

## INTER-CARRIER INTERFERENCE REDUCTION TECHNIQUE IN OFDM SYSTEM BASED ON SELF CANCELLATION TECHNIQUE

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### ABSTRACT:

Orthogonal frequency division multiplexing (OFDM) is considered as a one of the best modulation schemes in wireless communications. However, OFDM suffers from the sensitivity to frequency offset. This frequency offset introduces the problem of inter-carrier interference (ICI) in OFDM system. This paper presents a new self cancellation technique (modified self cancellation technique) using the self cancellation technique with zero – gab. In proposed technique a zero sub-carriers carries are used in order to mitigate the interference occurred by the main – lobe of the two adjacent sub-carriers, in addition to zeros sub-carrier, the proposed technique used *self-cancellation* in order to mitigate the effect of the side-lobes of other sub-carriers.

In terms of *carrier-to-interference ratio* (CIR) and *bit-error rate* (BER) the modified self cancellation technique gives good results compared to that using only the conventional self cancellation technique.

### Introduction:

An OFDM system represents special kind of Multicarrier techniques [1]. Multicarrier techniques, including OFDM – based wireless systems, will provide the solution for future-generation wireless communications [2]. It is thought that OFDM will be able to fulfill the three most important requirements of 4G mobile networks: higher *coverage* and *capacity*, with desired quality of service (QoS) at minimum *cost*. Most fourth-generation (4G) systems (including Mobile worldwide interoperability for microwave access (WiMAX) and long – term evolution (LTE) use OFDM and orthogonal frequency division multiple – access (OFDMA) [3].

The idea of the OFDM is to use parallel data and FDM with overlapping sub-channels. To realize this technique, it is needed to reduce cross talk between sub-carriers (SCs) which mean orthogonality between the different modulated carriers is desired [2].

By means of an inverse fast Fourier transform (IFFT), the transmitter transforms the frequency – domain data samples on several subcarriers, which are equidistantly distributed in the frequency domain [4]. The number of subcarriers is usually a power of two to allow efficient implementation of the IFFT/FFT (fast Fourier transform) [4].

Figure (1) shows an OFDM system block diagram. “The mapping function” The function of “*mapping*” is to add redundancy to the input symbol in a specific way in order to give the system immunity against frequency offset. On the other hand, in the “*de-mapping*” the redundancy is removed from the output of the IFFT, which means more immunity against frequency offset is provided.

It can be noticed from figure (1) that a simple channel is used. It shifts the signal in frequency and adds additive white Gaussian noise (AWGN) noise.

### Complex coefficient:

A discrete time situation is considered. In any symbol interval, N symbols  $X_0, X_0, \dots, X_{N-1}$  will be transmitted using N sub-carriers  $\phi_0[n], \phi_1[n], \dots, \phi_{N-1}[n]$ , the resulting signal is:

$$x[n] = X_0\phi_0[n] + X_1\phi_1[n] + \dots + X_{N-1}\phi_{N-1}[n] \quad (1)$$

In OFDM system this sub-carrier takes the following values:

$$\phi_l[n] = \frac{1}{N} * \exp\left(\frac{j2\pi ln}{N}\right) \quad (2)$$

N = number of symbols and sub-carriers.

l = discrete frequency.

n = time symbol.

By substituting eq. (2) in eq. (1) the result will be:

