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Distributed Formation Tracking Control of Multi-Agent Systems Based on Graph Rigidity

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Abstract

Multi-agent systems have increasingly become a key area of interest across diverse fields, ranging from civil to military applications, with distributed formation tracking control of multiple agents serving as the core for performing complex tasks. This paper investigates the formation tracking control problems for multi-agent systems within the framework of graph rigidity theory. A formation control protocol is developed to achieve the desired formation, to track and surround a dynamic target with unknown velocity. The control law is based on the graph rigidity matrix and is a function of the relative positions of agents within an infinitesimally and minimally rigid graph that models the formation. The leader-follower strategy is employed in the proposed scenario. Finally, the digital simulation is conducted to validate the effectiveness of the proposed protocol.

Keywords: Distributed formation tracking control, Coordination control, Multi-agent systems, Graph Rigidity, Graph theory, Leader-follower strategy.

1. Introduction

A multi-agent system (MAS) consists of a team of interacting agents that collaboratively perform large-scale, complex tasks. This approach offers several advantages over relying on a single large agent, such as enhanced task execution efficiency, scalability, adaptability, versatility, and robust performance in the event of agent failures [1,2]. In engineering applications, MAS is exemplified by groups of autonomous vehicles deployed for tasks such as tracking, surveillance, mapping, and disaster monitoring.

Multi-agent formation control refers to the design of control inputs for the agents to achieve and maintain a predefined geometric arrangement in space. The theory behind formation control for MAS is grounded in mathematical concepts from graph theory. This framework provides an effective method for describing the MAS formation shape, as well as the inter-agent sensing, communication, and control topology in distributed settings. A critical subfield of graph theory, rigid graph theory, plays a significant role in solving the formation control problem by addressing inter-agent distance constraints. In this context, the desired formation is enforced through graph rigidity, ensuring that agent collisions are avoided while maintaining the formation. Additionally, the concept of graph rigidity is applied to determine the minimum number of inter-agent distances required to ensure the stability and integrity of the MAS formation. Relevant literature on rigid graph theory and its application to multi-agent formation control is discussed in [3, 4].

A fundamental requirement for solving formation problems is the establishment and maintenance of a predefined geometric configuration in space [5,6]. However, in many practical applications, the formation control of multiple agents involves not only achieving the desired formation but also tracking a target. In [7], a target tracking approach was introduced for multi-agent systems, utilizing artificial potentials, and sliding mode control to track a target with a known velocity but unknown acceleration. Furthermore, an adaptive internal model controller was proposed for multi-agent dynamic systems with switched dynamics, maintaining a predefined formation along a reference trajectory [8]. In [9], the authors examined the effects of mismatches in the inter-agent distances on the gradient-based rigid formation control. Their results demonstrated that such mismatches lead to steady-state motion and distortions in the final formation shape. Regarding formation control strategies, these are generally categorized into behavior-based approaches, virtual structure methods, and leader-follower strategies [10]. The behavior-based approach does not provide a clear definition of the system's overall behavior and poses challenges in ensuring the stability of the formation control. The virtual structure method, while useful, lacks adaptability and flexibility. In [11], the concept of a rigidity graph was introduced, along with a formation control law that assumes the global positions of both leader and follower agents. Additionally, [12] provided an approach for analyzing the rigidity of directed formations under the leader-follower strategy. Therefore, the leader-follower strategy offers the advantage of simplicity in design, making it particularly well-suited for tracking tasks, especially when dealing with dynamic targets.

Motivated by the current research status and development trends, the realization of distributed formation tracking control for MAS using the graph rigidity approach holds significant engineering importance. In this work, we address the distributed formation problem where each agent only has locally sensed information about adjacent agents, obtained through sensors. The agents are tasked with both constructing and maintaining the desired formation while simultaneously tracking the moving target. For the purposes of this study, we employ the leader-follower strategy, assigning the leader role to one agent in the formation, who is primarily responsible for tracking the moving target. The relative position of the target with respect to the leader is known and can be communicated to the follower agents, while the target's velocity remains unknown to all agents. The rest of the paper is organized as follows. Section 2 provides the background on graph theory and presents some preliminary results related to graph rigidity, as well as infinitesimal and minimal rigidity. Section 3 outlines the problem statement. Section 4 proposes the design and stability analysis of the distributed formation tracking control. The visual simulation is described in Section 5. Finally, the conclusions and future developments are provided in Section 6.

