 0 \\ u_1 & -1 & q_{12} & 0 & q_{14} \\ u_2 & 0 & -1 & 0 & 0 \\ u_3 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{vmatrix} \quad (6.8)$$

$$N_3 = \mu_1 p_{01} + \mu_2 (p_{01} p_{12} + p_{02}) + \mu_3 p_{03} \quad (6.9)$$

## 7 Due To Preventive Maintenance

Let  $P_i(t)$  be the probability that the system is under preventive maintenance by a regular repair person at the time 't'. Now, by solving the above equations given for  $P_0^*(s)$ . we get a busy period of the server due to adaptive maintenance given by:-

$$P_0^* = \lim_{s \rightarrow 0} P_0^*(s) \quad (7.1)$$

$$= \frac{N_4(s)}{D_1(s)} \quad (7.2)$$

$$P_0(t) = q_{01}(t) \odot P_1(t) + q_{02}(t) \odot P_2(t) + q_{03}(t) \odot P_3(t) \quad (7.3)$$

$$P_1(t) = q_{10}(t) \odot P_0(t) + q_{12}(t) \odot P_2(t) + q_{14}(t) \odot P_4(t) \quad (7.4)$$

$$P_2(t) = q_{20}(t) \odot P_0(t) \quad (7.5)$$

$$P_3(t) = q_{30}(t) \odot P_0(t) \quad (7.6)$$

$$P_4(t) = w_4(t) + q_{40}(t) \odot P_0(t) \quad (7.7)$$

Where,

$$N_4 = \mu_4 p_{01} p_{14} \quad (7.8)$$

### Case studies with discussions :

(I) When shape parameter  $\eta = 0.5$  then failure of the unit due to abrupt, wear-out, and intermittent failures, adaptive maintenance, repair by regular repairman time distribution reduces to:-

$$f_1(t) = \frac{\lambda}{2\sqrt{t}} e^{(-\lambda\sqrt{t})}, \quad f_2(t) = \beta \eta t^{\eta-1} e^{-(\beta t^\eta)} dt = \frac{\beta}{2\sqrt{t}} e^{(-\beta\sqrt{t})}, \quad g_{re}(t) = \frac{h}{2\sqrt{t}} e^{(-h\sqrt{t})}, \quad f_3(t) = \frac{\gamma}{2\sqrt{t}} e^{(-\gamma\sqrt{t})}, \quad f(t) = \frac{\alpha}{2\sqrt{t}} e^{(-\alpha\sqrt{t})},$$

$$g_2(t) = \frac{\theta}{2\sqrt{t}} e^{(-\theta\sqrt{t})}, \quad g_r(t) = \frac{k}{2\sqrt{t}} e^{(-k\sqrt{t})}, \quad g_1(t) = \frac{l}{2\sqrt{t}} e^{(-l\sqrt{t})}.$$

(II) When shape parameter  $\eta = 1.0$  then repair/ failure of the unit, adaptive maintenance, time distribution reduces to exponential then:-

$$f_1(t) = \lambda e^{-(\lambda t)}, \quad f_2(t) = \beta e^{-(\beta t)}, \quad f_3(t) = \gamma e^{-(\gamma t)}, \quad f(t) = \alpha e^{-(\alpha t)}, \quad g_2(t) = \theta e^{-(\theta t)}, \quad g_r(t) = k e^{-(k t)}, \quad g_{re}(t) = h e^{-(h t)}, \quad g_1(t) = l e^{-(l t)}$$

(III) When shape parameter  $\eta = 2.0$  then failure /arrival time of the server/repair time distributions

reduces to rayleigh having the pdf:-

$$f_1(t) = 2\lambda e^{-(\lambda t^2)}, f_2(t) = 2\beta e^{-(\beta t^2)}, f_3(t) = 2\gamma e^{-(\gamma t^2)}, f(t) = 2\alpha e^{-(\alpha t^2)}, g_2(t) = 2\theta e^{-(\theta t^2)}, g_r(t) = 2k e^{-(k t^2)}, g_{re}(t) = 2h e^{-(h t^2)}, g_1(t) = 2l e^{-(l t^2)}$$

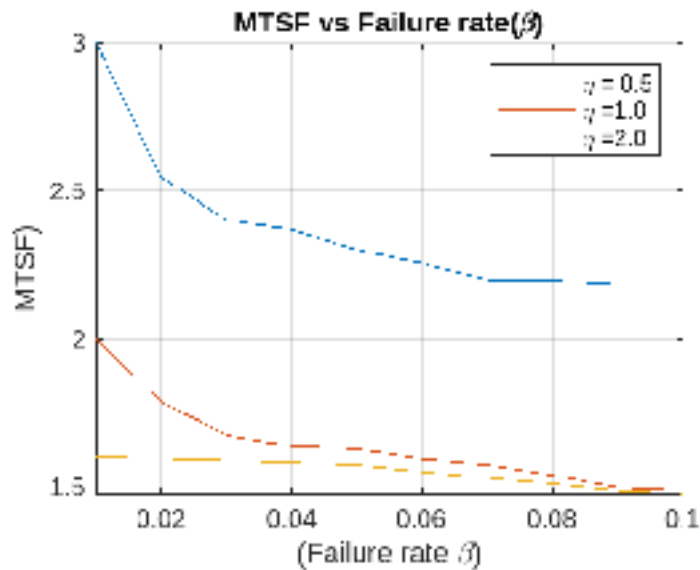


Figure 2: MTSF Vs FAILURE RATE

**Graphical analysis :** The curve for MTSF and Failure rate have been drawn for different values of shape parameters and values of repair rate of the repair person, preventive maintenance, and failure rates have been depicted. To See the behaviour of different parameter on the system, we take shape parameter  $\eta = 0.5, 1.0, 2.0$  from fig number 2 we observe that MTSF decreases as the failure  $\beta$  increases from 0.04 to 0.1.

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