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An Intuitionistic Fuzzy Topsis DSS Model with Weight Determining Methods

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***Abstract*−In this paper a TOPSIS method based on the theory of intuitionistic fuzzy set (IFS) is proposed which will suitably deal with vagueness and hesitancy. A fuzzy TOPSIS decision making model using entropy weight for dealing with multiple criteria decision making (MCDM)problems under intuitionistic fuzzy environment is proposed and also weight determining methods based on Regular Increasing Monotone (RIM) method and Gaussian methodare utilised. A numerical illustration is given for the application of the proposed model.**

***Key Words*−Entropy, Intuitionistic fuzzy set (IFS), Multiple criteria decision making (MCDM),TOPSIS, Criteria weights.**

1. INTRODUCTION

Decision making, by its nature, is a cognitive process, involving different cognitive tasks, like collecting information, evaluating situation, generating and selecting alternatives, and implementing solutions. Decision making is never error-proof, as decision makers are prone to their cognitive biases. Therefore, Decision Support Systems (DSS) are resorted to by decision makers to minimize cognitive errors and maximize the performance of actions. A properly-designed DSS can play an important role in compiling useful information from raw data, documents, personal knowledge, and business models to solve problems (Niu et al., 2009). It allows decision makers to perform many computations quickly. Therefore advanced models can be supported by DSS to solve complex decision problems. As many business decision problems involve large data sets stored in different databases, data warehouses, and even possibly websites outside the organization, DSS can retrieve processes and utilize data efficiently to assist decision making. A DSS is intended to support, not replace, the decision maker’s role in finding the optimum solution. Decision makers’ capabilities are extended through using DSS, particularly in ill-structured decision situations. In this case, a satisfied solution, instead of the optimal one, may be the goal of decision making. Solving ill-structured problems often relies on repeated interactions between the decision maker and the DSS. Some of the common Decision Support System (DSS) techniques for Multi-Attribute Decision-Making (MADM) are (Cheng, 2000):

- Simple Additive Weighted (SAW)
- Weighted Product Method (WPM)
- Cooperative Game Theory (CGT)
- Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)
- Elimination Et Choice Translating Reality with complementary analysis(ELECTRE)
- Preference Ranking Organization Method for Enrichment Evaluation(PROMETHEE)
- Analytical Hierarchy Process (AHP)

Decision support systems are built upon various decision support techniques, including models, methods, algorithms and tools. A cognition-based taxonomy for decision support techniques, includes six basic classes, as follows (Niu et al., 2009): Process models, Choice models, Information control techniques, Analysis and reasoning techniques, Representation aids and Human judgment amplifying/refining techniques. Multi-criteria decision making and Multi-attribute decision making comes under the category of Choice models.

Multiple Attribute Decision Making (MADM) proved to be an effective approach to rank a finite number of alternatives characterized by multiple attributes. One of the techniques for order preference is called **T**echnique for **O**rder **P**reference by **S**imilarity to **I**deal **S**olution and abbreviated as TOPSIS. This technique, based on fuzzy sets theory, has proved to be a powerful modelling tool for coping with imprecision in human judgments.Modelling using fuzzy sets proved to be an effective way to formulate decision problems where available information is subjective and imprecise. Many fuzzy TOPSIS methods were proposed to handle linguistic decision making problems. TOPSIS was first developed by Hwang &Yoon, (1981). Janic, (2003), Liu,P.D., (2009a), Chen & Tan, (1994), Hong & Choi, (2000,) Li, D.F., (2010), Li, D.F., & Nan, (2011), Cui & Yang, (2009) and Shih et al., (2007) worked on Group Decision Making for TOPSIS and the extension of TOPSIS on different types of fuzzy sets.Nayagam et al., (2008; 2011), Nayagam&Sivaraman, (2011) proposed a novel accuracy function for MCDM problems, which can also be utilized in TOPSIS algorithms. Most work done earlier in theTOPSIS literature presents different classes of score functions to identify positive and negative ideal solutions, and different forms of distance functions, similarity functions, and weighted distance and similarity functions to find the relative adjacent degree.

2. THE GENERAL TOPSIS METHOD

2.1. Decision Matrix:

Suppose there are m Alternative, n attributes (or criteria), then the given information can be represented as a matrix:

$$D=\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ . \\ . \\ . \\ A_m \end{array}\begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_3 & . & . & . & x_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & x_{13} & . & . & . & x_{1n} \\ x_{21} & x_{22} & x_{23} & . & . & . & x_{2n} \\ x_{31} & x_{32} & x_{33} & . & . & . & x_{3n} \\ . & . & . & & & & . \\ . & . & . & & & & . \\ . & . & . & & & & . \\ x_{m1} & x_{m2} & x_{m3} & . & . & . & x_{mn} \end{bmatrix} \end{array}$$

Hypothesis-1

Each Attribute in the decision matrix takes either monotonically decreasing utility.

Hypothesis-2

A set of weights for the attributes is required.

