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A New Approach to Automatic Generation of All Triangular Mesh For Finite Element Analysis

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Abstract

This paper presents a new mesh generation method for a convex polygonal domain. We first decompose the convex polygon into simple sub regions in the shape of triangles. These simple regions are then triangulated to generate a mesh of 6-node triangular elements. We propose then an automatic 6-node trianglar to 3-node triangular conversion scheme. Each isolated 6-node triangle is split into four triangles according to the usual scheme,that is,by adding three vertices in the middle of the edges and joining them by straight lines. To preserve the mesh conformity a similar procedure is also applied to every triangle of the domain to fully discretize the given convex polygonal domain into all triangles, thus propagating uniform refinement. This simple method generates a high quality mesh whose elements confirm well to the requested shape by refining the problem domain. Examples are presented to illustrate the simplicity and efficiency of the new mesh generation method for standard and arbitrary shaped polygonal domains and cracked polygonal domains. We have appended MATLAB programs which incorporate the mesh generation scheme developed in this paper. These programs provide valuable output on the nodal coordinates ,element connectivity and graphic display of the all triangular mesh for application to finite element analysis.

Keywords: finite elements, triangulation ,all triangular mesh generation, convex polygonal domain,cracked convex polygonal domain, uniform refinement

1.Introduction

The finite element method(FEM) is now used as a general purpose method applicable to all kinds of partial differential equations. The advent of modern computer technologies provided a powerful tool in numerical simulations for a range of problems in partial differential equations over arbitrary complex domains. A mesh is required for finite element method as it uses finite elements of a domain for analysis. Finite Element Analysis (FEA) is widely used in many fields including structures and optimization. The FEA in engineering applications comprises three phases: domain discretization, equation solving and error analysis. The domain discretization or mesh generation is the preprocessing phase which plays an important role in the achievement of accurate solutions.

FEM requires dividing the analysis region into many sub regions. These small regions are the elements which are connected with adjacent elements at their nodes. Mesh generation is a procedure of generating the geometric data of the elements and their nodes, and involves computing the coordinates of nodes, defining their connectivity and thus constructing the elements. Hence mesh designates aggregates of elements nodes and lines representing their connectivity. Though the FEM is a powerful and versatile tool, its usefulness is often hampered by the need to generate a mesh. Creating a mesh is the first step in a wide range of applications, including scientific and engineering computing and computer graphics. But generating a mesh can be very time consuming and prone to error if done manually. In recognition of this problem a large number of methods have been devised to automate the mesh generation task. An attempt to create a fully automatic mesh generator that is capable of generating a valid finite element meshes over arbitrary complex domains and needs only the information of the specified geometric boundary of the domain and the element size, started from the pioneering work [1] in the early 1970's. Since then many methodologies have been proposed and different algorithms have been devised in the development of automatic mesh generators [2-4]. In order to perform a reliable finite element simulation a number of researchers [5-7] have made efforts to develop adaptive FEA method which integrates with error estimation and automatic mesh modification. Traditionally adaptive mesh generation process is started from coarse mesh which gives large discretization error levels and takes a lot of iterations to get a desired final mesh. The research literature on the subject is vast and different techniques have been proposed [8]. As several engineering applications to real world problems cannot be defined on a rectangular domain or solved on a structured square mesh. The description and discretization of the design domain geometry, specification of the boundary conditions for the governing state equation, and accurate computation of the design response may require the use of unstructured meshes.

An unstructured simplex mesh requires a choice of mesh points (vertex nodes) and triangulation. Many mesh generators produce a mesh of triangles by first creating all the nodes and then connecting nodes to form of triangles. The question arises as to what is the 'best' triangulation on a given set of points. One particular scheme, namely Delaunay triangulation [8], is considered by many researchers to be most suitable for finite element analysis. If the problem domain is a subset of the Cartesian plane, triangular or quadrilateral meshes are typically employed.

In this paper, we present a novel mesh generation scheme of all triangular elements for convex polygonal domains. This scheme converts the elements in background triangular mesh into triangles through the operation of splitting. We first decompose the convex polygon into simple subregions in the shape of triangles. These simple subregions are then triangulated to generate a fine mesh of 6-node triangles. We propose then an automatic 6-node triangular to 3-node triangular conversion scheme in which each isolated triangle is split into four triangles according to the usual scheme, that is by adding three vertices in the middle of edges and then joining these three vertices by straight lines. Further, to preserve the mesh conformity a similar procedure is also applied to every triangle of the domain and this fully discretizes the given convex polygonal domain into all 3-node triangles, thus propogating uniform refinement. In section 2 of this paper, we present a scheme to discretize the arbitrary and standard triangles into a fine mesh of 6-node triangular elements. In section 3,we explain the procedure to split each of these 6-node triangles into four 3-node triangles. In section 4,we have presented a method of piecing together of all triangular subregions and eventually creating an all triangular mesh for the given convex polygonal domain. In section 5,we present several examples to illustrate the simplicity and efficiency of the proposed mesh generation method for standard and arbitrary triangles,rectangles,squares, convex and cracked convex polygonal domains.

