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# Controllability of mixed Volterra- Fredholm type Impulsive integro-differential inclusions in Banach spaces

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**Abstract**

The paper establishes a sufficient condition for the controllability of semilinear mixed Volterra-Fredholm type Impulsive integro-differential inclusions in Banach spaces. We use Bohnenblust-Karlin's fixed point theorem combined with a strongly continuous operator semigroup. Our main condition $(H_5)$ only depends upon the local properties of multivalued map on a bounded set. An example is also given to illustrate our main results.

## 1.INTRODUCTION

Controllability problems described as abstract differential equations or differential inclusions in infinite dimensional spaces has found wide applications in many branches of physics and technical sciences, such as heat flow in materials with memory, viscoelasticity and other physical phenomena[1]. These problems have been extensively studied by many authors, see for instance[2-9] and the references therein. Very recently, Chalishajar[10] studied the following mixed Volterra-Fredholm type impulsive integro-differential systems

$$x'(t)=Ax(t) + Bu(t) + f\left(t, x(t), \int_0^t g(t,s,x(s))ds, \int_0^t h(t,s,x(s))ds\right),\ t \in J = [0, b],$$

$$x(0) = x_0$$

in Banach spaces by applying a fixed point theorem due to Leray-Schauder alternative.

In this paper we are interested in the controllability of the following mixed Volterra-Fredholm type impulsive integro differential inclusions in Banach spaces:

$$x'(t)-Ax(t)\in Bu(t)+F\left(t,x(t), \int_0^t g(t,s,x(s))ds, \int_0^b h(t,s,x(s))ds\ ,t\in J\right) \tag{1.1}$$

$$x(0) = x_0 \tag{1.2}$$

$$\Delta x(t_k) = I_k(x(t_k)) \tag{1.3}$$

where the state $x(.)$ takes values in Banach space $X$ with the norm |.|. A generates a strongly continuous semigroup $T(t)$ in $X$, and the control function $u(.)$ is given in $L^2(J,U)$ , and a banach space of admissible control functions with $U$ as a Banach space. B is a bounded linear operator from $U$ into $X$. Here $g,h:\Delta\times X\rightarrow X$ are continuous functions and $F:JxXxXxX\rightarrow 2^X\setminus\{\varnothing\}$ is a multivalued map, $\Delta=(t,s):0\leq s\leq t\leq b, x_0\in X$ and b is a real constant, $I_k:X\rightarrow X$, $k=1,2,3,...m$ are appropriate functions and the symbol $\Delta\varepsilon(t)$ represents the jump of the function $\varepsilon$ at $t$, which is defined by $\Delta\varepsilon'(t)=\varepsilon'(t+)-\varepsilon'(t-)$

Based upon Bohnenblust-Karlin's fixed point theorem, we establish a controllability result for mild solutions of systems (1.1)-(1.3). Our main condition $(H_5)$ is only concerned with the local properties of $F$ on a bounded set.

## 2.PRELIMINARIES

In this section, we shall introduce some basic definitions, notations and lemmas which are used throughout this paper.

Let $PC(J,X)$ be the Banach space of piecewise continuous functions from $J$ into $X$ with the norm

$$\|y\|_\infty := sup\ \{|y(t)|:t\in J\}$$

A measurable function $y:J\to X$ is Bochner integrable if and only if $|y|$ is Lebesgue integrable.( For properties of the Bochner integral see Yosida[11].)

Let $L^1(J,X)$ be the Banach space of measurable functions $y:J\to X$ which are Bochner integrable and normed by

$$|y|_{L^1} = \int_0^b |y(t)|dt \quad \text{for all} \quad y\in L^1(J,X)$$

Let $(X,|.|)$ be a Banach space. Then a multivalued map $G : X \to 2^X\setminus\{\varnothing\}$ is convex(closed) valued if $G(x)$ is convex(closed) for all $x\in X$ .$G$ is bounded on bounded sets if $G(PC)=\bigcap_{x\in PC}G(x)$ is bounded in $X$ for any bounded set $PC$ of $X$ (i.e., $\sup_{x\in PC}$ {sup {|y| : $y \in G(x)$}} $< \infty$)

$G$ is called upper semicontinuous (u.s.c) on $X$ if for each $x_0\in X$, the set $G(x_0)$ is a nonempty closed subset of $X$, and if for each open set $PC$ of $X$ containing $G(x_0)$, there exists an open neighbourhood $V$ of $x_0$ such that $G(V)\subseteq PC$.

