

Capacity Analysis of MIMO Technology

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Abstract : Multiple-input multiple-output (MIMO) systems are today regarded as one of the most promising research areas of wireless communications. This is due to the fact that a MIMO channel can offer a significant capacity gain over a traditional single-input single-output (SISO) channel. Bandwidth is one important constraint in wireless communication. In wireless communication, high data transmission rates are essential for the services like triple play i.e. data, voice and video. At user end the capacity determines the quality of the communication systems. This paper aims to compare the different RF wireless communication systems like SISO, MISO, SIMO and MIMO systems on the capacity basis. Ergodic capacity and Outage capacity has also been discussed. Moreover, a computer simulation with Matlab is implemented.

Keywords: MIMO, SISO, SIMO, MISO, Ergodic Capacity

1. Introduction

The increasing demand for capacity in wireless systems has motivated considerable research aimed at achieving higher throughput on a given bandwidth. One important finding of this activity is the recent demonstration that for an environment sufficiently rich in multipath components, the wireless channel capacity can be increased using multiple antennas on both transmit and receive sides of the link [1].

During the last decade, the wireless communication industry has grown at exponential pace and people are taking more and better advantages of the technologies available from voice calling to video calling, from positioning to satellite television. The pace will continue in the upcoming future as well. For the users, the quality of the wireless communication can be defined by the availability and the data rates or capacity.

Mobile communication starting from 2G, 3G and now 4G with the data rates varying from 12kbps in 2g to 2Mbps in 3g and 100Mbps in 4g [2]. There has been significant in the data rates and the spectral efficiency of the radio wireless communication. The increase in data rates comes with the increase in the capacity of the systems. The very general, the mobile wireless communication systems transmit information bits information in the radio space to the receiver. We are here going to discuss and simulate the capacity of systems like SISO systems, SIMO and MISO systems.

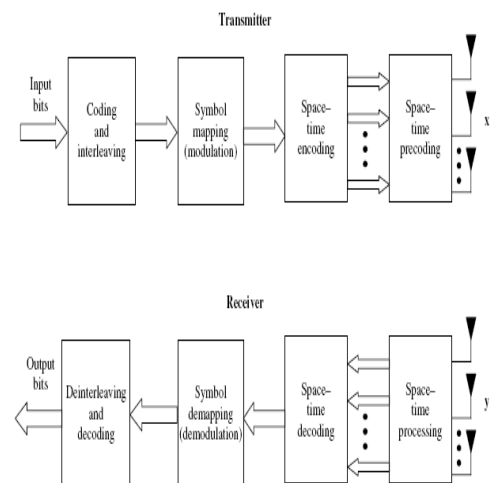


Fig1: A typical MIMO system including the signal processing subsystems.

2. SISO SYSTEMS

SISO Systems or the single input, single output communication systems are the simplest form of the communication system out of all four in which there is single transmitting antenna at the source and A single

receiving antenna at the destination[3]. SISO systems are used in multiple systems like Bluetooth, Wi-Fi, etc..



Fig2: SISO Communication System

The channel capacity C of a single-input single-output (SISO) system is given by [3]:

$$C = B \log_2 \left(1 + \frac{S}{N} \right) \text{ bit/s} \dots\dots\dots (1)$$

Where C is the capacity, B (in Hz) is the channel bandwidth, S (in Watt) is the signal power, and N (in Watt) is the noise power. Both S and N are measured at the output of the channel. The channel capacity is a measure of the maximum rate that information (in bits) can be transmitted through the channel with an arbitrarily small error after using a certain coding method. SISO are advantageous in terms of the simplicity. It does not require processing in terms of diversity schemes. The throughput of the system depends upon the channel bandwidth and signal to noise ratio. In some conditions, these systems are exposed to the issues like multipath effects. When an electromagnetic wave interacts with hills, buildings and other obstacles, waveform get scatter and takes many paths to reach the destination. Such issues are known as multipath. This causes several issues like fading, losses and attenuation also the reduction in data speed, packet loss and errors are increased.

