www.ijecs.in
International Journal Of Engineering And Computer Science ISSN: 2319-7242
Volume 6 Issue 2 Feb. 2017, Page No. 20362-20371
Index Copernicus Value (2015): 58.10, DOI: 10.18535/ijecs/v6i2.35

Mining Correlation Rules for Interval - Vague Sets

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Abstract−This paper discusses different classes of aggregation operators and their applications to Interval Vague Set (IVS) decision making problems.Correlation coefficient for interval vague sets is proposed and utilized in Multiple Attribute Group Decision Making (MAGDM) problems. Ordered Weighted Geometric (OWG) operators are used for the MAGDM models proposed for IVSs and the proposed correlation coefficient is used for ranking alternatives. A data mining algorithm is also utilized for reducing the number of alternatives for the final decision making process. Numerical illustrations are provided for MAGDM models for interval vague sets.

Key-words: MAGDM, Vague sets, Interval vague sets, Correlation coefficient of interval vague sets, Data mining.

1. INTRODUCTION

Various attempts are made by researchers on the study of data vagueness through intuitionistic fuzzy sets and Vague Sets (VSs). Gau&Buehrer, (1994) introduced the concept of vague sets, and it was shown that vague sets are indeed intuitionistic fuzzy sets (Bustince&Burillo, 1996). But Lu & Ng, (2004, 2005, 2009) based their research on the algebraic and graphical differences between vague sets and intuitionistic fuzzy sets. Combining interval-valued fuzzy sets and vague sets, Zhi-feng et al., (2001) introduced the concept of Interval Vague Sets (IVSs). Gau&Buehrer, (1994), Li &Rao, (2001), Liu, P.D., (2009a) and Liu, P.D., & Guan, (2008; 2009) have detailed the essential operations of vague sets and interval vague sets. Interval vague set is one of the higher order fuzzy sets and is being applied in various fields. The notion of truth membership function, false membership function and uncertainty function in interval vague sets describes the objective world more realistically and practically. Interval vague sets reflect people's understanding in three aspects comprehensively: Support degree, Negative degree and Uncertainty degree.

Correlation coefficient of Fuzzy sets, Interval-valued Fuzzy sets, Intuitionistic Fuzzy sets and Interval-valued Intuitionistic Fuzzy sets are already in the literature. Bustince&Burillo (1995) and Hong (1998) have focussed on the correlation degree of interval valued intuitionistic fuzzy sets. Park et al. (2009) have also worked on the correlation coefficient of interval valued intuitionistic fuzzy sets and applied in multiple-attribute group decision making problems. Robinson &Amirtharaj, (2011a; 2011b; 2012a; 2012b) defined the correlation coefficient of vague sets, interval vague sets. In this paper a different correlation coefficient of IVSs is proposed and utilized in MAGDM problems. Liu, et al, (2012) presented novel method for MCDM problems based on interval valued intuitionistic fuzzy sets(IVIFSs). Li, (2010) presented a TOPSIS-Based Nonlinear-Programming Methodology for Multi- attribute Decision Making with interval-valued intuitionistic fuzzy sets. Li& Nan, (2011) extended the TOPSIS method for Multi-attribute group decision making under Intuitionistic Fuzzy Sets (IFS) environments. Cui & Yong, (2009) developed a Fuzzy Multi-Attribute Decision Making model based on Degree of Grey Incidence and TOPSIS in the Open Tender of International Project about Contractor Prequalification Evaluation Process. Shih et al., (2001; 2007) worked on Group Decision Making for TOPSIS and its extension.

In many applications, ranking of IVSs plays a very important role in the decision making processes. Liu, (2009) presented a novel method of TOPSIS using a new type of score and precise function for choosing positive and negative ideal solutions in contrast to the score and accuracy functions defined by Chen & Tan, (1994), Hong & Choi, (2000), Wang et al., (2006) and Xu, (2007). However, Nayagam et al, (2011) proved the insufficiency of many of the score functions proposed in literature, and proposed a novel method of accuracy function for MCDM problems under IVIFS environment. In most of the previous TOPSIS techniques presented in literature, different forms of score and accuracy functions were used to identify positive and negative ideal solutions. In this work, a novel method is presented where the correlation coefficient of IVSs is used to identify positive and negative ideal solutions and for ranking alternatives based on the closeness coefficient. Comparison is made between the proposed TOPSIS and existing TOPSIS methods and some ranking functions proposed by Chen & Tan, (1994), Xu, (2007), Hong & Choi, (2000) and Liu, (2009).

Mining fuzzy association rule is an important task of fuzzy data mining which is often defined as finding the fuzzy item sets which frequently occur together in a given transaction data base. A popular application for fuzzy association rule mining is the market basket analysis which identifies the buying behaviors of customers. It is widely used to find the products which are frequently purchased together by same customers in transaction data bases. Of course, this kind of information is quite useful for many industries to make the marketing strategies. In a transaction data base, if a fuzzy item set almost occur in all of the records, then this fuzzy item sets may occur with fuzzy item sets frequently. Therefore, two fuzzy item sets which frequently occur together cannot imply that there is always an interesting relationship between them. The fuzzy simple correlation analysis has been used to discover the correlation relationship between two fuzzy item sets.

2. VAGUE SET

A vague set A in a universe of discourse U is characterized by a truth membership function, t_A, and a false membership function, f_A, as follows: $t_A : U \rightarrow [0,1]$, $f_A : U \rightarrow [0,1]$ and $t_A(u) + f_A(u) \leq 1$, where $t_A(u)$ is a lower bound on the grade of membership of u derived from the evidence for u, and $f_A(u)$ is a lower bound on the grade of membership of the negation of u derived from the evidence against u. Suppose $U = \{u_1, u_2, ..., u_n\}$. A vague set A of the universe of discourse U can be represented as: $A = \sum_{i=1}^{n} [t(u_i),\ 1 - f(u_i)] / u_i, \quad 0 \leq t(u_i) \leq 1 - f(u_i) \leq 1, \quad i = 1, 2, ..., n$. In other words, the grade of membership of u_i is bound to a subinterval $\left[t_A(u_i),\ 1 - f_A(u_i)\right]$ of [0,1]. Let $X = \{x_1, x_2, ..., x_n\}$ be the finite universal set, VS(X) be the collection of vague sets and A, $B \in$ VS(X) be given by $A = \left\{\left\langle x, \left[t_A(x), 1 - f_A(x)\right]\right\rangle / x \in X\right\}$, $B = \left\{\left\langle x, \left[t_B(x), 1 - f_B(x)\right]\right\rangle / x \in X\right\}$. And the length of the vague values are given by $\pi_A(x) = 1 - t_A(x) - f_A(x)$, $\pi_B(x) = 1 - t_B(x) - f_B(x)$.

