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Advantages of general parametric solution Approach in dynamics courses

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Abstract

Problems of dynamics are treated using a general parametric solution approach. The advantages of treating problems in the parametric way is discussed. The approach has many advantages over the numerical calculations starting from the beginning: 1) Solutions are expressed in the general form suitable for algorithmic programming 2) Unit check on the correctness of the results are easier 3) Effect of each input parameter on the desired output parameter can be easily observed 4) Results can be evaluated for the limiting cases to recognize the soundness of the solutions. 5) Mathematical principles can be more exploited on the solutions 6) Solutions give more insight on the design principles. The ideas are outlined using three sample problems. The advantages of the approach have been tested on Mechanical Engineering Sophomore students for over 16 years with success.

Keywords: Dynamics, Parametric Solution, Mechanical Engineering Education

1. Introduction

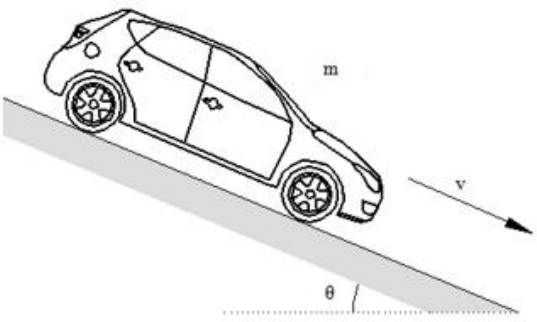
Dynamics is one of the most fundamental courses for mechanical engineering students in B.S. programs. The student has to understand the basic concepts of motion such as the displacement, velocity, acceleration etc. and their relationship among each other which is classified as kinematics. Then the effects of forces and moments on the motion are discussed which is classified as kinetics. Work-energy principle and its special case of energy conservation, impulse-momentum principle and conservation of momentum then follows. Usually all principles and solution methods are derived for the particle approach and then generalized to rigid-body approach. The course is well established and excellent textbooks are used worldwide to teach the principles. Some well-known examples are cited for brevity (Beer et al., 2012, Hibbeler, 2012, Meriam and Kraige, 2012).

Educational accreditation programs in engineering and physics usually require students to gain mathematical skills to be applied to real life problems and enhance design capabilities. To initiate and enhance such abilities, a slightly different approach of problem solving is discussed here. The mentioned approach may be called as "the general parametric solution approach". The method has been tested for over 16 years on the sophomore students of mechanical engineering in Manisa Celal Bayar University. All solved or homework problems of the course textbook (Pakdemirli, 2010) were carefully prepared following the parametric solution approach. I have, as the lecturer, observed that the students can adapt themselves to this approach without much difficulty, understand better the solutions, grasp the basic ideas better, can have a deeper insight into the consequences of the solutions, can check their results easier, comment on the influences of the input parameters to the output parameters and draw preliminary design principles from the solutions. They have more flexibility to apply calculus to the problems. Most of the time, the degenerate cases can be guessed without actually solving the problem and when a parametric solution is at hand, the solution can be reduced to those degenerate cases to check the preciseness. A unit check is also available if the problem is parametrically solved. It is observed that, in the exams, the problems formulated in the special parametric approach can be solved as successfully as the problems with purely numerical approach by the students. The ideas are discussed using the sample problems. This approach can be adapted if not fully, at least partially to other undergraduate courses in mechanical engineering also. I have thought for over a decade fluid mechanics course with partial implementation of the method.

2. Illustration of the Ideas

The ideas are exploited using three worked sample problems. The problem should be divided into parts with carefully asked questions to guide and stimulate the way of thinking of the students.