3. SIMO (Single Input, Multiple Output):

SIMO or the Single input and multiple output form of wireless communication scheme in which there are multiple antennas are present at the receiver and there is single transmitting antenna at the source.

In order to optimize the data scheme, various receive diversity schemes are employed at the receiver like selection diversity, maximum gain combining and equal gain and equal gain combining schemes. SIMO systems were used for short waves listening and receiving stations to counter the effects of ionosphere fading. The SIMO systems are acceptable in many applications but where the receiving system is located in the mobile device like mobile phone, the performance may be limited by size, cost and battery combining schemes.

4. MISO (Multiple Input,Single Output)

MISO or the multiple input and single output is a scheme of RF wireless communication system in which there are multiple transmitting antennas at the Source and single receiving antenna at the system like SIMO but at the destination, receiver has a single

antenna [4]. When we use two or more antenna at the receiving end or at destination, the effects of multipath wave propagation, delay, packet loss etc can be reduced. This scheme has various applications like in Digital television, MISO systems are advantageous because the redundancy and coding has been shifted from receiving end towards the transmitting end and hence say in examples of mobile phones, less power and processing is required at the user end or the receiver end[5].

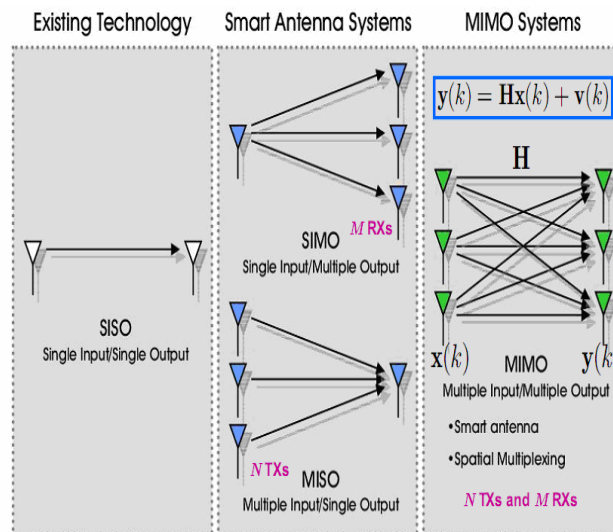


Fig 3: Comparison of various technologies

The capacity of MISO and SIMO systems can be expressed as

$$C = B \log_2 \left(1 + \frac{nS}{R} \right) \text{ bit/s} \dots\dots\dots (2)$$

Where n = number of transmit antenna in case of MISO systems and no. of receive antenna in case of SIMO systems. C = Capacity of the system, B = Bandwidth of the system and S/R = Signal to noise ratio

5. The MIMO (Multiple Input Multiple Output) System

Let us consider a single point-to-point MIMO system with arrays of t transmits and r receive antenna. The transmitted signals in each symbol period are denoted by a $t \times 1$ complex vector X , where the x_i refers to the transmitted signal from antenna

The total power of the complex transmitted signal X is constrained to P regardless of the number of transmitted antennas.[6][9]

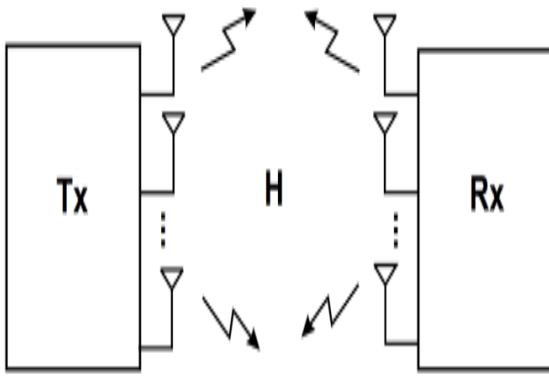


Fig4: n x n MIMO system

$$\text{So } \varepsilon[x^\dagger x] = \text{tr}(\varepsilon[xx^\dagger]) = p \dots\dots (3)$$

The transmitted signal bandwidth is narrow enough, so its frequency response can be considered as flat. The received signal y is given by $y = Hx + n \dots\dots\dots (4)$