Hypothesis-3

Any outcome which is expressed in a non-numerical way, should be quantified through the appropriate scaling technique.

Step-1

To transform the various attribute dimensions into non-dimensional attributes, which allows comparison across the attributes

$$r_{ij}=\frac{x_{ij}}{\sqrt{\sum_{i=1}^{m}x_{ij}^2}}$$

Step-2

Construct the Weighted Normalized Decision matrix

$$v=\begin{bmatrix} v_{11} & v_{12} & . & . & . & v_{1j} & . & . & . & v_{1n} \\ . & . & & & & . & & & & . \\ . & . & & & & . & & & & . \\ . & . & & & & . & & & & . \\ v_{i1} & v_{i2} & & & & v_{ij} & & & & v_{in} \\ . & . & & & & . & & & & . \\ . & . & & & & . & & & & . \\ . & . & & & & . & & & & . \\ v_{m1} & v_{m2} & . & . & . & v_{mj} & . & . & . & v_{mn} \end{bmatrix}=\begin{bmatrix} w_1r_{11} & w_2r_{12} & . & . & . & w_jr_{ij} & . & . & . & w_nr_{1n} \\ . & . & & & & . & & & & . \\ . & . & & & & . & & & & . \\ . & . & & & & . & & & & . \\ w_1r_n & w_2r_{12} & & & & w_jr_{ij} & & & & w_nr_{in} \\ . & . & & & & . & & & & . \\ . & . & & & & . & & & & . \\ . & . & & & & . & & & & . \\ w_1r_{m1} & w_2r_{m2} & . & . & . & w_jr_{mj} & . & . & . & w_nr_{mn} \end{bmatrix}$$

Step-3

Determine Ideal and Negative-Ideal Solutions

$$A^+=\left\{\left(\max_i v_{ij}\mid j\in J\right),\left(\min_i v_{ij}\mid j\in J\right)\mid i=1,2,...m\right\}$$

$$A^+=\left\{v_1^+,v_2^+,...,v_j^+,...,v_n^+\right\}$$

$$A^-=\left\{\left(\max_i v_{ij}\mid j\in J\right),\left(\min_i v_{ij}\mid j\in J\right)\mid i=1,2,...m\right\}$$

$$A^-=\left\{v_1^-,v_2^-,...,v_j^-,...,v_n^-\right\}$$

Where,

$J=\{j=1,2,...,n\mid j$, associated with benefit criteria}

$J'=\{j=1,2,...,n\mid j$, associated with cost criteria}

Step-4

Calculate the Separation Measure:

-Ideal Separation

$$S_i^+=\sqrt{\sum_{j=1}^{n}(v_{ij}-v_j^+)^2}\quad \text{i=1,2,…,m}$$

-Negative-Ideal separation

$$S_i^-=\sqrt{\sum_{j=1}^{n}(v_{ij}-v_j^-)^2}\quad \text{i=1,2,…,m}$$

Step-5

Calculate the Relative Closeness to the Ideal Solution

$$C_i^*=\frac{S_i^-}{(S_i^++S_i^-)},0<C_i^+<1,$$

$C_i^*=1$ if $A_i=A^+$; $C_i^*=0$ if $A_i=A^-$

Step-6

Rank the preference order.

A set of alternatives can now be preference ranked according to the descending order of C_i .

3. PRELIMINARIES

3.1. Intuitionistic fuzzy sets

Definition 1: An IFS *A* in the universe of discourse X is defined with the form

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle\mid x\in X\}$$

Were $\mu_A=X\to[0,1],V_A:X\to[0,1]$, With the condition $0\le\mu_A(x)+\nu_A(x)\le1,\forall x\in X.$

The number $\mu_A(x)$ and $\nu_A(x)$ denote the membership degree and the non-membership degree of x to A, respectively.

Obviously,each ordinary fuzzy set may be written as $\{\langle x,\mu_A(x),1-\mu_A(X)\rangle\mid x\in X\}.$

That is to say , fuzzy sets may be reviewed as the particular cases of IFSs.Note that *A*is a crisp set if and only if for ∀ $x\in X$, either $\mu_A(X)=0,\nu_A(x)=1 or\ \mu_A(X)=1,\nu_A(x)=0.$ For each IFS A in X, we will call $\pi_A(X)=1-\mu_A(x)-\nu_A(A)$ the intuitionistic index of x in A.It is measure of hesitancy degree of x to A[1]. It is obvious that $0\le\pi_A(x)\le1$ for each x ∈ X. For convenience of notation, IFSs(X) is denoted as the set of all IFSs in X.

