

Image Compression Using SVD Technique and Measurement of Quality Parameters

Rahul Samnotra¹, Randhir Singh², Javid Khan³

¹M.Tech, Department ECE, Shri Sai College of Engineering and Technology, Pathankot, India

²HOD, Dept ECE, Shri Sai College of Engineering and Technology, Pathankot, India

³Associate Analyst, Global Logic Technologies, Delhi, India

Javidk17@gmail.com

Abstract: Singular value decomposition (SVD) based algorithm is designed to compress the black and white images to evaluate the effect of information content of the image. In order to evaluate the effect second algorithm is developed to calculate the SNR, PSNR, MSE, and RMSE value of the compressed images with respect to reference image. It is interpreted from the results that high level of compression the information content of the image is degraded to higher level.

Keywords: SVD, compression, SNR, PSNR.

1. Introduction

Image compression is a very active and matured field of research. Vast redundancy and the ability to absorb the moderate errors in the reconstruction of image make the compression possible. Due to the popularity of this field standard algorithm that are wide spread [1]. The concept of image compression is not new and has been around almost as long as people have been saving images. Image compression has been a very important part of storing data due to the limitation of low storage media. Most of the compressions are lossy compressions, due to the fact that some of the image contents are lost during the compression. Image compression has been done using computer based algorithm [2].

In this research paper Singular Value Decomposition (SVD) method is implemented to achieve image compression and image quality parameters are calculated.

2. SVD Technique for Compression

Singular Value Decomposition (SVD) has been widely implemented in image processing. When image is considered as a matrix having a low rank, SVD can be implemented to approximate this matrix to represent much more compactable matrix than original image [3]. More specifically, suppose we are given an image A which we will assume for simplicity is an $N \times N$ real matrix. SVD representation is give by

$$A = U \Sigma V^T$$

where Σ is a diagonal matrix with entries along the diagonal ordered in a non-increasing order, and U, V are orthogonal matrix []. Then a rank r approximation to A is the matrix is given by

$$A_r = U_r \Sigma_r V_r^T$$

where Σ_r is the top-left $r \times r$ submatrix of Σ , U_r consists of the first r columns of U , and V_r^T the first r rows of V^T . The SVD decomposition is interesting because U_r, Σ_r, V_r^T provide the best rank r approximation to A in the sense of packing the maximum energy from A . Furthermore, for compression, the decomposition is interesting because unlike A which has N^2 entries, the total number of entries in U_r, Σ_r, V_r^T are only $2N_r + r$. Even with small values of r , the approximation A_r gets most of the energy of A , and is visually adequate. Compression is obtained from the value r columns of U , r rows of V , and $r \times r$ submatrix of Σ . The size of the representation is given by

$$r(d + 1)$$

where d is the sum of the dimensions of the matrix X . This representation will be good if X has low rank. It can only be achieved if the columns of X are similar.

The images in the ensemble are columns of the image in case of SVD, adjacent column to be similar. Notice however, that this similarity drops off as the distance between the columns increases. In case of SVD, the corresponding points on the columns can be as far apart as $N = n$.

Compressed image degrade the quality of the image which needed to be investigated. Root mean square error (RMSE) corresponds to pixels in the reference image I_r and the fused image I_f . If the reference image and fused image are alike give the RMSE value equal to zero and it will increase when the dissimilarity increases between the reference and fused image [10].

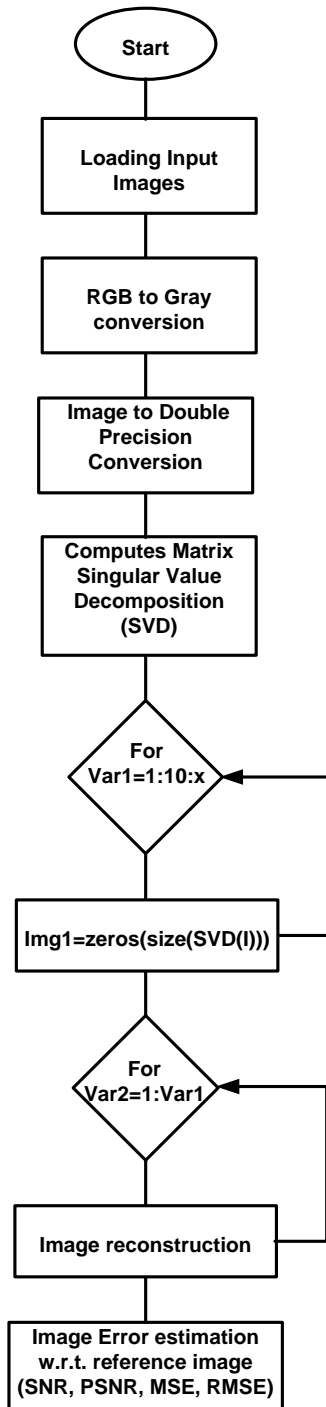


Figure 1: Algorithm designed for image compression and error estimation.