Where H is a $r \times t$ complex gain matrix. The ij th entry, h_{ij} of the matrix H represents the channel fading coefficient (gain) from j th transmit to i th receive antenna. And n is a $r \times 1$ noise vector which is statistically independent complex zero mean Gaussian random variables with independent and equal variance of real and imaginary parts. So the covariance matrix of n is given by

$$\varepsilon[nn^\dagger] = \sigma^2 I_r \dots\dots\dots (5)$$

Where σ^2 is the identical noise power at each of the receive antennas. The output of each receive antenna is given by P_r . The average SNR at each receive antenna is defined as

$$\text{SNR} = P_r / \sigma^2 \dots\dots\dots (6)$$

We can assume that the total power per receive antenna is equal to the total transmitted power. That means signal attenuation and amplification in the propagation process are ignored. It is assumed that H is perfectly known at the receiver, but not at the transmitter. The channel matrix can be estimated at the receiver by transmitting a training sequence. The channel state information can be reliably feed back to the transmitter.

5.1 Calculation of MIMO capacity

The channel matrix H is deterministic in nature. Using singular value decomposition theorem, H can be denoted as

$$H = UDV^\dagger \dots\dots\dots (7)$$

Where U and V are $r \times r$ and $t \times t$ unitary matrices respectively. The columns of U are the eigenvector of HH^\dagger in diagonal $D_{ii}, i=1,2,\dots,r$ are called the singular value of H and denoted by $\sqrt{\lambda_i}$ Where $i=1,2,\dots,r$

By substituting the above value into eqn (4), the result will be

$$y = UDV^\dagger x + n \dots\dots\dots (8)$$

Now consider

$$\tilde{y} = U^\dagger y, \tilde{x} = V^\dagger x, \tilde{n} = U^\dagger n \dots\dots\dots (9)$$

$$\text{Then } \tilde{y} = D\tilde{x} + \tilde{n} \dots\dots\dots (10)$$

So the covariance matrix for n is

$$\varepsilon[\tilde{n}\tilde{n}^\dagger] = \varepsilon[U^\dagger n n^\dagger U] = U^\dagger \varepsilon[nn^\dagger] U = \sigma^2 I_r \dots\dots\dots (11)$$

For the $r \times t$ matrix H, the rank r_0 is at most $\min(r, t)$.

$$\text{So } \tilde{y}_i = \{ \sqrt{\lambda_i} \tilde{x}_i + \tilde{n}_i, \dots, 1 \leq i \leq r_0 \} \\ \{ \tilde{n}_i, \dots, r_0 + 1 \leq i \leq r \} \dots\dots\dots (12)$$

so the above eqⁿ represent MIMO channel can be considered as r_0 uncoupled parallel. So the overall channel capacity is the sum of sub channel capacity, which is given by

$$C = \sum_{i=1}^{r_0} \ln(1 + P_{ri} / \sigma^2) \text{ nats/s/Hz} \dots\dots\dots (13)$$

Where P_{ri} is the received signal power at i th sub-channel

5.2 Equal Transmit Power allocation

The power allocated to sub channel I is given by

$$P_{ri} = \frac{\lambda_i P}{t} \dots\dots\dots (14)$$

Thus the channel capacity can be written as

$$C = \sum_{i=1}^{r_0} \ln(1 + P_{ri} / \sigma^2) = \ln \prod_{i=1}^{r_0} (1 + \lambda_i P / t \sigma^2) \dots\dots\dots (15)$$

5.3 Adaptive Transmit power allocation

When channel state information is known at the transmitter the capacity can be increased by allocating transmit power to different antennas using WATER filling algorithm

$$P_i = (\mu - \sigma^2 / \lambda_i)^+ \dots\dots\dots (16)$$

Where $i=1,2,3,\dots,r_0$...and μ is chosen so that $\sum_{i=1}^{r_0} P_i = P$ and a^+ denotes $\max(a, 0)$. So the received signal power at i th sub channel is given by

$$P_{ri} = (\lambda_i \mu - \sigma^2)^+ \dots\dots\dots (17)$$

Thus the channel capacity can be written as

$$C = \sum_{i=1}^{r_0} \ln [1 + (\mu \lambda_i - \sigma^2)^+ / \sigma^2] \dots \dots \dots (18)$$