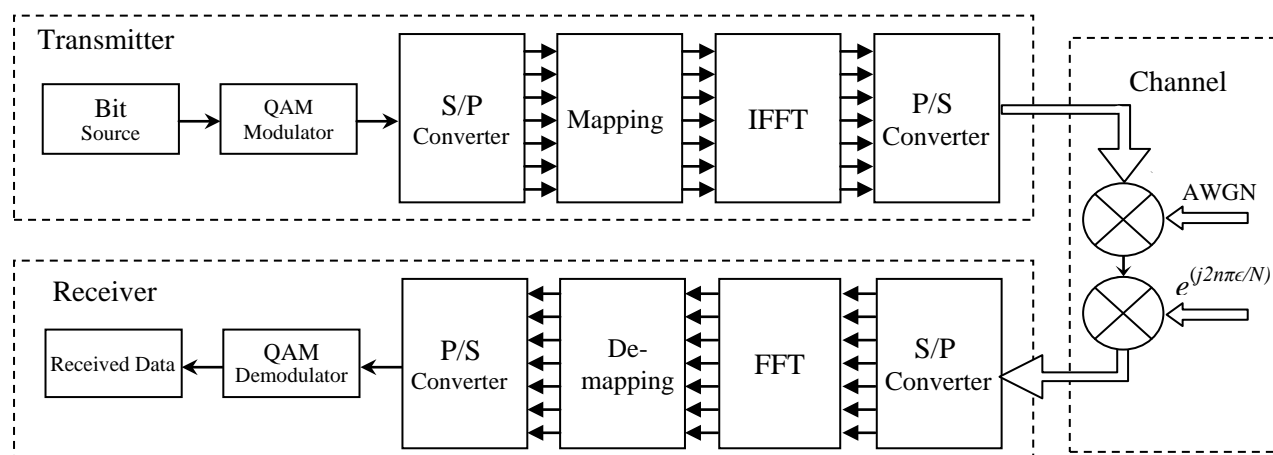


Figure (1): A block diagram of an OFDM system.

$$x[n] = \frac{1}{N} \sum_{l=0}^{N-1} X_l * \exp\left(\frac{j2\pi ln}{N}\right) \quad (3)$$

Eq. (3) represents the transmitted signal if the impulse response of the transmission filter is ignored. It is assumed that: the channel introduces only shift in frequency and ignoring the effects of random noise, impulse response of the channel and impulse response of the receiver filter. So the received signal will be:

$$y[n] = \frac{1}{N} \sum_{l=0}^{N-1} X_l * \exp\left(\frac{j2\pi n * (l + \epsilon)}{N}\right) \quad (4)$$

where  $\epsilon$  is normalized frequency shift which occurs due to frequency miss-match between local oscillator in the sender and receiver or the *Doppler shift*.

$\epsilon$  can be defined as:  $\epsilon = \Delta f T$ ,  $\epsilon = 0$  means there is no shifting in frequency,  $\epsilon = 1$  means that the frequency shift is equal to the difference between two adjacent sub-carrier [1/(Symbol period)]. For example, data sent in sub-carrier (0) will be received in sub-carrier (1). At the receiver, FFT is applied to convert the received signal from time-domain to frequency-domain. The received signal in sub-carrier  $k$  can be represented as follow:

$$Y_k = \frac{1}{N} \sum_{n=0}^{N-1} y[n] * \exp\left(\frac{-j2\pi kn}{N}\right) \quad (5)$$

by substituting eq. (4) in eq. (5), the result will be:

$$Y_k = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{l=0}^{N-1} X_l * \exp\left(\frac{j2\pi n * (l + \epsilon)}{N}\right) * \exp\left(\frac{-j2\pi kn}{N}\right) \quad (6)$$

then:

$$Y_k = \frac{1}{N} \sum_{l=0}^{N-1} X_l \sum_{n=0}^{N-1} \exp\left(\frac{j2\pi n * (l - k + \epsilon)}{N}\right) \quad (7)$$

Let:

$$S(l - k) = \frac{1}{N} \sum_{n=0}^{N-1} \exp\left(\frac{j2\pi n * (l - k + \epsilon)}{N}\right) \quad (8)$$

Using the properties of geometric series, eq. (8) can alternatively be expressed as:

$$S(l - k) = \frac{\exp(j\pi(l - k + \epsilon)(N - 1)/N) \sin(\pi(l - k + \epsilon))}{N \sin(\pi(l - k + \epsilon)/N)} \quad (9)$$

By substituting eq. (8) in eq. (7), the result will be:

$$Y_k = \sum_{l=0}^{N-1} X_l S(l - k) \quad (10)$$

By defining complex weighting coefficients  $S(1), S(2) \dots S(N - 1)$ , which give the contribution of each of the inputs  $X_1, X_2 \dots X_{m-1}$  to the output value  $Y_k$ , eq. (10) can be rewritten as follow:

$$Y_k = X \cdot S(0) + \sum_{\substack{l=0 \\ l \neq k}}^{N-1} X \cdot S(l - k) \quad (11)$$

The first term in the right side of eq. (11) represents the wanted signal and the second term represents unwanted signal (ICI components).

### Carrier to Interference Ratio:

Carrier to interference ratio (*CIR*) can be defined as a ratio between the wanted power and unwanted power. It increases when normalized frequency shift ( $\epsilon$ ) decreases and vice versa. It is used to determine the performance of the whole system. The goal of all ICI reduction algorithms is to get a larger value of *CIR*.