3. OPERATIONS IN VAGUE SETS

Let x, y be the two vague values in the universe of discourse U, $x = [t_x, 1 - f_x]$, $y = [t_y, 1 - f_y]$, where $t_x, f_x, t_y, f_y \in [0,1]$ and $t_x + f_x \leq 1$, $t_y + f_y \leq 1$;

The operation and relationship between vague values is defined as follows:

Definition: The minimum operation of vague values x and y is defined by

$$x \wedge y = \left[\min\left(t_x, t_y\right),\ \min\left(1 - f_x,\ 1 - f_y\right)\right] = \left[\min\left(t_x, t_y\right),\ 1 - \max\left(1 - f_x,\ 1 - f_y\right)\right] \quad (1)$$

Definition: The maximum operation of vague values x and y is defined by

$$x \vee y = \left[\max\left(t_x, t_y\right),\ \max\left(1 - f_x,\ 1 - f_y\right)\right] = \left[\max\left(t_x, t_y\right), 1 - \min\left(1 - f_x,\ 1 - f_y\right)\right] \quad (2)$$

Definition: The complement of vague value x is defined by

$$\overline{x} = \left[f_x,\ 1 - t_x\right] \quad (3)$$

Let A, B be two VSs in the universe of discourse $U = \{u_1, u_2, ..., u_n\}$,

$$A = \sum_{i=1}^{n} \left[t_A(u_i),\ 1 - f_A(u_i)\right] / u_i,$$

and $B = \sum_{i=1}^{n} \left[t_B(u_i),\ 1 - f_B(u_i)\right] / u_i$.

Then the operations between VSs are defined as follows.

The intersection of VSs A and B is defined by

$$A \cap B = \sum_{i=1}^{n} \left\{\left[t_A(u_i),\ 1 - f_A(u_i)\right] \wedge \left[t_B(u_i),\ 1 - f_B(u_i)\right]\right\} / u_i. \quad (4)$$

The union of vague sets A and B is defined by

$$A \cup B = \sum_{i=1}^{n} \left\{\left[t_A(u_i),\ 1 - f_A(u_i)\right] \vee \left[t_B(u_i),\ 1 - f_B(u_i)\right]\right\} / u_i. \quad (5)$$

The complement of vague set A is defined by

$$\overline{A} = \sum_{i=1}^{n} \left[f_A(u_i),\ 1 - t_A(u_i)\right] / u_i. \quad (6)$$

Definition: For the vague value $x = \left[t_x,\ 1 - f_x\right]$, define the de-fuzzification function to get the precise value as follows:

$$Dfzz\ (x) = \frac{t_x}{(t_x + f_x)}. \quad (7)$$

3.1. DIFFERENT CLASSES OF OPERATORS IN MAGDM PROBLEMS

3.1.1. Ordered Weighted Averaging (*OWA*) Operators

The *OWA* operator provides a parameterized family of aggregation operators which are used in many applications. The definition of the *OWA* operator was introduced by Yager, (1988).

Definition: An *OWA* operator of dimension n is a mapping $OWA: R^n \rightarrow R$ that has an associated weighting vector $w = (w_1, w_2, ..., w_n)^T$ of dimension n having the properties, $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$ and such that

$$OWA(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j b_j \quad (8)$$

Where b_j is the j^{th} largest of the a_i, for all i.

3.1.2. The Induced *OWA* (*I-OWA)* Operator

The *I-OWA* operator was introduced by Yager&Filev, (1999), and its main difference with the classical *OWA* operator is that the reordering step of the *I-OWA* is carried out with order-inducing variables, rather than depending on the values of the arguments a_i. It can be defined as follows:

Definition: An *I-OWA* operator of dimension n is a mapping

$I\text{-}OWA: R^n \times R^n \to R$ defined by an associated weighting vector w of dimension n with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$, and a set of order-inducing variables u_i, by a formula of the following form:

$$I\text{-}OWA\left(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, ..., \langle u_n, a_n\rangle\right) = \sum_{j=1}^{n} w_j b_j \quad (9)$$

Where a_i is the argument variable, u_i is the order-inducing variable and $(b_1,\ldots,b_n)$ is simply $(a_1,\ldots,a_n)$ reordered in ascending order of the values of the u_i.

3.1.3 The Hybrid Average (*HA*) Operator

The Hybrid Average (*HA*) operator is an aggregation operator that uses both the Weighted Averaging (*WA*) and the *OWA* operator in the same formulation. Thus, it is possible to consider the attitudinal character of the decision maker and the degree of importance of the variables in the same problem. One of its main characteristics is that it provides a parameterized family of aggregation operators that include the maximum, the minimum, the Arithmetic Mean (*AM*), the *WA* and the *OWA* operator. It can be defined as follows:

Definition: An *HA* operator of dimension n is a mapping $HA: R^n \to R$ that has an associated weighting vector w of dimension n with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$, such that

$$HA(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j b_j \quad (10)$$

where b_j is the j^{th} smallest of the $\hat{a}_i$, and $\hat{a}_i = n\omega_i a_i$, for all $i = 1, 2, ..., n.$

And $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is the weighting vector of the a_i, with $\omega_i \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1.$

3.1.4 The Fuzzy *OWA* Operator (*FOWA*)

The *FOWA* operator is an extension of the *OWA* operator that uses uncertain information in the arguments represented in the form of FNs. The reason for using this aggregation operator is that sometimes available information cannot be assessed with exact numbers and so it is necessary to use a parameterized family of aggregation operators that include the fuzzy maximum, the fuzzy minimum and the fuzzy average criteria.

Definition: Let ψ be the set of FNs. A *FOWA* operator of dimension n is a mapping $FOWA: \psi^n \to \psi$ that has an associated weighting vector w of dimension n with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that

$$FOWA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{n} w_j b_j \quad (11)$$

Where b_j is the j^{th} largest of the $\tilde{a}_i$, and the $\tilde{a}_i$ are FNs.

