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Profit analysis of a system having Three Main Unit in Fuzzy environment

Seema Sahu

Assistant Professor, BCS Govt. P.G. College, Dhamtari(C.G.),

Abstract

This paper presents a reliability analysis of a system having five units. First the proposed methodology is assumption of model. Then calculating Transition Probabilities, Mean Time to system failure, Availability analysis, Busy Period Analysis. Take particular Cases using fuzzy logic and Erlangian distributions. Calculate profit analysis and defuzzify the fuzzy profit analysis by using signed distance ranking method for defuzzification. It helps to allocate reliability of model before the actual system is built. It also helps to estimate the exact value of profit analysis.

Keywords: Triangular Fuzzy Number, MTSF, Availability, Busy Period, Signed Distance Ranking Method

Introduction

In the present time reliability has been important for our life. Any machine can have overall consistency of measurement . The problem of ensuring reliability of engineering systems is extremely complex and extends to all the stages of the service life of a system . In most of the system redundancy is used for improving the reliability of components. Earlier **Gopalan et-al [1975]** discussed availability and reliability of an n-unit system with standby and single repair facility using different exponential distributed parameter. **Arora et-al [1976]** has done 2-unit warm standby redundant system with random variable and also CDF is used TSF and MTSF using Markov renewal process. **S. Osaki[1980]** discussed a two-unit parallel redundant system with single repair facility in which the lifetimes of two unit obey a bivariate exponential distribution and the repair time of failed unit obeys an arbitrary distribution applying an extended Markov renewal process , obtained the quantities of interest thoroughly in reliability theory. **M.C.Rander et-al [1994]** have carried out a two unit cold standby system with major and minor failure and preparation time in the case of the major failure and also cost analysis of two dissimilar cold standby system by preventive maintenance and replacement of standby. Earlier **Pathak et-al [2013,2014]** studied reliability parameters of a main unit with its supporting units and also compared the results with two different distributions. This model particularly differs from the other models in the sense that the concept used in this model is based on the real situation. This kind of analysis is of immense help to the owners of small scale industry. Also the involvement of preventive maintenance in the model increases the reliability of the functioning units. At last fuzzy technique is used to access the cost of the system. Calculation work is done by C Language program.

Definitions and Preliminaries

A ***fuzzy set*** *A* is defined by a membership function $\mu_A(x)$ which maps each and every element of *X* to [0 1] i.e.,

$$\mu_A(x) \rightarrow [0\ 1]$$

[2.1]

where, *X* is the underlying ground set. In simple, a fuzzy set is a set whose boundary is not clear. On the other hand, a fuzzy set is a set whose elements are characterized by a membership function as above.

The ***α- cut of*** a fuzzy set *A* is the crisp subset of the ground set *X*, that contains all the elements whose membership grade is greater than or equal to α. It is denoted by A_α and is defined by

$$A_{\alpha} = \{x \mid \mu_A(x) \geq \alpha \ , x \in X\}$$

[2.2]

A ***triangular fuzzy number*** is a fuzzy set. It is denoted by $A = (a,b,c)$ and defined by the following membership function

$$\mu_A(x) = \begin{cases} 0; a \leq x \\ \dfrac{x-a}{b-a}; a \leq x \leq b \\ \dfrac{c-x}{c-b}; b \leq x \leq c \\ 0; x \geq c \end{cases}$$

[2.3]

where, $a, b, c \in$ R, $A \in F_N$,where F_N is the set of triangular fuzzy numbers and is represented graphically as

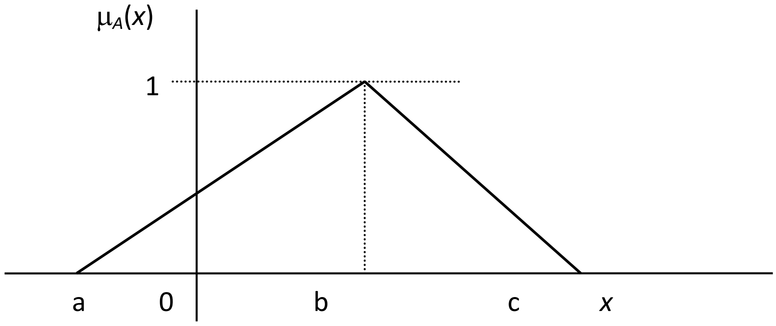


Fig.2. Variation of membership degree w.r.t. x

Properties of triangular fuzzy numbers

If *A* & *B* are two fuzzy numbers then their sum is also a fuzzy number. Suppose A = (a, b, c) & *B = (u, v, w)* then,

$$\begin{aligned} A \oplus B &= (a,b,c) \oplus (u,v,w) \\ &= (a+u, b+v, c+w) \end{aligned}$$

$$\begin{aligned} A - B &= (a,b,c) - (u,v,w) \\ &= (a,b,c) \oplus (-w,-v,-w) \\ &= (a-w, b-v, c-u) \end{aligned}$$

Example: Let *A = (5, 12, 13) & B = (-1, 2, 4)*

$$\begin{aligned} A \oplus B &= (5,12,13) + (-1,2,4) \\ &= (15-1, 12+2\ , 13+4) \\ &= (14,14,17) \end{aligned}$$

$$A - B = (5,12,13) - (-1,2,4)$$

$$= (5,12,13) \oplus (-4,2,1)$$
$$= (1,14,14)$$

Def 1: - Let d*(c, 0) = c; c, 0 $\in$ R

Geometrically, c>0 means that c lies to the right of the origin O and the distance between c and O is denoted by c = d*(c, 0). Similarly, c<0 means that c lies to the left of the origin O and the corresponding distance between c & o is denoted by –c = d*(c, 0). Therefore, d*(c, O) denotes the signed distance of c which is measured form O.

Def 2:- Signed-distance for $A = (a, b, c) \in F_N$, a triangular fuzzy number, the signed-distance of A measured from $\tilde{O}_1$ is defined by $d(A, \tilde{O}_1) = \frac{1}{4}(a+2b+c)$

Def 3:- Let $A=(a, b, c)$ & $B = (u, v, w) \in F_N$. Then the ranking of fuzzy numbers on F_N is defined by

$$A \prec B \quad \text{iff} \quad d(A, \tilde{O}_1) < d(B, \tilde{O}_1)$$

& $$A \approx B \quad \text{iff} \quad d(A, \tilde{O}_1) = d(B, \tilde{O}_1)$$

Fuzzy number are fuzzy subsets of set on real number satisfying some additional condition. Fuzzy number allow us to model non-probabilistic uncertainties in an easy way. Triangular and Trapezoidal fuzzy numbers are commonly used. Therefore here I am discuss about these two numbers only. Triangular and Trapezoidal fuzzy numbers can represented by (a, b, c) and (a, b, c, d) respectively. Triangular fuzzy numbers are special case of trapezoidal fuzzy numbers when b equal c.