2. Sample Problem 1

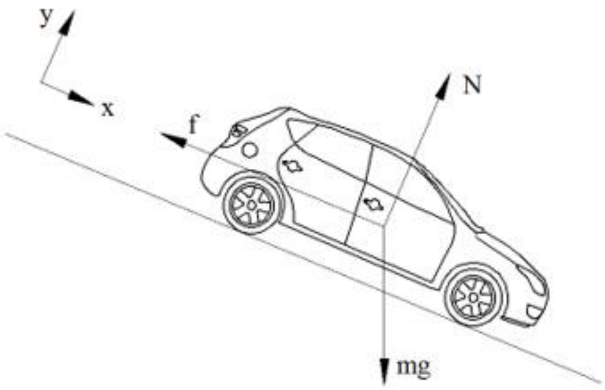


A car with mass *m* and velocity *v* is traveling down the road with a slope angle θ. The car suddenly applies brakes with a constant force off.

a. Find the stopping distance in terms of the given parameters.
b. Check your result with respect to units
c. Discuss the influence of given parameters on the stopping distance.
d. What is the minimum brake force that can stop the car? For a given *f*, what is the maximum slope for which the car can stop?
e. What is the stopping distance, if m=2000 kg, θ=50^0, f=7.5 kN, v=90 km/hr? What is the minimum brake force and maximum inclination angle?

Solution

a. Assume that the total distance traveled after brake force applied is x. The free body diagram of the car is shown. The forces that do work in the x direction are mgsin□ and f. f is against the motion and does negative work. Writing the principle of work-energy



$$T_1 + U_{1-2} = T_2$$

where T_1 is the initial kinetic energy and T_2 is the final

$$T_1 = \frac{1}{2}mv^2, \quad T_2 = 0$$

Since the forces are constant throughout the motion

$$U_{1-2} = \int_0^x (mg\sin\theta - f)dx$$

the work is

$$U_{1-2} = (mg\sin\theta - f)x$$

Inserting all into the work-energy equation

$$\frac{1}{2}mv^2 + (mg\sin\theta - f)x = 0$$

and solving for the stopping distance

$$x = \frac{mv^2}{2(f - mg\sin\theta)}$$

which is the desired result in terms of the input parameters. Since the problem has been solved in a general form, for different numerical values, the calculations need not be repeated. This type of solution is suitable for computer programming also.

b. A unit check is possible in this parametric solution form for the result. In SI units, the result should be in meters.

$$x = \frac{kg(m/s)^2}{N} = \frac{kgm^2/s^2}{kgm/s^2} = m$$

Assume that in the solution you write v instead of v2. This error can be realized from the unit check as the result will be seconds not meters.

c. In this general form of solution, the dependence of each parameter on the solution can be analyzed easily. We all know that as the mass increases, it is harder to stop. From the solution, this fact can be seen. As m increases, the denominator decreases and the numerator increases, both effecting to make x larger. This is why loaded trucks have difficulties in stopping down the road descents. If one can apply a harder brake, then the denominator increases and the stopping distance is less. If the road inclination increases, we know that it is harder to stop. This can be verified from the solution. As increases, sin is larger and the denominator is smaller which makes x larger. The stopping distance is very sensitive to the velocity of travel. When all other parameters are kept constant, it is proportional to the square of the velocity. That is, the double the speed, the quadruple the stopping distance. If one is travelling on the surface of the moon, the stopping distance is shorter because g will be smaller which makes the denominator larger. All these types of conclusions can be drawn if one solves the problem parametrically.

d. For the car to stop, x should be finite. So, the lower limit of the breaking force should be the one making the denominator zero.

$$f_{cr} - mg\sin\theta = 0$$

or

$$f_{cr} = mg\sin\theta$$

For a car to be able to stop then

$$f > f_{cr} = mg\sin\theta$$

With a similar reasoning, the maximum inclination angle for which the car can stop is

$$f - mg\sin\theta_{\max} = 0$$

or solving for the maximum angle

$$\theta_m = Arc\sin\left(\frac{f}{mg}\right)$$

Hence, the car can stop for a descent angle less than the maximum value

$$\theta < Arc\sin\left(\frac{f}{mg}\right)$$

e. If the problem had been solved from the beginning by introducing the numerical values, none of the above conclusions and comments could have been made. The numerical computation is thus done at the final step. For m=2000 kg, =500, f=7.5 kN, v=90 km/hr, the stopping distance is

$$x = \frac{2000\left(\frac{90}{3.6}\right)^2}{2(7500 - 2000x9.81\sin 5)} = 107.9 \text{ m}$$