2. Preliminaries

2.1 Basics of graph theory

A set of agents and their interactions are conceptually represented by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of vertices representing n agents in the group, $\mathcal{V} = \{1,2, \dots, n\}$, the edge set $\mathcal{E}$ represents the interaction between the agents, this interaction can be directed or undirected edge, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The cardinality of $\mathcal{V}$ and $\mathcal{E}$ are $|\mathcal{V}| = n$, and $|\mathcal{E}| = m$, respectively. In addition, the graph is also represented in the form $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$, and the set of weights $\mathcal{W}$ is used to represent various information such as the importance or priority of the links in the graph. The vertex i and vertex j are neighbors or adjacent if they are the ends of a common edge $ed = (i,j)^e \in \mathcal{E}, i,j \in \mathcal{V}$. The neighbors of the i^{th}agent is defined as $N_i = \{j \in \mathcal{V} | (i,j) \in \mathcal{E}\}$ [13].

The relationship of agents in MAS can be denoted by a directed or undirected graph. In this work, we will only be dealing with undirected graphs, so we will drop the undirected term when referring to a graph. For an edge $(i,j) \in \mathcal{E}$, the graph $\mathcal{G}$ has a weighting $w_{ij} \in \mathcal{W}$. The weighted adjacency matrix is defined as $\boldsymbol{\mathcal{A}} = [w_{ij}] \in \mathbb{R}^{n \times n}$. In the case where the weight is 1 or 0, then we can write the weight $a_{ij} \in \mathcal{W}$, and the weighted adjacency matrix is simply called adjacency matrix $\boldsymbol{\mathcal{A}} = [a_{ij}] \in \mathbb{R}^{n \times n}$. For the adjacency elements $a_{ij} = 1$ if and only if $(i,j) \in \mathcal{E}$, otherwise $a_{ij} = 0$.

2.2 Graph rigidity theory

A framework is a realization of a graph at given points in Euclidean space. Specifically, if $p_i \in \mathbb{R}^d$ (d-dimensional Euclidean space, $d \geq 2$) is the position values of agents i with respect to some fixed coordinate frame and $\boldsymbol{p} = [p_1^T, \dots, p_n^T]^T \in \mathbb{R}^{dn}$ is a configuration of n agents, then a framework F is a pair $(\mathcal{G}, \boldsymbol{p})$. The rigidity theory studies conditions for a unique realization of the framework when the constraints between neighboring agents of MAS are specified by some scalar or vector magnitude. If the magnitudes are distances in undirected edges of the graph, then this theory is called graph rigidity or distance rigidity. From the length of the edges in a framework, we define the edge function as:

$$\boldsymbol{h}_G(\boldsymbol{p}) \triangleq \left[\dots, \left\|p_i - p_j\right\|^2, \dots\right]^T, \qquad (i,j) \in \mathcal{E} \tag{1}$$

where $\|\cdot\|$ denotes the Euclidean norm (2-norm).

Consider two frameworks $(\mathcal{G}, \boldsymbol{p})$ and $(\mathcal{G}, \boldsymbol{q})$, where $\boldsymbol{q}$ is a configuration different from $\boldsymbol{p}$ in the same dimensional Euclidean space $\mathbb{R}^d$. If $\|p_j - p_i\| = \|q_j - q_i\|, \forall (i,j) \in \mathcal{E}$, then the two frameworks are called equivalent to each other, i.e., $\boldsymbol{h}_{\mathcal{G}}(\boldsymbol{p}) = \boldsymbol{h}_{\mathcal{G}}(\boldsymbol{q})$. If $\|p_j - p_i\| = \|q_j - q_i\|, \forall i,j \in \mathcal{V}, i \neq j$, the two frameworks are called congruent, i.e., all distances between agents are the same. In the case where two frameworks are congruent, then they are said to be isomorphic in $\mathbb{R}^d$, and the set of all frameworks that are isomorphic to F is denoted by the symbol $Iso(F)$. Additionally, if two frameworks are equivalent but not congruent, then they are said to be ambiguous, and the set of all ambiguities of framework F is denoted by $Amb(F)$ [14].

