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To Find the Comparison of Non-Split Domination Number, The Average Distance and The Diameter of Circular-Arc Graphs

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Abstract: A connected dominating set is used as a backbone for communications and vertices that are not in this set communicate by passing message through neighbors that are in the set. Among the various applications of the theory of domination and the distance, the most often discussed is a communication network. This network consists of communication links all distance between affixed set of sites. Circular-arc graphs are rich in combinatorial structures and have found applications in several disciplines such as Biology, Ecology, Psychology, Traffic control, Genetics, Computer sciences and particularly useful in cyclic scheduling and computer storage allocation problems etc. Suppose communication network does not work due to link failure. Then the problem is what is the fewest number of communication links such that at least one additional transmitter would be required in order that communication with all sites as possible. This leads to the introducing of the concept of the non-split domination number, average distance and diameter. In this paper we present the comparison of non-split domination number, the average distance and the diameter of circular-arc graphs.

Keywords: Circular-arc family, circular-arc graph, domination number, average distance, diameter.

1. Introduction

A graph G=(V,E) is called a circular-arc graph or an intersecting graph for a finite family A of a non empty set if there is a one to one correspondence between A and B . Such that two sets in A have non empty intersection if and only if there corresponding vertices in V are adjacent to each other. We call A an intersection model of G for an intersection model A we use G(A) to denote the intersection graph for G. If A is a family of arcs on a circle, then G is called circular-arc graph for A and A is called a circular-arc model of G[7]. Circular-arc graphs have many applications in different fields such as genetic research, traffic control, computer complier design etc.

Let A= {A_1, A_2, A_3,……A_n} be a family of n arcs on a circle C. Each end point of the arcs in assigned a positive integer called a coordinate. The end point of each arc are located on the circumference are C in the ascending order of the values of the coordinates in the clock wise direction. For convenience, each arcs are A_i, i=1,2,3…..n is represented as (h_i,t_i). Where h_i is the head point and t_i is the tail point respectively that starting and ending points of the arc when it is traversed in counter clock wise manner, starting with an arbitrary chosen point on C which is not an end of any arc in A. Without loss of generality, we will assume that the following conditions.

a) No two arcs share a common end point.
b) No single arc in A covers the entire circle C by itself otherwise the shortest path result becomes trivial and in this case the distance between any two arcs in either 1 or 2 unit.
c) $\bigcup_{i=1}^{n} Ci = C$, otherwise the result becomes one on trivial graph.
d) The end points of the arcs in A are already given and sorted, according to the order in which they are

visited during clockwise traversal along C by starting at arc a_1 .

e) The arcs are sorted in increasing values of h_i's that is $h_i > h_j$ for $i > j$.

f) The family of arcs A is said to be canonical if h_i's and t_i's for i = 1,2,….n are all distinct integers between 1 and 2n and the point 1 is the head of the arc a_1.

And again alternatively, the circular arc graph can be defined as follows an undirected graph G (V, E) is a circular arc graph if and only if its vertices circularly indexed $v_1, v_2, \ldots v_n$ and $(v_i,v_j) \in E$ provided a_i and a_j intersect with each other, where v_i and v_j are the vertices in the graph G corresponding to the arcs a_i and a_j in A respectively.

A subset D of v is said to be a dominating set of G if every vertex not in D is adjacent to vertex in D. The domination number γ(G) of a graph G is the minimum cardinality of a dominating set in G[3,4]. In a simple graph, there is only one edge between two consecutive vertices of a walk, so one could abbreviate the walk as $W=\{v_0, e_1,v_1,e_2,\ldots v_{n-1},e_n,v_n\}$. The length of a walk or directed walk is the number of edge steps in the walk sequence. A walk of length zero. i.e with one vertex and no edges is called a trivial walk. For a connected graph G the term distance we defined as

i. $d(a_i,a_j) \geq 0$, for all $a_i,a_j \in V(G)$,
ii. $d(a_i,a_j) = 0$, if and only if $a_i=a_j$,
iii. $d(a_i,a_j) = d(a_j,a_i)$, for all $a_i,a_j \in V(G)$,
iv. $d(a_i,a_k) \leq d(a_i,a_j) + d(a_j,a_k)$, for all $a_i,a_j,a_k \in V(G)$.