$G$ is said to be completely continuous if $G(PC)$ is relatively compact for every bounded subset $PC$ of $X$.

If the multivalued map $G$ is completely continuous with nonempty compact values, then $G$ is u.s.c if and only if $G$ has a closed graph i.e.,

$$x_n\to x_*,y_n\to y_*,y_n\in Gx_n \text{ imply } y_*\in Gx_*.$$

In the following BCC(X) denotes the set of all nonempty bounded,closed and convex subset of $X$.

$G$ has a fixed point if there is $x\in X$ such that $x\in G(x)$. For more details on multivalued maps see the books of Deimling[12] and Hu and Papageorgious[13].

Concerning the impulsive conditions in systems (1.1)-(1.3), it is convenient to introduce some additional concepts and notations. A function $u:[\sigma,\tau]\to X$ is said to be a normalized piecewise continuous function on $[\sigma,\tau]$,if $u$ is piecewise continuous and left continuous on $(\sigma,\tau]$. We denote by $PC([\sigma,\tau];X)$, the space of normalized piecewise continuous function from $[\sigma,\tau]$ into X. In particular, we introduce the space $PC$ formed by all normalized piecewise continuous $u:[0,a]\to X$ such that u is piecewise continuous at $t\neq t_i,i=1,2,3...n$ . It is clear that $PC$ enclosed with the norm $|u|_{PC}=sup_{s\in I}|u(s)|$ is a Banach space.

In what follows , for the case $I=[0,a]$, we set $t_0=0,t_{n+1}=a$ , and for $u\in PC$, we denote by $u_i\in C([t_k,t_{k+1}];X),i=1,2,...n$ , the function given by

$$u_i(t) =\begin{cases} u(t), \text{ for } t \in (t, ti+1] \\ u(t+ i ), \text{ for } t = t \end{cases}$$

To set the framework for our main controllability result, we will make use of the following definitions and lemma.

**DEFINITION 2.1.**

A piecewise continuous function $x(t)$ satisfying the following integral inclusion:

$$x(t)\in T(t)x_0+\int_0^t T(t-s)Bu(s)ds+\int_0^t T(t-s)F\left(s,x(s),\int_0^s g(t,\tau,x(\tau))d\tau,\int_0^b h(s,\tau,x(\tau))ds\right)$$
$$+\sum_{0<t_k<t} T(t-t_k)I_k(x(t_k))$$

is called a mild solution of problem (1.1)-(1.3) on I.

**DEFINITION 2.2**

System (1.1)-(1.3) is said to be piecewise controllable on the interval $J$ if for every initial function $x_0,x_1\in X$, there exists a control $u\in L^2(J,U)$ such that the mild solution $x(t)$ of Egs.(1.1)-(1.3) satisfies $x(b)=x_1$.

**Lemma 3.1** (Bohnenblust and Karlin[14]):

Let $X$ be a Banach space, $D$ a nonempty subset of $X$, which is bounded, closed, and convex. Suppose G : D → 2X\ {∅} is u.s.c with closed, convex values, and such that $G(D)\subset D$ and $\overline{G(D)}$ compact. Then $G$ has a fixed point.

Let us list the following hypothesis:

(H1) The strongly piecewise continuous semigroup of bounded linear operators $T(t)$ generated by $A$ is compact when $t>0$ and there exists a positive constant $M_1\geq 1$ such that $|T(t)|\leq M_1$.

(H2) The linear operator $W:L^2(J,U)\to X$ define by

$$Wu=\int_0^b T(b-s)Bu(s)ds$$

has an induced inverse operator $W^{-1}$ which takes values in $L^2(J,U)/kerW$ , and there exist positive constants $M_2,M_3$ such that

$$|B|\leq M_2,\ \ |W^{-1}|\leq M_3$$

(H3) For each $(t,s)\in\Delta$ , the functions $g(t,s,.,),h(t,s,.):X\to X$ are piecewise continuous and for each $x\in X$ the functions $g(.,.,x,),h(.,.,x):\Delta\to X$ are strongly measurable.

(H4) $F:J\times X\times X\times X\to BCC(X)$ is measurable to $t$ for each $(x,y,z)\in X\times X\times X$, u.s.c. with respect to $(x,y,z)$ for each $x\in C(J,X)$ the set

$$S_{F,x}=\{ f\in L^1(J,X) : f(t)\in F( t, x(t), \int_0^t g(t, s, x(s))\, ds, \int_0^b h(t, s, x(s)ds) ) , t\in J\}$$

is nonempty.