The theoretical *CIR* for standard OFDM can be written as:

$$CIR = \text{abs}(S(0)) \div \sum_{\substack{l=0 \\ l \neq k}}^{N-1} \text{abs}(S(l - k)) \quad (12)$$

This equation can be applied for all kinds of modulation and any number of subcarriers; but the derivation assumes that: the standard transmitted data has zero mean, the symbols transmitted on the different subcarriers are statistically independent, and the additive noise is omitted. This equation is verified by simulation results as shown later.

### Contribution (Adjacent sub-carriers):

Figures (2) and (3) show how each sub-carrier contributes in sub-carrier 0 and the figures (4) and (5) shows the case for sub-carrier 3.

In the both cases, the desired signal decreases and the undesired signals increases when the normalized frequency shift ( $\epsilon$ ) is increased. And the desired signal increases and the undesired signals decreases when the normalized frequency shift ( $\epsilon$ ) is decreased.

It can also be noticed that the adjacent carrier has the maximum contribution in the ICI. When normalized shift ( $\epsilon$ ) is negative, the maximum contribution in the ICI component comes from the left sub-carrier as shown in the figures (2) and (4). When ( $\epsilon$ ) is positive, the maximum contribution to the ICI component comes from the right sub-carrier as shown in the figures (3) and (5).

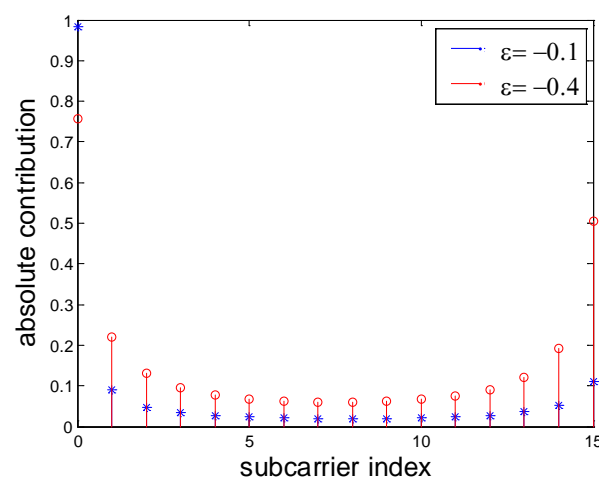
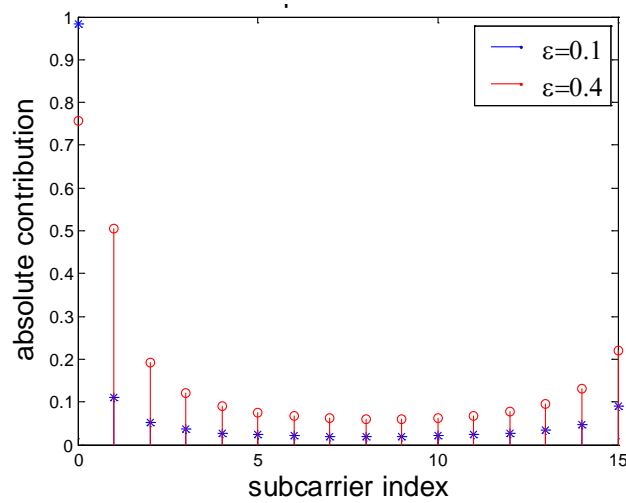
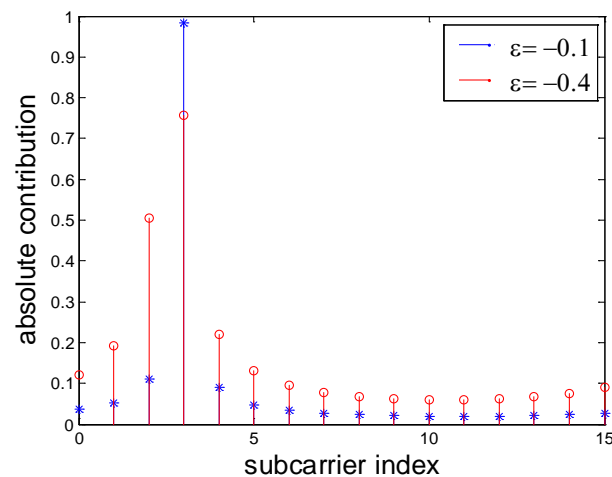
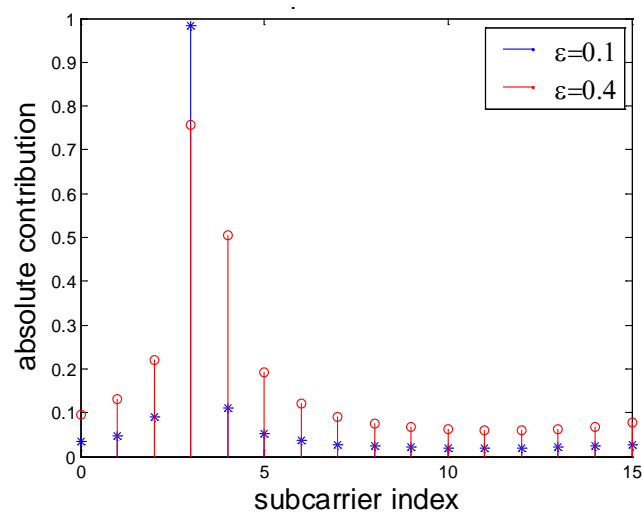