Let A and B be two triangular fuzzy numbers, parameterized by (a1,a2,a3) and (b1,b2,b3). Their arithmetic can be described following

A+B=(a1+b1, a2+b2, a3+b3); A-B=(a1-b3, a2-b2, a3-b1);

A*B=(a1*b1, a2*b2, a3*b3); A/B=(a1/b3, a2/b2, a3/b1)

Let A and B be two triangular fuzzy numbers, parameterized by (a1,a2,a3,a4) and (b1,b2,b3,b4). Their arithmetic can be described following

A+B=(a1+b1, a2+b2, a3+b3, a4+b4); A-B=(a1-b4, a2-b3, a3-b2, a4-b1)

A*B=(a1*b1, a2*b2, a3*b3, a4*b4); A/B=(a1/b4, a2/b3, a3/b2, a4/b1)

These are the operations performed on fuzzy numbers. However, these values need to be mapped to real values for the calculation. Process of converting fuzzy numbers into crisp numbers is called defuzzification. Formula for performing defuzzification operations on triangular and trapezoidal fuzzy numbers .These formulas are given below. Let A(a1, a2, a3) and B(b1,b2,b3) are two triangular fuzzy numbers. Their defuzzification formula is given as $G(t) = \frac{1}{4}(a+2b+c)$

System Description about the model:
The system consists of five units namely three main units F in which two are standby mode and two associate units T and U. Here the associate units depend upon main units for operation . Associate units T and U work with the help of one of the main units. As soon as a job arrives, all the units work with load. There is a single repairman who repairs the failed units on first come first served basis. Using regenerative

point technique several system characteristics such as transition probabilities, mean sojourn times, availability and busy period of the repairman are evaluated.

Assumptions used in the model:

a. The system consists of three main units in which two are in standby mode and two associate units.

b. The associate unit T and U work with the help of main units.

c. There is a single repairman which repairs the failed units on priority basis.

d. After random period of time the whole system goes to preventive maintenance.

e. All units work as new after repair.

f. The failure rates of all the units are taken to be exponential whereas the repair time distributions are arbitrary.

g. Switching devices are perfect and instantaneous.

Symbols and Notations:

p_{ij} =Transition probabilities from S_i to S_j

μ_i =Mean sojourn time at time t

E_0 =State of the system at epoch t=0

E=set of regenerative states $S_0 - S_6$

$q_{ij}(t)$ =Probability density function of transition time from S_i to S_j

$Q_{ij}(t)$ =Cumulative distribution function of transition time from S_i to S_j

$\pi_i(t) =$ Cdf of time to system failure when starting from state $E_0 = S_i \in E$

$\mu_i(t)$ =Mean Sojourn time in the state $E_0 = S_i \in E$

$B_i(t)$ =Repairman is busy in the repair at time t / $E_0 = S_i \in E$

$r_1 / r_2 / r_3$ =Constant repair rate of Main unit F /Unit T/Unit U

$\alpha / \beta / \gamma$ =Failure rate of Main unit F /Unit T/Unit U

$g_1(t)/g_2(t)/g_3(t)$ =Probability density function of repair time of Main unit F/Unit T/UnitU

$\overline{G_1}(t)/\overline{G_2}(t)/\overline{G_3}(t)$ =Cumulative distribution function of repair time of Main unit F/UnitT/UnitU

a(t) = Probability density function of preventive maintenance .

b(t) = Probability density function of preventive maintenance completion time.

$\overline{A}(t)$ = Cumulative distribution functions of preventive maintenance.

$\overline{B}(t)$ = Cumulative distribution functions of preventive maintenance completion time.

$\boxed{s}$ =Symbol for Laplace-stieltjes transforms.

$\boxed{c}$ =Symbol for Laplace-convolution.

Symbols used for states of the system:

$F_0 / F_S / F_g / F_r / F_{wr}$ -- Main unit 'F' under operation/standby / good and non-operative mode/ repair/waiting for repair

$T_0 / T_r / T_g$ -- Associate Unit 'T' under operation/repair/ good and non-operative mode

$U_0 / U_r / U_g$ -- Associate Unit 'U' under operation/repair/good and non-operative mode

P.M. -- System under preventive maintenance.

Up states: $S_0 = (F_0, F_S, F_S, T_0, U_0); S_1 = (F_r, F_0, F_S, T_0, U_0); S_2 = (F_{wr}, F_r, F_0, T_0, U_0)$

Down States: $S_3 = (F_g, F_S, F_S, T_g, U_r); S_4 = (F_{wr}, F_g, F_S, T_g, U_r); S_5 = (F_g, F_S, F_S, T_r, U_g);$
$S_6 = (F_{wr}, F_g, F_S, T_r, U_g); S_7 = (P.M.); S_8 = (S.D.)$

Transition Probabilities:

Simple probabilistic considerations yield the following non-zero transition probabilities:

1. $Q_{01}(t) = \int_0^t \alpha e^{-x_1 t} \overline{A}(t) dt$
2. $Q_{03}(t) = \int_0^t \gamma e^{-x_1 t} \overline{A}(t) dt$
3. $Q_{05}(t) = \int_0^t \beta e^{-x_1 t} \overline{A}(t) dt$
4. $Q_{07}(t) = \int_0^t e^{-x_1 t} a(t) dt$
5. $Q_{10}(t) = \int_0^t e^{-x_1 t} g_1(t) dt$
6. $Q_{12}(t) = \int_0^t \alpha e^{-x_1 t} \overline{G}_1(t) dt$
7. $Q_{14}(t) = \int_0^t \gamma e^{-x_1 t} \overline{G}_1(t) dt$
8. $Q_{16}(t) = \int_0^t \beta e^{-x_1 t} \overline{G}_1(t) dt$
9. $Q_{21}(t) = \int_0^t e^{-x_1 t} g_1(t) dt$
10. $Q_{28}(t) = \int_0^t x_1 e^{-x_1 t} \overline{G}_1(t) dt$
11. $Q_{30}(t) = \int_0^t g_3(t) dt = Q_{41}(t)$
12. $Q_{50}(t) = \int_0^t g_2(t) dt = Q_{61}(t)$
13. $Q_{70}(t) = \int_0^t b(t) dt$
14. $Q_{80}(t) = \int_0^t g_4(t) dt$

Where $x_1 = \alpha + \beta + \gamma$, Now letting $t \to \infty$, we get $\lim_{t \to \infty} Q_{ij}(t) = p_{ij}$

15. $p_{01} = \int_0^\infty \alpha e^{-x_1 t} \overline{A}(t) dt = \frac{\alpha}{x_1}[1 - a^*(x_1)]$,
16. $p_{03} = \int_0^\infty \gamma e^{-x_1 t} \overline{A}(t) dt = \frac{\gamma}{x_1}[1 - a^*(x_1)]$

17. $p_{05} = \int_0^\infty \beta e^{-x_1 t} \overline{A}(t)dt = \frac{\beta}{x_1}[1 - a^*(x_1)]$, 18. $p_{07} = \int_0^\infty e^{-x_1 t} a(t)dt = a^*(x_1)$

