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Collocation Method for Fifth Order Boundary Value Problems by Using Quintic B-splines

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Abstract: ***A finite element method involving collocation method with Quintic B-splines as basis functions has been developed to solve fifth order boundary value problems. The fifth order derivative for the dependent variable is approximated by the finite differences. The basis functions are redefined into a new set of basis functions which in number match with the number of collocated points selected in the space variable domain. The proposed method is tested on three linear and two non-linear boundary value problems. The solution to a nonlinear problem has been obtained as the limit of a sequence of solutions of linear problems generated by the quasilinearization technique. Numerical results obtained by the present method are in good agreement with the exact solutions available in the literature.***

Keywords: Collocation method, Quintic B-spline, Fifth order boundary value problem, Absolute error.

1. Introduction

In this paper, we consider a general fifth order boundary value problem

$$a_0(x)y^{(5)}(x)+a_1(x)y^{(4)}(x)+a_2(x)y'''(x)+a_3(x)y''(x)$$
$$+a_4(x)y'(x)+a_5(x)y(x)=b(x), c<x<d$$

(1)

subject to boundary conditions

$$y(c)=A_0, y(d)=C_0, y'(c)=A_1, y'(d)=C_1, y''(c)=A_2 \quad (2)$$

where A_0, A_1, A_2, C_0, C_1 are finite real constants and $a_0(x), a_1(x), a_2(x), a_3(x), a_4(x), a_5(x)$ and $b(x)$ are all continuous functions defined on the interval $[c, d]$.

The fifth order boundary value problems occur in the mathematical modelling of the viscoelastic flows and other branches of mathematical, physical and engineering sciences [1], [2]. The existence and uniqueness of the solution for these types of problems have been discussed in Agarwal [3]. Finding the analytical solutions of such type of boundary value problems in general is not possible. Over the years, many researchers have worked on fifth order boundary value problems by using different methods for numerical solutions. Wazwaz [4] developed the solution of special type of fifth order boundary value problems by using the modified Adomian decomposition method. Siddiqi and Ghazala [5] presented the solution of a special case of linear fifth order boundary value problems by using quartic spline functions and sextic spline functions techniques respectively. Azam et al. [6], Siddiqi and Ghazala [7], Siddiqi et al. [8] have presented the solution of a special case of linear fifth order boundary value problems by using non polynomial spline functions. Noor and Syed [9] applied the Homotopy perturbation method for solving fifth order boundary value problems. Caglar and Caglar [10] presented the Local polynomial regression method to solve the special case of fifth order boundary value problems. Gamel [11] presented the solution of fifth order boundary value problems by Sinc-Galerkin method. Noor and Syed [12], Zhao [13] have developed the solution of fifth order boundary value problems by variational iteration method. Lamnii et al. [14] developed the sextic spline collocation method to solve a special case of fifth order boundary value problems. Kasi Viswanadham and Showri Raju [15] developed the quartic B-spline collocation method to solve a general fifth order boundary value problem. Kasi Viswanadham and Murali [16] presented quintic B-spline Galerkin method to solve a special case of fifth order boundary value problem. Kasi Viswanadham and Sreenivasulu [17] developed quartic B-spline Galerkin method to solve a general fifth order boundary value problem. Doha et al. [18] presented dual Petrov-Galerkin method with Jacobi polynomials to solve fifth order boundary value problem. Ali et al. [19] developed reproducing kernel method
for the solution of fifth order boundary value problem. So far, fifth order boundary value problems have not been solved by using collocation method with quintic B-splines as basis functions.

In this paper, we try to present a simple finite element method which involves collocation approach with quintic B-splines as basis functions to solve the fifth order boundary value problem of the type (1)-(2). This paper is organized as follows. In section 2 of this paper, the justification for using the collocation method has been mentioned. In section 3, the definition of quintic B-splines has been described. In section 4, description of the collocation method with quintic B-splines as basis functions has been presented and in section 5, solution procedure to find the nodal parameters is presented. In section 6, numerical examples of both linear and non-linear boundary value problems are presented. The solution to a nonlinear problem has been obtained as the limit of a sequence of solution of linear problems generated by the quasilinearization technique [20]. Finally, the last section is dealt with conclusions of the paper.