Definition 2: For every A∈IFSs(X).the IFS λA for any positive real number λ is defined as follows:

$$\lambda A=\{\langle x,1-(1-\mu_A(x))^\lambda,(\nu_A(X))^\lambda\rangle\mid x\in X\}.$$

3.2. Entropy of IFS

In 1948,Shannon proposed the entropy function, $H=(p_1,p_2.......p_n)=-\sum_{n-1}^{n}p_i\log(p_i),$ as a measure of uncertainty in a discrete distribution based on Boltsmann entropy of classical statistical mechanics, where $p_i(i=1,2,.....n)$ are the probabilities of random variable computed from a probability mass function P. Later,De Luca &

Termini, (1972) defined on non-probabilistic entropy formula of a fuzzy set based on Shannon's function on a finite universal set $X = \{x_1, x_2, ..., x_n\}$ as Eq. (2)

$$E_{LT}(A) = -k\sum_{i=1}^{n}[\mu_A(x_i) In\mu_A(x_i) + (1-\mu_A(x_i)) In(1-\mu_A(x_i))], k>0 \quad (2)$$

Szmidt&Kacprzyk, (2000) extended De Luca &Termini, (1972) axioms presenting the four definitions with regard to entropy measure on IFSs(X). Recently, Vlachos &Sergiadis, (2007) presented Eq (3)as the measure of intuitionistic fuzzy entropy which was proved to satisfy the four axiomatic requirements.

$$E_{LT}^{IFS}(A) = -\frac{1}{n\,In2}\sum_{i=1}^{n}\mu_A(x_i) In\mu_A(x_i) + \nu_A(x_i) In\nu_A(x_i) - (1-\pi_A(x_i)) In(1-\pi_A(x_i)) - \pi_A(x_i)\, In2]. \quad (3)$$

It is noted that $E_{LT}^{IFS}(A)$ is composed of the hesitancy degree and the fuzziness degree of the IFS A.

4. PROPOSED FUZZY TOPSIS DECISION MAKING MODEL

The procedures of calculation for this proposed model can be described as follows:

Step 1. Construct an intuitionistic fuzzy decision matrix.

A MCDM problem can be concisely expressed in matrix format as

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & \cdots & C_n \end{matrix} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{array}$$

$$W = (w_1, w_2, \cdots, w_n)$$

(4)

Let $A = \{A_1, A_2, \cdots, A_m\}$ be a set of alternatives which consists of *m* non-inferior decision –making alternatives. Each alternatives is assessed on *n* criteria, and the set of all criteria is denoted $C = \{C_1, C_2, \cdots C_n\}$ .Let $W = (W_1, W_2, \cdots\cdots, W_n)$ be the weighting vector of criteria, where $w_j \geq 0\, and \sum_{j=1}^{n} w_j = 1.$

In this study , the characteristics of the alternatives A_i are respected by the IFS as: $A_i = \{\langle C_j, \mu_{A_i}(C_j), \nu_{A_i}(C_j)\rangle \mid C_j \in C\}, i = 1, 2, ..., m,$

(5)

Where $\mu_{A_i}(C_j)$ and $\nu_{A_i}(C_j)$ indicates the degrees that the alter natives A_i satisfies and does not satisfy the criterion C_j ,respectively, and $\mu_{A_i}(C_j) \in [0,1]$, $\nu_{A_i}(C_j) \in [0,1]$, $\mu_{A_i}(C_j), +\nu_{A_i}(C_j) \in [0,1]$. The intuitionistic index $\pi_{A_i}(C_j) = 1 - \mu_{A_i}(C_j), -\nu_{A_i}(C_j)$ is such that the larger $\pi_{A_i}(C_j)$ the higher a hesitation margin of the DM about the alternative A_i with respect to the criterion C_j .

Step 2 : Determine the criteria weights using the entropy –based method .

The well-known entropy method [6 , 13] can obtain the objective weights , i.e. called entropy weights . The smaller entropy values to which all alternatives A_i (i = 1,2, ... ,m) with littler similar criteria values with respect to a set of criteria can be obtained . According to the idea mentioned as above, for the decision matrix $\tilde{D} = \left[\tilde{x}_{ij}\right]_{m\times n}$, i =1 , ... ,m , j = 1 , ... ,n , under intuitionistic fuzzy environment , the expected information content emitted from each criterion C_j can be measured by the entropy value , denoted as $E_{LT}^{IFS}(C_j)$, as

$$E_{LT}^{IFS}(C_j) = -\frac{1}{m\ln 2}\sum_{i=1}^{m}\begin{bmatrix} \mu_{ij}(C_j)\ln\mu_{ij}(C_j) + \nu_{ij}(C_j)\ln\nu_{ij}(Cj) - \\ (1-\pi_{ij}(C_j))\ln(1-\pi_{ij}(C_j)) - \pi_{ij}(C_j)\ln 2 \end{bmatrix}$$

(6)where j = 1 , 2 , ... , n and 1/(m ln2) is a constant which assures $0 \leq E_{LT}^{IFS}(C_j) \leq 1.$

Therefore, the degree of divergence (d_j) of the average intrinsic information provided by the corresponding performance ratings on criterion C_j can be defined as

$$d_j = 1 - E_{LT}^{IFS}(C_j), j = 1, 2, ..., n. \quad (7)$$

The value of d_j represents the inherent contrast intensity of criterion C_j ,then the entropy weight of the *jth* criterion is

$$w_j = d_j \Big/ \sum_{j=1}^{n} d_j \quad (8)$$

Step 3. Construct the weighted intutionistic fuzzy decision matrix.

