www.ijecs.in
International Journal Of Engineering And Computer Science ISSN:2319-7242
Volume 3 Issue 11 November, 2014 Page No. 9219-9222

Solutions to the Bargaining Problem

Saksham Kashyap, Dipit Malhotra

Information Technology
MSIT, New Delhi sakshamkashyap@yahoo.com
Information Technology
MSIT, New Delhi
dipitmalhotra@gmail.com

ABSTRACT

We briefly describe the intersection of game theory and bargaining problem as a topic of study, and explain the term *Nash bargaining game.* Very less work has been recorded in the field of game theory with bargaining problem. This paper presents the survey of various approaches which are used in the field of game theory with bargaining problem and elaborate the Nash bargaining game. Finally, we propose a future scope of bargaining problem in the field of zoology.

General Terms

Nash Bargaining problem, Game Theory, Algorithms, Two-person games, Artificial Intelligence, Nash Equilibrium.

1. INTRODUCTION

Game theory is a study of strategic decision making. Specifically, it is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers"[12]. An alternative term suggested "as a more descriptive name for the discipline" is *interactive decision theory* [11]. Game theory is mainly used in economics, political science, and psychology, as well as logic, computer science, and biology. The subject first addressed zero-sum games, such that one person's gains exactly equal net losses of the other participant or participants. Today, however, game theory applies to a wide range of behavioral relations, and has developed into an umbrella term for the logical side of decision science, including both humans and non-humans (e.g. computers, insects/animals).

Modern game theory began with the idea regarding the existence of mixed-strategy equilibria in two-person zero-sum games and its proof by John von Neumann. Von Neumann's original proof used Brouwer fixed-point theorem on continuous mappings into compact convex sets, which became a standard method in game theory and mathematical economics. His paper was followed by the 1944 book *Theory of Games and Economic Behavior*, co-written with Oskar Morgenstern, which considered cooperative games of several players. The second edition of this book provided an axiomatic theory of expected utility, which allowed mathematical statisticians and economists to treat decision-making under uncertainty.

This theory was developed extensively in the 1950s by many scholars. Game theory was later explicitly applied to biology in the 1970s, although similar developments go back at least as far as the 1930s. Game theory has been widely recognized as an important tool in many fields. Ten game-theorists have won the Nobel Memorial Prize in Economic Sciences and John Maynard Smith was awarded the Crafoord Prize for his application of game theory to biology.

The two person **bargaining problem** is a problem of understanding how two agents should cooperate when non-cooperation leads to Pareto-inefficient results. It is in essence an equilibrium selection problem; many games have multiple equilibria with varying payoffs for each player, forcing the players to negotiate on which equilibrium to target. The quintessential example of such a game is the ultimatum game. The underlying assumption of bargaining theory is that the resulting solution should be the same solution an impartial arbitrator would recommend. Solutions to bargaining come in two flavors: an axiomatic approach where desired properties of a solution are satisfied and a strategic approach where the bargaining procedure is modeled in detail as a sequential game. The **bargaining game** or **Nash bargaining game** is a simple two-player game used to model bargaining interactions. In the Nash bargaining game, two players demand a portion of some good (usually some amount of money). If the total amount requested by the players is less than that available, both players get their request. If their total request is greater than that available, neither player gets their request.

Table 1: Battle of the Sexes

Opera	**opera**	**Football**
opera	3,2	0,0
football	0,0	2,3

The battle of the sexes, as shown, is a two player coordination game. Both opera/opera and football/football are Nash equilibria. Any probability distribution over these two Nash equilibria is a correlated equilibrium. The question then becomes which of the infinitely many possible equilibria should be chosen by the two players. If they disagree and choose different distributions, they are likely to receive 0 payoffs. In this symmetric case the natural choice is to play opera/opera and football/football with equal probability. Indeed all bargaining solutions described below prescribe this solution. However, if the game is asymmetric---for example, football/football instead yields payoffs of 2,5--- the appropriate distribution is less clear. The problem of finding such a distribution is addressed by the bargaining theory.