2. Justification for using Collocation method

In finite element method (FEM) the approximate solution can be written as a linear combination of basis functions which constitute a basis for the approximation space under consideration. FEM involves variational methods like Ritzs approach, Galerkins approach, least squares method and collocation method etc. The collocation method seeks an approximate solution by requiring the residual of the differential equation to be identically zero at N selected points in the given space variable domain where N is the number of basis functions in the basis [21]. That means, to get an accurate solution by the collocation method one needs a set of basis functions which in number match with the number of collocation points selected in the given space variable domain. Further, the collocation method is the easiest to implement among the variational methods of FEM. When a differential equation is approximated by mth order B-splines, it yields $(m + 1)^{th}$ order accurate results [22]. Hence this motivated us to solve a fifth order boundary value problem of type (1)-(2) by collocation method with quintic B-splines as basis functions.

3. Definition of quintic B-spline

The quintic B-splines are defined in [22]-[24]. The existence of quintic spline interpolate s(x) to a function in a closed interval [c, d] for spaced knots (need not be evenly spaced) of a partition $c = x_0 < x_1 < \ldots < x_{n-1} < x_n = d$ is established by constructing it. The construction of $s(x)$ is done with the help of the quintic B-splines. Introduce ten additional knots x_{-5}, x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_{n+1}, x_{n+2}, x_{n+3}, x_{n+4} and x_{n+5} in such a way that $x_{-5}<x_{-4}<x_{-3}<x_{-2}<x_{-1}<x_0$ and $x_n<x_{n+1}<x_{n+2}<x_{n+3}<x_{n+4}<x_{n+5}$.
Now the quintic B-splines $B_i(x)'s$ are defined by

$$B_i(x)=\begin{cases}\sum\limits_{r=i-3}^{i+3}\dfrac{(x_r-x)_+^5}{\pi'(x_r)}, & x\in[x_{i-3},\ x_{i+3}]\\ 0, & otherwise\end{cases}$$

where

$$(x_r-x)_+^5=\begin{cases}(x_r-x)^5, & if\ x_r\ge x\\ 0, & if\ x_r\le x\end{cases}$$

and $$\pi(x)=\prod_{r=i-3}^{i+3}(x-x_r)$$

where $\{B_{-2}(x), B_{-1}(x), B_0(x), B_1(x),\ldots,B_n(x), B_{n+1}(x), B_{n+2}(x)\}$ forms a basis for the space $s_5(\pi)$ of quintic polynomial splines. Schoenberg [24] has proved that quintic B-splines are the unique nonzero splines of smallest compact support with the knots at
$x_{-5}<x_{-4}<x_{-3}<x_{-2}<x_{-1}<x_0<x_1<\ldots<x_{n-1}<x_n<x_{n+1}<x_{n+2}<x_{n+3}<x_{n+4}<x_{n+5}$.

4. Description of the method

To solve the boundary value problem (1) subject to boundary conditions (2) by the collocation method with quintic B-splines as basis functions, we define the approximation for $y(x)$ as

$$y(x)=\sum_{j=-2}^{n+2}\alpha_j B_j(x) \tag{3}$$

Where $\alpha_j's$ are the nodal parameters to be determined and $B_j(x)'s$ are quintic B-spline basis functions. In the present method, the mesh points $x_1, x_2, \ldots, x_{n-1}, x_n$ are selected as the collocation points. In collocation method, the number of basis functions in the approximation should match with the number of collocation points [21]. Here the number of basis functions in the approximation (3) is n+5, where as the number of selected collocation points is n. So, there is a need to redefine the basis functions into a new set of basis functions which in number match with the number of selected collocation points. The procedure for redefining the basis functions is as follows:

Using the definition of quintic B-splines, the Dirichlet, Neumann and second order derivative boundary condition of (2), we get the approximate solution at the boundary points as

$$y(c)=y(x_0)=\sum_{j=-2}^{2}\alpha_j B_j(x_0)=A_0 \tag{4}$$

$$y(d)=y(x_n)=\sum_{j=n-2}^{n+2}\alpha_j B_j(x_n)=C_0 \tag{5}$$

$$y'(c)=y'(x_0)=\sum_{j=-2}^{2}\alpha_j B'_j(x_0)=A_1 \tag{6}$$

$$y'(d)=y'(x_n)=\sum_{j=n-2}^{n+2}\alpha_j B'_j(x_n)=C_1 \tag{7}$$

$$y''(c)=y''(x_0)=\sum_{j=-2}^{2}\alpha_j B''_j(x_0)=A_2 \tag{8}$$

Eliminating $\alpha_{-2},\alpha_{-1},\alpha_0,\alpha_{n+1}$ and α_{n+2} from the equations (3) to (8), we get the approximation for y(x) as

$$y(x)=w(x)+\sum_{j=1}^{n}\alpha_j R_j(x) \tag{9}$$

where

$$w(x)=w_2(x)+\frac{A_2-w_2''(x_0)}{Q_0''(x_0)}Q_0(x)$$

$$w_2(x)=w_1(x)+\frac{A_1-w_1'(x_0)}{P_{-1}'(x_0)}P_{-1}(x)+\frac{C_1-w_1'(x_n)}{P_{n+1}'(x_n)}P_{n+1}(x)$$

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$$w_1(x)=\frac{A_0}{B_{-2}(x_0)}B_{-2}(x)+\frac{C_0}{B_{n+2}(x_n)}B_{n+2}(x)$$

$$R_j(x)=\begin{cases}Q_j(x)-\dfrac{Q_j''(x_0)}{Q_0''(x_0)}Q_0(x), & j=1,2\\ Q_j(x), & j=3,4,\dots,n\end{cases} \qquad (10)$$

$$Q_j(x)=\begin{cases}P_j(x)-\dfrac{P_j'(x_0)}{P_{-1}'(x_0)}P_{-1}(x), & j=0,1,2\\ P_j(x), & j=3,4,\dots,n-3\\ P_j(x)-\dfrac{P_j'(x_n)}{P_{n+1}'(x_n)}P_{n+1}(x), & j=n-2,n-1,n\end{cases}$$

$$P_j(x)=\begin{cases}B_j(x)-\dfrac{B_j(x_0)}{B_{-2}(x_0)}B_{-2}(x), & j=-1,0,1,2\\ B_j(x), & j=3,4,\dots,n-3\\ B_j(x)-\dfrac{B_j(x_n)}{B_{n+2}(x_n)}B_{n+2}(x), & j=n-2,n-1,n,n+1\end{cases}$$

Now the new basis functions for the approximation y(x) are {$R_j(x)$, j=1, 2,…, n} and they are in number match with the number of selected collocated points. Since the approximation for y(x) in (9) is a quintic approximation, let us approximate $y^{(5)}$ at the selected collocation points with finite differences as

$$y_i^{(5)}=\frac{y_{i+1}^{(4)}-y_{i-1}^{(4)}}{2h} \quad \text{for } i=1,2,\dots,n-1$$

(11)

$$y_i^{(5)}=\frac{y_i^{(4)}-y_{i-1}^{(4)}}{h} \quad \text{for } i=n$$

(12)

where

$$y_i=y(x_i)=w(x_i)+\sum_{j=1}^{n}\alpha_jR_j(x_i)$$

(13)

Now applying the collocation method to (1), we get

$$a_0(x_i)y_i^{(5)}+a_1(x_i)y_i^{(4)}+a_2(x_i)y_i'''+a_3(x_i)y_i''+a_4(x_i)y_i'+a_5(x_i)y_i=b(x_i) \quad \text{for } i=1,2,\cdots,n. \qquad (14)$$