A weighted intuitionistic fuzzy decision matrix $\tilde{Z}$ be obtained by aggregating the weight vector W and the intuitionistic decision matrix $\tilde{D}$ as:

$$\tilde{Z} = W^T \otimes \tilde{D} = W^T \otimes \left[\tilde{x}_{ij}\right]_{m\times n} = \left[\tilde{x}_{ij}\right]_{m\times n}, \quad (9)$$

Where $W = (w_1, w_2, \cdots, w_j, \cdots w_n)$; $\hat{x}_{ij} = \langle \hat{\mu}_{ij}, \hat{\nu}_{ij}\rangle = \langle 1-(1-\mu_{ij})^{w_j}, \nu_{ij}^{w_j}\rangle, w_j > 0.$

Step 4.Determine intuitionistic fuzzy positive-ideal solution (IFPIS, A^+) and intuitionistic fuzzy negative-ideal solution (IFPIS, A^-) .

In general, the evaluation criteria can be categorized into two kinds, benefit and cost. Let G be a collection of benefit criteria and B be a collection of cost criteria. According to IFS theory and the principle of classical TOPSIS method, IFPIS and IFNIS can be defined as:

$$A^- = \{\langle C_j, \left((\min_i \hat{\mu}_{ij}(C_j) \mid j \in G), (\max_i \hat{\mu}_{ij}(C_j) \mid j \in B)\right), \left((\max_i \hat{\nu}_{ij}(C_j) \mid j \in G), (\min_i \hat{\nu}_{ij}(C_j) \mid j \in B)\right)\rangle \mid i \in m\} \quad (10a)$$

$$A^{+}=\{\langle C_j,\Big((\max_i \hat{\mu}_{ij}(C_j)\mid j\in G),(\min_i \hat{\mu}_{ij}(C_j)\mid j\in B)\Big),$$
$$\Big((\min_i \hat{\nu}_{ij}(C_j)\mid j\in G),(\max_i \hat{\nu}_{ij}(C_j)\mid j\in B\Big)\rangle \mid i\in m\}. \quad (10b)$$

Step 5.Calculate the distance measure of each alternative A_i from IFPIS and IFNIS.

We use the measure of intuitionistic Euclidean Distance (refer to Szmidt and Kacprzyk [8]) to help determining the ranking of all alternatives.

$$d_{IFS}(A_i,A^-)=\sqrt{\sum_{j=1}^{n}\Big[\big(\mu_{A_i}(C_j)-\mu_{A^-}(C_j)\big)^2+\big(\nu_{A_i}(C_j)-\nu_{A^-}(C_j)\big)^2+\big(\pi_{A_i}(C_j)-\pi_{A^-}(C_j)\big)^2\Big]}$$
$$d_{IFS}(A_i,A^+)=\sqrt{\sum_{j=1}^{n}\Big[\big(\mu_{A_i}(C_j)-\mu_{A^+}(C_j)\big)^2+\big(\nu_{A_i}(C_j)-\nu_{A^+}(C_j)\big)^2+\big(\pi_{A_i}(C_j)-\pi_{A^+}(C_j)\big)^2\Big]} \quad (11a)$$

Step 6. Calculate the relative closeness coefficient (CC) of each alternaive and rank the preference order of all alternatives.

The relative closeness coefficient (CC) of each alternative with respect to the intutionistic fuzzy ideal solution is called as:

$$CC_i = d_{IFS}(A_i,A^-)\big/d_{IFS}(A_i,A^+)+d_{IFS}(A_i,A^-),$$

Where $0\le CC_i\le 1, i=1,2,...,m.$

The larger the value of CC indicates that an alternative is closer to IFPIS and farther from IFNIS simultaneously. Therefore, the ranking order of all the alternatives can be determined according to the descending order of CC values. The most preferred alternative is the one with the highest CC value.

5. NUMERICALILLUSTRATION

In this section, an example is provided as follows. An investment company wants to invest a sum of money in the best choice. There are five possible companies $A_i (i=1,2,\cdots,5)$ in which to invest the money (1) A_1 is a car company; (2) A_2 is a food company;(3) A_3 is a computer company ;(4) A_4 is an arms company ; and (5) A_5 is a TV company. Each possible company will be evaluated across three criteria which are: (1) C_1 is economical benefit ;(2) C_2 is social benefit ;(3) C_3 is environment pollution , where $C_1 and\, C_2$ are benefit criteria, and C_3 is cost criterion.