2. THE BARGAINING PROBLEM

The earliest solution to the bargaining problem dates back to as early as 1930. F. Zeuthen in his book [1] proposed a solution to the bargaining problem. This Nash's bargaining solution was shown by John Harsanyi to be the same as Zeuthen's solution of the bargaining problem [2].

Between 1950 and 1953, John Nash published two papers on the bargaining problem. His first paper, *The Bargaining Problem* [3], was a new treatment of a classical economic problem, which occurred in many forms, as bargaining, bilateral monopoly etc. He stated a two person bargaining situation where two individuals have the opportunity to collaborate for mutual benefit in more than one way. In this paper, no action taken by one of the individuals without the consent of the other can affect the well being of the other one was considered. He considered situations of monopoly versus monopsony, of state trading between two nations, and of negotiation between employer and labor union as bargaining problems. The main aim of the paper is to determine the amount of satisfaction each individual should expect to get from the situation, or, rather, a determination of how much it should be worth to each of these individuals to have this opportunity to bargain. A different approach to this was suggested by von Neumann and Morgenstern in their paper [5], which permits the identification of this typical exchange situation with a nonzero sum two-person game. Nash idealized the bargaining problem by assuming that two individuals are highly rational and each has full knowledge of preferences of each other, having equal bargaining skills. He applied a numerical utility to express the preferences of each individual engaged in bargaining [3].

The theory developed in his second paper *Two-Person Cooperative Games* [4], was to treat economic (or other) situations involving two individuals whose interests are neither completely opposed nor completely coincident. He stated it was conventional to call these situations 'games' as they were being studied from an abstract pint of view. He applied the theory of von Neumann and Morgenstern [5] to the games referred in his paper. In this paper, there was no assumption about side-payments. The von Neumann and Morgenstern approach, on the other hand, was rather incomplete as it leaves the final situation only determined up to a side-payment. Nash's paper also analyzed the problem for the cooperative case, which was worked out for just two players. He gave two independent derivations of his solution of the two-person cooperative game. On first, the cooperative game is reduced to a non-cooperative game. To do this, one makes the players' steps of negotiation in the cooperative game become moves in the non-cooperative model. The second approach is by axiomatic method. One states as axioms several properties that it would seem natural for the solution to have and then one discovers that the axioms actually determine the solution uniquely. Both approaches to the problem, are complementary to each other [4]. These papers provided the first execution of the Nash Program.

In 1964, the idea of bargaining Set was introduced and discussed in the paper by R. J. Aumann and M. Maschler in their paper, *The Bargaining Set for Cooperative Games* [6]. This paper was made to translate mathematical formulas when a cooperative n-person game is described by a characteristic function. To solve this problem, these authors made an assumption that all 'players' can bargain together, and settle at a stable outcome which is present on the 'threats' and 'counter threats' that they possess. The bargaining sets for 2- and 3-person games were fully described and some special cases of 4-person games are also treated. Moreover, some counter examples and possible modifications are also suggested.

Around this time, B. Peleg proved that for each coalition structure there exists at least one payoff which makes it stable in the sense of the bargaining set [7]. He proposed the idea of decomposing the bargaining set into various subsets- each of which represents a specific "way of thinking" that may cause the players to end up within a particular set of outcomes. Continuing the previous works on bargaining sets in 1965, M. Davis and M. Maschler in *The Kernel of a cooperative Game* [8], introduced the concept of kernel. The subset of bargaining set is known as 'kernel' of the game. They studied some of its properties and proved its existence. They first studied the kernel for the 3-person game and generalized it to the games in which all sets other than the n-l and the n-person coalitions are flat. At the end, they describe the analysis of a certain weighted majority game according to which one arrives at outcomes, which seem unintuitive. This analysis after being presented to several experts on game theory, is re-iterated along with the answers from these experts.