(H5) For each positive number $r$ and $x\in C(J,X)$ with $|x|_\infty\leq r$,there exists a function $l_r\in L^1(J,R_+)$ such that

$$\mathrm{Sup}\{|f| : sup\left\{|f|:f(t)\in F\left(t,x(t),\int_0^t g\,(t,s,x(s)\ ds)\int_0^b h\,(t,s,x(s)ds\right)\leq l_r(t)\right\}\}$$

for a.e.$t\in J$.

(H6) $\lim\inf_{r\to+\infty}\int_0^b l_r(t)dt/r=\alpha<\infty$

(H7) ∃ a positive constant $L_k$, such that

$$|I_k(x)-I_k(y)|\leq L_k|x-y|,\quad x,y\in X$$

**Remark 2.1**

The construction of the operator $W$ and its inverse is studied by Quinn and Carmichael in Ref.[15].

## 3. CONTROLLABILITY RESULTS:

In this section, we shall present and prove our main results

**Theorem 3.1.** Suppose that (H1)-(H7) are satisfied. Then system(1.1)-(1.3) is controllable on J provided that

$$(1+bM_1M_2M_3)M_1(\alpha+L_k)<1. \tag{3.1}$$

**Proof:**

Using hypothesis (H2) for an arbitrary function $x(.)$, define the control

$$u_x(t) = W^{-1}\left[x_1-T(b)x_0-\int_0^b T(b-s)f(s)ds-\sum_{k=1}^m T(b-t_k)I_k(x(t_k))\right](t),$$

where $f\in S_{F,x}$ .It shall be shown that when using this control the operator $\Gamma:PC(J,X)\to 2^{PC(J,X)}$ defined by

$$\Gamma(x)=\left\{\phi\in PC(J,X):\phi(t)=T(t)x_0+\int_0^t T(t-s)\left[f(s)+Bu_x(s)\right]ds \quad +\sum_{k=1}^{m} T(t-t_k)I_k(x(t_k)):f\in S_{F,x}\right\}$$

has a fixed point. This fixed point is then a mild solution of system (1.1)-(1.3). Clearly, $x_1\in(\Gamma x)(b)$,which implies that the system is controllable.

We now show that Γ satisfies all the conditions of Lemma 3.1. The proof will be given in several steps.

**Step 1:** $\Gamma(x)$ is convex for each $x\in PC(J,X)$.

In fact, if $\phi_1,\phi_2$ belong to $\Gamma(x)$, then there exist $f_1,f_2\in S_{F,x}$ such that for each $t\in J$, we have

$$\phi_i(t) = T(t)x_0+\int_0^t T(t-s)f_i(s)ds+\sum_{k=1}^{m} T(t-t_k)I_k(x(t_k))$$

$$+\int_0^t T(t-\eta)BW^{-1}\times\left[x_1-T(b)x_0-\int_0^b T(b-s)f_i(s)-\sum_{k=1}^{m} T(b-t_k)I_k(x(t_k)\right](\eta)d\eta$$

Let $\lambda\in[0,1]$. Then for each $t\in J$, we get

$$(\lambda\phi_1+(1-\lambda)\phi_2)(t) = T(t)x_0+\int_0^t T(t-s)[\lambda f_1(s)+(1-\lambda)f_2(s)]ds$$

$$+\sum_{k=1}^{m} T(t-t_k)I_k(x(t_k))+\int_0^t T(t-\eta)BW^{-1}\times[x_1$$

$$-T(b)x_0-\int_0^b T(b-s)[\lambda f_1(s)+(1-\lambda)f_2(s)]ds$$

$$-\sum_{k=1}^{m} T(b-t_k)I_k(x(t_k)](\eta)d\eta$$

Since $S_{F,x}$ is convex (F has convex values), thus

$$\lambda\phi_1+(1-\lambda)\phi_2\in\Gamma(x).$$

**Step 2:** For each constant $r>0$, let $B_r=.\{x\in C(J,X):||x||_\infty\le r\}$. Then $B_r$ is a bounded closed convex set in $C(J,X)$. We claim that there exists a positive number $r$ such that $\Gamma(B_r)\subseteq B_r$. If it is not true, then for each positive number $r$, there exists a function $x_r\in B_r$ but $|\Gamma(x_r)|:=sup\left\{|\phi_r|_\infty:\phi_r\in\Gamma(x_r)\right\}>r$ and

$$\phi_r(t) = T(t)x_0+\int_0^t T(t-s)f_r(s)ds+\sum_{k=1}^{m} T(t-t_k)I_k(x(t_k))+\int_0^t T(t-\eta)BW^{-1}X$$