19. $p_{10} = \int_0^\infty e^{-x_1 t} g_1(t)dt = g_1^*(x_1)$ 20. $p_{12} = \int_0^\infty \alpha e^{-x_1 t} \overline{G}_1(t)dt = \frac{\alpha}{x_1}[1 - g_1^*(x_1)]$

21. $p_{14} = \int_0^\infty \gamma e^{-x_1 t} \overline{G}_1(t)dt = \frac{\gamma}{x_1}[1 - g_1^*(x_1)]$ 22. $p_{16} = \int_0^\infty \beta e^{-x_1 t} \overline{G}_1(t)dt = \frac{\beta}{x_1}[1 - g_1^*(x_1)]$

23. $p_{21} = \int_0^\infty e^{-x_1 t} g_1(t)dt = g_1^*(x_1)$, 24. $p_{28} = \int_0^\infty x_1 e^{-x_1 t} \overline{G}_1(t)dt = \frac{x_1}{x_1}[1 - g_1^*(x_1)] = 1 - g_1^*(x_1)$

25. $p_{30} = p_{41} = p_{50} = p_{61} = p_{70} = p_{80} = 1$ [7.1-7.25]

It is easy to see that

$p_{01} + p_{03} + p_{05} + p_{07} = 1$, $p_{10} + p_{12} + p_{14} + p_{16} = 1$, $p_{21} + p_{28} = 1$ [7.26-7.28]

And mean sojourn time are given by

29. $\mu_0 = \frac{1}{x_1}[1 - a^*(x_1)]$, 30. $\mu_1 = \frac{1}{x_1}[1 - g_1^*(x_1)] = \mu_2$,

31. $\mu_3 = \int_0^\infty \overline{G}_3(t)dt = \mu_4$, 32. $\mu_5 = \int_0^\infty \overline{G}_2(t)dt = \mu_6$,

33. $\mu_7 = \int_0^\infty \overline{B}(t)dt$ 34. $\mu_8 = \int_0^\infty \overline{G}_4(t)dt$ [7.29-7.34]

We note that the Laplace-stieltjes transform of $Q_{ij}(t)$ is equal to Laplace transform of $q_{ij}(t)$ i.e.

$$\tilde{Q}_{ij}(s) = \int_0^\infty e^{-st} Q_{ij}(t)dt = L\{Q_{ij}(t)\} = q_{ij}^*(s)$$

[7.35]

36. $\tilde{Q}_{01}(s) = \int_0^\infty \alpha e^{-(s+x_1)t} \overline{A}(t)dt = \frac{\alpha}{s + x_1}[1 - a^*(s + x_1)]$ 37. $\tilde{Q}_{03}(s) = \int_0^\infty \gamma e^{-(s+x_1)t} \overline{A}(t)dt = \frac{\gamma}{s + x_1}[1 - a^*(s + x_1)]$

38. $\tilde{Q}_{05}(s) = \int_0^\infty \beta e^{-(s+x_1)t} \overline{A}(t)dt = \frac{\beta}{s + x_1}[1 - a^*(s + x_1)]$ 39. $\tilde{Q}_{07}(s) = \int_0^\infty e^{-(s+x_1)t} a(t)dt = a^*(s + x_1)$

40. $\tilde{Q}_{10}(s) = \int_0^\infty e^{-(s+x_1)t} g_1(t)dt = g_1^*(s + x_1)$ 41. $\tilde{Q}_{12}(s) = \int_0^\infty \alpha e^{-(s+x_1)t} \overline{G}_1(t)dt = \frac{\alpha}{s + x_1}[1 - g_1^*(s + x_1)]$

42. $\tilde{Q}_{14}(s) = \int_0^\infty \gamma e^{-(s+x_1)t} \overline{G}_1(t)dt = \frac{\gamma}{s + x_1}[1 - g_1^*(s + x_1)]$

43. $\tilde{Q}_{16}(s)=\int_0^\infty \beta e^{-(s+x_1)t}\overline{G}_1(t)dt=\frac{\beta}{s+x_1}[1-g_1^{\ *}(s+x_1)]$

44. $\tilde{Q}_{21}(s)=\int_0^\infty e^{-(s+x_1)t}g_1(t)dt=g_1^{\ *}(s+x_1)$

45. $\tilde{Q}_{28}(s)=\int_0^\infty x_1\, e^{-(s+x_1)t}\overline{G}_1(t)dt=\frac{x_1}{s+x_1}[1-g_1^{\ *}(s+x_1)]$ define

[7.36-7.45]

$m_{i\,j}$ as follows:-

$$m_{i\,j}=-[\frac{d}{ds}\tilde{Q}_{i\,j}(s)]_{s=0}\ =-Q_{i\,j}^{\ /}(0)$$

[7.46]

It can to show that $m_{01}+m_{03}+m_{05}+m_{07}=\mu_0;m_{10}+m_{12}+m_{14}+m_{16}=\mu_1;m_{21}+m_{28}=\mu_2$

Where $\alpha+\beta+\gamma=x_1$

[7.47-7.50]

Mean time to System failure-

Time to system failure can be regarded as the first passage time to the failed state. To obtain it we regard the down state as absorbing. Using the argument as for the regenerative process, we obtain the following recursive relations.

$$\pi_0(t)=Q_{01}(t)\ \boxed{s}\ \pi_1(t)+Q_{03}(t)+Q_{05}(t)+Q_{07}(t)$$

$$\pi_1(t)=Q_{10}(t)\ \boxed{s}\ \pi_0(t)+Q_{12}(t)\ \boxed{s}\ \pi_2(t)+Q_{14}(t)+Q_{16}(t)$$

$$\pi_2(t)=Q_{21}(t)\ \boxed{s}\ \pi_1(t)+Q_{28}(t)$$

[8.1-8.3]

Taking Laplace - stieltjes transform of above equations and writing in matrix form.