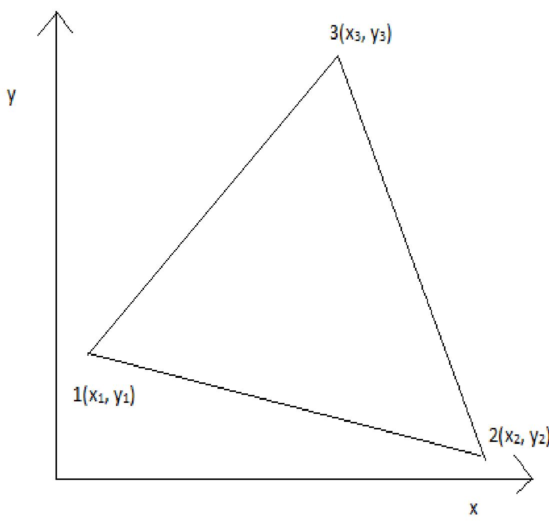
2. Division of an Arbitrary Triangle

We can map an arbitrary triangle with vertices $((x_i, y_i),\ i = 1, 2, 3)$ into a right isosceles triangle in the (u, v) space as shown in Fig. 1a, 1b. The necessary transformation is given by the equations.

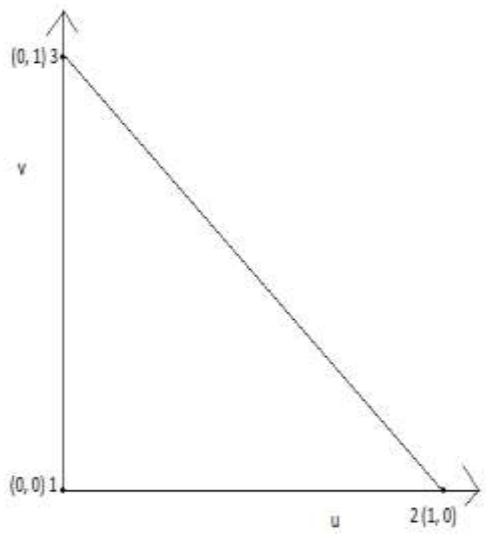
$$x = x_1 + (x_2 - x_1)u + (x_3 - x_1)v$$

$$y = y_1 + (y_2 - y_1)u + (y_3 - y_1)v \qquad (1)$$

The mapping of eqn.(1) describes a unique relation between the coordinate systems. This is illustrated by using the area coordinates and division of each side into three equal parts in Fig. 2a Fig. 2b. It is clear that all the coordinates of this division can be determined by knowing the coordinates $((x_i, y_i),\ i = 1, 2, 3)$ of the vertices for the arbitrary triangle. In general , it is well known that by making 'n' equal divisions on all sides and the concept of area coordinates, we can divide an arbitrary triangle into n^2 smaller triangles having the same area which equals Δ/n^2 where Δ is the area of a linear arbitrary triangle with vertices $((x_i, y_i),\ i = 1, 2, 3)$ in the Cartesian space.



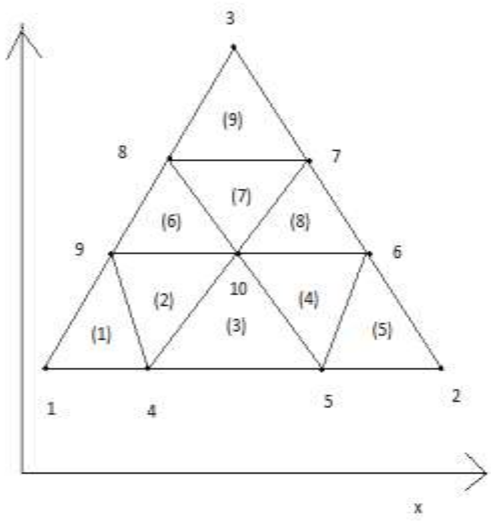
1.a

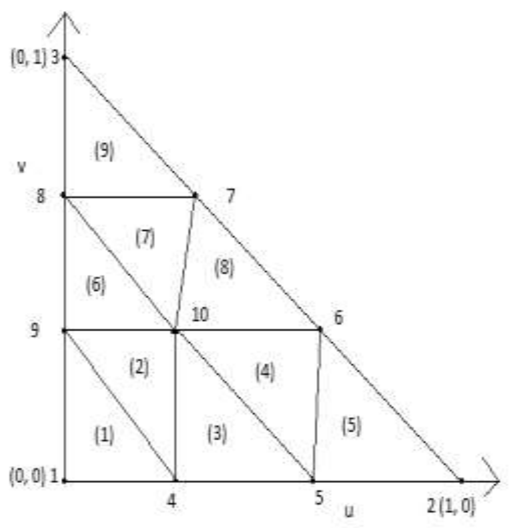


1. b

Fig. 1a An Arbitrary Linear Triangle in the (x, y) space

Fig. 1b A Right Isosceles Triangle in the (u, v)





2(a) 2(b)

Fig. 2a Division of an arbitrary triangle into Nine triangles in Cartesian space Type equation here.