Suppose an interval graph V(G) is not a connected and then G_1 and G_2 are two graphs of G such that $G= G_1 \cup G_2$ and $E(G) = E(G_1) \cup E(G_2)$.

And $G_1 \cap G_2 = \phi$ that is $V(G1) \cap V(G2) = \phi$ and

$E(G_1) \cap E(G_2) = \phi$. Then $d(a_i,a_j) = \infty$ for $a_i \in V(G_1).a_j \in V(G_2)$. Then G must be connected.The diameter and radius are two of the most basic graph parameters. The diameter of a graph is the largest distance between its vertices. The distance orientations of graphs by V.Chvatal [8]. The diameter of a graph G is the maximum of eccentricity of all its vertices and is denoted by Diam (G) that is Diam (G) = max {e (v): v ∈ V (G)}, where the maximum distance from vertex u to any vertex of G is called eccentricity of the vertex v and is denoted by ecc (v) that is ecc (v) = max {d (u,v): u ∈ V (G)},where as the distance between two vertices u and v of a graph is the length of the shortest path between them and is denoted by d(u,v) or d (v,u). In this section we discuss about the computation of average distance of circular arc graph G. The average distance μ(G) of a connected circular arc graph is defined to be the average of all distances in G[5,7].

$$\mu(G)=\frac{1}{2\ {}^{n}c_2}\sum_{\substack{x,y_j \in v(G)\\ x\neq y}}\delta(x,y).$$

Where δ(x, y) denotes the length of shortest path joining the vertices x and y . The average distance can be used as a tool in analytic networks where the performance time is proportions to the distance between any two nodes. It is a measure of the time needed in the average case as opposed to the diameter, which indicates the maximum performance time and also the formulated of the walk, length of a walk, eccentricity, radius and diameter of the graph.

A walk from v_0 to v_n is an alternating sequence $W= \{v_0, e_1,v_1,e_2,\ldots v_{n-1},e_n,v_n\}$ of vertices and edges such that end points $(e_i)=\{v_{i-1},v_i\}$; for $i=\{1,2,\ldots n\}$.

In a simple graph, there is only one edge between two consecutive vertices of a walk, so one could abbreviate the walk as $W=\{v_0, e_1,v_1,e_2,\ldots v_{n-1},e_n,v_n\}$.

The length of a walk or directed walk is the number of edge steps in the walk sequence. A walk of length zero. i.e with one vertex and no edges is called a trivial walk.

2. MAIN THEOREMS

Theorem.1: Let $\gamma_{ns}(D)$ be a domination number of the given circular-arc graph. If a_i, a_j are any two arcs in A such that a_j

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is contained in a_i and $a_i \in$ D and if p is an arc in A, which is to the left of a_j in clock wise direction, such that p< a_j and p intersects a_j and if there is at least one $a_k > a_j$, such that a_k intersects a_j, then the non-split domination number $\gamma_{ns}(D)$ is greater than the average distance μ(G) is greater than the diameter of G.

i.e. $\gamma_{ns}(D) > \mu(G) > Diam(G)$

Proof: Let {a_1, a_2, ….., a_n}be a family of n-arcs on a circle A and let G be a circular-arc graph corresponding to circular-arc family I. First we will prove that $\gamma_{ns}(D)$ is a minimal domination number og G. Then for every vertex v in $\gamma_{ns}(D)$, $\gamma_{ns}(D)$ – {v} is not a domination number . Thus some vertex u in V - $\gamma_{ns}(D) \cup$ {v} is not dominated by any vertex in $\gamma_{ns}(D)$ – {v}. Now either u = v or u $\in$ V - $\gamma_{ns}(D)$. If u = v, then v is an isolated vertex of $\gamma_{ns}(D)$. If u $\in$ V - $\gamma_{ns}(D)$ and u is not dominated by $\gamma_{ns}(D)$ – {v}, but is dominated by $\gamma_{ns}(D)$, then u is adjacent to any two vertices v in $\gamma_{ns}(D)$, that is N(u) ∩ $\gamma_{ns}(D)$ = {v}.