The proposed fuzzy TOPSIS decision making model is applied to solvethe problem, and the computational procedure is described step by step as below :

Step 1 . The ratings for five possible companies with respect to three criteria are represented by IFSs. The intuitionistic fuzzy decision matrix D is constructed by the investment company can be expressed as follows:

Intuitionistic Fuzzy matrix

	C_1	C_2	C_3
A_1	$\langle 0.70, 0.20\rangle$	$\langle 0.85, 0.10\rangle$	$\langle 0.30, 0.50\rangle$
A_2	$\langle 0.90, 0.05\rangle$	$\langle 0.70, 0.25\rangle$	$\langle 0.40, 0.50\rangle$
A_3	$\langle 0.80, 0.10\rangle$	$\langle 0.85, 0.21\rangle$	$\langle 0.30, 0.60\rangle$
A_4	$\langle 0.90, 0.00\rangle$	$\langle 0.80, 0.10\rangle$	$\langle 0.20, 0.70\rangle$
A_5	$\langle 0.80, 0.15\rangle$	$\langle 0.75, 0.20\rangle$	$\langle 0.50, 0.40\rangle$

Step 2. Determine the criteria weights . Using Eq. (6)

$$r_1=-\frac{1}{5\ln 2}\begin{bmatrix}(0.70)\ln(0.70)+(0.20)\ln(0.20)-(1-0.1)\ln(1-0.1)-0.1\ln 2\\ +(0.90)\ln(0.90)+(0.05)\ln(0.05)-(1-0.05)\ln(1-0.05)-0.05\ln 2\\ +(0.80)\ln(0.80)+(0.10)\ln(0.10)-(1-0.1)\ln(1-0.1)-0.1\ln 2\\ +(0.90)\ln(0.90)+(0.90)\ln(0.90)-(1-0.1)\ln(1-0.1)-0.1\ln 2\\ +(0.80)\ln(0.80)+(0.15)\ln(0.15)-(1-0.05)\ln(1-0.05)-0.05\ln 2\end{bmatrix} \quad (11b)$$

$$=\begin{bmatrix}-0.249672-0.321888+0.094824-0.069315\\ -0.094824-0.149787+0.048729-0.034657\\ -0.178515-0.230259+0.094824-0.069315\\ -0.094824-0.000000+0.094824-0.069315\\ -0.178515-0.284568+0.048729-0.034657\end{bmatrix} \quad (12)$$

$$=\big[-0.546051-0.230539-0.383265-0.069315-0.449011\big]$$
$$5\ln 2=3.4657$$
$$=[-1.678181][-0.288539]$$
$$=0.484221$$

$$r_2=-\frac{1}{5\ln 2}\begin{bmatrix}(0.85)\ln(0.85)+(0.10)\ln(0.10)-(1-0.05)\ln(1-0.05)-0.05\ln 2\\ +(0.70)\ln(0.70)+(0.25)\ln(0.25)-(1-0.05)\ln(1-0.05)-0.05\ln 2\\ +(0.85)\ln(0.85)+(0.10)\ln(0.10)-(1-0.05)\ln(1-0.05)-0.05\ln 2\\ +(0.80)\ln(0.80)+(0.10)\ln(0.10)-(1-0.1)\ln(1-0.1)-0.1\ln 2\\ +(0.75)\ln(0.75)+(0.20)\ln(0.20)-(1-0.05)\ln(1-0.05)-0.05\ln 2\end{bmatrix}$$

$$=\begin{bmatrix}-0.138141-0.230259+0.014072-0.034657\\ -0.249672-0.346574+0.014072-0.034657\\ -0.138141-0.230259+0.014072-0.034657\\ -0.178515-0.230259+0.025509-0.069315\\ -0.215762-0.321888+0.014072-0.034657\end{bmatrix}$$

$$=\big[-0.354328-0.582174-0.354328-0.383265-0.523578\big]$$
$$5\ln 2=3.4657$$
$$=[-2.197673][-0.288539]$$
$$=0.634114$$

$$r_3=-\frac{1}{5\ln 2}\begin{bmatrix}(0.30)\ln(0.30)+(0.50)\ln(0.50)-(1-0.20)\ln(1-0.20)-0.20\ln 2\\ +(0.40)\ln(0.40)+(0.50)\ln(0.50)-(1-0.1)\ln(1-0.1)-0.1\ln 2\\ +(0.30)\ln(0.30)+(0.60)\ln(0.60)-(1-0.1)\ln(1-0.1)-0.1\ln 2\\ +(0.20)\ln(0.20)+(0.70)\ln(0.70)-(1-0.1)\ln(1-0.1)-0.1\ln 2\\ +(0.50)\ln(0.50)+(0.40)\ln(0.40)-(1-0.1)\ln(1-0.1)-0.1\ln 2\end{bmatrix}$$

$$=\begin{bmatrix}-0.361192-0.346574+0.039885-0.138629\\ -0.366516-0.346574+0.025509-0.069315\\ -0.361192-0.306495+0.025509-0.069315\\ -0.321888-0.249672+0.025509-0.069315\\ -0.346574-0.366516+0.025509-0.069315\end{bmatrix}$$