$$= \sum_{i=1}^{r_0} \ln [1 + (\lambda_i \mu / \sigma^2)^+]$$

$$= \sum_{i=1}^{r_0} \ln [(\mu \lambda_i / \sigma^2)^+] \text{ nats/s/Hz}$$

6. Ergodic Capacity and Outage Capacity

This is the time-averaged capacity of a stochastic channel. It is found by taking the mean of the capacity values obtained from a number of independent channel realization. Figure4 below shows the Ergodic capacity over different system configurations as a function of SNR (dB) . A plot of the capacity versus SNR for SISO, MIMO systems is plotted in MATLAB. We note that Ergodic capacity increases with increasing SNR (dB) and with increasing N_t or N_r . It is observed that at SNR=20dB the capacity varies from 7 bits/s/ Hz for SISO to 21 bits/s/Hz for MIMO($N_t=N_r=4$). Hence it is concluded that the capacity growth achieved by MIMO system is the highest compared to other systems yielding remarkable improvement (especially for High SNR).

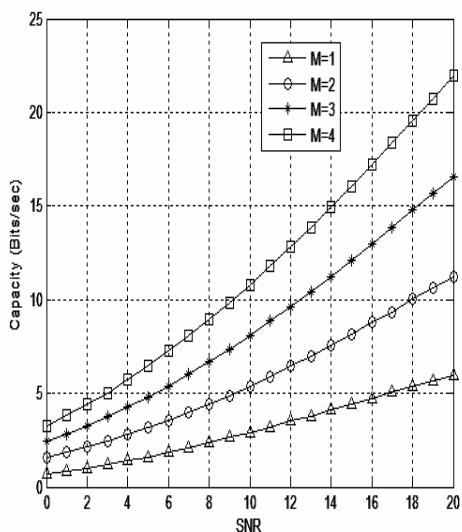


Fig5: Ergodic capacity based Water Filling algorithm for different antenna configurations MIMO system in a Rayleigh Fading Channel

7. Mean and Outage Capacity

Since the channel \mathbf{H} is random, the capacity of the MIMO channel is a random variable. The capacity of fading channels can be defined in a number of ways. In practice, mean capacity and outage capacity are the two most commonly used statistical measures. The mean capacity \bar{C} of a MIMO channel is the ensemble average

of the information rate over the all realizations of the channel matrix \mathbf{H} [11]. The outage capacity defines the level of capacity that is guaranteed with a certain level of reliability, and we can define the $q\%$ outage capacity $C_{out,q}$ as the information rate that is guaranteed for $(100-q)\%$ of the channel realization, i.e. [11]

$$q\% = \Pr\{C \leq C_{outage}\} \dots \dots \dots (19)$$

This can also be expressed as

$$1 - q = \Pr\{C > C_{outage}\} \dots \dots \dots (20)$$

A capacity of 20bps/Hz with 1% outage probability means that the capacity will remain at least 20bps/Hz for 99% of the time. FIG 6 shows the cumulative distribution function (CDF) of the capacity of a flat fading MIMO channel with $N_t = N_r = 2$ and $\rho = 10$ dB when the channel is unknown to the transmitter where N_t is the number of the transmit antennas and N_r is the no of receive antennas. The mean capacity of this channel is 5.5593 bps/Hz. While 10% outage capacity is about 3.896 bps/Hz. The significance of the mean capacity is that in an Ergodic channel, we can transmit the signal at the rate given by mean capacity without errors. In this sense, the mean capacity is the right metric when the channel is known to the transmitter. Outage capacity is a useful characterization when the channel is unknown to the transmitter and \mathbf{H} is random but held constant for each use of the channel [11]. It must be noted that while absolute values are different, the trends shown by outage capacity are the same as mean capacity [11]