Next we will find the non-split domination number of G. Suppose there is atleast one arc k ≠ i, k > j such that k intersects j. Then it is obvious that in <V - $\gamma_{ns}(D)$> such that p intersects j. Further by hypothesis there is atleast one p < j in <V - $\gamma_{ns}(D)$> such that p intersect j. Hence j is connected to its left as well as to its right, so that there will not be any disconnection in <V - $\gamma_{ns}(D)$>. Again we will prove that let G be a circular-arc graph corresponding to circular-arc family A= {a_1, a_2, ….., a_n} be a circular-arc family on a circle. Where each $a_i$is an arc. Without lot of generality assume that the end points of all arcs are distinct and no arcs covers the entire circle. We discuss about the comparison of average distance of a circular-arc graph. The average distance μ(G) of a connected circular-arc graph is defined to be the average of all distances in G.

Since $\mu(G) = \frac{1}{2\ {}^{n}c_{2}} \sum_{\substack{ai,aj \in v(G) \\ i \neq j}} \delta(ai, aj).$

Where $\delta(a_i, a_j)$ denotes the length of a distance joining the vertices ai and a_j. The average distance can be used as a tool ina analytical network where the performance time is proportional to the distance between two vertices (a_i, a_j). Now we will prove that the diameter of G. in this connection first we prove that the eccentricity $e(a_i)$ of a vertex a_i to a vertex furthest from a_i. Vertex a_j is said to be a furthest neighbor of the vertex a_i of $\delta(a_i, a_j) = (a_i)$. The diameter of a graph Gis the minimum among all eccentricities. The eccentricity $e(a_i)$ = { $\delta(a_i, a_j)$: $a_j \in V$ }. Infact we have to find the diameter of G. Since Diam(G) = max { $e(a_i)$: $a_j \in V$ }. By the theorem our proportion to find $\gamma_{ns}(D) > \mu(G) >$ Diam(G).There fore the theorem is hold.

3. EXPERIMENTAL PROBLEM FOR THEOREM.1:

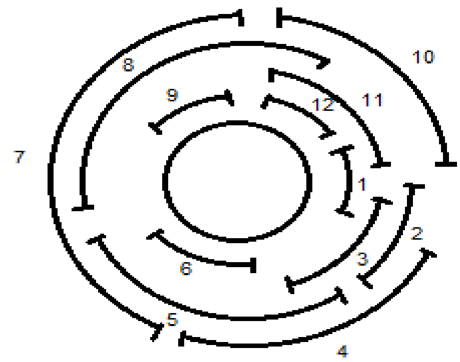
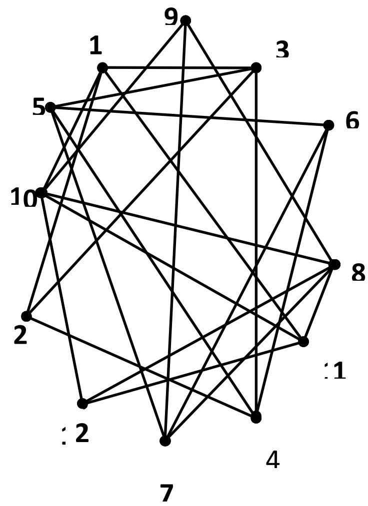


Fig. 1: Circular-arc family A



4. To find the distances from G

d(1,1)=0	d(2,1)=1	
d(1,2)=1	d(2,2)=0	d(3,1)=1
d(1,3)=1	d(2,3)=1	d(3,2)=1
d(1,4)=2	d(2,4)=1	d(3,3)=0
d(1,5)=2	d(2,5)=2	d(3,4)=1
d(1,6)=3	d(2,6)=2	d(3,5)=1
d(1,7)=3	d(2,7)=3	d(3,6)=2
d(1,8)=2	d(2,8)=3	d(3,7)=2
d(1,9)=2	d(2,9)=3	d(3,8)=3
d(1,10)=1	d(2,10)=2	d(3,9)=3
d(1,11)=1	d(2,11)=2	d(3,10)=2
d(1,12)=2	d(2,12)=3	d(3,11)=2
		d(3,12)=3

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Fig. 2: Circular-arc graph G

Dominating Set = {1,4,8}, $\gamma_{ns}(G)$=3

d(5,1)=2
d(5,2)=2
d(5,3)=1
d(5,4)=1
d(5,5)=0
d(5,6)=1
d(5,7)=1
d(5,8)=2
d(5,9)=2
d(5,10)=3
d(5,11)=3
d(5,12)=3