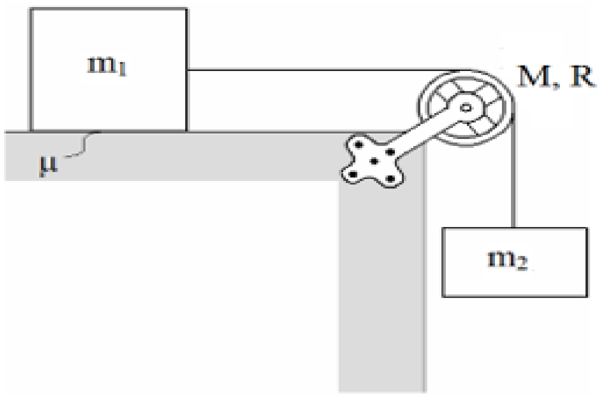
The brake force should be larger than

f>2000x9.81xsin5=1710 N

and the inclination angle should be smaller than

$$\theta < Arc\sin\left(\frac{7500}{2000x9.81}\right) = 22.4^0$$

2.2. Sample Problem 2

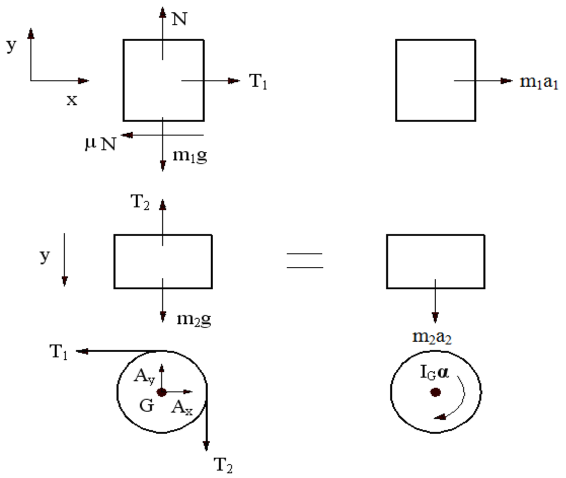


The pulley mass-system starts moving from rest. The rope does not slip over the pulley during the motion

a) Find the accelerations of each mass, the tension forces in the ropes and the angular acceleration of the pulley in terms of the given quantities.
b) Check your results with respect to units.
c) Discuss the influence of each parameter on the solutions? What is the condition for the system to start moving?
d) Check your results for the limiting cases of m1 very large, m2 very large or M very large compared to other masses.
e) If m1=100kg, m2=80kg, M=3kg, R=20 cm and □=0.2, what are the accelerations and tensions?

Solution

a) The free body diagrams of each mass and pulley is drawn first



First mass

$$\sum F_x = m_1 a_1 \quad \Rightarrow \quad T_1 - \mu N = m_1 a_1 \quad (1)$$

$$\sum F_y = 0 \quad \Rightarrow \quad N - m_1 g = 0 \quad (2)$$

Second mass

$$\sum F_y = m_2 a_2 \;\Rightarrow\; m_2 g - T_2 = m_2 a_2 \quad (3)$$

Pulley

$$\sum M_G = I_G \alpha \;\Rightarrow\; T_2 R - T_1 R = \frac{1}{2} M R^2 \alpha \quad (4)$$

For the six unknowns T_1, T_2, a_1, a_2, N and α two more equations are needed. They are provided from the kinematical conditions such that both masses travel at the same acceleration and the rope does not slip over the pulley during motion

$a_1 = a_2 = a$ (5)

$a = \alpha R$ (6)