A framework $F = (\mathcal{G}, \boldsymbol{p})$ is rigid in $\mathbb{R}^d$ if all of its motions satisfy $p_i(t) = \mathcal{T}(p_i), \forall i \in \mathcal{V}$, and $\forall t \in [0,1]$, where $\mathcal{T}$ accounts for rotation and/or translation of the vector p_i, it means that the family of frameworks $F(t)$ is isomorphic [15].

In general, if the MAS has many agents, it is difficult to determine rigidity from the definition. Thus, we will consider a subset of rigidity called infinitesimal rigidity [16], in which the first-order preservation of distances has remained during an infinitesimal motion. The infinitesimal rigidity can be easily verified via a rigidity matrix rank. The infinitesimal rigidity implies rigidity, but the rigidity does not imply infinitesimal rigidity. The rigidity matrix $\boldsymbol{\mathcal{R}}(\boldsymbol{p}) \in \mathbb{R}^{m \times dn}$ of the framework $(\mathcal{G}, \boldsymbol{p})$ is defined as:

$$\boldsymbol{\mathcal{R}}(\boldsymbol{p}) = \frac{1}{2}\frac{\partial \boldsymbol{h}_{\mathcal{G}}(\boldsymbol{p})}{\partial \boldsymbol{p}} \tag{2}$$

where $\boldsymbol{h}_{\mathcal{G}}(\boldsymbol{p})$ was given in (1).

Result 1 [16]: Based on the rigidity matrix, a framework $F = (\mathcal{G}, \boldsymbol{p})$ is infinitesimally rigid in $\mathbb{R}^d$ if and only if

$$rank\big(\boldsymbol{\mathcal{R}}(\boldsymbol{p})\big) = dn - \frac{d(d+1)}{2} \tag{3}$$

Result 2 [16]: Two frameworks, $F = (\mathcal{G}, \boldsymbol{p})$ and $F' = (\mathcal{G}, \boldsymbol{p}')$, have the same graph. If framework F is infinitesimally rigid and $dist(\boldsymbol{p}', Iso(F)) \leq \sigma$, where σ is a sufficiently small positive constant, then framework F' is also infinitesimally rigid.

This result leads to the corollary 2.1, which will be very useful for the control designs in this work and its proof can be found in [16].

Corollary 2.1: Consider $F = (\mathcal{G}, \boldsymbol{p})$ and $F' = (\mathcal{G}, \boldsymbol{p}')$ be two frameworks sharing the same graph $\mathcal{G}$, and let the function:

$$\Upsilon(F, F') = \sum_{(i,j)\epsilon\mathcal{E}} \left(\left\|p_i' - p_j'\right\| - \left\|p_i - p_j\right\|\right)^2 \tag{4}$$

If framework F is infinitesimally rigid and $\Upsilon(F, F') \leq \varepsilon$, where ε is a sufficiently small positive constant, then framework F' is also infinitesimally rigid.

The minimum number of edges that ensures rigidity has a crucial role in practice, as it guarantees that the formation of MAS is rigid with the minimum number of communication and sensing links. A graph is considered minimally rigid if it is rigid, and the removal of any edge causes the graph to lose its rigidity [14]. Similar to infinitesimal rigidity, we can verify the condition for minimal rigidity of the graph using the condition presented in Result 3.

Result 3 [14]: A rigid graph $\mathcal{G}$ is minimally rigid in $\mathbb{R}^d$ if and only if

$$m = rank(\boldsymbol{\mathcal{R}}(\boldsymbol{p})) = dn - \frac{d(d+1)}{2} \tag{5}$$

The recurrence of equations (3) and (5) leads to an important following corollary and its proof can be found in [14].