1)=1
2)=2

d(4,1)=2
d(4,2)=1
d(4,3)=1
d(4,4)=0
d(4,5)=1
d(4,6)=1
d(4,7)=2
d(4,8)=3
d(4,9)=3
d(4,10)=3
d(4,11)=3
d(4,12)=3

d(6,1)=3
d(6,2)=2
d(6,3)=2
d(6,4)=1
d(6,5)=1
d(6,6)=0
d(6,7)=1
d(6,8)=2
d(6,9)=2
d(6,10)=3
d(6,11)=3
d(6,12)=3

d(7,1)=3
d(7,2)=3
d(7,3)=2
d(7,4)=2
d(7,5)=1
d(7,6)=1
d(7,7)=0
d(7,8)=1
d(7,9)=1
d(7,10)=2
d(7,11)=2
d(7,12)=2

d(8,1)=2
d(8,2)=3
d(8,3)=3
d(8,4)=3
d(8,5)=2
d(8,6)=2
d(8,7)=1
d(8,8)=0
d(8,9)=1
d(8,10)=1
d(8,11)=1
d(8,12)=1

d(9,1)=2
d(9,2)=3
d(9,3)=3
d(9,4)=3
d(9,5)=2
d(9,6)=2
d(9,7)=1
d(9,8)=1
d(9,9)=0
d(9,10)=1
d(9,11)=2
d(9,12)=2

d(10,3)=2
d(10,4)=3
d(10,5)=3
d(10,6)=3
d(10,7)=2
d(10,8)=1
d(10,9)=1
d(10,10)=
d(10,11)=
d(10,12)=

d(11,3)=2
d(11,4)=3
d(11,5)=3
d(11,6)=3
d(11,7)=2
d(11,8)=1
d(11,9)=2
d(11,10)=1
d(11,11)=0
d(11,12)=1

d(12,3)=3
d(12,4)=4
d(12,5)=3
d(21,6)=3
d(12,7)=2
d(12,8)=1
d(12,9)=2
d(12,10)=1
d(12,11)=1
d(12,12)=0

5. Eccentricity:

e(1) = max {0, 1, 1, 2, 2, 3, 3, 2, 2, 1, 1, 2}=3

e(2) = max {1, 0, 1, 1, 2, 2, 3, 3, 3, 2, 2, 3}=3

e(3) = max {1, 1, 0, 1, 1, 2, 2, 3, 3, 2, 2, 3}=3

e(4) = max {2, 1, 1, 0, 1, 1, 2, 3, 3, 3, 3, 4}=4

e(5) = max {2, 2, 1, 1, 0, 1, 1, 2, 2, 3, 3, 3}=3

e(6) = max {3, 2, 2, 1, 1, 0, 1, 2, 2, 3, 3, 3}=3

e(7) = max {3, 3, 2, 2, 1, 1, 0, 1, 1, 2, 2, 2}=3

e(8) = max {2, 3, 3, 3, 2, 2, 1, 0, 1, 1, 1, 1}=3

e(10) = max {1, 2, 2, 3, 3, 3, 2, 1, 1, 0, 1, 1}=3

e(11) = max {1, 2, 2, 3, 3, 3, 2, 1, 2, 1, 0, 1}=4

e(12) = max {2, 3, 3, 4, 3, 3, 2, 1, 2, 1, 1, 0}=4

e(v) = max {3, 3, 3, 4, 3, 3, 3, 3, 3, 3, 3, 4}=4

6. Diameter:

diam(G) = max {3, 3, 3, 4, 3, 3, 3, 3, 3, 3, 3, 4}=4

Therefore diam(G) = 4

7. To find the average distance of G

V	1	2	3	4	5	6	7	8	9	10	11	12
1	0	1	1	2	2	3	3	2	2	1	1	2
2	1	0	1	1	2	2	3	3	3	2	2	3
3	1	1	0	1	1	2	2	3	3	2	2	3
4	2	1	1	0	1	1	2	3	3	3	3	4
5	2	2	1	1	0	1	1	2	2	3	3	3
6	3	2	2	1	1	0	1	2	2	3	3	3
7	3	3	2	2	1	1	0	1	1	2	2	2
8	2	3	3	3	2	2	1	0	1	1	1	1
9	2	3	3	3	2	2	1	1	0	1	2	2
10	1	2	2	3	3	3	2	1	1	0	1	1
11	1	2	2	3	3	3	2	1	2	1	0	1
12	2	3	3	4	3	3	2	1	2	1	1	0
Total	20	23	21	24	21	23	20	20	22	20	21	25

Table-1

Therefore Average distance

$$\mu(G)=\frac{1}{2\ {}^{n}c_2}\sum_{\substack{ai,aj\in v(G)\\ i\neq j}}\delta(ai,aj).$$

$$\mu(G)=\frac{1}{12\times 11}260$$

$$\mu(G)=1.96$$

Therefore Diam(G) > γ_{ns}(D) > μ(G).