We get

$$\begin{bmatrix}1 & -\tilde{Q}_{01} & 0\\ -\tilde{Q}_{10} & 1 & -\tilde{Q}_{12}\\ 0 & -\tilde{Q}_{21} & 1\end{bmatrix}\begin{bmatrix}\pi_0\\ \pi_1\\ \pi_2\end{bmatrix}=\begin{bmatrix}\tilde{Q}_{03}+\tilde{Q}_{05}+\tilde{Q}_{07}\\ \tilde{Q}_{14}+\tilde{Q}_{16}\\ \tilde{Q}_{28}\end{bmatrix}$$

$$D_1(s)=\begin{vmatrix}1 & -\tilde{Q}_{01} & 0\\ -\tilde{Q}_{10} & 1 & -\tilde{Q}_{12}\\ 0 & -\tilde{Q}_{21} & 1\end{vmatrix}=1-\tilde{Q}_{12}\tilde{Q}_{21}-\tilde{Q}_{01}\tilde{Q}_{10} \qquad [8.4]$$

And $$N_1(s)=\begin{vmatrix}\tilde{Q}_{03}+\tilde{Q}_{05}+\tilde{Q}_{07} & -\tilde{Q}_{01} & 0\\ \tilde{Q}_{14}+\tilde{Q}_{16} & 1 & -\tilde{Q}_{12}\\ \tilde{Q}_{28} & -\tilde{Q}_{21} & 1\end{vmatrix}=(\tilde{Q}_{03}+\tilde{Q}_{05}+\tilde{Q}_{07})(1-\tilde{Q}_{12}\tilde{Q}_{21})+\tilde{Q}_{01}(\tilde{Q}_{14}+\tilde{Q}_{16}+\tilde{Q}_{12}\tilde{Q}_{28})$$

Now letting $s\to 0$ we get [8.5]

$$D_1(0)=1-p_{12}p_{21}-p_{01}p_{10} \qquad [8.6]$$

The mean time to system failure when the system starts from the state S_0 is given by

MTSF= $E(T) = -[\frac{d}{ds}\tilde{\pi}_0(s)]_{s=0} = \frac{D_1^{/}(0) - N_1^{/}(0)}{D_1(0)}$ [8.7]

To obtain the numerator of the above equation, we collect the coefficients of relevant of m_{ij} in $D_1^{/}(0) - N_1^{/}(0)$.

Coeff. of ($m_{01} = m_{03} = m_{05} = m_{07}$) $=1 - p_{12}p_{21}$ Coeff. of ($m_{21} = m_{28}$) $= p_{01}p_{12}$

Coeff. of ($m_{10} = m_{12} = m_{14} = m_{16}$) $= p_{01}$
[8.8-8.10]

From equation [7.7]

MTSF= MTSF= $E(T) = -[\frac{d}{ds}\tilde{\pi}_0(s)]_{s=0} = \frac{D_1^{/}(0) - N_1^{/}(0)}{D_1(0)}$

$$= \frac{\mu_0(1 - p_{12}p_{21}) + \mu_1 p_{01} + \mu_2 p_{01}p_{12}}{1 - p_{12}p_{21} - p_{01}p_{10}}$$

[8.11]

Availability Analysis:
Let $M_i(t)(i = 0,1,2)$ denote the probability that system is initially in regenerative state $S_i \in E$ is up at time t without passing through any other regenerative state or returning to itself through one or more non regenerative states .i.e. either it continues to remain in regenerative S_i or a non regenerative state including itself . By probabilistic arguments, we have the following recursive relations

$M_0(t) = e^{-(\alpha+\beta+\gamma)t}\overline{A}(t),\qquad M_1(t) = e^{-(\alpha+\beta+\gamma)t}\overline{G}_1(t) = M_2(t)$ [9.1-9.2]

Recursive relations giving point wise availability $A_i(t)$ given as follows:

$A_0(t) = M_0(t) + \sum_{i=1,3,5,7} q_{0i}(t) \boxed{c} A_i(t)$; $A_1(t) = M_1(t) + \sum_{i=0,2,4,6} q_{1i}(t) \boxed{c} A_i(t)$

$A_2(t) = M_2(t) + \sum_{i=1,8} q_{2i}(t) \boxed{c} A_i(t)$; $A_3(t) = (\boxed{c}) A_0(t)$;

$A_4(t) = q_{41}(t) \boxed{c} A_1(t)$; $A_5(t) = \boxed{c}t) A_0(t)$;

$A_6(t) = q_{61}(t) \boxed{c} A_1(t)$; $A_7(t) = q_{70}(t) \boxed{c} A_0(t)$;

$A_8(t) = q_{80}(t) \boxed{c} A_0(t)$ [9.3-9.11]

Taking Laplace - stieltjes transformation of above equations; and writing in matrix form, we get

$q_{9X9}[A_0^*, A_1^*, A_2^*, A_3^*, A_4^*, A_5^*, A_6^*, A_7^*, A_8^*]^{/} = [M_0^*, M_1^*, M_2^*, 0,0,0,0,0,0]^{/}$ [9.12]

$$\text{Where } q_{9X9} = \begin{bmatrix} 1 & -q_{01}^* & 0 & -q_{03}^* & 0 & -q_{05}^* & 0 & -q_{07}^* & 0 \\ -q_{10}^* & 1 & -q_{12}^* & 0 & -q_{14}^* & 0 & -q_{16}^* & 0 & 0 \\ 0 & -q_{21}^* & 1 & 0 & 0 & 0 & 0 & 0 & -q_{28}^* \\ -q_{30}^* & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q_{41}^* & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -q_{50}^* & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -q_{61}^* & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -q_{70}^* & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -q_{80}^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{9X9}$$

$$\text{Therefore } D_2(s) = \begin{vmatrix} 1 & -q_{01}^* & 0 & -q_{03}^* & 0 & -q_{05}^* & 0 & -q_{07}^* & 0 \\ -q_{10}^* & 1 & -q_{12}^* & 0 & -q_{14}^* & 0 & -q_{16}^* & 0 & 0 \\ 0 & -q_{21}^* & 1 & 0 & 0 & 0 & 0 & 0 & -q_{28}^* \\ -q_{30}^* & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q_{41}^* & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -q_{50}^* & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -q_{61}^* & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -q_{70}^* & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -q_{80}^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$= (1 - q_{12}^* q_{21}^* - q_{14}^* q_{41}^* - q_{16}^* q_{61}^*)[1 - q_{03}^* q_{30}^* - q_{05}^* q_{50}^* - q_{07}^* q_{70}^*] - q_{01}^* q_{10}^* - q_{01}^* q_{12}^* q_{28}^* q_{80}^*$$

If $s \to 0$ we get $D_2(0) = 0$ which is true
[9.13]

$$\text{Now } N_2(s) = \begin{vmatrix} M_0^* & -q_{01}^* & 0 & -q_{03}^* & 0 & -q_{05}^* & 0 & -q_{07}^* & 0 \\ M_1^* & 1 & -q_{12}^* & 0 & -q_{14}^* & 0 & -q_{16}^* & 0 & 0 \\ M_2^* & -q_{21}^* & 1 & 0 & 0 & 0 & 0 & 0 & -q_{28}^* \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q_{41}^* & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -q_{61}^* & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

Solving this Determinant, we get

$$N_2(s) = M_0^*(1 - q_{12}^* q_{21}^* - q_{14}^* q_{41}^* - q_{16}^* q_{61}^*) + M_1^* q_{01}^* + M_2^* q_{01}^* q_{12}^* \qquad [9.14]$$

If $s \to 0$ we get

$N_2(0) = \mu_0(p_{10} + p_{12}p_{28}) + \mu_1 p_{01} + \mu_2 p_{01}p_{12} \quad [\because 1 - p_{14} - p_{16} = p_{10} + p_{12}]$ [9.15]

To find the value of $D_2^{/}(0)$ we collect the coefficient m_{ij} in $D_2(s)$ we get