1. Problem statement

In this work, we consider a MAS composed of *n* agents. Each agent in the system can exchange information about states and control inputs with its neighbors. The desired formation for the agents is an infinitesimally and minimally rigid framework $F^* = (\mathcal{G}^*, \boldsymbol{p}^*)$, In which, $\mathcal{G}^* = (\mathcal{V}^*, \mathcal{E}^*)$, $\boldsymbol{p}^* = \left[p_1^{*T}, \dots, p_n^{*T}\right]^T$. The distance between agents is $d_{ij}(t) = \|p_i(t) - p_j(t)\|$. The desired distance between agents is $d_{ij}^* = \|p_i^* - p_j^*\| > 0, i, j \in \mathcal{V}^*$.

The specific design tasks of this study are as follows:

(i) ***Design a formation control protocol*** to ensure that the agents form and maintain a predefined geometric configuration in space. The control objective for this formation problem serves as the common and foundational objective for the other tasks. This objective is to design u_i such that:

$$d_{ij}(t) \to d_{ij}^* \text{ as } t \to \infty \;\; i, j \in \mathcal{V}^* \tag{6}$$

(ii) ***The agents track and encircle a moving target with a predefined formation***. In this problem, we employ the leader–followers strategy by taking the n^{th} agent to be the leader while the other agents are followers. The control protocol includes: Acquiring a desired formation F^*, the leader agent chasing the moving target, and the followers tracking and surrounding the leader while maintaining the desired formation F^*. Let the target position is denoted as $p_T(t) \in \mathbb{R}^d$, and the second objective is then expressed as:

$$p_T(t) \in \mathrm{conv}\{p_1(t), p_2(t), \dots, p_{n-1}(t)\} \text{ as } t \to \infty \tag{7}$$

2. Distributed Formation Tracking Control Design

Formation control problems generally emphasize the coordination between multiple agents and the control laws that regulate their interactions. In this study, to simplify the analysis, we adopt a point-like kinematic model. This approach allows for a more tractable examination of the control dynamics, providing insights into the fundamental principles of coordination without the complexity of higher-order models. Consider a system of n agents in $\mathbb{R}^d$, formulated with the single-integrator model:

$$\dot{p}_i = u_i, \quad i = 1,2, \dots, n \tag{8}$$

where $p_i \in \mathbb{R}^d$ represents the position and $u_i \in \mathbb{R}^d$ is the velocity-level control input of the i^{th} agent. The control objective for the first problem which acquires and maintains formation is to design $u_i = f(p_i - p_j, d_{ij}^*)$, $i = 1,2, \dots, n$, $\forall j \in N_i$. This control input will control the distances d_{ij}, $(i,j) \in \mathcal{E}$, such that:

$$d_{ij}(t) \longrightarrow d_{ij}^* \text{ as } t \longrightarrow \infty, \;\; (i,j) \in \mathcal{E} \tag{9}$$

Eq. (9) is equivalent to:

$$\boldsymbol{h}_{\mathcal{G}}(\boldsymbol{p}(t)) \longrightarrow \boldsymbol{h}_{\mathcal{G}}(\boldsymbol{p}^*) \text{ as } t \longrightarrow \infty \tag{10}$$

Define the relative position z_{ij} as

$$z_{ij} = p_i - p_j \tag{11}$$

The distance error e_{ij} is given by

$$e_{ij} = \|z_{ij}\| - d_{ij}^* = d_{ij} - d_{ij}^* \quad (12)$$

The error $\bar{e}_{ij}$ is given by

$$\bar{e}_{ij} = \|z_{ij}\|^2 - \left(d_{ij}^*\right)^2 = d_{ij}^2 - \left(d_{ij}^*\right)^2 \quad (13)$$

$\boldsymbol{z} = \left[\dots, z_{ij}^T, \dots\right]^T \in \mathbb{R}^{dm}$, $\boldsymbol{e} = \left[\dots, e_{ij}, \dots\right]^T \in \mathbb{R}^m$, $\bar{\boldsymbol{e}} = \left[\dots, \bar{e}_{ij}, \dots\right]^T \in \mathbb{R}^m$, $(i, j) \in \mathcal{E}$