Theorem.2: Let A= {a_1, a_2, ….., a_n} be a circular-arc family and let G be a circular-arc graph corresponding to circular-arc family. If a_j = 2 is the arc contained in a_i = 1 and $a_i \in \gamma_{ns}$(D) and there is atleast one arc that intersect a_j to its right other than a_i. Then the non-split domination number is greater than the average distance is greater than the diameter of G.i.e. γ_{ns}(D) > μ(G) > Diam(G)

Proof: In this theorem we have already proved the domination number of G in theorem.1. Now we have to find the non-split domination number γ_{ns}(D). From Theorem let a_j = 2 be the arc a_i = 1 and $a_i \in \gamma_{ns}$(D). Suppose a_k is any arc, $a_k \neq a_j$, $a_k > a_j$ such that a_k intersects a_j.Then <V - γ_{ns}(D)> does not contain a_i. But the induced subgraph <V - γ_{ns}(D)>, a_j is connected to a_k, so that there is no disconnection for a_j to its right in clock wise direction. Then we have a connected graph G. Next we will find the average distance of G as well as the diameter already we proved in theorem.1.

8. EXPERIMENTAL PROBLEM FOR THEOREM.2

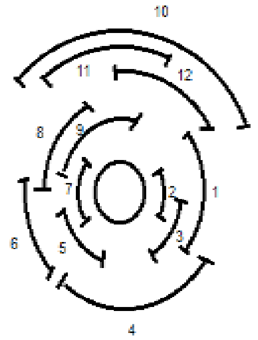

Fig. 4: Circular-arc family A

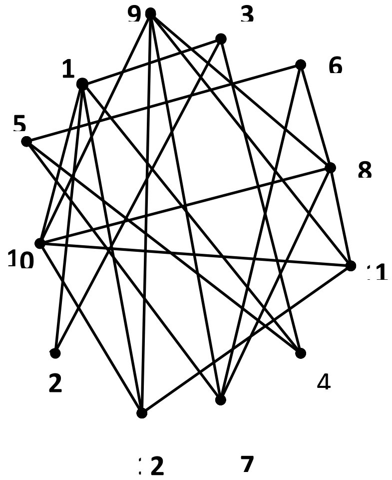

Fig. 4: Circular-arc graph G

Dominating Set = {4,7,9}, γ_{ns}(G)=3

9. To find the distances from G

d(1,1)=0	d(2,1)=1	d(3,1)=1
d(1,2)=1	d(2,2)=0	d(3,2)=1
d(1,3)=1	d(2,3)=1	d(3,3)=0
d(1,4)=1	d(2,4)=2	d(3,4)=1
d(1,5)=2	d(2,5)=3	d(3,5)=2
d(1,6)=3	d(2,6)=4	d(3,6)=3
d(1,7)=3	d(2,7)=4	d(3,7)=3
d(1,8)=2	d(2,8)=3	d(3,8)=3
d(1,9)=2	d(2,9)=3	d(3,9)=3
d(1,10)=1	d(2,10)=2	d(3,10)=2
d(1,11)=2	d(2,11)=3	d(3,11)=3
d(1,12)=1	d(2,12)=2	d(3,12)=2