Figure (2): Coefficient contribution in sub-carrier 0 for negative ( $\epsilon$ ).


 Figure (3): Coefficient contribution in sub-carrier 0 for positive ( $\epsilon$ ).

 Figure (4): Coefficient contribution in sub-carrier 3 for negative ( $\epsilon$ ).

 Figure (5) Coefficient contribution in sub-carrier 3 for positive ( $\epsilon$ ).

### Self Cancellation:

The self cancellation technique was introduced by Zhao and Sven-Gustav Häggmanin [5]. In this method the input symbol is assigned in a pair of subcarriers, where in the standard OFDM each symbol is mapped into a single carrier. For example, if symbol  $X$  is transmitted, then it will be mapped into two adjacent sub-carriers with  $(X, -X)$  values. By this substitution; the received signal will be:

$$Y'_k = \sum_{l=0,2,4,\dots}^{N-2} X(l)[S(l-k) - S(l+1-k)] + n_k \quad (13)$$

Where  $n_k$  denotes the additive noise symbol introduced in sub-carrier  $k$ , and  $Y'_k$  denotes the received symbol in sub-carrier  $k$ .

The received symbol in sub-carrier  $k+1$  is represented by:

$$Y'_{k+1} = \sum_{l=0,2,4,\dots}^{N-2} X(l)[S(l-k-1) - S(l-k)] + n_{k+1} \quad (14)$$

And the ICI coefficient  $S'(l-k)$  is referred to as:

$$S'(l-k) = S(l-k) - S(l+1-k) \quad (15)$$

which is better than the original one. Better coefficient can be achieved by subtracting the two adjacent carriers, the result will be:

$$Y''(k) = Y'(k) - Y'(k+1) = \sum_{l=0,2,4,\dots}^{N-2} X(l)[-S(l-k-1) + 2S(l-k) - S(l-k+1) + n_k - n_{k+1}] \quad (16)$$

Subsequently, the ICI coefficients for the received signal become:

$$S''(l-k) = -S(l-k-1) + 2S(l-k) - S(l-k+1) \quad (17)$$

The coefficient in eq. (17) provides a better value. Eq. (16) can be rewritten so as the received signal becomes:

$$Y''(k) = x(k)(-S(-1) + 2S(0) - S(1)) +$$

$$\sum_{\substack{l=0,2,4,6,\dots \\ l \neq k}}^{N-2} X(l)(-S(l-k-1) + 2S(l-k) - S(l+1-k)) + n_k - n_{k+1} \quad (18)$$

Eq. (18) represents the received signal at sub-carrier  $k$ . In the right side of Eq. (18) there are three parts: first part represents the wanted signal, the second part represents the unwanted signal, and the last part represents the Gaussian noise. From Eq. (18) it can be derived that the CIR for self cancellation is given by:

$$CIR = \frac{-S(-1) + 2S(0) - S(1)}{\sum_{\substack{l=0,2,4,6,\dots \\ l \neq k}}^{N-2} (-S(l-k-1) + 2S(l-k) - S(l+1-k))} \quad (19)$$

The derivation assumes that: the standard transmitted data has zero mean, the symbols transmitted on the different sub-carriers are statistically independent, and the additive noise is omitted.

In addition, this equation (Eq. 19) will be verified by simulation as shown later.