In 1975, E. Kalai and M. Smorodinsky published a paper titled *Other Solutions To Nash's Bargaining Problem* [9]. This paper takes into consideration four axioms provided by Nash [4], and shows there is a unique solution to a problem where the axioms describe the behavior of players. These axioms are different from those suggested by Nash. This paper considers a two-person bargaining problem and solves it by using their own alternate axioms. It also states that experiments conducted by H. W. Crott [10] led to the solution implied by their axioms rather than to Nash's solution. At the end of the paper, the idea of classifying all the possible continuous solutions is proposed.

Ariel Rubinstein in 1982, published a paper on, *Perfect equilibrium in a Bargaining model* [14]. He referred to a class of bargaining games that feature alternating offers through an infinite time horizon. Two players have to reach an agreement on the partition of a pie size I. Each has to make in turn, a proposal as to how it should be divided. After one player has made and offer, the other must decide either to accept it, or to reject it and continue bargaining. Several properties which the payers' preferences possess are assumed. The Perfect Equilibrium Partitions are characterized in all models satisfying these assumptions. He provided mathematical proves to the theorems he proposed. For a long time, the solution to this type of game was a mystery; thus, Rubinstein's solution is one of the most influential findings in game theory.

The most attractive feature of the Rubinstein's model lies in the fact that the natural restriction of subgame-perfectness suffices to ensure the uniqueness of equilibrium. This result relies on a positive time cost due to discounting [15]. Unfortunately, uniqueness does not carry over when the Rubinstein model is generalized in a natural way to n-person bargaining. The Rubinstein model is most appealing for discount factors close to one because then the first player advantage to the initial offerer is small. However, it is just in this case that the Rubinstein model loses its ability to predict the outcome of the n-person bargaining process. To overcome this problem, Geir B. Ashiem in 1990 published *A Unique Solution to n-Person Sequential Bargaining* [15]. He offered and alternative route to the uniqueness of the stationary division. Ashiem established three important propositions along with their mathematical proofs which were not only optimized, but were also very stable.

In 1992, Benett and Houba [16] continued the Nash program by extending the Nash Bargaining solution and the Rubinstein alternating offer model to a class of three-player bargaining problems. In the simple bargaining studied by Nash, a pair of players bargain over the division of a 'cake'. Their new cooperative model in the paper, *Bargaining among Three Players* [16], considered three-player/three-cake problems as a set of interrelated two-player bargaining problems. Each pair of players faces a bargaining problem: division of the cake they control. Within this bargaining problem, the role of the third player is indirect: either player could threaten to abandon the current negotiations and take up negotiation with the third player. They capture this threat by endogenously determined outside option vector; given this vector they assumed that division of cake is that specified by the Nash Bargaining solution [3]. A multi-lateral Nash solution specifies a 'division' of the cake for each potential coalition that is consistent both with the bargaining within each coalition and with the evaluation of each player's outside option. This paper then provides various theorems and address the questions of which coalitions might form, and what coalitions might be, supposing that they do form. Analysis of both cooperative and non-cooperative model has been done in this regard.

By extending the similar approach, Akira Okada in 2006, presented an new concept called *Nash Core* in his paper, *The Nash Bargaining Solution in General n-person Cooperative Games* [17]. He presented a non-cooperative foundation of the Nash Bargaining solution for an n-person cooperative game in strategic form in which coalitions exert externalities. Nash Core, for a cooperative game, was a concept in which any deviating coalition anticipates the Nash Bargaining Solution behavior of the complimentary coalition. He proved that the ANsh bargaining solution can be supported (in every subgame) by a stationary subgame perfect equilibrium of the bargaining model if and only if the Nash bargaining solution belongs to the Nash Core. The weights of players for the asymmetric Nash solution are determined by their likelihood to make proposals. The purpose of the paper was to extend the Nash bargaining solution to a geneal n-person cooperative game and to present a non-cooperative foundation of it.