3.1.5 The Probabilistic *OWA* (*POWA*) Operator

The *POWA* operator is an aggregation operator that unifies the probability and the *OWA* operator in the same formulation considering the degree of importance that each concept has in the analysis and providing a parameterized family of aggregation operators between the minimum and the maximum, defined as follows:

Definition: A *POWA* operator of dimension n is a mapping $POWA: R^n \to R$ that has an associated weighting vector w of dimension n such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1,$ according to the formula:

$$POWA(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} \hat{p}_j b_j \quad (12)$$

Where b_j is the j^{th} largest of the a_i, and its corresponding probability is p_j. Each argument a_i has an associated probability p_i with $\sum_{i=1}^{n} p_i = 1$ and $p_i \in [0,1]$, $\hat{p}_j = \beta w_j + (1-\beta) p_j$ with $\beta \in [0,1].$

3.1.6 The Fuzzy Probabilistic *OWA* (*FPOWA*) Operator

The *FPOWA* operator is an aggregation operator that provides a parameterized family of aggregation operators between the fuzzy maximum and the fuzzy minimum whose main advantage is unifying fuzzy probabilistic aggregation and the *FOWA* operator in the same formulation and considering the degree of importance of each concept in the aggregation. The probabilistic information and the attitudinal character of the decision maker can be used in the same formulation.

Definition: Let ψ be the set of FNs. A *FPOWA* operator of dimension n is a mapping $FPOWA: \psi^n \to \psi$ that has associated a weighting vector w of dimension n such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$FPOWA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{n} \hat{p}_j b_j \quad (13)$$

Where b_j is the j^{th} largest of the $\tilde{a}_i$, with corresponding probability $\tilde{p}_j$, the $\tilde{a}_i$ are FNs and each one has an associated probability $\tilde{p}_i$ with $\sum_{i=1}^{n} \tilde{p}_i = 1$ and

$\tilde{p}_i \in [0,1],\ \hat{p}_j = \tilde{\beta} w_j + (1-\tilde{\beta})\tilde{p}_j$ with $\tilde{\beta} \in [0,1]$ and $\tilde{p}_j$ is the $\tilde{p}_i$ ordered according to b_j, that is, according to the j^{th} largest of the $\tilde{a}_i$.

3.1.7 Geometric Mean (*GM*) Operator:

Definition: A Geometric Mean (*GM*) operator of dimension *m* is a mapping $GM: R^m \to R$ and is defined as:

$$GM(a_1, a_2, ..., a_m) = \prod_{j=1}^{m} \left(a_j\right)^{1/m} \tag{14}$$

3.1.8 Ordered Weighted Geometric (*OWG*) Operators:

The Ordered Weighted Geometric (*OWG*) operator is based on the *OWA* operator and the Geometric Mean (*GM*) operator and provides a parameterized family of aggregation operators used in many applications. The definition of the *OWG* operator is as follows:

Definition: An *OWG* operator of dimension *m* is a mapping $OWG: R^m \to R$ that has an associated weighting vector $w = (w_1, w_2, ..., w_m)^T$ of dimension *m* having the properties, $w_j \in [0,1],\ \sum_{j=1}^{m} w_j = 1$ and such that

$$OWG(a_1, a_2, ..., a_m) = \prod_{j=1}^{m} b_j^{\,w_j} \tag{15}$$

Where b_j is the j^{th} largest of the a_i.

4. INTERVAL VAGUE SET

The set of all IVSs in *X* is denoted by IVS(*X*). Then for each $x \in X$, $t_A(x)$ and $f_A(x)$ are closed intervals and their lower and upper end points are denoted by: $t_{AL}(x),\ f_{AL}(x),\ t_{AU}(x),\ f_{AU}(x)$. Then,

$A = \left\{ \left\langle x, \left[t_{AL}(x), t_{AU}(x)\right], \left[f_{AL}(x), f_{AU}(x)\right] \right\rangle / x \in X \right\}$,

where $0 \le t_{AU}(x) + f_{AU}(x) \le 1,\ x \in X$.

For each $A \in$ IVS(*X*), the hesitancy degree of a vague interval of *X* in *A* is defined and denoted as: $\pi_A(x) = 1 - t_A(x) - f_A(x) = \left[1 - t_{AU}(x) - f_{AU}(x),\ 1 - t_{AL}(x) - f_{AL}(x)\right]$.

4.1 Induced Vague Ordered Weighted Geometric (*I-VOWG*) Operator

Definition: Let $\tilde{a}_j = (t_j, f_j)$, for all *j* = 1,2,…,*n* be a collection of vague values. The Vague Weighted Geometric (*VWG*) operator $VWG: Q^n \to Q$ is defined as:

$$VWG_\omega(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{n} \tilde{a}_j^{\,\omega_j} = \left(\prod_{j=1}^{n} t_j^{\omega_j}, 1 - \prod_{j=1}^{n} (1 - f_j)^{\omega_j} \right) \tag{16}$$

Where $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ be the weight vector of $\tilde{a}_j\ (j = 1, 2, ..., n)$, and $\omega_j > 0$, $\sum_{j=1}^{n} \omega_j = 1$.

Definition: Let $\tilde{a}_j = (t_j, f_j)$, for all *j* = 1,2,…,*n* be a collection of vague values. The Vague Ordered Weighted Geometric (*VOWG*) operator, $VOWG: Q^n \to Q$ is defined as:

$$VOWG_w(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{n} \tilde{a}_{\sigma(j)}^{w_j} = \left(\prod_{j=1}^{n} t_{\sigma(j)}^{w_j}, 1 - \prod_{j=1}^{n} (1 - f_{\sigma(j)})^{w_j} \right) \tag{17}$$

Where $w = (w_1, w_2, ..., w_n)^T$ is the associated weighting vector such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Furthermore, $(\sigma(1), \sigma(2), ..., \sigma(n))$ is a permutation of (1,2,…,n), such that $a_{\sigma(j-1)} \ge a_{\sigma(j)}$ for all *j* = 2,…, n.

Definition: Let $\tilde{a}_j = (t_j, f_j)$, for all *j* = 1,2,…,*n* be a collection of vague values. An Induced Vague Ordered Weighted Geometric (*I-VOWG*) operator, $I-VOWG: Q^n \to Q$ is defined as:

$$I\text{-}VOWG_w\left(\langle u_1, \tilde{a}_1\rangle, \langle u_2, \tilde{a}_2\rangle, ..., \langle u_n, \tilde{a}_n\rangle\right) = \sum_{j=1}^{n} a_{\sigma(j)}^{w_j} = \left(\prod_{j=1}^{n} t_{\sigma(j)}^{w_j}, 1 - \prod_{j=1}^{n} (1 - f_{\sigma(j)})^{w_j} \right) \tag{18}$$

where $w = (w_1, w_2, ..., w_n)^T$ is a weighting vector, such that $w_j \in [0,1],\ \sum_{j=1}^{n} w_j = 1,\ j = 1, 2, ..., n,$

$a_{\sigma(j)} = (t_{\sigma(j)}, f_{\sigma(j)})$ is the $\tilde{a}_i$ value of the *VOWG* pair $\langle u_i, \tilde{a}_i \rangle$ having the j^{th} largest u_i, $u_i \in [0,1]$, and u_i in $\langle u_i, \tilde{a}_i \rangle$ is referred to as the order inducing variable and $\tilde{a}_i$, $\tilde{a}_i = (t_i, f_i)$ are the vague values.