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d(4,1)=1	d(5,1)=2	d(6,1)=3
d(4,2)=2	d(5,2)=3	d(6,2)=4
d(4,3)=1	d(5,3)=2	d(6,3)=3
d(4,4)=0	d(5,4)=1	d(6,4)=2
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=2	d(5,6)=1	d(6,6)=0
d(4,7)=2	d(5,7)=1	d(6,7)=1
d(4,8)=3	d(5,8)=2	d(6,8)=1
d(4,9)=3	d(5,9)=2	d(6,9)=2
d(4,10)=2	d(5,10)=3	d(6,10)=2
d(4,11)=3	d(5,11)=3	d(6,11)=2
d(4,12)=2	d(5,12)=3	d(6,12)=3

d(7,1)=3	d(8,1)=2	d(9,1)=2
d(7,2)=4	d(8,2)=3	d(9,2)=3
d(7,3)=3	d(8,3)=3	d(9,3)=3
d(7,4)=2	d(8,4)=3	d(9,4)=3
d(7,5)=1	d(8,5)=2	d(9,5)=2
d(7,6)=1	d(8,6)=2	d(9,6)=2
d(7,7)=0	d(8,7)=1	d(9,7)=1
d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=1	d(8,9)=1	d(9,9)=0
d(7,10)=2	d(8,10)=1	d(9,10)=1
d(7,11)=2	d(8,11)=1	d(9,11)=1
d(7,12)=2	d(8,12)=1	d(9,12)=1

d(10,1)=1	d(11,1)=2	d(12,1)=1
d(10,2)=2	d(11,2)=3	d(12,2)=2
d(10,3)=2	d(11,3)=3	d(12,3)=2
d(10,4)=2	d(11,4)=3	d(12,4)=2
d(10,5)=3	d(11,5)=3	d(12,5)=3
d(10,6)=2	d(11,6)=2	d(21,6)=3
d(10,7)=2	d(11,7)=2	d(12,7)=2
d(10,8)=1	d(11,8)=1	d(12,8)=2
d(10,9)=1	d(11,9)=1	d(12,9)=1
d(10,10)=	d(11,10)=1	d(12,10)=1
d(10,11)=	d(11,11)=0	d(12,11)=1
d(10,12)=	d(11,12)=	d(12,12)=0

10. Eccentricity:

e(1) = max {0, 1, 1, 1, 2, 3, 3, 2, 2, 1, 2, 1}=3

e(2) = max {1, 0, 1, 2, 3, 4, 4, 3, 3, 2, 3, 2}=4

e(3) = max {1, 1, 0, 1, 2, 3, 3, 3, 3, 2, 3, 2}=3

e(4) = max {1, 2, 1, 0, 1, 2, 2, 3, 3, 2, 3, 2}=3

e(5) = max {2, 3, 2, 1, 0, 1, 1, 2, 2, 3, 3, 3}=3

e(6) = max {3, 4, 3, 2, 1, 0, 1, 1, 2, 2, 2, 3}=4

e(7) = max {3, 4, 3, 2, 1, 1, 0, 1, 1, 2, 2, 2}=4

e(8) = max {2, 3, 3, 3, 2, 1, 1, 0, 1, 1, 1, 2}=3

e(9) = max {2, 3, 3, 3, 2, 2, 1, 1, 0, 1, 1, 1}=3

e(10) = max {1, 2, 2, 2, 3, 2, 2, 1, 1, 0, 1, 1}=3

e(11) = max {2, 3, 3, 3, 3, 2, 2, 1, 1, 1, 0, 1}=3

e(12) = max {1, 2, 2, 2, 3, 3, 2, 2, 1, 1, 1, 0}=3

e(v) = max {3, 4, 3, 3, 3, 4, 4, 3, 3, 3, 3, 3}=4

11. Diameter:

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diam(G) = max {3, 4, 3, 3, 3, 4, 4, 3, 3, 3, 3, 3}=4

Therefore diam(G) = 4

12. To find the average distance of G

V	1	2	3	4	5	6	7	8	9	10	11	12
1	0	1	1	1	2	3	3	2	2	1	2	1
2	1	0	1	2	3	4	4	3	3	2	3	2
3	1	1	0	1	2	3	3	3	3	2	3	2
4	1	2	1	0	1	2	2	3	3	2	3	2
5	2	3	2	1	0	1	1	2	2	3	3	3
6	3	4	3	2	1	0	1	1	2	2	2	3
7	3	4	3	2	1	1	0	1	1	2	2	2
8	2	3	3	3	2	1	1	0	1	1	1	2
9	2	3	3	3	2	2	1	1	0	1	1	1
10	1	2	2	2	3	2	2	1	1	0	1	1
11	2	3	3	3	3	2	2	1	1	1	0	1
12	1	2	2	2	3	3	2	2	1	1	1	0
Total	19	28	24	22	23	24	22	20	20	17	22	20

Table-3

Therefore Average distance

$$\mu(G) = \frac{1}{2\ {}^{n}c_{2}} \sum_{\substack{ai,aj \in v(G) \\ i \neq j}} \delta(ai, aj).$$

$$\mu(G) = \frac{1}{12 \times 11} 261$$

$$\mu(G) = 1.97$$

Therefore Diam(G) > γ_{ns}(D) > μ(G).