From Eq. (1) and Eq. (13), we have that

$$\bar{\boldsymbol{e}} = \left[\dots, \bar{e}_{ij}, \dots\right]^T = \boldsymbol{h}_{\mathcal{G}}(\boldsymbol{p}) - \boldsymbol{h}_{\mathcal{G}}(\boldsymbol{p}^*) \quad (14)$$

Eq. (13) can be rewritten by using Eq. (12) as

$$\bar{e}_{ij} = e_{ij}\left(e_{ij} + 2d_{ij}^*\right) \quad (15)$$

It is not difficult to see that $e_{ij} \geq -d_{ij}^*$ and $\bar{e}_{ij} = 0$ if and only if $e_{ij} = 0$.

Consider the potential function using the Lyapunov function candidate [17]:

$$V(\boldsymbol{e}) = \frac{1}{4} \sum_{(i,j)\epsilon\varepsilon^*} \bar{e}_{ij}^2 \quad (16a)$$

Using Eq. (14), Eq.(16a) is equivalent to

$$V(\boldsymbol{e}) = \frac{1}{4}\|\bar{\boldsymbol{e}}\|^2 = \frac{1}{4}\left\|\boldsymbol{h}_{\mathcal{G}}(\boldsymbol{p}) - \boldsymbol{h}_{\mathcal{G}}(\boldsymbol{p}^*)\right\|^2 \quad (16b)$$

This function is positive definite in $\boldsymbol{e}$, continuously differentiable, and radially unbounded. The time derivative of $V(\boldsymbol{e})$, we have that

$$\begin{aligned} \dot{V} = \frac{1}{4}\frac{d}{dt}\left(\left\|\boldsymbol{h}_{\mathcal{G}}(\boldsymbol{p}) - \boldsymbol{h}_{\mathcal{G}}(\boldsymbol{p}^*)\right\|^2\right) &= \frac{1}{2}\bar{\boldsymbol{e}}^T \frac{d\left(\boldsymbol{h}_{\mathcal{G}}(\boldsymbol{p})\right)}{dt} \\ &= \bar{\boldsymbol{e}}^T \boldsymbol{\mathcal{R}}(\boldsymbol{p})\boldsymbol{u} \end{aligned} \quad (17)$$

where Eq. (2) and Eq. (8) were used, $\boldsymbol{u} = [u_1, \dots, u_n]^T \in \mathbb{R}^{dn}$.

Given the formation framework $F(t) = (\mathcal{G}^*, \boldsymbol{p}(t))$ and let the initial conditions be such that $e(0) = \mathcal{S}_1 \cap \mathcal{S}_2$, $e \in \mathbb{R}^m$, where

$$\mathcal{S}_1 = \{\Upsilon(F, F^*) \leq \varepsilon\} \quad (18)$$

$$\mathcal{S}_2 = \left\{dist\left(\boldsymbol{p}, Iso(F^*)\right) < dist\left(\boldsymbol{p}, Amb(F^*)\right)\right\} \quad (19)$$

The formation control law:

$$\boldsymbol{u}_f = -\nabla\left(V(\boldsymbol{e})\right)^T = -\left[\frac{\partial V(e)}{\partial \boldsymbol{p}}\right] = -k_p \boldsymbol{\mathcal{R}}^T(\boldsymbol{p})\bar{\boldsymbol{e}} \quad (20)$$

where $k_p > 0$ is a control gain.

This control law makes the time derivative of $V(e)$ in Eq. (17) negative definite and will be embedded in the tracking control law. From corollary 2.1, the condition in Eq. (18) is a sufficient constraint for the formation framework $F(t)$ to remain infinitesimally rigid. The condition in Eq. (19) ensures that $F(t)$ is closer to a formation framework in $Iso(F^*)$ at $t = 0$ than to one in $Amb(F^*)$ to prevent converging to an ambiguous framework.