The *I-VOWG* operator has the following properties similar to those of the *I-OWA* operator.

a. **Commutativity**

$$I\text{-}VOWG_w\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,...,\langle u_n,\tilde{a}_n\rangle\right)$$
$$=I\text{-}VOWG_w\left(\langle u_1,\tilde{a}'_1\rangle,\langle u_2,\tilde{a}'_2\rangle,...,\langle u_n,\tilde{a}'_n\rangle\right)$$

where $\left(\langle u_1,\tilde{a}'_1\rangle,\langle u_2,\tilde{a}'_2\rangle,...,\langle u_n,\tilde{a}'_n\rangle\right)$ is any permutation of $\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,...,\langle u_n,\tilde{a}_n\rangle\right)$.

b. Idempotency

If $\tilde{a}_j=\tilde{a},$ where $\tilde{a}_j=(\mu_j,\gamma_j)$ and $\tilde{a}=(\mu,\gamma)$ for all *j*, then

$$I\text{-}VOWG_w\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,...,\langle u_n,\tilde{a}_n\rangle\right)=\tilde{a}.$$

c. Monotonicity

If $\tilde{a}_j\le\tilde{a}'_j$ for all *j*, then

$$I\text{-}VOWG_w\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,...,\langle u_n,\tilde{a}_n\rangle\right)$$
$$\le I\text{-}VOWG_w\left(\langle u_1,\tilde{a}'_1\rangle,\langle u_2,\tilde{a}'_2\rangle,...,\langle u_n,\tilde{a}'_n\rangle\right).$$

4.2 Induced Interval-Vague Ordered Weighted Geometric (*I-IVOWG*) Operator

Definition: Let $\tilde{a}_j=([a_j,b_j],[c_j,d_j]),$ for all $j=1,2,...,n$ be a collection of interval-vague values. The Interval-vague Weighted Geometric (*IVWG*) operator $IVWG:\ Q^n\to Q$ is defined as:

$$IVWG_{\omega}(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n)=\sum_{j=1}^{n}\tilde{a}_j^{\omega_j}$$
$$=\left(\left[\prod_{j=1}^{n}a_j^{\omega_j},\prod_{j=1}^{n}b_j^{\omega_j}\right],\left[1-\prod_{j=1}^{n}(1-c_j)^{\omega_j},1-\prod_{j=1}^{n}(1-d_j)^{\omega_j}\right]\right)\quad(19)$$

Where $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ be the weight vector of $\tilde{a}_j\ (j=1,2,...,n),$ and $\omega_j>0,\ \sum_{j=1}^{n}\omega_j=1.$

Definition: Let $\tilde{a}_j=([a_j,b_j],[c_j,d_j]),$ for all $j=1,2,...,n$ be a collection of interval-vague values. The Interval-Vague Ordered Weighted Geometric (*IVOWG*) operator, $IVOWG:\ Q^n\to Q$ is defined as:

$$IVOWG_w(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n)=\sum_{j=1}^{n}\tilde{a}_{\sigma(j)}^{w_j}$$
$$=\left(\left[\prod_{j=1}^{n}a_{\sigma(j)}^{w_j},\prod_{j=1}^{n}b_{\sigma(j)}^{w_j}\right],\left[1-\prod_{j=1}^{n}(1-c_{\sigma(j)})^{w_j},1-\prod_{j=1}^{n}(1-d_{\sigma(j)})^{w_j}\right]\right)\quad(20)$$

where $w=(w_1,w_2,...,w_n)^T$ is the associated weight vector such that $w_j>0$ and $\sum_{j=1}^{n}w_j=1$. Furthermore, $(\sigma(1),\sigma(2),...,\sigma(n))$ is a permutation of (1,2,…, n), such that $a_{\sigma(j-1)}\ge a_{\sigma(j)}$ for all *j*=2,…, n.

Definition: Let $\tilde{a}_j=([a_j,b_j],[c_j,d_j]),$ for all $j=1,2,...,n$ be a collection of interval-vague values. An Induced Interval-Vague Ordered Weighted Geometric (*I-IVOWG*) operator, $I\text{-}IVOWG:\ Q^n\to Q$ is defined as:

$$I\text{-}IVOWG_w\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,...,\langle u_n,\tilde{a}_n\rangle\right)=\sum_{j=1}^{n}a_{\sigma(j)}^{w_j}$$
$$=\left(\left[\prod_{j=1}^{n}a_{\sigma(j)}^{w_j},\prod_{j=1}^{n}b_{\sigma(j)}^{w_j}\right],\left[1-\prod_{j=1}^{n}(1-c_{\sigma(j)})^{w_j},1-\prod_{j=1}^{n}(1-d_{\sigma(j)})^{w_j}\right]\right)\quad(21)$$

Where $w=(w_1,w_2,...,w_n)^T$ is a weighting vector, such that $w_j\in[0,1],\ \sum_{j=1}^{n}w_j=1,\ j=1,2,...,n,$ $a_{\sigma(j)}=([a_{\sigma(j)},b_{\sigma(j)}],[c_{\sigma(j)},d_{\sigma(j)}]),$ is the $\tilde{a}_i$ value of the *I-IVOWG* pair $\langle u_i,\tilde{a}_i\rangle$ having the *j*th largest u_i, $u_i\in[0,1]$, and u_i in $\langle u_i,\tilde{a}_i\rangle$ is referred to as the order inducing variable and $\tilde{a}_i$, $\tilde{a}_i=([a_i,b_i],[c_i,d_i])$ are interval-vague values.