### Modified Self Cancellation:

In the OFDM spectrum, each carrier consists of a *main-lobe* followed by a number of *side-lobes* with degraded amplitudes, as shown in figure (6).

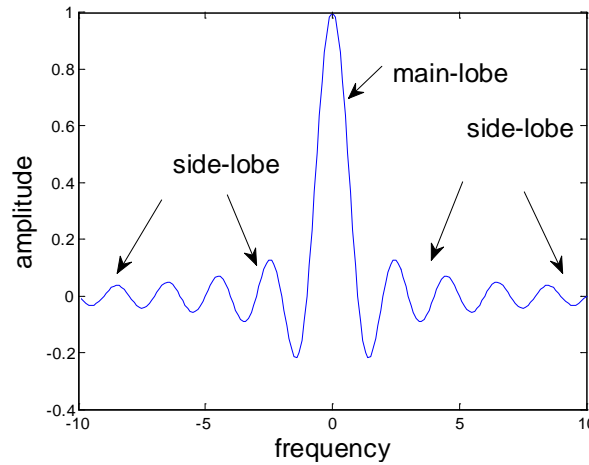


Figure (6): Frequency domain of a single carrier

The proposed method (modified self cancellation) using a zero gap in order to mitigate the noise that comes from the main-lobe (of adjacent sub-carrier) and using the conventional self cancellation technique in order to mitigate the effect of side-lobes. Using these two techniques together achieve better CIR and BER results as it will be shown in the next section.

The input symbol is modulated into three sub-carriers, one sub-carrier carries zeros so as to cancel the main-lobe power comes from adjacent sub-carrier. The role of the remaining two sub-carriers is to reduce the ICI components come from side-lobes.

The main disadvantage of this method is the bandwidth inefficiency, but it can be overcome by increasing the level of QAM, or by increasing the number of subcarriers (reducing frequency separation between sub-carriers). This method introduces redundancy in the symbol since each three of subcarriers transmit only one data symbol. Then the following symbols are generated:

$$X_0 = -X_1, X_2 = 0, X_3 = -X_4, X_5 = 0 \dots \dots, X_{N-3} = -X_{N-2}, X_{N-1} = 0$$

If the signal is shifted in frequency and affected by an AWGN noise then the received signal at sub-carrier  $k$  is expressed as follows:

$$Y'_k = \sum_{l=0,3,6,\dots}^{N-3} X(l)(S(l-k) - S(l+1-k)) + n_k \quad (20)$$

and the received signal at sub-carrier sub-carrier  $k+1$  is:

$$Y'_{k+1} = \sum_{l=0,3,6,\dots}^{N-3} X(l)(S(l-k-1) - S(l-k)) + n_{k+1} \quad (21)$$

At this point the effect of main-lobe noise which consumes in carrier  $k+2$  and  $k-1$  is mitigated and neglected at the receiver. The main-lobe noise represents a large component in the unwanted signal. Moreover, the effect of the side-lobe can be reduced by subtracting  $Y'_k$  from  $Y'_{k+1}$ :

$$Y''_k = Y'_k - Y'_{k+1} \quad (22)$$

By substituting the values of  $Y'_k$  and  $Y'_{k+1}$  in eq. (22)

$$Y''_k = \sum_{l=0,3,6,\dots}^{N-3} X_l(-S(l-k-1) + 2S(l-k) - S(l-k+1)) + n_k - n_{k+1} \quad (23)$$

Eq. (23) can be rewritten as follow:

$$Y''_k = X_k(-S(-1) + 2S(0) - S(1)) + \sum_{l=3,6,\dots}^{N-3} X_l(-S(l-k-1) + 2S(l-k) - S(l-k+1)) + n_k - n_{k+1} \quad (24)$$

Eq. (24) represents the received signal after *de-mapping*. In the right side of this equation there are three parts: the first one represents the wanted signal, the second part represents the unwanted signal and the last part represents the Gaussian noise. In this method, the unwanted components are reduced by removing  $S(-1)$  &  $S(1)$  from the unwanted signal, which are the largest complex component. Moreover, this represents the effect of the main-lobe.

The coefficient in eq. (24) is similar to the coefficient in the self cancellation, but there are two differences: the coefficient  $S(1)$  and  $S(-1)$  (it can be noted that  $S(-1) = S(N-1)$  where  $N$  is the number of sub-carriers) does not appear in the unwanted signal. The second difference is that the total number of interferer signals is one third of which standard OFDM has.