Coeff. of $(m_{01} = m_{03} = m_{05} = m_{07}) = p_{10} + p_{12}p_{28} = L_0$

Coeff. of $(m_{10} = m_{12} = m_{14} = m_{16}) = p_{01} = L_1$ Coeff. of $(m_{21} = m_{28}) = p_{01}p_{12} = L_2$

Coeff.of $m_{30} = p_{03}(p_{10} + p_{12}p_{28}) = L_3$ Coeff.f $m_{41} = p_{14}p_{01} = L_4$

Coeff. of $m_{50} = p_{05}(p_{10} + p_{12}p_{28}) = L_5$ Coeff. of $m_{61} = p_{16}p_{01} = L_6$

Coeff.of $m_{70} = p_{07}(p_{10} + p_{12}p_{28}) = L_7$ Coeff. of $m_{80} = p_{01}p_{12}p_{28} = L_8$ [9.16-9.24]

Thus the solution for the steady-state availability is given by

$$A_0^*(\infty) = \underset{t\to\infty}{Lim} A_0^*(t) = \underset{S\to 0}{Lim}\, sA_0^*(s) = \frac{N_2(0)}{D_2^{/}(0)} = \frac{\mu_0 L_0 + \mu_1 L_1 + \mu_2 L_2}{\sum_{i=0,1,2,3,4,5,6,7,8} \mu_i L_i} \qquad [9.25]$$

Busy Period Analysis:

(a) Let $W_i(t)$ $(i = 1,2,3,4,5,6)$ denote the probability that the repairman is busy initially with repair in regenerative state S_i and remain busy at epoch t without transiting to any other state or returning to itself through one or more regenerative states.

By probabilistic arguments we have

$$W_3(t) = \overline{G}_3(t) = W_4(t), W_1(t) = \overline{G}_1(t) = W_2(t), W_5(t) = \overline{G}_2(t) = W_6(t) \qquad [10.1\text{-}10.3]$$

Developing similar recursive relations as in availability, we have

$B_0(t) = \sum_{i=1,3,5,7} q_{0i}(t) \boxed{c}\ B_i(t)$; $B_1(t) = W_1(t) + \sum_{i=0,2,4,6} q_{1i} \boxed{c}\ B_i(t)$;

$B_2(t) = W_2(t) + \sum_{i=1,8} q_{2i}(t) \boxed{c}\ B_i(t)$; $B_3(t) = W_3(t) + q_{30}(t \boxed{c}\ B_0(t)$;

$B_4(t) = W_4(t) + q_{41}(t) \boxed{c}\ B_1(t)$; $B_5(t) = W_5(t) + q_{50}(\boxed{c}\ B_0(t)$;

$B_6(t) = W_6(t) + q_{61}(t) \boxed{c}\ B_1(t)$; $B_7(t) = q_{70}(\boxed{c}\ B_0(t)$;

$B_8(t) = q_{80}(t) \boxed{c}\ B_0(t)$

[10.4-10.12]

Taking Laplace - stieltjes transformation of above equations; and writing in matrix form, we get

$$q_{9X9}[B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*, B_6^*, B_7^*, B_8^*]^{/} = [0, W_1^*, W_2^*, W_3^*, W_4^*, W_5^*, W_6^*, 0, 0]^{/}$$

Where q_{9x9} is denoted by [8.13] and therefore $D_2^{/}(s)$ is obtained as in the expression of availability.

Now $N_3(s) = \begin{vmatrix} 0 & -q_{01}^{*} & 0 & -q_{03}^{*} & 0 & -q_{05}^{*} & 0 & -q_{07}^{*} & 0 \\ W_1^{*} & 1 & -q_{12}^{*} & 0 & -q_{14}^{*} & 0 & -q_{16}^{*} & 0 & 0 \\ W_2^{*} & -q_{21}^{*} & 1 & 0 & 0 & 0 & 0 & 0 & -q_{28}^{*} \\ W_3^{*} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ W_4^{*} & -q_{41}^{*} & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ W_5^{*} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ W_6^{*} & -q_{61}^{*} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$

Solving this Determinant, In the long run, we get the value of this determinant after putting $s \to 0$ is

$$N_3(0) = (\mu_1 p_{01} + \mu_2 p_{01} p_{12} + \mu_4 p_{01} p_{14} + \mu_6 p_{01} p_{16}) + (\mu_3 p_{03} + \mu_5 p_{05})(p_{10} + p_{12} p_{28})$$

$$= \mu_1 L_1 + \mu_2 L_2 + \mu_3 L_3 + \mu_4 L_4 + \mu_5 L_5 + \mu_6 L_6 = \sum_{i=1,2,3,4,5,6} \mu_i L_i \qquad [10.13]$$

Thus the fraction of time for which the repairman is busy with repair of the failed unit is given by:

$$B_0^{1^*}(\infty) = \underset{t\to\infty}{Lim} B_0^{1^*}(t) = \underset{s\to 0}{Lim}\, s B_0^{1^*}(s) = \frac{N_3(0)}{D_2^{/}(0)} = \frac{\sum_{i=1,2,3,4,5,6} \mu_i L_i}{\sum_{i=0,1,2,3,4,5,6,7,8} \mu_i L_i} \qquad [10.14]$$

(b) Busy period of the Repairman in preventive maintenance in time (0, t]: By probabilistic arguments we have

$$W_7(t) = \overline{B}(t)$$
[10.15]

Similarly developing similar recursive relations as in 9(a), we have

$B_0(t) = \sum_{i=1,3,5,7} q_{0i}(t) \boxed{c}\ B_i(t)$; $B_1(t) = \sum_{i=0,2,4,6} q_{1i}(\boxed{c}\ B_i(t)$;

$B_2(t) = \sum_{i=1,8} q_{2i}(t) \boxed{c}\ B_i(t)$; $B_3(t) = q_{30} \boxed{c}\ B_0(t)$;

$B_4(t) = q_{41}(t) \boxed{c}\ B_1(t)$; $B_5(t) = q_{50} \boxed{c}\ B_0(t)$;

$B_6(t) = q_{61}(t) \boxed{c}\ B_1(t)$; $B_7(t) = W_7(t) + q_{7(} \boxed{c}\ B_0(t)$;

$B_8(t) = q_{80}(t) \boxed{c}\ B_0(t)$ [10.16-10.24]

Taking Laplace - stieltjes transformation of above equations; and writing in matrix form, we get

$$q_{9X9}[B_0^{*}, B_1^{*}, B_2^{*}, B_3^{*}, B_4^{*}, B_5^{*}, B_6^{*}, B_7^{*}, B_8^{*}]^{/} = [0,0,0,0,0,0,0,W_7^{*},0]^{/}$$

Where q_{9x9} is denoted by [8.13] and therefore $D_2^{/}(s)$ is obtained as in the expression of availability.