The *I-IVOWG* operator has the following properties similar to those of the *I-OWA* operator.

a. Commutativity

$$I\text{-}IVOWG_w\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,...,\langle u_n,\tilde{a}_n\rangle\right)$$
$$=I\text{-}IVOWG_w\left(\langle u_1,\tilde{a}'_1\rangle,\langle u_2,\tilde{a}'_2\rangle,...,\langle u_n,\tilde{a}'_n\rangle\right)$$

Where $\left(\langle u_1,\tilde{a}'_1\rangle,\langle u_2,\tilde{a}'_2\rangle,...,\langle u_n,\tilde{a}'_n\rangle\right)$ is any permutation of $\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,...,\langle u_n,\tilde{a}_n\rangle\right)$.

b. Idempotency

If $\tilde{a}_j=\tilde{a}$ for all *j*, where $\tilde{a}_j=([a_j,b_j],[c_j,d_j]),\ \ \tilde{a}=([a,b],[c,d]),$ then

$$I\text{-}IVOWG_w\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,...,\langle u_n,\tilde{a}_n\rangle\right)=\tilde{a}.$$

c. Monotonicity

If $\tilde{a}_j\le\tilde{a}'_j$ for all j, then

$$I\text{-}IVOWG_w\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,...,\langle u_n,\tilde{a}_n\rangle\right)$$
$$\le I\text{-}IVOWG_w\left(\langle u_1,\tilde{a}'_1\rangle,\langle u_2,\tilde{a}'_2\rangle,...,\langle u_n,\tilde{a}'_n\rangle\right).$$

4.3 CORRELATION COEFFICIENT OF INTERVAL-VAGUE SETS

Let $X=\{x_1,x_2,...,x_n\}$ be the finite universal set and *A*, *B*∈ IVS(*X*) be given by

$$A=\left\{\left\langle x,\left[t_{AL}(x),t_{AU}(x)\right],\left[f_{AL}(x),f_{AU}(x)\right]\right\rangle/x\in X\right\},$$

$$B = \left\{ \left\langle x, \left[t_{BL}(x), t_{BU}(x) \right], \left[f_{BL}(x), f_{BU}(x) \right] \right\rangle / x \in X \right\},$$

For each $A \in$ IVS(X), the informational vague energy of A is defined as:

$$E_{IVS}(A) = \frac{1}{2}\sum_{i=1}^{n}\left[t^2{}_{AL}(x_i) + t^2{}_{AU}(x_i) + f^2{}_{AL}(x_i) + f^2{}_{AU}(x_i) + \pi^2{}_{AL}(x_i) + \pi^2{}_{AU}(x_i) \right]. \quad (22)$$

The correlation of A and B is given by the formula:

$$C_{IVS}(A,B) = \frac{1}{2}\sum_{i=1}^{n}\left[\begin{array}{l} t_{AL}(x_i)t_{BL}(x_i) + t_{AU}(x_i)t_{BU}(x_i) + f_{AL}(x_i)f_{BL}(x_i) \\ \quad + f_{AU}(x_i)f_{BU}(x_i) + \pi_{AL}(x_i)\pi_{BL}(x_i) + \pi_{AU}(x_i)\pi_{BU}(x_i) \end{array} \right]. \quad (23)$$

For $A, B \in$ IVS(X), the correlation has the following properties:

(1) $C_{IVS}(A, A) = E(A)$.

(2) $C_{IVS}(A, B) = C_{IVS}(B, A)$.

Furthermore, the correlation coefficient of A and B is defined by the formula

$$K_{IVS}(A,\ B) = \frac{C_{IVS}(A,\ B)}{\sqrt{E_{IVS}(A)\ .\ E_{IVS}(B)}} \quad (24)$$

4.4 Algorithm for Mining Fuzzy Correlation Rules

4.4.1 Frequent Fuzzy Item-Sets, Closed Fuzzy Item-Sets and Fuzzy Association Rules

Since most of the real time databases are fuzzy in nature, it is necessary to explore and discover association rules and correlation rules in a fuzzy environment. To this end many researchers have proposed methods for mining fuzzy association rules, from various fuzzy datasets. If a fuzzy item set almost occurs in all records, then it may frequently occur with other fuzzy item-sets also. In order to find out useful relationships between the fuzzy item-sets based on fuzzy statistics, fuzzy correlation rules are generated. By using the fuzzy correlation analysis, the fuzzy correlation rules for fuzzy numbers are generated to see that two fuzzy sets not only frequently occur together in same records, but also are related to each other.

Let $I = \{I_1, I_2, ..., I_m\}$ be a set of fuzzy items. Let D, the task-relevant fuzzy data, be a set of database transactions where each transaction T is a set of fuzzy items such that $T \subseteq I$. Let A be a set of fuzzy items. A transaction T is said to contain A if and only if $A \subseteq T$. A fuzzy association rule is an implication of the form $A \Rightarrow B$, where $A \subset I, B \subset I$ and $A \cap B = \phi$. The rule $A \Rightarrow B$ holds in the transaction set D with fuzzy support s, where s is the percentage of transactions in D that contain $A \cup B$ (i.e., both A and B). This is taken to be the probability $P(A \cup B)$. The rule $A \Rightarrow B$ has fuzzy confidence c in the transaction set D, where c is the percentage of transactions in D containing A that also contain B. This is taken to be the conditional probability, $P(B/A)$.

$$\textit{Fuzzy support}\,(A \Rightarrow B) = P(A \cup B),$$
$$\textit{fuzzy confidence}\,(A \Rightarrow B) = P(B/A) \quad (25)$$

A set of items is referred to as an item-set. An item-set that contains k items is called a k-item-set. The occurrence frequency of an item-set is the number of transactions that contain the item-set. The fuzzy item-set support defined in equation (1) is called relative support, whereas the occurrence frequency is called the absolute support. If the relative support of an item-set I, satisfies a pre-specified minimum support threshold, then I is a frequent fuzzy item-set. From (1) we have:

fuzzy confidence

$$(A \Rightarrow B) = P(B/A) = \frac{\sup port(A \cup B)}{\sup port(A)} = \frac{\sup port\_count(A \cup B)}{\sup port\_count(A)} \quad (26)$$

Equation (26) shows that the fuzzy confidence of rule $A \Rightarrow B$ can be easily derived from the support counts of A and $A \cup B$. Once the support counts of A, B and $A \cup B$ are found, it is straightforward to derive the corresponding association rules $A \Rightarrow B$ and $B \Rightarrow A$, and check whether they are strong. Thus the problem of mining fuzzy association rules can be reduced to that of mining frequent fuzzy item-sets. A fuzzy item-set X is closed in a fuzzy data set S if there exists no proper super-itemset Y such that Y has the same support count as X in S. An item-set X is a closed frequent fuzzy item-set in set S if X is both closed and frequent in S.

The fuzzy item-sets which frequently occur together in large databases are found using fuzzy association rules. All the methods used for mining fuzzy association rules are based upon a support-confidence framework where fuzzy support and fuzzy confidence are used to identify the fuzzy association rules. Let $F = \{f_1, f_2, ..., f_m\}$ be a set of fuzzy items, $T = \{t_1, t_2, ..., t_n\}$ be a set of fuzzy records, and each fuzzy record t_i is represented as a vector with m values, $(f_1(t_i), f_2(t_i), ..., f_m(t_i))$, where $f_j(t_i)$ is the degree that f_j appears in record t_i, $f_j(t_i) \in [0,1]$. Then a fuzzy association rule is defined as an implication form such as $F_X \Rightarrow F_Y$, where $F_X \subset F, F_Y \subset F$ are two fuzzy item-sets.