Theorem.3: Let A= {a_i, a_j, ….., a_n} be a circular-arc family. If a_i, a_j, a_k are three consecutive forward arcs in A. Where A is a proper circular-arc family such that $a_i < a_j < a_k$, $a_j \neq 1$ and a_i and a_j intersect, a_j and a_k intersect and $a_j \in \gamma_{ns}$(D) and there is at least backward arc in <V - γ_{ns}(D)>. Then the non-split domination number is greater than the average distance of G is greater than the diameter of G.

γ_{ns}(D) > μ(G) > Diam(G)

Proof: We consider A= {a_i, a_j, ….., a_n} be a circular arc family and let G be a circular-arc graph corresponding to circular arc family A. Let a_i, a_j, a_k are three consecutive forward arcs ina proper arc family A satisfying the hypothesis. Since $a_j \in \gamma_{ns}$(D), a_i and a_k are disconnected in clock wise direction in <V - γ_{ns}(D)>. However if there is a backward arc in <V - γ_{ns}(D)>, then there is a path between a_k and a_i in clockwise direction, namely (a_k , a_i) – path, so that a_i and a_k are connected in <V - γ_{ns}(D)>. Hence if there is atleast one backward arc in <V - γ_{ns}(D)>, then there is no disconnection in <V - γ_{ns}(D)>. There fore the induced subgraph is connected.

Now we have to prove that the average distance μ(G) is greater than diameter of G and also we proved in theorem.1.Where $\mu(G) = \frac{1}{2\ {}^{n}c_{2}} \sum_{\substack{ai,aj \in v(G) \\ i \neq j}} \delta(ai, aj).$ And

Diam(G) = max { e(a_i) : $a_j \in$ V}

The above set we got γ_{ns}(D) > μ(G) > Diam(G).

13. EXPERIMENTAL PROBLEM FOR THEOREM.3

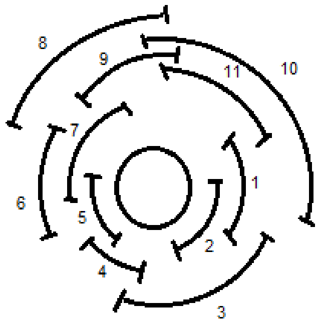

Fig. 6: Circular-arc family A

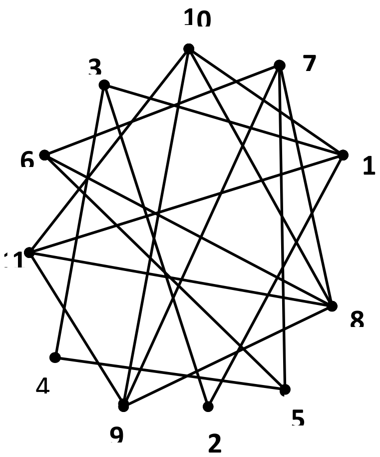

Fig. 7: Circular-arc graph G

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Dominating Set = {1,4,7}, $\gamma_{ns}(G)$=3

14. To find the distances from G

d(1,1)=0	d(2,1)=1	d(3,1)=1
d(1,2)=1	d(2,2)=0	d(3,2)=1
d(1,3)=1	d(2,3)=1	d(3,3)=0
d(1,4)=2	d(2,4)=2	d(3,4)=1
d(1,5)=3	d(2,5)=3	d(3,5)=2
d(1,6)=3	d(2,6)=4	d(3,6)=3
d(1,7)=3	d(2,7)=4	d(3,7)=3
d(1,8)=2	d(2,8)=3	d(3,8)=3
d(1,9)=2	d(2,9)=3	d(3,9)=3
d(1,10)=	d(2,10)=2	d(3,10)=2
d(1,11)=	d(2,11)=2	d(3,11)=2