$$\text{Now } N_4(s) = \begin{vmatrix} 0 & -q_{01}^{*} & 0 & -q_{03}^{*} & 0 & -q_{05}^{*} & 0 & -q_{07}^{*} & 0 \\ 0 & 1 & -q_{12}^{*} & 0 & -q_{14}^{*} & 0 & -q_{16}^{*} & 0 & 0 \\ 0 & -q_{21}^{*} & 1 & 0 & 0 & 0 & 0 & 0 & -q_{28}^{*} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q_{41}^{*} & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -q_{61}^{*} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ W_7^{*} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

Solving this Determinant, In the long run, we get the value of this determinant after putting $s \to 0$ is $N_4(0) = \mu_7 p_{07}(p_{10} + p_{12} p_{28}) = \mu_7 L_7$ [10.25]

Thus the fraction of time for which the system is under preventive maintenance is given by:

$$B_0^{2^*}(\infty) = \underset{t\to\infty}{Lim} B_0^{2^*}(t) = \underset{s\to 0}{Lim}\, sB_0^{2^*}(s) = \frac{N_4(0)}{D_2^{/}(0)} = \frac{\mu_7 L_7}{\sum_{i=0,1,2,3,4,5,6,7,8} \mu_i L_i} \quad [10.26]$$

(c) Busy period of the Repairman in Shut Down repair in time (0, t]: By probabilistic arguments we have

$W_8(t) = \overline{G}_4(t)$
[10.27]

Similarly developing similar recursive relations as in 9(b), we have

$B_0(t) = \sum_{i=1,3,5,7} q_{0i}(t) \boxed{c}\ B_i(t)$; $B_1(t) = \sum_{i=0,2,4,6} q_{1i}(\boxed{c}\ B_i(t)$;

$B_2(t) = \sum_{i=1,8} q_{2i}(t) \boxed{c}\ B_i(t)$; $B_3(t) = q_{30}(\boxed{c}\ B_0(t)$;

$B_4(t) = q_{41}(t) \boxed{c}\ B_1(t)$; $B_5(t) = q_{50}\boxed{c}\ B_0(t)$; \

$B_6(t) = q_{61}(t) \boxed{c}\ B_1(t)$; $B_7(t) = q_{70}\boxed{c}\ B_0(t)$;

$B_8(t) = W_8(t) + q_{80}(t)\boxed{c}\ B_0(t)$
[10.28-10.36]

Taking Laplace- stieltjes transformation of above equations; and writing in matrix form, we get

$$q_{9X9}[B_0^{*}, B_1^{*}, B_2^{*}, B_3^{*}, B_4^{*}, B_5^{*}, B_6^{*}, B_7^{*}, B_8^{*}]^{/} = [0,0,0,0,0,0,0,0,W_8^{*}]^{/}$$

Where q_{9x9} is denoted by [8.13] and therefore $D_2^{/}(s)$ is obtained as in the expression of availability.

Now $N_5(s) = \begin{vmatrix} 0 & -q_{01}^{\ *} & 0 & -q_{03}^{\ *} & 0 & -q_{05}^{\ *} & 0 & -q_{07}^{\ *} & 0 \\ 0 & 1 & -q_{12}^{\ *} & 0 & -q_{14}^{\ *} & 0 & -q_{16}^{\ *} & 0 & 0 \\ 0 & -q_{21}^{\ *} & 1 & 0 & 0 & 0 & 0 & 0 & -q_{28}^{\ *} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q_{41}^{\ *} & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -q_{61}^{\ *} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0^* & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ W_8^{\ *} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$

In the long run, we get the value of this determinant after putting $s \to 0$ is

$$N_4(0) = \mu_8 p_{01} p_{12} p_{28} = \mu_8 L_8 \qquad [10.37]$$

Thus the fraction of time for which the system is under shut down is given by:

$$B_0^{3^*}(\infty) = \underset{t\to\infty}{Lim} B_0^{3^*}(t) = \underset{s\to 0}{Lim}\, s B_0^{3^*}(s) = \frac{N_5(0)}{D_2^{/}(0)} = \frac{\mu_8 L_8}{\sum\limits_{i=0,1,2,3,4,5,6,7,8} \mu_i L_i} \qquad [10.38]$$

Particular cases: When all repair time distributions are n-phase Erlangian distributions i.e.

Density function $g_i(t) = \dfrac{nr_i(nr_i t)^{n-1} e^{-nr_i t}}{n-1!}$ And Survival function $\overline{G}_j(t) = \sum\limits_{j=0}^{n-1} \dfrac{(nr_i t)^j e^{-nr_i t}}{j!}$ [11.1-11.2]

And other distributions are negative exponential

$$a(t) = \theta e^{-\theta t}, b(t) = \eta e^{-\eta t}, \overline{A}(t) = e^{-\theta t}, \overline{B}(t) = e^{-\eta t} \qquad [11.3\text{-}11.6]$$

For n=1 $g_i(t) = r_i e^{-r_i t}$, $\overline{G}_i(t) = e^{-r_i t}$ If i=1, 2, 3, 4

$$g_1(t) = r_1 e^{-r_1 t}, g_2(t) = r_2 e^{-r_2 t}, g_3(t) = r_3 e^{-r_3 t}, g_4(t) = r_4 e^{-r_4 t}$$

$$\overline{G}_1(t) = e^{-r_1 t}, \overline{G}_2(t) = e^{-r_2 t}, \overline{G}_3(t) = e^{-r_3 t}, \overline{G}_4(t) = e^{-r_4 t} \qquad [11.7\text{-}11.14]$$

Also

$$p_{01} = \frac{\alpha}{x_1+\theta}, p_{03} = \frac{\gamma}{x_1+\theta}, p_{05} = \frac{\beta}{x_1+\theta}, p_{07} = \frac{\theta}{x_1+\theta},$$

$$p_{10} = \frac{r_1}{x_1+r_1}, p_{12} = \frac{\alpha}{x_1+r_1}, p_{14} = \frac{\gamma}{x_1+r_1}, p_{16} = \frac{\beta}{x_1+r_1},$$

$$p_{21} = \frac{r_1}{x_1+r_1}, p_{26} = \frac{x_1}{x_1+r_1}, \mu_0 = \frac{1}{x_1+\theta}, \mu_1 = \frac{1}{x_1+r_1} = \mu_2$$

$$\mu_3 = \frac{1}{r_3} = \mu_4, \mu_5 = \frac{1}{r_2} = \mu_6, \mu_7 = \frac{1}{\eta}, \mu_8 = \frac{1}{r_4} \qquad [11.15\text{-}11.30]$$