The fuzzy association rule $F_X \Rightarrow F_Y$ holds in T with the fuzzy support ($fsupp(\{F_X, F_Y\})$) and the fuzzy confidence ($fconf(F_X \Rightarrow F_Y)$). The fuzzy support and fuzzy confidence are given as follows:

$$fsupp(\{F_X, F_Y\}) = \frac{\sum_{i=1}^{n} \min(f_j(t_i) / f_j \in \{F_X, F_Y\})}{n} \quad (27)$$

$$fconf(F_X \Rightarrow F_Y) = \frac{fsupp(\{F_X, F_Y\})}{fsupp(\{F_X\})} \quad (28)$$

If the $fsupp(\{F_X, F_Y\})$ is greater than or equal to a predefined threshold, minimal fuzzy support (s_f), and the $fconf(F_X \Rightarrow F_Y)$ is also greater than or equal to a predefined threshold, minimum fuzzy confidence (c_f), then $F_X \Rightarrow F_Y$ is considered as an interesting fuzzy association rule, and it means that the presence of the fuzzy item-set F_X in a record can imply the presence of the fuzzy item sets F_Y in the same record.

4.5 Mining Fuzzy Association and Fuzzy Correlation Rules

Mining fuzzy association rules is better done by finding frequent fuzzy item-sets using candidate generation method (Agrawal et al. 1993; Agrawal&Srikanth, 1994). Apriori is a seminal algorithm proposed for mining frequent fuzzy item-sets. The algorithm uses prior knowledge of frequent fuzzy item-set properties. Apriori employs an iterative approach known as level-wise search, where k-item sets are used to explore (k+1) –item sets. First, the set of 1-item sets is found by scanning the fuzzy database to accumulate the count for each item, and collecting those items that satisfy minimum support. The resulting set is denoted by L_1. Next L_1 is used to find L_2, the set of frequent fuzzy 2-itemset, which is used to find L_3, and so on, until no more frequent fuzzy k-item-sets can be found. The finding of each L_k requires one full scan of the database. The algorithm consists of two steps, namely (i) the join step and (ii) the prune step, for candidate generation.

The fuzzy correlation rules mining method already proposed by Thiagarasu&Umasankar, (2016)Will be utilized in this section.

Assume that $F=\{f_1, f_2,..., f_m\}$ be a set of fuzzy items; $T=\{t_1,t_2,...,t_n\}$ be a random sample with n fuzzy data records, and each sample record t_i is represented as a vector with m values, $\left(f_1(t_i), f_2(t_i),...., f_m(t_i)\right)$, where $f_j(t_i)$ is the degree that fuzzy item f_j occurs in record t_i, $f_j(t_i)\in[0,1]$.

And next, three predefined thresholds are needed to be defined. Here, s_f is the minimal fuzzy support; c_f is the minimal fuzzy confidence; r_f is the minimal fuzzy correlation coefficient. The procedure of mining fuzzy correlation rules is described as the follows:

Step 1: Transform the Vague dataset into a fuzzy set using any of the transformation technique.

Step 2: The fuzzy support of each fuzzy item $f_i \in F$, $f\text{supp}(f_i)$ is computed by using formula (eqn. 27 & 28).

Step 3: Let $L_1=\left\{ f_p \,\middle|\, f_p \in F,\ f\text{supp}(f_p)\geq s_f \right\}$ be the set of frequent fuzzy item sets whose size is equal to 1.

Step 4: Let $C_2=\{(F_A,F_B)\}$ be the set of all combinations of two elements belong to L_1, where $F_A, F_B \in L_1$, $F_A \neq F_B$. That is, C_2 is generated by L_1 joint with L_1. Because F_A and F_B are the element of L_1, the size of each element of C_2 is 2.

Step 5: For each element of C_2, (F_A,F_B) the fuzzy support, $f\text{supp}(\{F_A,F_B\})$ is computed by using formula (27) and then the fuzzy correlation between F_A and $F_B, r_{A,B}$, is computed by using formula discussed by Thiagarasu&Umasankar, (2016). Since $r_{A,B}$ is computed from the random sample T, $r_{A,B}$ is needed to be tested to determine if it is really greater than the minimal fuzzy correlation coefficient r_f, The formula for testing is as follows:

$$t=\frac{r_{A,B}-r_f}{\sqrt{\frac{1-r_{A,B}^2}{n-2}}} \tag{29}$$

Compare the computed t value to $t_{1-\alpha(n-2)}$, where $t_{1-\alpha(n-2)}$ is the $(1-\alpha)^{th}$ percentile in the t distribution with degree of freedom n-1. If we obtain the t value which is greater than the predefined minimal fuzzy correlation coefficient.

Step 6: For each element, whose fuzzy support is greater than or equal to s_f and fuzzy correlation coefficient passes the test, of C_2, then it is an element of L_2. Hence, L_2 is the set of the frequent combinations of two fuzzy item sets, and still, the size of each element of L_2 is 2.

Step 7: Next, each C_k, $k\geq 3$, is generated by L_{k-1} joint with L_{k-1}. Assume that (F_W,F_X) and (F_Y,F_Z) are two elements of L_{k-1}, where $F_X=F_Y$. If the size of the combinations $(F_X,\{F_W,F_Z\})$ is k, and (F_W,F_Z) is also a frequent combination of two fuzzy item sets, then the combination $(F_X,\{F_W,F_Z\})$ is a element with size k of C_k. For each element of C_k, its fuzzy support and fuzzy correlation coefficient are still used to find the elements of L_k.

Step 8: When each L_k, $k\geq 2$, is obtained, for each element of L_k, (F_G,F_H), two candidate fuzzy correlation rules, $F_G \rightarrow F_H$ and $F_H \rightarrow F_G$ can be generated. If the fuzzy confidence of a rule is greater than or equal to c_f, then it is considered as an interesting fuzzy correlation rule.

The algorithm won't stop until no next C_{k+1} can be generated.

4.6 ALGORITHM FOR MAGDM USING *I-IVOWG* OPERATOR AND CORRELATION COEFFICIENT OF IVSS

Let $A=\{A_1,A_2,\ldots,A_m\}$ be a discrete set of alternatives, and $G=\{G_1,G_2,\ldots,G_n\}$ be the set of attributes, $\omega=(\omega_1,\omega_2,...,\omega_n)$ is the weighting vector of the attribute G_j(j=1,2,…,n), where $\omega_j \in[0,1]$, $\sum_{j=1}^{n}\omega_j=1$. Let $D=\{D_1,D_2,\ldots,D_t\}$ be the set of decision makers, $w=(w_1,w_2,...,w_t)^T$ be the weighting vector of decision makers, with $w_k\in[0,1],\ \sum_{k=1}^{n} w_k=1$.