In the latter problem, we assume that the target velocity $v_T := \dot{p}_T$ is unknown to all agents in MAS. However, the leader (n^{th} agent) can measure the target's relative position $z_T = p_T - p_n$ and communicate this information to the followers. The control objective for tracking the moving target with a desired formation is to design $u_i = f\left(z_{ij}, d_{ij}^*, z_T, \hat{v}_T\right)$, where $\hat{v}_T$ is the target velocity estimate value. This value is generated by the continuous dynamic estimation mechanism [18]

$$\hat{v}_T(t) = \int_0^t \left[k_1 z_T(\tau) + k_2 sgn\left(z_T(\tau)\right)\right] d\tau \quad (21)$$

where $k_1, k_2 > 0$ are control gains.

Consider the formation framework $F(t) = (G^*, \boldsymbol{p}(t))$ and let the initial conditions be such that $\boldsymbol{e}(0) = \mathcal{S}_1 \cap \mathcal{S}_2$ given in Eq. (18) and Eq. (19). Then, the tracking formation control law $\boldsymbol{u} \in \mathbb{R}^{dn}$ is designed as

$$\boldsymbol{u} = \boldsymbol{u}_f + \mathbf{1}_n \otimes \boldsymbol{\kappa} \tag{22}$$

where $\boldsymbol{u}_f$ was defined in Eq. (20), term $\boldsymbol{\kappa} \in \mathbb{R}^d$ is given by

$$\boldsymbol{\kappa} = k_1 z_T + \hat{v}_T - u_{fn} \tag{23}$$

The control law in Eq. (22) has two components: $\boldsymbol{u}_f$ ensures the formation problem while the term $\boldsymbol{\kappa}$ guarantees tracking and encircling a moving target. This render $\boldsymbol{e} = 0$ exponentially stable and ensures Eq. (6) and Eq. (7) are satisfied.

From Eq. (22) and Eq. (23), we have that the leader agent control input $u_n \in \mathbb{R}^d$ is

$$u_n = k_1 z_T + \hat{v}_T \tag{24}$$

The follower i^{th} agent control input $u_i \in \mathbb{R}^d$ is

$$u_i = u_{fi} + \boldsymbol{\kappa} \quad (25)$$

$$= -k_p \sum_{j \epsilon N_i} z_{ij}\, \bar{e}_{ij} + k_1 z_T + \int_0^t \left[k_1 z_T(\tau) + k_2 sgn\big(z_T(\tau)\big)\right] d\tau - u_{fn} \tag{25}$$

where $u_{fi} = -k_p \sum_{j \epsilon N_i} z_{ij}\, \bar{e}_{ij}$ $(i = 1, \dots, n-1)$ and $u_{fn} = -k_p \sum_{j \epsilon N_n} z_{nj}\, \bar{e}_{nj}$

We can see that the follower control law is distributed since it depends on its relative position to neighboring agents the target's relative position, and the formation control of the leader agent.

3. Simulation results

In this section, we tested the control laws defined by equations (20), (22), (24), and (25) to evaluate their effectiveness in satisfying equations (6) and (7) in $\mathbb{R}^2$. The experiment involved a Multi-Agent System (MAS) with 6 agents: Agent 6 acts as the leader, responsible for tracking a moving target, while the remaining agents function as followers, tasked with encircling the leader and maintaining the desired formation. In the desired formation, the five vertices of the pentagon represent the followers and the vertex at the center represents the leader agent. The desired formation was designed infinitesimally and minimally rigid framework. According to Eq. (5), we can derive the minimal number of edges $m = 9$, that are five followers are connected however only four followers need to be connected to the leader agent. Accordingly, the desired topology was established up by the adjacency matrix $\boldsymbol{\mathcal{A}} \in \mathbb{R}^{6\times 6}$, as follows:

$$\boldsymbol{\mathcal{A}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \tag{26}$$

The desired formation of MAS in two-dimensional space (2D) is shown in Fig.1.