$$[x_1-T(b)x_0-\int_0^b T(b-s)\,fr(s)\sum T(b-tk)Ik(x(tk))](\eta)d\eta,\ \ i=1,2.$$

for some $f_r\in S_{F,x_r}$. However,on the other hand, we have from (H1),(H2) and (H5)

$$r \le |\Gamma(x_r)|$$

$$\le M_1|x_0|+M_1\int_0^b l_r(s)ds+M_1\sum_{k=1}^{m} I_k(x(t_k))+bM_1M_2M_3\left[|x_1|+M_1|x_0|\right.$$

$$+M_1 \sum_{k=1}^{m} I_k(x(t_k))\Bigg] + b(M_1)^2 M_2 M_3 \int_0^b l_r(s)ds.$$

Dividing both sides by $r$,and take the lower limit as $r\to\infty$,we obtain

$$(1+bM_1M_2M_3)M_1(\alpha+L_k)<1$$

which contradicts Eq.(3.1).Hence there exists a positive number $r$ such that $\Gamma(B_r)\subseteq B_r$.

**Step 3:** $\Gamma$ sends bounded sets into equicontinuous sets of $C(J,X)$.

Let $0<t_1<t_2\le b$ and $\varepsilon>0$. For each $x\in B_r$,$\phi\in\Gamma(x)$, there exists $f\in S_{F,x}$ such that

$$\phi(t) = T(t)x_0 + \int_0^t T(t-s)fi(s)ds + \sum_{k=1}^{m} T(t-t_k)I_k(x(t_k)) + \int_0^t T(t-\eta)BW^{-1}$$

$$\text{X } [x_1 - T(b)x_0 - \int_0^b T(b-s)\, fr(s) \sum T(b-tk) Ik(x(tk))\,](\eta)d\eta,\ \ i=1,2. \quad (3.2)$$

Clearly,

$$|u_x(t)| \le M_3\left(|x_1|+M_1|x_0|+M_1\int_0^b l_r(t)dt + M_1\sum_{k=1}^{m} I_k(x(t_k))\right). \quad (3.3)$$

From $(H1)-(H5)$ and $Eq.(3.3)$ we have

$$|\phi(t_1)-\phi(t_2)| \le |T(t_1)-T(t_2)|\,|x_0| + |\int_o^{t_1-\varepsilon} (T(t_1-s)-T(t_2-s))f(s)ds|$$

$$+|\int_{t_1-\varepsilon}^{t_1} (T(t_1-s)-T(t_2-s))f(s)ds| + |\int_{t_1}^{t_2} T(t_2-s)f(s)ds|$$

$$+|\int_o^{t_1-\varepsilon} (T(t_1-\eta)-T(t_2-\eta))Bu_x(\eta)d\eta|$$

$$+|\int_{t_1-\varepsilon}^{t_1} (T(t_1-\eta)-T(t_2-\eta))Bu_x(\eta)d\eta|$$

$$+|\int_{t_1}^{t_2} T(t_2-\eta))Bu_x(\eta)d\eta| + \sum_{k=1}^{m}\left[T(t_1-t_k)-T(t_2-t_k)\right]I_k(x(t_k)$$

$$\le |T(t_1)-T(t_2)|\,|x_0| + \int_o^{t_1-\varepsilon} |(T(t_1-s)-T(t_2-s))|l_r(s)ds$$

$$+\int_{t_1-\varepsilon}^{t_1} |T(t_1-s)-T(t_2-s)|l_r(s)ds + M_1\int_{t_1}^{t_2} l_r(s)ds$$

$$+M_2\int_o^{t_1-\varepsilon} |T(t_1-\eta)-T(t_2-\eta)||u_x(\eta)|d\eta$$

$$+M_2 \int_{t_1-\varepsilon}^{t_1} |T(t_1-\eta)-T(t_2-\eta)||u_x(\eta)|d\eta$$

$$+M_1M_2 \int_{t_1}^{t_2} |u_x(\eta)|d\eta+ \sum_{k=1}^{m} \left[T(t_1-t_k)-T(t_2-t_k)\right]I_k(x(t_k))$$

The right-hand side of the above inequality tends to zero independently of $x\in B_r$ as $(t_1-t_2)\to 0$ and ε sufficiently small, since the compactness of $T(t)(t>0)$ implies the continuity in the uniform operator topology. Thus Γ sends $B_r$ into equicontinuous family of functions.