Suppose that $\tilde{R}_k=\left(\tilde{r}_{ij}^{(k)}\right)_{m\times n}=\left([a_{ij}^{(k)},b_{ij}^{(k)}],[c_{ij}^{(k)},d_{ij}^{(k)}]\right)_{m\times n}$ is the interval-vague decision matrix, where $[a_{ij}^{(k)},b_{ij}^{(k)}]$ indicates the degree that the alternative A_i satisfies the attribute G_j given by the decision maker D_k, $[c_{ij}^{(k)},d_{ij}^{(k)}]$ indicates the degree that the alternative A_i does not satisfy the attribute G_j given by the decision maker D_k, $[a_{ij}^{(k)},b_{ij}^{(k)}]\subset[0,1],\quad [c_{ij}^{(k)},d_{ij}^{(k)}]\subset[0,1],\quad b_{ij}^{(k)}+d_{ij}^{(k)}\le 1,$

$$i=1,2,...,m,\quad j=1,2,...,n,\quad k=1,2,...,t.$$

In the following, the *I-IVOWG* and *IVWG* operator are applied to MAGDM based on interval-vague information.

Step-1: Utilize the decision information given in matrix $\tilde{R}_k$, and the *I-IVOWG* operator with the weighting vector of the decision makers as $w=(w_1,w_2,...,w_n)^T$.

$$\tilde{r}_{ij}=([a_{ij},b_{ij}],[c_{ij},d_{ij}])=I\text{-}IVOWG_w\left(\left\langle u_1,\tilde{r}_{ij}^{(1)}\right\rangle,\left\langle u_2,\tilde{r}_{ij}^{(2)}\right\rangle,...,\left\langle u_t,\tilde{r}_{ij}^{(t)}\right\rangle\right)$$

i=1,2…,m, j=1,2,….,n, to aggregate all the decision matrices $\tilde{R}_k,\ (k=1,2,...,t)$ into a collective decision matrix $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}$.

Step-2: Utilize the decision information given in matrix $\tilde{R}$, and the *IVWG* operator $\tilde{r}_i=([a_i,b_i],[c_i,d_i])=IVWG_\omega\left(\tilde{r}_{i1},\tilde{r}_{i2},...,\tilde{r}_{in}\right),\quad i=1,2,...,m,$ to derive the collective overall preference values $\tilde{r}_i$, for $i=1,2,...,m$ of the alternative A_i, where $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ is the weighting vector of the attributes.

Step-3: Calculate the correlation coefficient $K_{PK}(A,\ B)=\dfrac{C_{PK}(A,\ B)}{\sqrt{E_{PK}(A)\ .\ E_{PK}(B)}}$, for interval-vague preference value $\tilde{r}_i$, for $i=1,2,...,m$ and the positive ideal vague value $\tilde{r}^{+}=([1,1],[0,0])$.

Step-4: Rank all the alternatives A_i, for i=1, 2,…,m, and select the best one(s) in accordance with the calculated correlation coefficient $K_{IVS}(A,\ B)$.

Step-5: Utilize the above discussed algorithm for mining fuzzy correlation rules and reduce the number of alternatives from the final decision system.

4.7 NUMERICAL ILLUSTRATION

Suppose there are five possible alternatives A_i for i=1,2,3,4,5, to be evaluated using the interval-vague numbers by the three decision makers, whose weighting vector $w=(0.35,0.40,0.25)^T$ under the above four attributes used in section 2.5.4, whose weighting vector $\omega=(0.2,0.1,0.3,0.4)^T$, and construct, respectively, the decision matrices as listed in the following matrices $\tilde{R}_k=\left(\tilde{r}_{ij}^{(k)}\right)_{5\times 4},k=1,2,3;\ i=1,2,3,4,5;\ j=1,2,3,4,$ as follows:

$$\tilde{R}_1=\begin{pmatrix}([0.3,0.4],[0.4,0.5]) & ([0.5,0.6],[0.1,0.3]) & ([0.4,0.5],[0.3,0.4]) & ([0.4,0.6],[0.2,0.4])\\ ([0.3,0.6],[0.3,0.4]) & ([0.4,0.7],[0.1,0.2]) & ([0.5,0.6],[0.2,0.3]) & ([0.6,0.7],[0.2,0.3])\\ ([0.2,0.5],[0.4,0.5]) & ([0.2,0.3],[0.4,0.6]) & ([0.3,0.5],[0.3,0.4]) & ([0.1,0.3],[0.5,0.6])\\ ([0.4,0.5],[0.3,0.5]) & ([0.5,0.8],[0.1,0.2]) & ([0.2,0.5],[0.3,0.4]) & ([0.4,0.7],[0.1,0.2])\\ ([0.5,0.6],[0.2,0.4]) & ([0.6,0.7],[0.1,0.3]) & ([0.3,0.4],[0.1,0.3]) & ([0.6,0.7],[0.1,0.3])\end{pmatrix}$$

$$\tilde{R}_2=\begin{pmatrix}([0.4,0.5],[0.3,0.4]) & ([0.4,0.6],[0.2,0.4]) & ([0.1,0.3],[0.5,0.6]) & ([0.3,0.4],[0.3,0.5])\\ ([0.6,0.7],[0.2,0.3]) & ([0.6,0.7],[0.2,0.3]) & ([0.4,0.7],[0.1,0.2]) & ([0.5,0.6],[0.1,0.3])\\ ([0.3,0.6],[0.3,0.4]) & ([0.5,0.6],[0.3,0.4]) & ([0.5,0.6],[0.1,0.3]) & ([0.4,0.5],[0.2,0.4])\\ ([0.7,0.8],[0.1,0.2]) & ([0.6,0.7],[0.1,0.3]) & ([0.3,0.4],[0.1,0.2]) & ([0.3,0.7],[0.1,0.2])\\ ([0.3,0.4],[0.2,0.3]) & ([0.3,0.5],[0.1,0.3]) & ([0.2,0.5],[0.4,0.5]) & ([0.3,0.4],[0.5,0.6])\end{pmatrix}$$

$$\tilde{R}_3=\begin{pmatrix}([0.2,0.5],[0.3,0.4]) & ([0.4,0.5],[0.1,0.2]) & ([0.3,0.6],[0.2,0.3]) & ([0.3,0.7],[0.1,0.3])\\ ([0.2,0.7],[0.2,0.3]) & ([0.3,0.6],[0.2,0.4]) & ([0.4,0.7],[0.1,0.2]) & ([0.5,0.8],[0.1,0.2])\\ ([0.1,0.6],[0.3,0.4]) & ([0.1,0.4],[0.3,0.5]) & ([0.2,0.6],[0.2,0.3]) & ([0.2,0.4],[0.1,0.5])\\ ([0.3,0.6],[0.2,0.4]) & ([0.4,0.6],[0.2,0.3]) & ([0.1,0.4],[0.3,0.6]) & ([0.3,0.7],[0.1,0.2])\\ ([0.4,0.7],[0.1,0.3]) & ([0.5,0.6],[0.3,0.4]) & ([0.2,0.5],[0.3,0.4]) & ([0.5,0.6],[0.2,0.4])\end{pmatrix}$$

Then, utilize the approach developed to get the most desirable alternative(s).