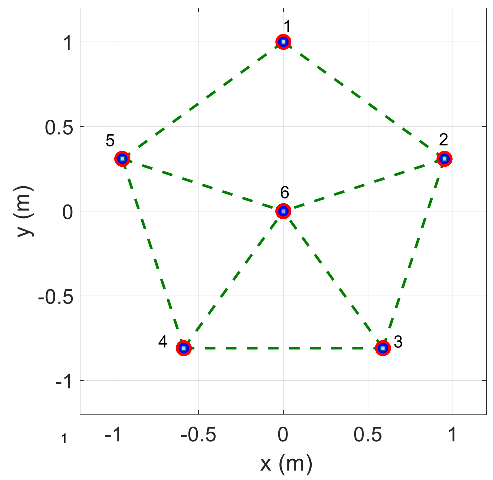

Fig. 1 The desired formation in two-dimensional space

The rigidity matrix $\mathcal{R}(\boldsymbol{p}) \in \mathbb{R}^{9\times 12}$ was derived from Eq. (2) and applied to the control laws.

The desired inter-follower distances of five agents are:

$$d_{12}^{*} = d_{23}^{*} = d_{34}^{*} = d_{45}^{*} = d_{15}^{*} = \sqrt{2(1 - cos(2\pi/5)} \qquad (27)$$

The desired leader-follower distances are

$$d_{26}^{*} = d_{36}^{*} = d_{46}^{*} = d_{56}^{*} = 1 \qquad (28)$$

The initial conditions of all agents are randomly selected by $p_i(0) = p_i^{*} + (\text{rand}(0,1) - I_2 0.5)$. The control gains: $k_p = 1$, $k_1 = 2$, and $k_2 = 2$. The unknown velocity of the target was set up $v_T = [1, sin\, t]$ with initial position $p_T(0) = [1, -1]$.

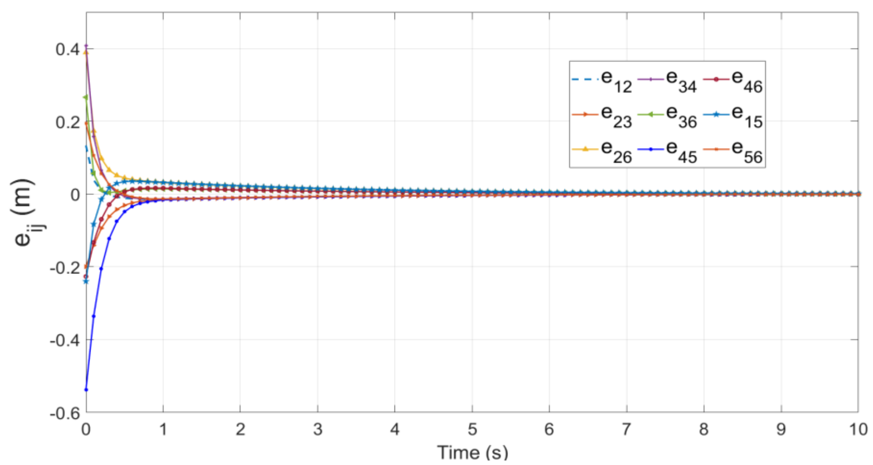

Fig. 2 Distance errors e_ij (t)

The simulation results shown in Fig. 2 demonstrate that the distance errors between the agents, e_ij (t), converge to zero. this indicates that the multi-agent system successfully forms and maintains the desired formation throughout the process of tracking and encircling the moving target. The convergence of these distance errors confirms the effectiveness of the proposed control laws in ensuring the stability of the formation during dynamic operations.