Substituting (11), (12) and (13) in (14), we get

$$\frac{a_0(x_i)}{2h}\left[w^{(4)}(x_{i+1})+\sum_{j=1}^{n}\alpha_jR_j^{(4)}(x_{i+1})-w^{(4)}(x_{i-1})-\sum_{j=1}^{n}\alpha_jR_j^{(4)}(x_{i-1})\right]$$
$$+a_1(x_i)\left[w^{(4)}(x_i)+\sum_{j=1}^{n}\alpha_jR_j^{(4)}(x_i)\right]+a_2(x_i)\left[w'''(x_i)+\sum_{j=1}^{n}\alpha_jR_j'''(x_i)\right]$$
$$+a_3(x_i)\left[w''(x_i)+\sum_{j=1}^{n}\alpha_jR_j''(x_i)\right]+a_4(x_i)\left[w'(x_i)+\sum_{j=1}^{n}\alpha_jR_j'(x_i)\right]$$
$$+a_5(x_i)\left[w(x_i)+\sum_{j=1}^{n}\alpha_jR_j(x_i)\right]$$

for i=1, 2,…, n-1.

(15)

$$\frac{a_0(x_i)}{h}\left[w^{(4)}(x_i)+\sum_{j=1}^{n}\alpha_jR_j^{(4)}(x_i)-w^{(4)}(x_{i-1})-\sum_{j=1}^{n}\alpha_jR_j^{(4)}(x_{i-1})\right]$$
$$+a_1(x_i)\left[w^{(4)}(x_i)+\sum_{j=1}^{n}\alpha_jR_j^{(4)}(x_i)\right]+a_2(x_i)\left[w'''(x_i)+\sum_{j=1}^{n}\alpha_jR_j'''(x_i)\right]$$
$$+a_3(x_i)\left[w''(x_i)+\sum_{j=1}^{n}\alpha_jR_j''(x_i)\right]+a_4(x_i)\left[w'(x_i)+\sum_{j=1}^{n}\alpha_jR_j'(x_i)\right]$$
$$+a_5(x_i)\left[w(x_i)+\sum_{j=1}^{n}\alpha_jR_j(x_i)\right]$$

for i=n. (16)

Rearranging the terms and writing the system of equations (15) and (16) in the matrix form, we get

$$\mathbf{A}\alpha=\mathbf{B}$$

(17)

where $\mathbf{A}=[a_{ij}]$;

$$a_{ij}=\frac{a_0(x_i)}{2h}\left[R_j^{(4)}(x_{i+1})-R_j^{(4)}(x_{i-1})\right]+a_1(x_i)R_j^{(4)}(x_i)+a_2(x_i)R_j'''(x_i)+a_3(x_i)R_j''(x_i)+a_4(x_i)R_j'(x_i)+a_5(x_i)R_j(x_i)$$

for i= 1, 2, …., n-1; j=1, 2,…, n.

(18)

$$a_{ij}=\frac{a_0(x_i)}{h}\left[R_j^{(4)}(x_i)-R_j^{(4)}(x_{i-1})\right]+a_1(x_i)R_j^{(4)}(x_i)+a_2(x_i)R_j'''(x_i)+a_3(x_i)R_j''(x_i)+a_4(x_i)R_j'(x_i)+a_5(x_i)R_j(x_i)$$

for i= n; j=1, 2,…, n. (19)

$$\mathbf{B}=[b_i];$$

$$b_i=\frac{a_0(x_i)}{2h}\left[w^{(4)}(x_{i+1})-w^{(4)}(x_{i-1})\right]+a_1(x_i)w^{(4)}(x_i)+a_2(x_i)w'''(x_i)+a_3(x_i)w''(x_i)+a_4(x_i)w'(x_i)+a_4(x_i)w(x_i)$$

for i= 1, 2, …, n-1.

(20)

$$b_i=\frac{a_0(x_i)}{h}\left[w^{(4)}(x_i)-w^{(4)}(x_{i-1})\right]+a_1(x_i)w^{(4)}(x_i)+a_2(x_i)w'''(x_i)+a_3(x_i)w''(x_i)+a_4(x_i)w'(x_i)+a_4(x_i)w(x_i)$$

for i=n.

(21)

and $\alpha=[\alpha_1,\alpha_2,\dots,\alpha_n]^T$.