From eq. (24) it can be derived that the CIR for this proposed method is given by:

$$CIR = \frac{-S(-1) + 2S(0) - S(1)}{\sum_{\substack{l=3,6,\dots \\ l \neq k}}^{N-3} (-S(l-k-1) + 2S(l-k) - S(l-k+1))} \quad (25)$$

The derivation assumes that: the standard transmitted data has zero mean, the symbols transmitted on the different subcarriers are statistically independent, and the additive noise is omitted. In addition, eq. (25) will be verified by simulation as shown later.

**Results and Discussion:**

In order to compare between the proposed scheme and the *self cancellation* scheme, the *BER* curves were used to evaluate the performance of each schemes. The OFDM transceiver shown in figure (1) was employed and simulated using MATLAB. Modulation scheme of quadrature amplitude modulation (QAM) was chosen, different values of normalized frequency (0.1, 0.5, 0.8 and 1.1) were used and the results are shown in figures (7), (8), (9) and (10) respectively. In these figures the blue dashed curve represent the standard OFDM system, the red dot-dashed curve represent the OFDM system using *self cancellation* technique and the black solid curve represent the OFDM system using the proposed scheme.

It can be observed from these figures the *BER* performance of the proposed system is outperforms the *BER* performance of the two other systems. While the *BER* performance for the OFDM using the *self cancellation* technique is outperforms the *BER* performance of the standard OFDM system. Moreover, it can be seen from these figures that as frequency offset increases the *BER* performance is degraded.

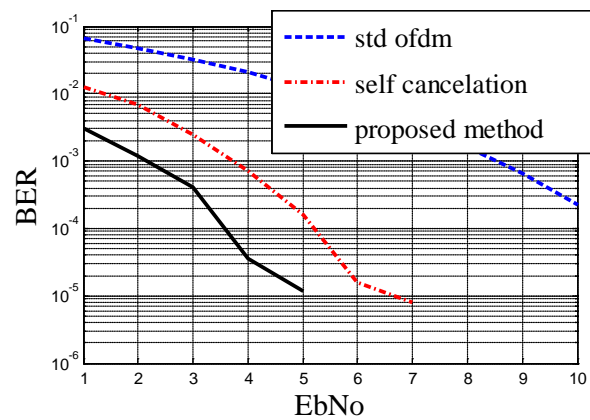


Figure (7): *BER* performance of the considered systems using:  $\epsilon = 0.1, M = 4$

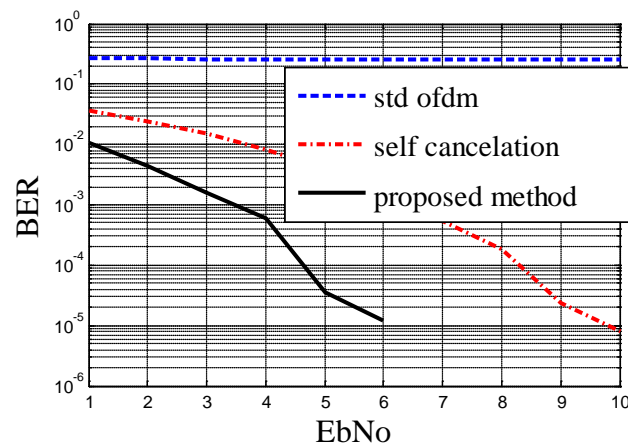


Figure (8): *BER* performance of the considered systems using:  $\epsilon = 0.5, M = 4$

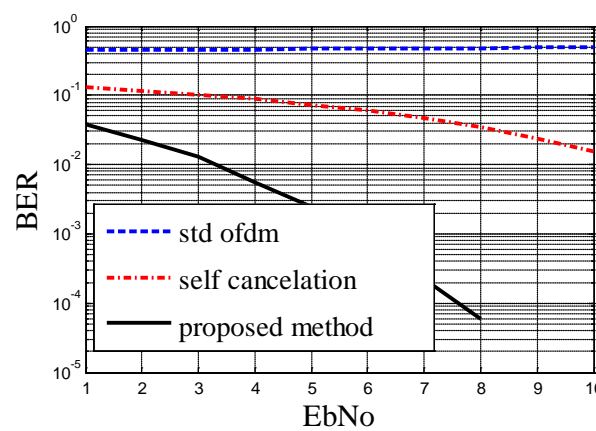


Figure (9): *BER* performance of the considered systems using:  $\epsilon = 0.8, M = 4$