MTSF=$\dfrac{\mu_0(1-p_{12}p_{21})+\mu_1 p_{01}+\mu_2 p_{01}p_{12}}{1-p_{12}p_{21}-p_{01}p_{10}}$, $A_0(\infty)=\dfrac{\mu_0 L_0+\mu_1 L_1+\mu_2 L_2}{\sum_{i=0,1,2,3,4,5,6,7,8}\mu_i L_i}$,

$$B_0^{1^*}(\infty)=\frac{\sum_{i=1,2,3,4,5,6}\mu_i L_i}{\sum_{i=0,1,2,3,4,5,6,7,8}\mu_i L_i},\ B_0^{2^*}(\infty)=\frac{\mu_7 L_7}{\sum_{i=0,1,2,3,4,5,6,7,8}\mu_i L_i},\ B_0^{3^*}(\infty)=\frac{\mu_8 L_8}{\sum_{i=0,1,2,3,4,5,6,7,8}\mu_i L_i} \qquad [11.31\text{-}11.35]$$

Where $L_0=p_{10}+p_{12}p_{28}; L_1=p_{01}; L_2=p_{01}p_{12};\ L_3=p_{03}(p_{10}+p_{12}p_{28})\,; L_4=p_{01}p_{14};$

$$L_5=p_{05}(p_{10}+p_{12}p_{28});\ L_6=p_{01}p_{16}\,; L_7=p_{07}(p_{10}+p_{12}p_{28}); L_8=p_{01}p_{12}p_{28} \qquad [11.36\text{-}11.44]$$

Profit Analysis:-

The profit analysis of the system can be carried out by considering the expected busy period of the repairman in repair of the unit in (0,t].

Therefore, G(t) = Expected total revenue earned by the system in (0,t] -Expected repair cost of the failed units

-Expected repair cost of the repairman in preventive maintenance -Expected repair cost of the

Repairman in shut down

$$=C_1\mu_{up}(t)-C_2\mu_{b1}(t)-C_3\mu_{b2}(t)-C_4\mu_{b3}(t)$$

$$=C_1A_0-C_2B^1{}_0-C_3B^2{}_0-C_4B^3{}_0 \qquad [12.1]$$

Where $\mu_{up}(t)=\int_0^t A_0(t)\,dt\,; \mu_{b1}(t)=\int_0^t B_0^1(t)\,dt\,; \mu_{b2}(t)=\int_0^t B_0^2(t)\,dt\,; \mu_{b3}(t)=\int_0^t B_0{}^3(t)dt$ [12.2-12.5]

C_1 is the revenue per unit time and C_2, C_3, C_4 are the cost per unit time for which the system is under simple repair, preventive maintenance and shut down repair respectively.

Apply fuzzy concept in [12.1]

$$\tilde{G}(t)=\tilde{C}_1\mu_{up}(t)-\tilde{C}_2\mu_{b1}(t)-\tilde{C}_3\mu_{b2}(t)-\tilde{C}_4\mu_{b3}(t) \qquad [12.6]$$

Taking triangle fuzzy number

$\tilde{C}_1=(2,8,18);\quad \tilde{C}_2=(2,5,8);\quad \tilde{C}_3=(5,7,9);\quad \tilde{C}_4=(-3,4,5);$

$\mu_{up}=1.027;\quad \mu_{b1}=0.174;\quad \mu_{b2}=0.022;\quad \mu_{b3}=1.151;$

$$\tilde{G}(t)=(-5.291,2.588,21.481) \qquad [12.7]$$

Applying defuzzification in eq. [12.7],we get

$$G(t)=\frac{1}{4}(a+2b+c)$$

$$=\frac{1}{4}(-5.291+2*2.588+21.481)$$

$$= 5.341\ (rounding \quad off)$$

Discussion and result:

It is seen in the fig. 1 and 2 that value of MTSF decreases with increase in the failure rate of main unit . The same can be predicated in the case of Availability and Profit . It is also seen that application of Preventive Maintenance technique increases the Availability to some extent. This concludes the reliability and profit of system also increases as the repair rate increases. The observation draw fig.3 for a value of probability of unit after inspection, the profit of system decreases if the failure rate is greater ,profit is less or nil. If failure rate is less and profit is positive value. The use of fuzzy theory in profit analysis removes uncertainty in the cost of various parameters and gives the exact value of profit of any system.