$= [0.667881 - 0.687581 - 0.642178 - 0.546051 - 0.687581]$

$5\ln 2 = 3.4657$

$= [-3.231272][-2.88539]$

$= 0.932348$

$$d_j = [(1-0.4842),(1-0.6341),(1-0.9323)]$$
$$= 0.5158, 0.3659, 0.0677$$

the entropy values for criteria C_1, C_2 and C_3 , respectively ,are: 0.4842 , 0.6341 , and 0.9323. The degree of divergence d_j on each criterion $C_j\ (j=1,2,3)$ may be obtained by Eq.(7) as 0.5158, 0.3659,0.0677, respectively. Therefore, the criteria weighting vector can be expressed as W=(0.543, 0.385,0.071) applying Eq.(8).

$$w_1 = \frac{0.5158}{0.5158+0.3659+0.0677} = 0.5432$$

$$w_2 = \frac{0.3659}{0.5158+0.3659+0.0677} = 0.3854$$

$$w_3 = \frac{0.0677}{0.5158+0.3659+0.0677} = 0.0713$$

Step 3. After determining criteria weighting vector, using Eq.(9),

$$x_{ij} = \left(\mu_{ij}, \nu_{ij}\right) = \left(1-(1-\mu_{ij})^{w_j}, \nu_{ij}^{w_j}\right)$$

C_1

$$(0.70,0.20) = (1-(1-0.70)^{0.543},(0.20)^{0.543}) = (0.479912, 0.417310)$$

$$(0.90,0.05) = (1-(1-0.90)^{0.543},(0.05)^{0.543}) = (0.713582, 0.196581)$$

$$(0.70,0.20) = (1-(1-0.70)^{0.543},(0.20)^{0.543}) = (0.582690, 0.286418)$$

$$(0.90,0.00) = (1-(1-0.90)^{0.543},(0.00)^{0.543}) = (0.713582, 0.000000)$$

$$(0.80,0.15) = (1-(1-0.80)^{0.543},(0.15)^{0.543}) = (0.582690, 0.356958)$$

C_2

$$(0.85,0.10) = (1-(1-0.85)^{0.385},(0.10)^{0.385}) = (0.518279, 0.412098)$$

$$(0.70,0.25) = (1-(1-0.70)^{0.385},(0.25)^{0.385}) = (0.370941, 0.586417)$$

$$(0.85,0.10) = (1-(1-0.85)^{0.385},(0.10)^{0.385}) = (0.518279, 0.412098)$$

$$(0.80,0.10) = (1-(1-0.80)^{0.385},(0.10)^{0.385}) = (0.461858, 0.412078)$$

$$(0.75,0.20) = (1-(1-0.75)^{0.385},(0.20)^{0.385}) = (0.4135831, 0.538142)$$

C_3

$$(0.30,0.50) = (1-(1-0.30)^{0.071},(0.50)^{0.071}) = (0.025006, 0.951978)$$

$$(0.40,0.50) = (1-(1-0.40)^{0.071},(0.50)^{0.071}) = (0.035619, 0.951978)$$

$$(0.30,0.60) = (1-(1-0.30)^{0.071},(0.60)^{0.071}) = (0.025006, 0.964381)$$

$$(0.20,0.70) = (1-(1-0.20)^{0.071},(0.70)^{0.071}) = (0.015718, 0.974994)$$

$$(0.50,0.40) = (1-(1-0.50)^{0.071},(0.40)^{0.071}) = (0.048022, 0.937014)$$

the weighted intutionistic fuzzy decision matrix $\tilde{Z}$ is then obtained as Table 2.

Table 2. Weighted intuitionistic decision matrix $\tilde{Z}$

	C_1	C_2	C_3
A_1	$\langle 0.4799, 0.4173\rangle$	$\langle 0.5183, 0.4121\rangle$	$\langle 0.0250, 0.9520\rangle$
A_2	$\langle 0.7136, 0.1966\rangle$	$\langle 0.3709, 0.5864\rangle$	$\langle 0.0356, 0.9520\rangle$
A_3	$\langle 0.5827, 0.2864\rangle$	$\langle 0.5183, 0.4121\rangle$	$\langle 0.0250, 0.9644\rangle$
A_4	$\langle 0.7136, 0.0000\rangle$	$\langle 0.4619, 0.4121\rangle$	$\langle 0.0157, 0.9750\rangle$
A_5	$\langle 0.5827, 0.3570\rangle$	$\langle 0.4136, 0.5381\rangle$	$\langle 0.0480, 0.9370\rangle$

Step 4. In this case, criteria C_1 and C_2 belong to benefit criteria, and criterion C_3 belong to cost criterion. Using Eqs.(10a) and (10b), each alternative's IFPIS(A^+) and IFNIS(A^-) with respect to criteria can be determined as

$$A^+ = [(0.7136, 0.0000)(0.5183, 0.4121)(0.0157, 0.9750)]$$

$$A^- = [(0.4799, 0.4173)(0.3709, 0.5864)(0.0480, 0.9370)]$$

Step 5:

Calculate the distance between alternatives and intuitionistic fuzzy ideal solutions (IFPIS and INFNIS) using Eqs. (11a) and (11b).