Step-1: Utilizing the decision information given in matrix $\tilde{R}_k$, and the *I-IVOWG* operator:

$$\tilde{r}_{ij}=([a_{ij},b_{ij}],[c_{ij},d_{ij}])=I\text{-}IVOWG_w\left(\left\langle u_1,\tilde{r}_{ij}^{(1)}\right\rangle,\left\langle u_2,\tilde{r}_{ij}^{(2)}\right\rangle,...,\left\langle u_t,\tilde{r}_{ij}^{(t)}\right\rangle\right)$$

$$I\text{-}IVOWG_w\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,...,\langle u_n,\tilde{a}_n\rangle\right)=\sum_{j=1}^{3} a_{\sigma(j)}{}^{w_j}$$

$$=\left(\left[\prod_{j=1}^{3} a_{\sigma(j)}^{w_j},\prod_{j=1}^{3} b_{\sigma(j)}^{w_j}\right],\left[1-\prod_{j=1}^{3}(1-c_{\sigma(j)})^{w_j},1-\prod_{j=1}^{3}(1-d_{\sigma(j)})^{w_j}\right]\right)$$

which has the decision makers weight as $w=(0.35,0.40,0.25)^T$, then a collective decision matrix $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}$ is obtained as follows:

$$\tilde{R}=\begin{pmatrix}([0.28,0.45],[0.35,0.45]) & ([0.45,0.57],[0.12,0.29]) & ([0.28,0.48],[0.32,0.42]) & ([0.35,0.58],[0.19,0.39])\\ ([0.31,0.65],[0.25,0.35]) & ([0.40,0.67],[0.15,0.29]) & ([0.45,0.65],[0.15,0.25]) & ([0.55,0.71],[0.15,0.27])\\ ([0.18,0.55],[0.35,0.45]) & ([0.20,0.38],[0.35,0.54]) & ([0.29,0.55],[0.23,0.35]) & ([0.16,0.36],[0.34,0.54])\\ ([0.41,0.58],[0.23,0.42]) & ([0.48,0.71],[0.13,0.25]) & ([0.18,0.45],[0.26,0.44]) & ([0.35,0.70],[0.10,0.20])\\ ([0.42,0.58],[0.17,0.35]) & ([0.49,0.62],[0.17,0.29]) & ([0.24,0.45],[0.23,0.38]) & ([0.49,0.60],[0.23,0.40])\end{pmatrix}$$

Step-2: Utilizing the *IVWG* operator, the collective overall preference values $\tilde{r}_i$ of the alternatives A_i(i=1, 2, …,5) are obtained as follows:

$$\tilde{r}_i=([a_i,b_i],[c_i,d_i])=IVWG_\omega\left(\tilde{r}_{i1},\tilde{r}_{i2},...,\tilde{r}_{in}\right),\quad i=1,2,...,m$$

DOI: 10.18535/ijecs/v6i2.35

$$IVWG_{\omega}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{4} \tilde{a}_j^{\;\omega_j}$$
$$= \left(\left[\prod_{j=1}^{4} a_j^{\omega_j}, \prod_{j=1}^{4} b_j^{\omega_j} \right], \left[1 - \prod_{j=1}^{4} (1-c_j)^{\omega_j}, 1 - \prod_{j=1}^{4} (1-d_j)^{\omega_j} \right] \right)$$

Hence, the collective values are calculated as follows:

$$\tilde{r}_1 = ([0.319, 0.518], [0.260, 0.405]),$$
$$\tilde{r}_2 = ([0.444, 0.673], [0.173, 0.284]),$$
$$\tilde{r}_3 = ([0.201, 0.447], [0.316, 0.470]),$$
$$\tilde{r}_4 = ([0.303, 0.591], [0.182, 0.329]),$$
$$\tilde{r}_5 = ([0.388, 0.547], [0.211, 0.373]).$$

Step-3: The correlation coefficient $K_{IVS}(A, B)$ for interval-vague sets between the collective overall preference values $\tilde{r}_i$ and the interval-vague positive ideal solution $\tilde{r}^+ = ([1,1],[0,0])$ is given as:

$$K_{IVS}(\tilde{r}_1, \tilde{r}^+) = 0.6681,\; K_{IVS}(\tilde{r}_2, \tilde{r}^+) = 0.8284,$$
$$K_{IVS}(\tilde{r}_3, \tilde{r}^+) = 0.5119,$$
$$K_{IVS}(\tilde{r}_4, \tilde{r}^+) = 0.6840,\; K_{IVS}(\tilde{r}_5, \tilde{r}^+) = 0.7389$$

Step-4: Rank all the alternatives A_i (i=1, 2, 3, 4, 5) in accordance with the correlation coefficient $K_{IVS}(A, B)$ of the collective overall interval-vague preference values $\tilde{r}_i (i = 1, 2, ..., 5)$:

$A_2 > A_5 > A_4 > A_1 > A_3$,

And thus the most desirable alternative is A_2.

Step-5: Utilizing the algorithm for mining fuzzy correlation rules and reducing the number of alternatives from the final decision system, we get the ranking of alternatives as follows:

$$A_2 > A_5 > A_4.$$

5. CONCLUSION

This paper discussed different classes of aggregation operators and their applications to MAGDM problems with IVSs. Correlation coefficient for interval vague sets was proposed and utilized in decision making together with some data mining techniques already proposed in our previous work. Ordered Weighted Geometric (OWG) operators were used for the MAGDM models proposed for IVSs and the proposed correlation coefficient is used for ranking alternatives. Data mining algorithm was utilized for reducing the number of alternatives and removing the less important decision variables from the final decision making process. Numerical illustrations were provided for MAGDM models for interval vague sets.

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