2.1 Conclusion

While Bargaining Problem has various solutions in the field of Computer Science and Economics, not much work has been done on the problem in recent years. The articles mentioned here have been researched upon so as to identify the most efficient solution to the bargaining problem and get an insight to the mathematically unproven work done on the field. Among the various solutions to the inherent bargaining problem, Nash's conceptualization provides an internally consistent framework. Along with the strong foundation of NH Theory, it is also a convenient analytical model. This till date is considered the most efficient solution to the problem.

3. FUTURE SCOPE

The primary purpose of this paper is to understand the solutions to the already prevailing bargaining problem and extend the hypothesis to various fields of study, other than economics and computer science. In zoology, the bargaining problem can be extended to the behavior of animals during hunting or during fight for territory. Consider a situation where two lions are fighting for supremacy over a territory. Both the lions exist in the same habitat and they posses similar behavior which satisfies the Nash equilibrium. These lions are in equilibria. By further research on hunting patterns and psychological behavior of both of them, we can deduce the steps or patterns involved when one of them attacks the other. Similarly, this approach can be used to observe behavior in other animals of similar habitat. We regard this work as the fundamental step in the analysis of solutions to the bargaining problem. We expect the insights obtained here to be of value in more general, particularly in practical situations. For this purpose the theoretical base has been defined. However, the problem can be tested on various aspects for further research.

4. ACKNOWLEDGEMENT

We would like to thank our Head of Department (IT), **Ms. Prabhjot Kaur** for guiding us through the development of this research paper and clarification of doubts wherever we encountered.

5. REFERENCES

1. Zeuthen, F. (1930), Problems of Monopoly and Economic Warfare. London: George Routledge and Sons.

2. Harsanyi, J. C. (1956), Approaches to the Bargaining Problem Before and After the Theory of Games: A Critical Discussion of Zeuthen's, Hicks', and Nash's Theories, Econometrica 24, 144-157.

3. Nash, J. F. (1950), The Bargaining Problem, Econometrica 18, 155-162.

4. Nash, J. F. (1953), Two Person Cooperative Games, Econometrica 21, 128-140.

5. von Neumann, J., and O. Morgenstern (1944), Theory of Games and Economic Behavior. Princeton: Princeton University Press.

6. Aumann, R. J. and M. Maschler (1964), The Bargaining Set for Cooperative Games, pp. 443-476 in Advances in Game Theory (Annals of Mathematics Studies, 52) (M. Dresher, L. S. Shapley and A. W. Tucker, eds.), Princeton: Princeton University Press.

7. Peleg, Bezalel. Existence theorem for the bargaining set . Bulletin of the American Mathematical Society 69 (1963), no. 1, 109--110.

8. Davis, M. and M. Maschler (1965), The Kernel of a Cooperative Game, Naval Research Logistics Quarterly 12, 223-259.

9. Kalai, E. and M. Smorodinsky (1975), Other Solutions to Nash's Bargaining Problem, Econometrica 43, 513-518.

10. Crott. H. W. : "Experiment Untersuchung zum Verhandlungsverhalten in Kooperativen Spielen." Zeitschrift fur Sozialpsychologie, 2 (1971), 61-74.

11. R. J. Aumann ([1987] 2008). "game theory," Introduction, The New Palgrave Dictionary of Economics, 2nd Edition. Abstract.

12. Roger B. Myerson (1991). Game Theory: Analysis of Conflict, Harvard University Press, p. 1. Chapter-preview links, pp. vii–xi.

13. http://www.econ.canterbury.ac.nz/personal_pages/paul_walker/gt/hist.htm

14. Rubinstein, Ariel (1982). "Perfect Equilibrium in a Bargaining Model". *Econometrica* **50** (1): 97–109

15. Ashiem, Gier B. (1990). "A Unique Solution to n-Person Sequential Bargaining" The Norwegian School of Economics and Business Administration, Journal of Economic Literature Classification. Number: C72, C78.

16. Elaine Bennett and Harold Houba (1992) "Bargaining Among Three Players", UCLA Department of Economics #662.

17. Akira Okada (2006) "The Nash Bargaining Solution in General N-Person Cooperative Games", Graduate School of Economics, Hitotsubashi University, Tokyo.