Step 6:

Using eq.(12), the relative closeness coefficient (cc) can be obtained.The distance, relative closeness coefficient and corresponding ranking of five possible companies are tabulated in Table-1. Therefore,we can see that the order of rating among five alternatives is $A_4 \succ A_2 \succ A_3 \succ A_1 \succ A_5$, and the best choice would be A_4 (arms company).

5.1. Calculation:

$$d_{IFS}(A_i, A^+)$$

$$C_1$$

$$= \begin{bmatrix} (0.4799-0.7136)^2+(0.4173-0.0000)^2+(0.1028-0.2864)^2 \\ (0.7136-0.7136)^2+(0.1966-0.0000)^2+(0.0898-0.2864)^2 \\ (0.5827-0.7136)^2+(0.2864-0.0000)^2+(0.1309-0.2864)^2 \\ (0.7136-0.7136)^2+(0.0000-0.0000)^2+(0.2864-0.2864)^2 \\ (0.5827-0.7136)^2+(0.3570-0.0000)^2+(0.0603-0.2864)^2 \end{bmatrix} = \begin{bmatrix} 0.0546+0.1741+0.0337 \\ 0.0000+0.0387+0.0387 \\ 0.1741+0.0820+0.0242 \\ 0.0000+0.0000+0.0000 \\ 0.0509+0.1274+0.0337 \end{bmatrix} = \begin{bmatrix} 0.2624 \\ 0.0774 \\ 0.1233 \\ 0.0000 \\ 0.2120 \end{bmatrix}$$

Similarly all the other values are calculated.

Table-1. The distance measure, relative closeness coefficient and rank.

Alternatives	$d_{IFS}(A_i, A^+)$	$d_{IFS}(A_i, A^-)$	CC_i	*Rank*
A_1	0.5127	0.2315	0.3111	4
A_2	0.3618	0.4252	0.5403	2
A_3	0.3513	0.2874	0.4500	3
A_4	0.0800	0.5571	0.8744	1
A_5	0.4914	0.1806	0.2688	5

5.2. Linguistic (RIM) Quantifiers for Determining Unknown Expert-Weights

The problem of determining weights for an operator can be addressed in different ways, for example with the use of the so-called 'Linguistic Quantifiers', introduced by Zadeh, (1983). A relative linguistic quantifier Q, such as 'most', 'few', 'many', and 'all', can be represented as a fuzzy subset of the unit interval, where for a given proportion $r \in [0,1]$ of the total of the values to aggregate, $Q(r)$ indicates the extent to which this proportion satisfies the semantics defined in Q. For example, given Q = 'most', if $Q(0.7) = 1$ then it would mean that a proportion of 70% totally satisfies the idea conveyed by the quantifier 'most', whereas $Q(0.55) = 0.25$ indicates that the proportion 55% is barely compatible with this concept (i.e., only 25%).

Regular Increasing Monotone (RIM) quantifiers are especially interesting for their use in operators. These quantifiers present the following properties:

i. $Q(0) = 0$

ii. $Q(1) = 1$

iii. If $r_1 < r_2$ then $Q(r_1) \geq Q(r_2)$.

Yager, (1988) suggested the following method to compute weights w_i, with the use of a RIM quantifier Q:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), \quad i = 1, 2, ..., n. \tag{13}$$

Where the membership function of a linear RIM quantifier $Q(r)$ is defined by two parameters $a, b \in [0,1]$ as:

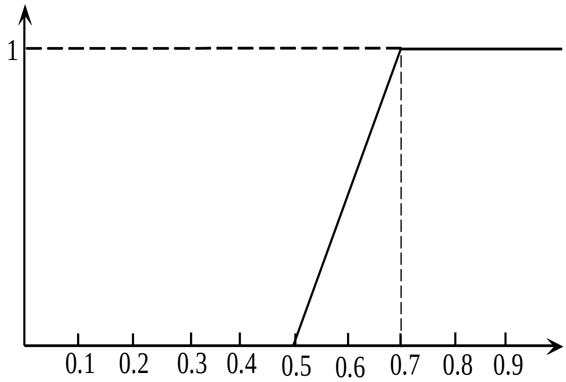
$$Q(r) = \begin{cases} 0 & \text{if } r < a \\ \dfrac{r-a}{b-a} & \text{if } a \leq r \leq b \\ 1 & \text{if } r > b \end{cases} \tag{14}$$

An example of RIM quantifier Q = 'most', with $a = 0.5$ and $b = 0.7$ is given as:

$$Q(r) = \begin{cases} 0 & \text{if } r < 0.5 \\ 5r - 2.5 & \text{if } 0.5 \leq r \leq 0.7 \\ 1 & \text{if } r > 0.7 \end{cases} \tag{15}$$

Since the use of RIM quantifiers captures the notion of the soft consensus correctly, they can be adopted for the purpose of studying the effect of different aggregation operators on the resolution of a consensus problem with many experts, and expressing a desired group's attitude.