**Step 4:** The set $\Psi(t)=\left\{\phi(t):\phi\in\Gamma(B_r)\right\}$ is pre-compact in $X$.

Let $t\in(0,b]$ be fixed and ε a real number satisfying $0<\varepsilon<t$. For $x\in B_r$, we define

$$\phi_\varepsilon(t)=T(t)x_0+ \int_0^{t-\varepsilon} T(t-s)f(s)ds+ \int_0^{t-\varepsilon} T(t-\eta)Bu_x(\eta)d\eta+ \sum_{k=1}^{m} T(t-t_k)I_k(x(t_k))$$

Since $T(t)(t>0)$ is a compact operator, the set $\Psi_\varepsilon(t)=\left\{\phi_\varepsilon(t):\phi_\varepsilon\in\Gamma(B_r)\right\}$ is pre-compact in $X$ for each $\varepsilon, 0<\varepsilon<t$. Moreover, for each $0<\varepsilon<t$, we have

$$|\phi(t)-\phi_\varepsilon(t)|\le M_1 \int_{t-\varepsilon}^{t} l_r(s)ds+M_1M_2 \int_{t-\varepsilon}^{t} |u_x(\eta)|d\eta+ \sum_{k=1}^{m} T(t-t_k)I_k(x(t_k))$$

Hence there exist pre-compact sets arbitrarily close to the set $\Psi(t)=\left\{\phi(t):\phi_\varepsilon\in\Gamma(B_r)\right\}$, and the set $\Psi(t)$ is pre-compact in $X$.

**Step 5:** Γ has a closed graph.

Let $x_n\to x_*(n\to\infty),\phi_n\in\Gamma(x_n)$, and $\phi_n\to\phi_*(n\to\infty)$. We shall show that $\phi_*\in\Gamma(x_*)$. The relation $\phi_n\in\Gamma(x_n)$ means that there exists $f_n\in S_{F,x_n}$ such that

$$\phi_n(t)=T(t)x_0+ \int_0^{t} T(t-s) \left[f_n(s)+Bu_{x_n}(s)\right]ds+ \sum_{k=1}^{m} T(t-t_k)I_k(x(t_k))$$

where

$$u_{x_n}(t)=W^{-1}\left[x_1-T(b)x_0- \int_0^{b} T(b-s)f_n(s)ds- \sum_{k=1}^{m} T(b-t_k)I_k(x(t_k))\right](t)$$

We must prove that there exists $f_*\in S_{F,x_*}$ , such that

$$\phi_*(t)=T(t)x_0+ \int_0^{t} T(t-s) \left[f_*(s)+Bu_{x_*}(s)\right]ds+ \sum_{k=1}^{m} T(t-t_k)I_k(x(t_k))$$

$$u_{x_*}(t)=W^{-1}\left[x_1-T(b)x_0- \int_0^{b} T(b-s)f_*(s)ds- \sum_{k=1}^{m} T(b-t_k)I_k(x(t_k))\right](t)$$

$$\bar{u}_x(t)=W^{-1} \left[x_1-T(b)x_0\right](t).$$

Since $W^{-1}$ is continuous, then

$$\bar{u}_{x_n}(t)\to\bar{u}_{x_*}(t) for t\in J \ \ as \ \ n\to\infty.$$

Clearly, we have

$$\left\| \left( \phi_n - T(t)x_0 - \int_0^t T(t-s)B\bar{u}_{x_n}(s)ds - \sum_{k=1}^{m} T(t-t_k)I_k(x(t_k)) \right) \right.$$
$$\left. - \left( \phi_* - T(t)x_0 - \int_0^t T(t-s)B\bar{u}_{x_*}(s)ds \ \sum_{k=1}^{m} T(t-t_k)I_k(x(t_k)) \right) \right\|_\infty \to 0, as n\to\infty$$

Consider the operator

$$F \quad : \quad L^1(J,X)\to C(J,X),$$
$$f\mapsto F(f)(t) \quad = \quad \int_0^t T(t-s)\left[ f(s) - BW^{-1}\left( \int_0^b T(b-\tau)f(\tau)d\tau \right) - \sum_{k=1}^{m} T(b-t_k)I_k(x(t_k)) \right](s)ds$$

We can see that the operator $F$ is linear and continuous. From (H4) and Lasota-Opial in [16], it follows that $FoS_F$ is a closed graph operator. Moreover, we obtain that

$$\phi_n(t) - T(t)x_0 - \int_0^t T(t-s)B\bar{u}_{x_n}(s)ds - \sum_{k=1}^{m} T(t-t_k)I_k(x(t_k))$$

In view of $x_n\to x_*(n\to\infty)$, it follows again from Lasota-Opial in [16] that

$$\phi_* - T(t)x_0 - \int_0^t T(t-s)B\bar{u}_{x_*}(s)ds$$
$$= \int_0^t T(t-s)\left[ f(s) - \tilde{B}W^{-1}\left( \int_0^b T(b-\tau)f(\tau)d\tau \right) - \sum_{k=1}^{m} T(b-t_k)I_k(x(t_k)) \right](s)ds$$

for some $f_*\in S_{F,x_*}$ .