Fig.1 Availability vs Failure Rate

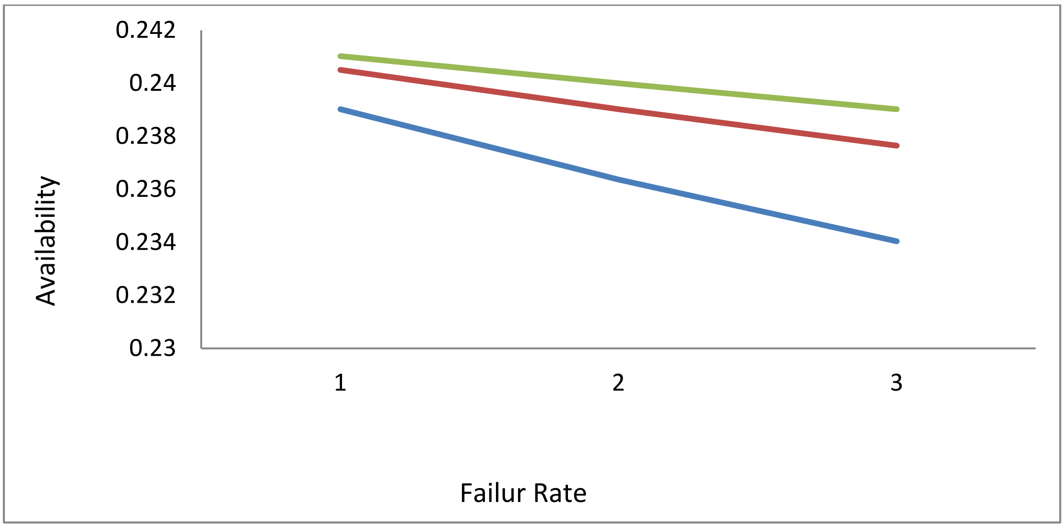


Fig.2 MTSF vs Failure Rate

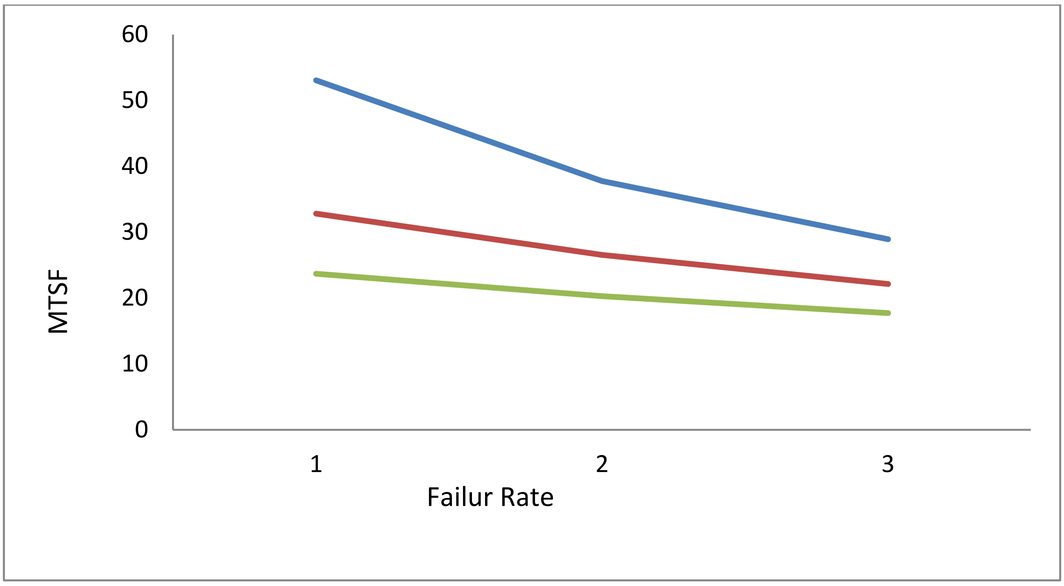
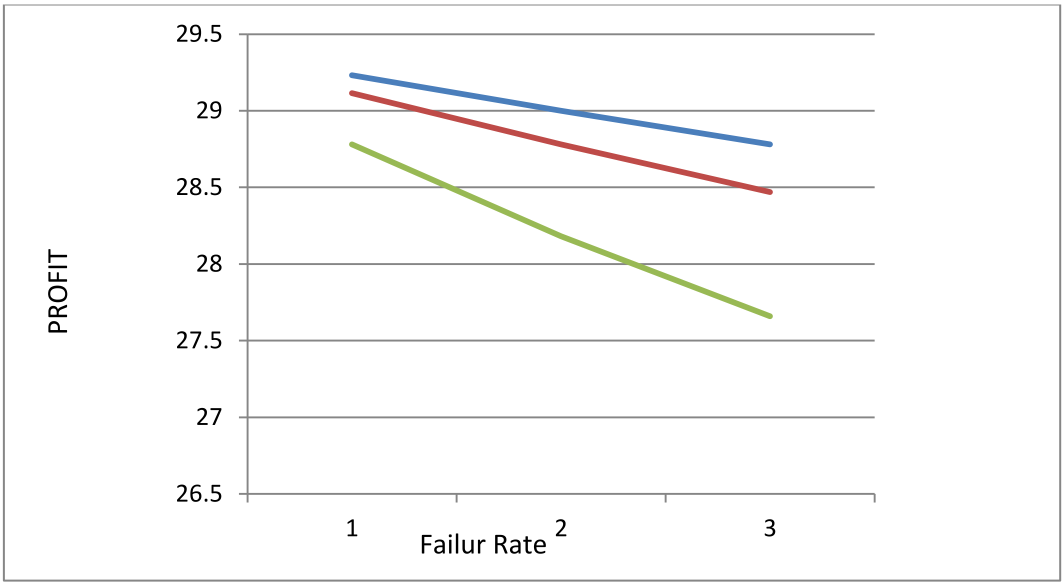


Fig.3 Profit Vs Failure Rate


29.5
29
28.5
28
27.5
27
26.5
PROFIT
1
2
3
Failur Rate

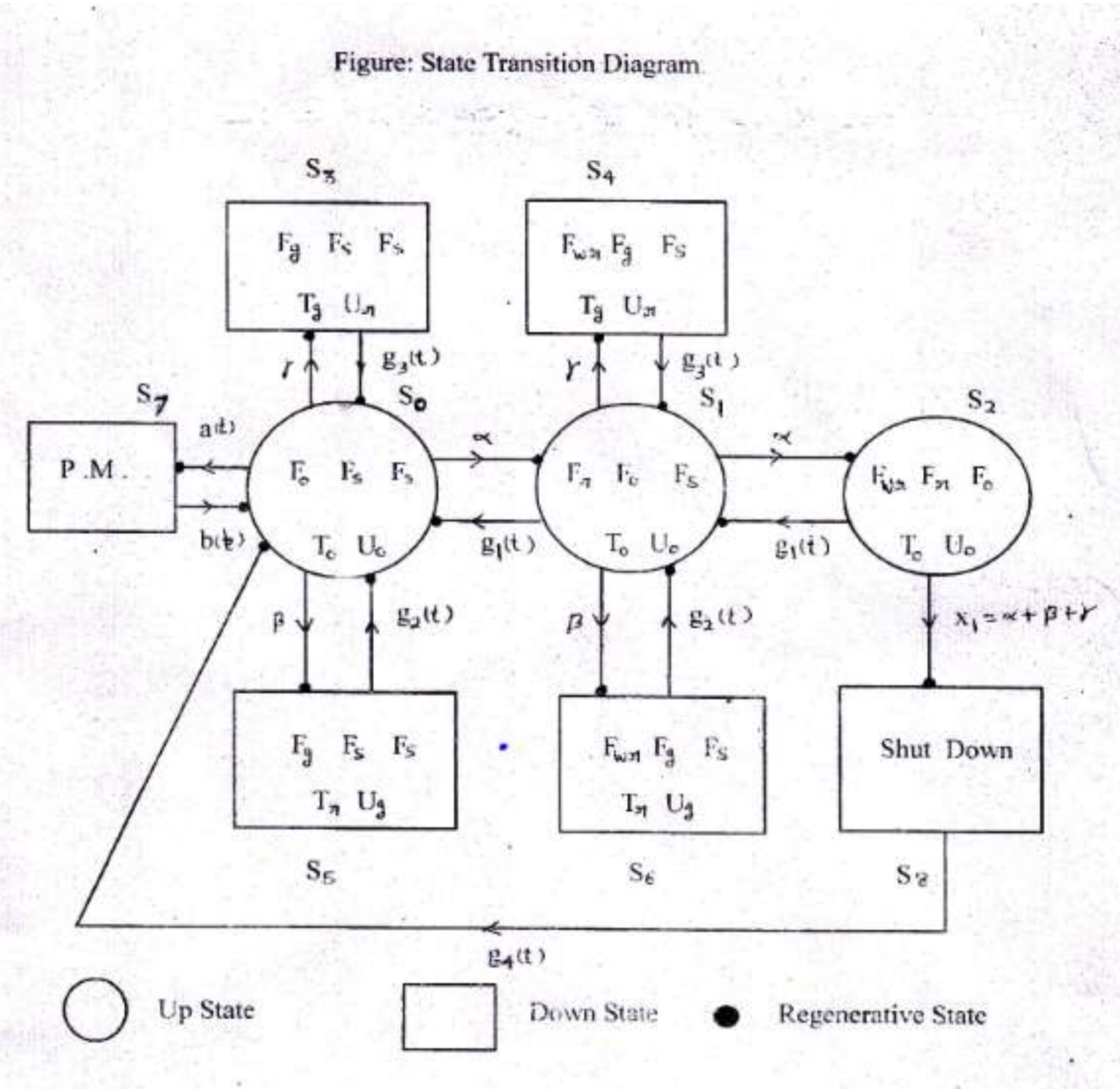


Figure 1

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