5. Solution procedure to find the nodal parameters

The basis function $R_i(x)$ is defined only in the interval $[x_{i-3},x_{i+3}]$ and outside of this interval it is zero. Also at the end points of the interval $[x_{i-3},x_{i+3}]$ the basis function $R_i(x)$ vanishes. Therefore, $R_i(x)$ is having non-vanishing values at the mesh points $x_{i-2},x_{i-1},x_i,x_{i+1},x_{i+2}$ and zero at the other mesh points. The first four derivatives of $R_i(x)$ also have the same nature at the mesh points as in the case of $R_i(x)$. Using these facts, we can say that the Thus the stiff matrix $\mathbf{A}$ is a seven diagonal band matrix. Therefore, the system of equations (17) is a seven band system in $\alpha_i\,'s$. The nodal parameters $\alpha_i\,'s$ can be obtained by using band matrix solution package. We have used the FORTRAN-90 programming to solve the boundary value problem (1)-(2) by the proposed method

6. Numerical results

To demonstrate the applicability of the proposed method for solving the fifth order boundary value problems of the type (1) and (2), we considered three linear and two nonlinear boundary value problems. The obtained numerical results for each problem are presented in tabular forms and compared with the exact solutions available in the literature.

Example 1: Consider the linear boundary value problem

$$y^{(5)} + xy = (1-x)cosx - 5sinx + xsinx - x^2 sinx, \quad 0 < x < 1 \quad (22)$$

subject to

$$y(0) = 0,\ y(1) = 0,\ y'(0) = 1,\ y'(1) = -sin1,\ y''(0) = -2.$$

The exact solution for the above problem is $y = (1-x)sinx.$

The proposed method is tested on this problem where the domain [0, 1] is divided into 10 equal subintervals. The obtained numerical results for this problem are given in Table 1. The maximum absolute error obtained by the proposed method is 8.642673×10^{-7}.

Table 1: Numerical results for Example 1

x	Absolute error by the proposed method
0.1	2.756715E-07
0.2	7.748604E-07
0.3	2.175570E-06
0.4	3.933907E-06
0.5	5.364418E-06
0.6	5.602837E-06
0.7	5.811453E-06
0.8	4.291534E-06
0.9	2.518296E-06

Example 2: Consider the linear boundary value problem

$$y^{(5)} + y^{(4)} + e^{-2x} y = e^{-x}\big[-4e^{2x}(-3+x)cosx$$
$$-(1-x+4e^{2x}(5+2x))sinx\big], \quad 0 \le x \le 1$$

(23)

subject to

$$y(0) = 0,\ y(1) = 0,\ y'(0) = -1,\ y'(1) = esin1,\ y''(0) = 0.$$

The exact solution for the above problem is

$$y = e^x(x-1)sinx.$$

The proposed method is tested on this problem where the domain [0, 1] is divided into 10 equal subintervals. The obtained numerical results for this problem are given in Table 2. The maximum absolute error obtained by the proposed method is 5.811453×10^{-6}.

Table 2: Numerical results for Example 2

x	Absolute error by the proposed method
0.1	1.788139E-07
0.2	4.023314E-07
0.3	4.619360E-07
0.4	7.301569E-07
0.5	7.003546E-07
0.6	7.152557E-07
0.7	8.642673E-07
0.8	8.195639E-07
0.9	4.917383E-07

Example 3: Consider the linear boundary value problem

$$y^{(5)} + sinx\, y^{(4)} - y = (1+sinx)e^x, \quad 0 < x < 1 \quad (24)$$

subject to

$$y(0) = 1, y(1) = e, y'(0) = 1, y'(1) = e, y''(0) = 1.$$

The exact solution for the above problem is $y = e^x$.

The proposed method is tested on this problem where the domain [0, 1] is divided into 10 equal subintervals. The obtained numerical results for this problem are given in Table 3. The maximum absolute error obtained by the proposed method is 7.104874×10^{-5}.