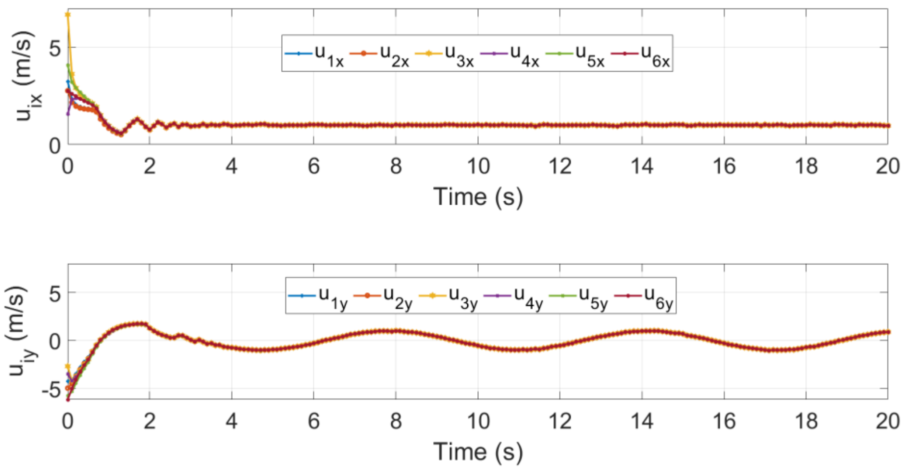

Fig. 3 Control inputs u_ix (t) and u_iy (t)

Fig 3 shows that the control inputs of each agent u_i=[u_ix,u_iy] converge to the value v_T=[1,sin t], even though all agents in the formation do not have direct information about the target velocity. This result demonstrates the ability of the proposed control algorithm to effectively estimate and follow the target's trajectory, with the agents adjusting their control inputs to converge toward the target's motion profile. The convergence of the control inputs, despite the lack of velocity information, underscores the robustness of the control strategy in achieving the desired formation and target tracking.

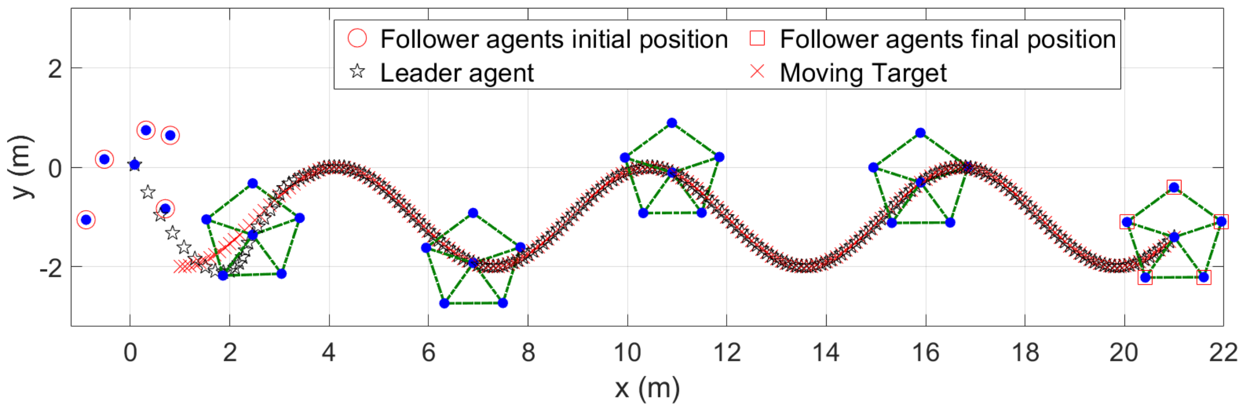

Fig. 4 The formation F(t) tracks and encircles the moving target while maintaining the desired formation

The results in Fig. 4 depict the process of forming a formation, tracking, and encircling a moving target when applying the proposed control laws. The leader follows the trajectory of the moving target, while the follower agents track and encircle the target according to the desired formation. Throughout the movement, the formation remains undistorted, indicating that the proposed control laws effectively maintain the formation without any deformation during the process.

6. Conclusions

This study introduces a formation tracking control algorithm for MAS based on rigidity graph. The proposed method consists of a formation protocol and a target encirclement mechanism. The desired formation is modeled as an infinitesimally and minimally rigid framework. Consequently, the control laws are formulated using the rigidity matrix to stabilize the inter-agent distance dynamics to the desired distances. In the target encirclement problem, the control algorithm incorporates an additional term to estimate the unknown target velocity, with a leader-follower strategy employed to address this issue. The effectiveness of the proposed algorithm was verified through MATLAB simulations. The control law design follows a standard approach, assuming that agent motion is governed by a single-integrator model, where each agent is treated as a kinematic point, with its state defined by position and the control input by velocity. In the future, further research will focus on the dynamic models of real-world multi-agent systems and their integration into the control framework

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