Performing the algebraic manipulations and solving for the unknowns

$$a = \frac{m_2 - \mu m_1}{m_1 + m_2 + \frac{1}{2}M} g$$

$$T_1 = \frac{m_1 m_2 (1+\mu) + \frac{1}{2}\mu m_1 M}{m_1 + m_2 + \frac{1}{2}M} g$$

$$T_2 = \frac{m_1 m_2 (1+\mu) + \frac{1}{2} m_2 M}{m_1 + m_2 + \frac{1}{2}M} g$$

$$\alpha = \frac{m_2 - \mu m_1}{m_1 + m_2 + \frac{1}{2}M} \frac{g}{R}$$

Note that, this general form of expressing the solution as mentioned is more convenient and does not need a repeat of all solution steps for different set of input parameter values.

b) The linear acceleration should have units of m/s2, angular acceleration rad/ s2 and forces N.

$$a = \frac{kg}{kg}\frac{m}{s^2} = \frac{m}{s^2}$$

$$T_{1,2} = \frac{kg^2}{kg}\frac{m}{s^2} = \frac{kgm}{s^2} = N$$

$$\alpha = \frac{kg}{kg}\frac{m}{s^2}\frac{1}{m} = \frac{1}{s^2}$$

c) a>0 for the condition of movement

$$m_2 - \mu m_1 > 0$$

or

$$m_2 > \mu m_1$$

As can be seen from the acceleration solution, the friction coefficient has an effect of slowing down the motion and increasing up the tension in both portions of the rope. m2 has an effect to increase accelerations and m1 has an effect to decrease them. As the pulley mass increases, from the solutions, the linear and angular accelerations decrease because the denominators are larger. This fact can be guessed without solving, because for a pulley with more mass, it will become harder to rotate it. Increasing m1 results in an increase in T1 since the numerator is growing faster than the denominator. Similarly, m2 tends to increase T2.

d) If m1 is large, from the condition of movement, the motion is impossible. If m2 is very large, one expects the free fall solution for the acceleration since the other parts of the system do not contribute much to the motion. This can be verified by taking the limit

$$a = \lim_{m_2 \to \infty} \frac{m_2 - \mu m_1}{m_1 + m_2 + \frac{1}{2}M} g = g$$

If M is relatively large, the acceleration tends to zero

$$a = \lim_{M \to \infty} \frac{m_2 - \mu m_1}{m_1 + m_2 + \frac{1}{2}M} g = 0$$

as expected because it becomes hard to rotate the pulley by negligible masses. For this case, the tension forces are

$$T_1 = \lim_{M \to \infty} \frac{m_1 m_2 (1+\mu) + \frac{1}{2}\mu m_1 M}{m_1 + m_2 + \frac{1}{2}M} g = \mu m_1 g$$

$$T_2 = \lim_{M \to \infty} \frac{m_1 m_2 (1+\mu) + \frac{1}{2} m_2 M}{m_1 + m_2 + \frac{1}{2}M} g = m_2 g$$

which are the expected static results, since there is no motion. If one neglects the mass of the pulley, the well-known solutions for a pulley-mass system with effect of rotary inertia neglected case is retrieved

$$a = \lim_{M \to 0} \frac{m_2 - \mu m_1}{m_1 + m_2 + \frac{1}{2}M} g = \frac{m_2 - \mu m_1}{m_1 + m_2} g$$

$$T_1 = \lim_{M \to 0} \frac{m_1 m_2 (1+\mu) + \frac{1}{2}\mu m_1 M}{m_1 + m_2 + \frac{1}{2}M} gT_1 = \frac{m_1 m_2 (1+\mu)}{m_1 + m_2} g$$

$$T_2 = \lim_{M \to 0} \frac{m_1 m_2 (1+\mu) + \frac{1}{2} m_2 M}{m_1 + m_2 + \frac{1}{2}M} g = \frac{m_1 m_2 (1+\mu)}{m_1 + m_2} g$$

Note that for this case, the tension forces in each part of the rope turns out to be identical.

e) The solutions in the general form are obtained, the influences of input parameters on the output parameters are discussed, unit check and limiting case checks are performed and numerical computations can now be done as a final step. For the given values of m1=100kg, m2=80kg, M=3kg, R=20 cm and □=0.2

$$a = \frac{80 - 0.2x100}{100 + 80 + \frac{3}{2}} 9.81 = 3.24 \text{ m/s}^2$$

$$T_1 = \frac{100x80(1+0.2) + \frac{1}{2}x0.2x100x3}{100 + 80 + \frac{3}{2}} 9.81 =$$