Table 3: Numerical results for Example 3

x	Absolute error by the proposed method
0.1	3.337860E-06
0.2	1.335144E-05
0.3	2.872944E-05
0.4	4.959106E-05
0.5	6.484985E-05
0.6	7.104874E-05
0.7	6.508827E-05
0.8	4.506111E-05
0.9	1.311302E-05

Example 4: Consider the nonlinear boundary value problem

$$y^{(5)} + 24e^{-5y} = \frac{48}{(1+x)^5}, \quad 0 \le x \le 1$$

(25)

subject to

$$y(0) = 0,\ y(1) = ln2,\ y'(0) = 1,\ y'(1) = 0.5,\ y''(0) = -1.$$

The exact solution for the above problem is $y = \ln(1+x)$.

The nonlinear boundary value problem (25) is converted into a sequence of linear boundary value problems generated by quasilinearization technique [20] as

$$y^{(5)}_{(n+1)} - 120e^{-5y_{(n)}} y_{(n+1)} = \frac{48}{(1+x)^5} - 120 y_{(n)} e^{-5y_{(n)}} \qquad (26)$$

$$-24e^{-5y_{(n)}}, \ n = 0,1,2,...$$

Subject to

$$y_{(n+1)}(0) = 0,\ y_{(n+1)}(1) = ln2,\ y'_{(n+1)}(0) = 1,\ y'_{(n+1)}(1) = 0.5,$$
$$y''_{(n+1)}(0) = -1.$$

Here $y_{(n+1)}$ is the $(n+1)^{th}$ approximation for $y(x)$. The domain [0, 1] is divided into 10 equal subintervals and the proposed method is applied to the sequence of linear problems (26). The obtained numerical results for this problem are presented in Table 5. The maximum absolute error obtained by the proposed method is 1.892447x10^{-5}.

Table 4: Numerical results for Example 4

x	Absolute error by the proposed method
0.1	9.164214E-07
0.2	4.559755E-06
0.3	9.953976E-06
0.4	1.585484E-05
0.5	1.892447E-05
0.6	1.847744E-05
0.7	1.513958E-05
0.8	9.357929E-06
0.9	2.384186E-06

Example 5: Consider the nonlinear boundary value problem

$$y^{(5)} + y^{(4)} + e^{-2x} y^2 = 2e^x + 1, \quad 0 < x < 1$$

(27)

subject to

$$y(0) = 1,\ y(1) = e,\ y'(0) = 1,\ y'(1) = e,\ y''(0) = 1.$$

The exact solution for the above problem is $y = e^x$.

The nonlinear boundary value problem (29) is converted into a sequence of linear boundary value problems generated by quasilinearization technique [20] as

$$y^{(5)}_{(n+1)} + y^{(4)}_{(n+1)} + 2e^{-2x} y_{(n)} y_{(n+1)}$$
$$= 2e^x + e^{-2x} y^2_{(n)} + 1, \quad n = 0,1,2,...$$

(28)

subject to

$$y_{(n+1)}(0) = 1,\ y_{(n+1)}(1) = e,\ y'_{(n+1)}(0) = 1,\ y'_{(n+1)}(1) = e,\ y''_{(n+1)}(0) = 1.$$

Here $y_{(n+1)}$ is the $(n+1)^{th}$ approximation for $y(x)$. The domain [0, 1] is divided into 10 equal subintervals and the proposed method is applied to the sequence of linear problems (30). The obtained numerical results for this problem are presented in Table 5. The maximum absolute error obtained by the proposed method is 2.121925x10^{-5}.

Table 5: Numerical results for Example 5

x	Absolute error by the proposed method
0.1	1.430511E-06
0.2	7.152557E-07
0.3	2.861023E-06
0.4	7.152557E-06
0.5	1.430511E-05
0.6	2.110004E-05
0.7	2.121925E-05
0.8	1.573563E-05
0.9	9.298325E-06

7. Conclusions

In this paper, we have developed a collocation method with quintic B-splines as basis functions to solve fifth order boundary value problems. Here we have taken mesh points $x_1, x_2, ..., x_{n-1}, x_n$ as the collocation points. The quintic B-spline basis set has been redefined into a new set of basis functions which in number match with the number of selected collocation points. The proposed method is applied to solve several number of linear and non-linear problems to test the efficiency of the method. The numerical results obtained by the proposed method are in good agreement with the exact solutions available in the literature. The objective of this paper is to present a simple method to solve a fifth order boundary value problem and its easiness for implementation

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