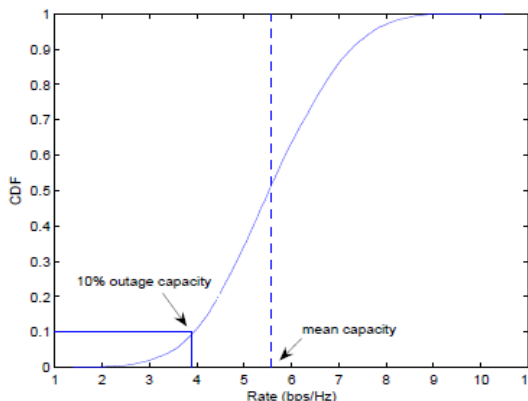


Fig6: CDF of capacity for the i.i.d. MIMO channel with $N_T = N_R = 2$ and SNR = 10 dB.

8. Discussion and Simulation Result of comparison of SISO, MISO, SIMO AND MIMO

The capacity for SISO system and MIMO systems are shown and compared in the following fig 6. The

equations defined above for capacity of the systems i.e. equation are implemented in Matlab 7.10.0 for the simulation. For the sake of simulations, the value of B is taken as 1.graph:

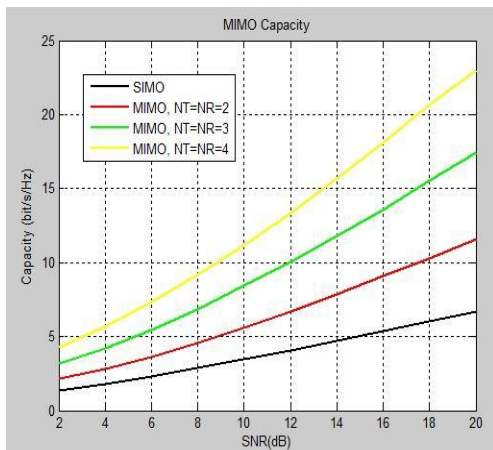


Fig7: Capacity of SISO and MIMO systems

We can see that as the number of transmit and receive antenna increases, capacity is increasing. The black line is showing the capacity of SISO system and red, green and yellow lines show the system capacity for 2x2, 3x3 and 4x4 MIMO systems respectively. Thus, at high SNR, the capacity increases linearly with the number of antennas at both transmitter and receiver side. MIMO system is approximately three times the capacity of the SISO system. Thus, at high SNR, the capacity increases linearly with the number of antennas at both transmitter and receiver side

In the following figure, all the three schemes are compared in different configurations i.e. SISO systems as 1x1, MISO or SIMO systems as 2x1 and 3x1 or 1x2 and 1x3 respectively and MIMO systems with 2x2 and 3x3 configurations.

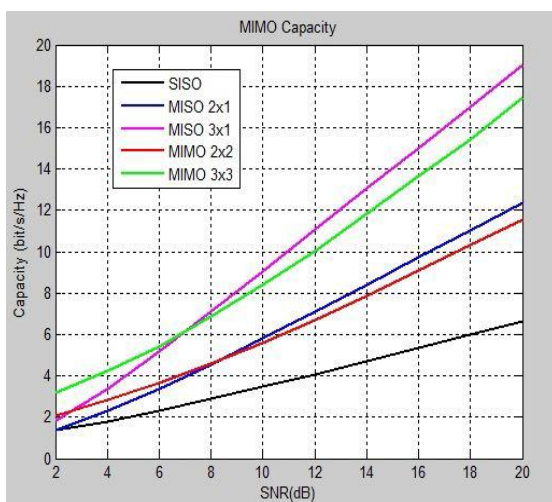


Fig8: SISO vs MISO vs MIMO System

The capacity of MIMO systems is better than SISO and other systems. MISO or SIMO systems show higher capacity but at the cost of the high SNR which is undesirable in wireless communication system

9. Conclusion

(1)MIMO inherently possess spatial diversity, which increases robustness of the system by eliminating fades .Using MIMO the effective SNR of the system, thereby system throughput can be increased with the aid of spatial multiplexing.

(2) The MIMO system shows the maximum capacity theoretically which has been proved by simulations as well. Also as the number of transmit and receive antenna increase in MIMO systems there capacity increases as the green line is showing in the figure 7.

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