$$RMSE = \sqrt{\frac{1}{MN} \sum_{x=1}^M \sum_{y=1}^N (I_r(x, y) - I_f(x, y))^2}$$

Peak signal to noise ratio (PSNR) value will be high when the fused and reference images are alike and higher value implies better fusion. PSNR is calculated by follow equation [11] [12]

$$PSNR = 20 \log_{10} \left(\frac{L^2}{\sqrt{\frac{1}{MN} \sum_{x=1}^M \sum_{y=1}^N (I_r(x, y) - I_f(x, y))^2}} \right)$$

Signal to noise ratio (SNR) is calculated using following [13]

$$SNR = 10 \log_{10} \left(\frac{\sum_{x=1}^M \sum_{y=1}^N (I_r(x, y) - I_f(x, y))^2}{\sum_{x=1}^M \sum_{y=1}^N I_r(x, y)} \right)$$

3. Methodology

Research work is carried out to compress the black and white image using SVD technique. The work is divided into two part. In the first part computer based algorithm is developed to compress the image and second part comprises of image error estimation by calculating SNR, PSNR, MSE, and MSE parameters of the image with respect to the reference image. Computer based algorithm for SVD based image compression and error estimation shown in Fig. 1.

4. Result and Discussion

SVD based image compression is carried out on the black and white shown in Fig. 2. After applying SVD approach to compress the image with different matrix rank value. Figure 3 and 4 shows the compressed image with minimum and maximum matrix rank value implemented in the experiment. Different rank matrix size was taken to reconstruct the image, giving images with varying size. The reconstructed images were used in second algorithm to calculate SNR, PSNR, MSE, and RMSE values with respect to the reference image. Table 1 shows the calculated values of these parameters.

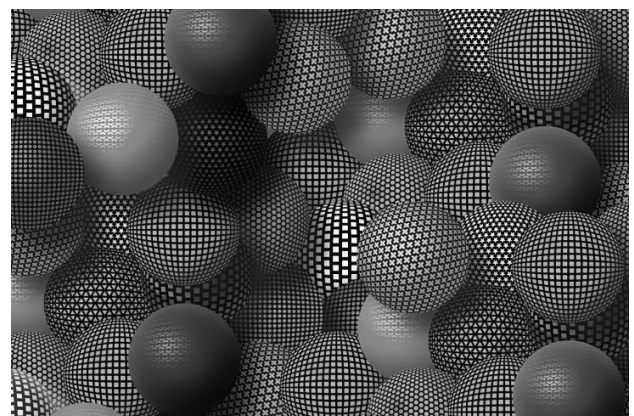


Figure 2: Reference black and white image.

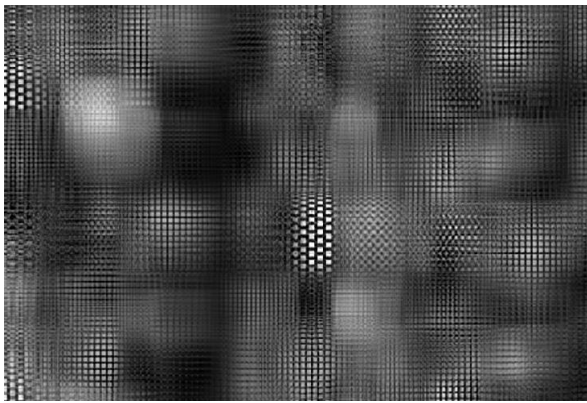


Figure 3: Reconstructed image with minimum matrix rank used in algorithm.

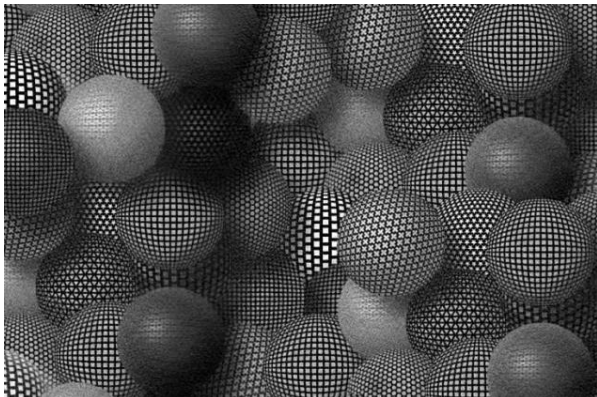


Figure 4: Reconstructed image with maximum matrix rank used in algorithm.

Table 1. Image size with obtained with different matrix rank.

Image Error Estimation			
SNR (dB)	PSNR (dB)	MSE	RMSE
-0.37546	15.43870	1873.291	43.28154
-0.31579	16.43220	1490.254	38.60381
-0.27340	17.25285	1233.659	35.12348
-0.24329	17.98234	1042.907	32.29408
-0.21848	18.66909	890.3701	29.83907
-0.19491	19.31202	767.8522	27.71015
-0.17314	19.93055	665.9240	25.80550
-0.15581	20.54340	578.2825	24.04751
-0.14064	21.13271	504.9047	22.47008
-0.12708	21.71723	441.3237	21.00771

Figure 5 shows the plot for SNR values calculate for reconstructed images. Increasing the matrix level gives the SNR value less negative.