d(4,1)=2	d(5,1)=3	d(6,1)=3
d(4,2)=2	d(5,2)=3	d(6,2)=4
d(4,3)=1	d(5,3)=2	d(6,3)=3
d(4,4)=0	d(5,4)=1	d(6,4)=2
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=2	d(5,6)=1	d(6,6)=0
d(4,7)=2	d(5,7)=1	d(6,7)=1
d(4,8)=3	d(5,8)=2	d(6,8)=1
d(4,9)=3	d(5,9)=2	d(6,9)=2
d(4,10)=3	d(5,10)=3	d(6,10)=2
d(4,11)=3	d(5,11)=3	d(6,11)=2

d(7,1)=3	d(8,1)=2	d(9,1)=2
d(7,2)=4	d(8,2)=3	d(9,2)=3
d(7,3)=3	d(8,3)=3	d(9,3)=3
d(7,4)=2	d(8,4)=3	d(9,4)=3
d(7,5)=1	d(8,5)=2	d(9,5)=2
d(7,6)=1	d(8,6)=1	d(9,6)=2
d(7,7)=0	d(8,7)=1	d(9,7)=1
d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=1	d(8,9)=1	d(9,9)=0
d(7,10)=2	d(8,10)=1	d(9,10)=1
[illegible]		

d(10,1)=1	d(11,1)=1
d(10,2)=2	d(11,2)=2
d(10,3)=3	d(11,3)=2
d(10,4)=3	d(11,4)=3
d(10,5)=3	d(11,5)=3
d(10,6)=2	d(11,6)=2
d(10,7)=2	d(11,7)=2
d(10,8)=1	d(11,8)=1
d(10,9)=1	d(11,9)=1
d(10,10)=0	d(11,10)=1
d(10,11)=1	d(11,11)=0

15. Eccentricity:

d(8,5)=2

e(1) = max {0, 1, 1, 2, 3, … 3

d(8,6)=1

e(2) = max {1, 0, 1, 2, 3, … 4

d(8,7)=1

e(3) = max {1, 1, 0, 1, 2, … 3

d(8,8)=0

e(4) = max {2, 2, 1, 0, 1, … 3

d(8,9)=1

e(5) = max {3, 3, 2, 1, 0, … 3

d(8,10)=1

e(6) = max {3, 4, 3, 2, 1, … 4

d(8,11)=1

e(7) = max {3, 4, 3, 2, 1, 1, 0, 1, 1, 2, 2}=4

e(8) = max {2, 3, 3, 3, 2, 1, 1, 0, 1, 1, 1}=3

e(9) = max {2, 3, 3, 3, 2, 2, 1, 1, 0, 1, 1}=3

e(10) = max {1, 2, 3, 3, 3, 2, 2, 1, 1, 0, 1}=3

e(11) = max {1, 2, 2, 3, 3, 2, 2, 1, 1, 1, 0}=3

e(v) = max {3, 4, 3, 3, 3, 4, 4, 3, 3, 3, 3}=4

16. Diameter:

diam(G) = max {3, 4, 3, 3, 3, 4, 4, 3, 3, 3, 3}=4

Therefore diam(G) = 4

17. To find the average distance of G

V	1	2	3	4	5	6	7	8	9	10	11
1	0	1	1	2	2	3	3	3	2	1	1
2	1	0	1	2	3	4	4	3	3	2	2
3	1	1	0	1	2	3	3	3	3	2	2
4	2	2	1	0	1	2	2	3	3	3	3
5	3	3	2	1	0	1	1	2	2	3	3
6	3	4	3	2	1	0	1	1	2	2	2
7	3	4	3	2	1	1	0	1	1	2	2
8	2	3	3	3	2	1	1	0	1	1	1
9	2	3	3	3	2	2	1	1	0	1	1
10	1	2	3	3	3	2	2	1	1	0	1
11	1	2	2	3	3	2	2	1	1	1	0
Total	19	25	22	22	21	21	20	18	19	18	18

Table-1

Therefore Average distance

$$\mu(G)=\frac{1}{2\ {}^{n}c_{2}}\sum_{\substack{ai,aj\in v(G)\\ i\neq j}}\delta(ai,aj).$$

$$\mu(G)=\frac{1}{11\times 10}223$$

$$\mu(G)=2.027$$

Therefore Diam(G) > γ_{ns}(D) > μ(G).

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