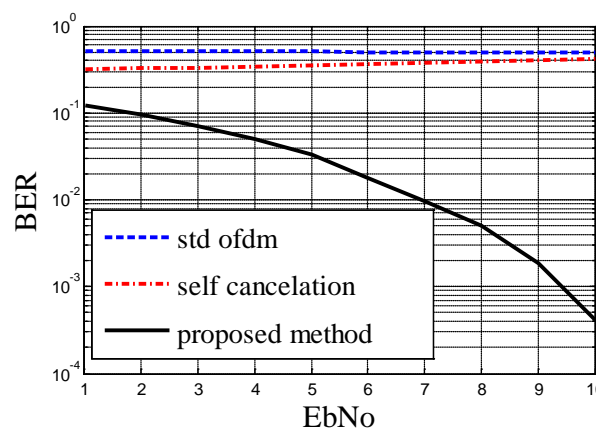


Figure (10): *BER* performance of the considered systems using:  $\epsilon = 1.1, M = 4$

The *CIR* is used in order to compare and evaluate the performance of the two different cancellation schemes. The results of the *CIR* curves are shown in figure (11) below.

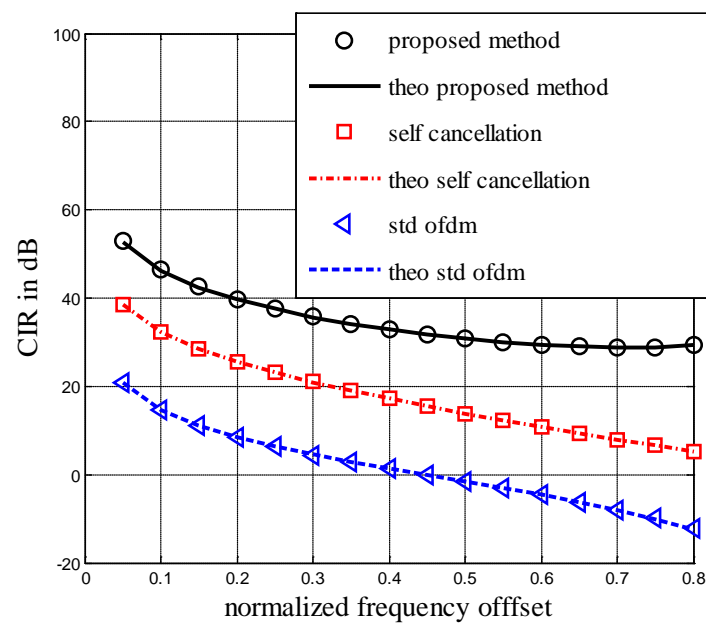

 Figure (11): CIR curve for different schemes. (Theo  $\equiv$  theoretical).

Figure (11) shows the theoretical *CIR* curves for standard OFDM, self cancellation and the proposed method which are calculated by eq. (12), (19) and (25) respectively. Also figure (11) shows that all the theoretical and simulation results are identical, so these results verify the simulation and verify the theoretical expression for the proposed method.

From figure (11) and table (1), it is noted that the best method which gives better *CIR* value is the proposed method followed by the conventional self cancellation, and finally the standard OFDM. It has been found that when the frequency offset is increased, the *CIR* values will decrease. At frequency offset ( $\epsilon$ ) = 0.15, *self cancellation* gives about 17 dB improvement over the standard OFDM, and the proposed method gives more than 31 dB improvement over the standard OFDM.

 Table (1): *CIR* values for different schemes.

Frequency offset	Standard OFDM (dB)	Self cancellation (dB)	Proposed method (dB)
0.0500	20.7907	38.5019	52.9680
0.2500	6.2203	23.1406	37.6127
0.4500	-0.3891	15.5657	31.7751

### Conclusion:

This paper proposed a new self cancellation technique (modified self cancellation). In this paper, a comparison was done between standard OFDM, OFDM using conventional self cancellation technique and OFDM using modified self cancellation technique. In term of *CIR*, the proposed scheme gives better results than standard OFDM about 31 dB improvements and 17 dB improvements over *the conventional self-cancellation* at frequency offset ( $\epsilon$ ) = 0.15 no matter what the type of the amplitude modulation is used. Also in term of *BER* the proposed method outperform the the conventional self cancellation and standard OFDM. Moreover, the theoretical *CIR* equations for the proposed method, the conventional self cancellation and standard OFDM were also verified.

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