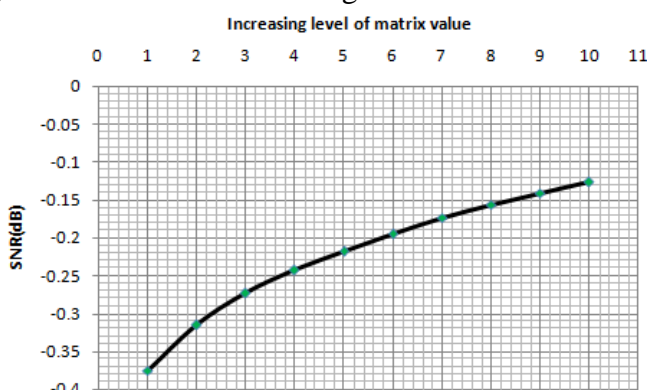


Figure 5: Calculated value of SNR.

Figure 6 shows the PSNR graphical representation of the calculated values. Since the higher the value of PSNR with respect to reference image means the identical images.

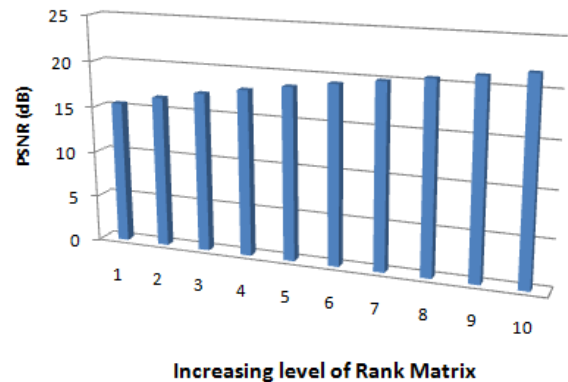


Figure 6: Calculated value of PSNR.

5. Conclusion

Experiment is carried out to compress the black and white images with SVD algorithm. The information degrading rate of the compressed image has been calculated i.e. various parameters such as SNR, PSNR, MSE, and RMSE. It is seen that with higher rate of compression the noises in the image increases i.e. lower value of SNR and PSNR.

References

- [1] Ori Bryt, Michael Elad, Compression of facial images using the K-SVD algorithm, *Journal of Visual Communication and Image Representation*, vol. 19, pp. 270–282, 2008.
- [2] D.S. Taubman, M.W. Marcellin, *JPEG2000: Image Compression Fundamentals, Standards and Practice*, Kluwer Academic Publishers, Norwell, MA, USA, 2001.
- [3] H. C. Andrews and C. L. Patterson, “Singular Value Decomposition (SVD) Image Coding,” *IEEE Transactions on Communications*, vol. 24, pp. 425–432, 1976.
- [4] J. F. Yang and C. L. Lu. “Combined Techniques of Singular Value Decomposition and Vector Quantization for Image Coding,” *IEEE Transactions on Image Processing*, vol. 4, pp. 1141–1146, 1995.
- [5] C.S.M. Goldrick, W.J. Dowling, A. Bury, Image coding using the singular value decomposition and vector quantization,” *Image Processing and its Applications*, pp. 296–300, 1995.
- [6] P. Waldemar, T.A. Ramstad, Image compression using singular value decomposition with bit allocation and scalar quantization, in: *Proceedings of NORISG Conference*, 1996, pp. 83–86.
- [7] J.-F. Yang, C.-L. Lu, Combined techniques of singular value decomposition and vector quantization for image coding, *IEEE Transactions on Image Processing* 4 (8) (1995) 1141–1146.
- [8] G.H. Golub, C.F.V. Loan, *Matrix Computations*, The John Hopkins University Press, 1983.
- [9] J. J. Gerbrands, On the relationships between SVD, KLT and PCA, *Pattern Recognition* 14 (6) (1981) 375–381.

- [10] Ashino, R.; Morimoto, A. Nagase, M. & Vaillancourt, R. Image compression with multiresolution singular value decomposition and other methods. CRM-2939, ibid. 01/2004.
- [11] Naidu, V.P.S. Discrete Cosine Transform-based Image Fusion. Def. Sci. J., 2010, 60(1), 48-54.
- [12] Arce, gonzalo R. Nonlinear Signal Processing – A statistical approach. Wiley-Interscience Inc., Publication, USA, 2005.
- [13] Mohamed A. Mohamed, Abdul-Fattah Ibrahim Abdul-Fattah, Aziza S. Asem, Abeer S.El-Bashbishy, “Medical image filtering, fusion and classification techniques,” Egyptian Journal of Bronchology, Vol. 5, No2, pp. 142-151, 2011.