Figure – 1
Representation of the Linguistic Quantifier *Q*(r)



The five possible alternativesA_i, where i= 1,2,…,m, are to be evaluated by three decision makers whose weighting vector is completely unknown. The membership function for the linguistic quantifier$Q$ = 'most' is given as follows:

$$\mu_{most} = \begin{cases} 0 & \text{if } x \leq 0.5 \\ \dfrac{x-0.5}{0.9-0.5} & \text{if } 0.5 < x < 0.9 \\ 1 & \text{if } x \geq 0.9 \end{cases}$$

$$\mu_{most} = \begin{cases} 0 & \text{if } x \leq 0.5 \\ 2x - 0.4 & \text{if } 0.5 < x < 0.9 \\ 1 & \text{if } x \geq 0.9 \end{cases}$$

The unknown weights are computed by the RIM quantifier Q as follows:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), \; i = 1, \; 2, ..., n.$$

Which gives the weights as $V = (0.26, 0.68, 0.06)^T$.

5.3. Gaussian Distribution for Determining Unknown Expert-Weights

Let us consider a situation where there is an unfair argument among the experts in fixing the weights in a decision making problem. In that case we need to relieve the influence of unfair arguments on the decision variables. Xu, (2005) introduced a procedure for generating the weights based on the use of the Gaussian distribution. They are referred as Gaussian weights which are given as follows:

Consider a Gaussian distribution $G(\mu_n, \sigma_n)$, where μ_n is the mean of the collection and σ_n is the deviation of the collection, and given by:

$$\mu_n = \frac{1}{n}\sum_{j=1}^{n} j = \frac{n+1}{2} \text{ and } \sigma_n = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(j-\mu_n)^2} \quad (16)$$

Let $G(j) = \frac{1}{\sqrt{2\pi\sigma_n}} e^{-(j-\mu_n)^2/2\sigma_n^2}$. Then the associated *OWA* weights are defined as:

$$w_j = \frac{G_j}{\sum_{j=1}^{n} G(j)} = \frac{e^{-(j-\mu_n)^2/2\sigma_n^2}}{\sum_{j=1}^{n} e^{-(j-\mu_n)^2/2\sigma_n^2}} \text{ where } w_i \in [0, 1] \text{ and } \sum_j w_j = 1. \quad (17)$$

It can be noted that the closer j is to $\mu_n = \frac{n+1}{2}$, the larger w_j. Furthermore, if n is odd, the maximal value of w_j occurs for $j = \frac{n+1}{2}$. If n is even, the maximal value of w_j occurs for $j = \frac{n}{2}$ and $j = \frac{n}{2}+1$. It can also be shown that the weighting vector generated using this approach is symmetric, i.e., $w_j = w_{n+1-j}$.

The five possible alternatives A_i, where $i = 1,2,\ldots,m$, are to be evaluated by three decision makers whose weighting vector is completely unknown. The mean and the deviation of the collection $1,2,\ldots,n$ are given by equation (16) as follows:

$$\mu_n = \frac{1}{n}\sum_{j=1}^{n} j = \frac{n+1}{2} \text{ and } \sigma_n = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(j-\mu_n)^2}$$

$$\mu_1 = 1, \quad \mu_2 = 1.5, \quad \mu_3 = 2$$

$$\sigma_1 = 0, \quad \sigma_2 = 0.5, \quad \sigma_3 = 0.8164$$

Then the weights are calculated using equation(17) as follows:

$$w_j = \frac{G_j}{\sum_{j=1}^{n} G(j)} = \frac{e^{-(j-\mu_n)^2/2\sigma_n^2}}{\sum_{j=1}^{n} e^{-(j-\mu_n)^2/2\sigma_n^2}} \text{ where } w_j \in [0,1]$$

and $\sum_j w_j = 1$.

$w_1 = 0.2429$, $w_2 = 0.5142$,
$w_3 = 0.2429$

Hence the weights of the experts are taken as $V = (0.2429, 0.5142, 0.2429)^T$.

When applying the above two weight determining methods, it is observed that the final ranking remains the same as the entropy method, $A_4 \succ A_2 \succ A_3 \succ A_1 \succ A_5$.

6. CONCLUSION

In this work, an entropy-based MCDM model is proposed,in which the characteristics of the alternatives are represented by IFSs.In information theory,the entropy is related with the average information quality of a source. Based on the principle, the optimal criteria weights can be obtained by the proposed entropy-based model.The difference of the method from classical TOPSIS consists in the introduction of the objective entropy weight under intuitionstic fuzzy environment.Different weight determining methods are also utilised for the MCDM model and it can be observed that the final ranking of the alternatives remain unchanged.

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