As a consequence of Steps 1-5 together with the Arzela-Ascoli theorem, we conclude that Γ is a compact multivalued map, u.s.c with convex closed values. As a consequence of Lemma 2.1, we deduce that Γ has a fixed point $x$ which is a mild solution of problem (1.1)-(1.3). Therefore,system (1.1)-(1.3) is controllable on $J$.

**Remark 3.2.** Let $F$ take its single-valued form in[10], then condition (H5) is reduced to the condition (H8) in [10]. So,Theorem 3.1 gives a new sufficient condition for the controllability of the impulsive integro-differential systems by dropping assumptions (H5)-(H7) and (H9) in [10].

**4. An Example:** Consider the partial impulsive integro-differential equation of the form

$$\left.\begin{array}{l} \omega_t(t;y)\in\omega_{yy}(t;y)+\mu(t;y)+P\left( t;\omega(t;y); \int_0^t a(t;s;\omega(s;y))ds; \int_0^b b(t;s;\omega(s;y))ds \right); \omega(t;0)=\omega(t;\pi)=0; \quad t\in J=[0;1] \\ \omega(0;y)=\omega_0(y); \quad 0<y<\pi \qquad \Delta W(.;t_k)=W(.;t_k^+)-W(.;t_k^-)= \int_0^\pi P_i(s;\omega(\varepsilon;t_i))ds \end{array}\right\} \quad (4.1)$$

where $\mu:(0,\pi)\times J\to(0,\pi)$ is continuous in $t$ and $(ti)_{i\in N}$ is a strictly increasing sequence.

Let $X=L^2[0,\pi]$ and let $A:X\to X$ be defined by

$$Aw=w'', w\in D(A),$$

where D(A) = {w ∈ X: w is absolutely continuous, w' ∈ X, w(0)=w( $\pi$ )=0}. Then

$$Aw=\sum_{n=1}^{\infty} n^2(w,w_n)w_n,\quad w\in X,$$

where $w_n(s)=\sqrt{2/\pi}\,\sin ns, n=1,2,3,\ldots$ is the orthogonal set of eigenfunctions of $A$. It can be easily shown that $A$ is the infinitesimal generator of an analytic semigroup $S(t), t>0$ in $X$ and is given by

$$S(t)w=\sum_{n=1}^{\infty} exp(-n^2t)(w,w_n)w_n,\quad w\in X,$$

where $S(t)$ satisfies hypothesis (H1).

Let $Bu:[0,1]\to X$ be defined by

$$(Bu)(t)(y)=\mu(t,y),\quad y\in(0,\pi).$$

Here, $P:[0,1]\times X\times X\times X\to 2^X\setminus\{\varnothing\}$, $a:[0,1]\times[0,1]\times X\to X$, and $b:[0,1]\times[0,1]\times X\to X$.

Let

$$g(t,s,w)(x)=a(t,s,w(x)),$$

$$h(t,s,w)(x)=b(t,s,w(x)),$$

$$F(t,w,\sigma_1,\sigma_2)(x)=P(t,w(x),\sigma_1(x),\sigma_2(x))\quad and$$

$$I_k(m)(\varepsilon)=\int_0^{\pi} p_i(s,x(\varepsilon))ds$$

and we assume that these functions $P,a,b$ satisfy (H3)-(H6).

With the choice of $A,B$ and $f$, Eqs.(1.1)-(1.3) is the abstract formulation of (4.1). Now the linear operator $W$ is given by

$$(Wu)(y)=\sum_{n=1}^{\infty}\int_0^1 exp(-n^2(1-s)(\mu(s,y),w_n))w_n ds,\quad y\in(0,1).$$

Assume that this operator has a bounded invertible operator $\tilde{W}^{-1}$ in $L^2(J,U)/kerW$ . Thus all the conditions of the above theorem are satisfied.Hence system (4.1) is controllable on $J$.

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