520.5 N

$$T_2 = \frac{100x80(1+0.2) + \frac{1}{2}x80x3}{100 + 80 + \frac{3}{2}} 9.81 = 525.4 \text{ N}$$

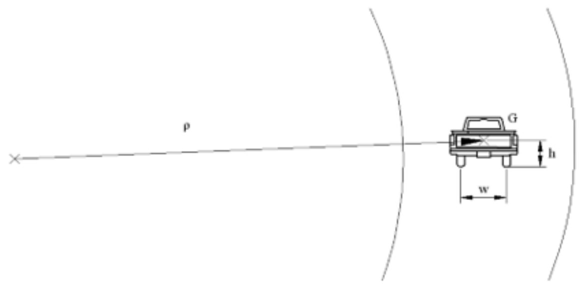
$$\alpha = \frac{80 - 0.2x100}{100 + 80 + \frac{3}{2}} \frac{9.81}{0.2} = 16.2 \text{ rad/s}^2$$

2.3. Sample Problem 3

The center of mass height is h and the width are w for the pickup truck shown. In a curved road with radius of curvature ρ,

a) What is the maximum velocity the pickup can attain not to turnover in terms of the given parameters?

b) Check your results with respect to units.
c) Discuss the influence of each parameter on the solutions?
d) What are the design criteria for not being turnover?
e) If ρ=100 m, w=1.6 m, h=0.5 m, what is the maximum velocity?



Solution

a) Using the D'Alambert's principle of dynamic equilibrium and taking moments with respect to point A which is the critical point of rotation of the pickup

$$\sum M_A = 0 \quad \Rightarrow \quad mg\frac{w}{2} - m\frac{v^2}{\rho}h = 0$$

and solving for the velocity

$$v = \sqrt{\frac{\rho g w}{2h}}$$

which is the desired maximum velocity.

b) The solution should have velocity units if it is correctly calculated.

$$v = \sqrt{\frac{m\frac{m}{s^2}m}{m}} = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s}$$

Note that unit check is a necessary condition for correctness but not a sufficient condition. It may happen that in the result, the number 2 is missing and this error cannot be identified by a unit check.

c) The maximum velocity is proportional to the square root of the radius of the curvature and the width, that is, if you increase those values, the maximum speed can be increased proportional to the square root of the values. In contrast, velocity is inversely proportional to the square root of the height. To increase the velocity, one has to decrease the height.

d) The design criteria are to increase the width and decrease the height as much as possible or in a more compact form increase width/height ratio as much as possible. The high-speed racing cars have such extreme width/height ratios in their designs.

e) For ρ=100 m, w=1.6 m, h=0.5 m

$$v = \sqrt{\frac{100x9.81x1.6}{2x0.5}} = 39.6 \text{ m/s} = 142.6$$

km/hr

is the maximum safe speed against turnover

Concluding Remarks

In the mechanisms and machine theory course which has to be taken after dynamics, parametric solutions cannot always be done for complex systems. The number of coupled nonlinear equations are hard to solve in terms of the input parameters. However, in dynamics, the systems treated are much simpler and parametric solutions are almost always available. These types of solutions, as discussed, has many advantages over the numerical computations of the problem from the beginning: the easy implementation of the solutions to algorithmic approach, a better understanding of the design principles, easy checking of the solutions for units and limiting cases, better interpretation of the influence of the input parameters on the output parameters, enabling the application of the calculus principles on the results. It may lead to a more involved algebra, but the advantages go far beyond this disadvantage. In the exams, the questions raised by the parametric approach and pure numerical approach has almost the same success ratios, so once the student is adapted to this style, the grades are unaffected by the way the problem is posed. Based on the past successful implementations, I recommend development of dynamics problems in this style.

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