Fig. 2b Division of a right isosceles triangle into Nine right isosceles triangles in (u, v) space Type equation here.

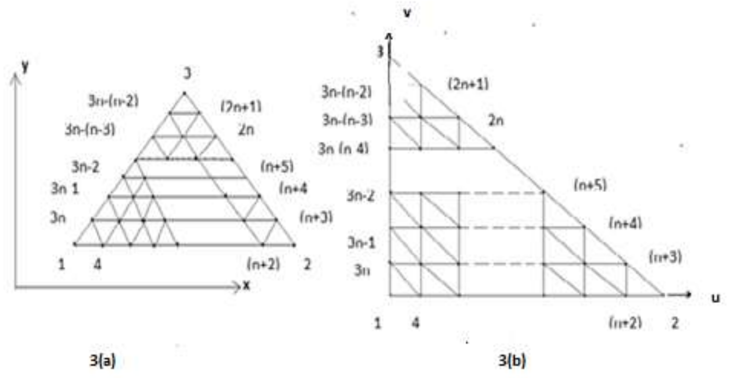


Fig. 3a Division of an arbitrary triangle with vertices$((x_i, y_i\ ), i = 1,2,3)$ into n^2 triangle in Cartesian space (x, y), where each side is divided into n divisions of equal length

Fig. 3b Division of a right isosceles triangle with vertices{1(0,0),2(1,0),3(0,1)} into n^2 right isosceles triangle in (u, v) space, where each side is divided into n divisions of equal length

We have shown the division of an arbitrary triangle in Fig. 3a , Fig. 3b, We divided each side of the triangles (either in Cartesian space or natural space) into n equal parts and draw lines parallel to the sides of the triangles. This creates (n+1) (n+2)/2 nodes. These nodes are numbered from triangle base line l_{12} (letting l_{ij} as the line joining the vertex (x_i, y_i) and (x_j, y_j)) along the line $v = 0$ and upwards up to the line $v = 1$. The nodes 1, 2, 3 are numbered anticlockwise and then nodes 4, 5, ------, (n+2) are along line $v = 0$ and the nodes (n+3), (n+4), ------, 2n, (2n+1) are numbered along the line l_{23} i.e. $u + v = 1$ and then the node (2n+2), (2n+3), -------, 3n are numbered along the line $u = 0$. Then the interior nodes are numbered in increasing order from left to right along the line $v = \frac{1}{n}, \frac{2}{n}, ---, \frac{n-1}{n}$ bounded on the right by the line

$+v = 1$. Thus the entire triangle is covered by (n+1) (n+2)/2 nodes. This is shown in the $\underline{rr}$ matrix of size $(n+1)\times(n+1)$, only nonzero entries of this matrix refer to the nodes of the triangles

$$\underline{rr}=\begin{bmatrix} 1, & 4, & 5, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, & (n+2), & 2 \\ 3n, & (3n+1), \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, & 3n+(n-2), & (n+3), & 0 \\ 3n-1, & 3n+(n-1), \ldots\ldots\ldots\ldots\ldots, & 3n+(n-2)+(n-3), & (n+4), \quad 0, & 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 3n-(n-3), & \frac{(n+1)(n+2)}{2}, & 2n, & 0, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & 0 \\ 3n-(n-2), & (2n+1), & 0, & 0, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & 0 \\ 3, & 0, & 0, & 0, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & 0 \end{bmatrix}$$

-----------(2)

3. Triangulation of an Arbitrary Triangle

We now consider the quadrangulation of an arbitrary triangle. We first divide the arbitrary triangle into a number of equal size six node triangles. Let us define l_{ij} as the line joining the points (x_i, y_i) and (x_j, y_j) in the Cartesian space (x, y). Then the arbitrary triangle with vertices at $((x_i, y_i), i = 1,2,3)$ is bounded by three lines $l_{12},\ l_{23},$ and l_{31} . By dividing the sides $l_{12},\ l_{23},\ l_{31}$ into $n = 2m$ divisions (m, an integer) creates m^2 six node triangular divisions. Then joining by straight lines the midpoints of the sides, we obtain four new triangles for each of these six node triangles. We have illustrated this process for the two and four divisions of $l_{12},\ l_{23},$ and l_{31} sides of the arbitrary and standard triangles in Figs. 4 and 5

3.1 Two Divisions of Each side of an Arbitrary Triangle and Standard Triangle

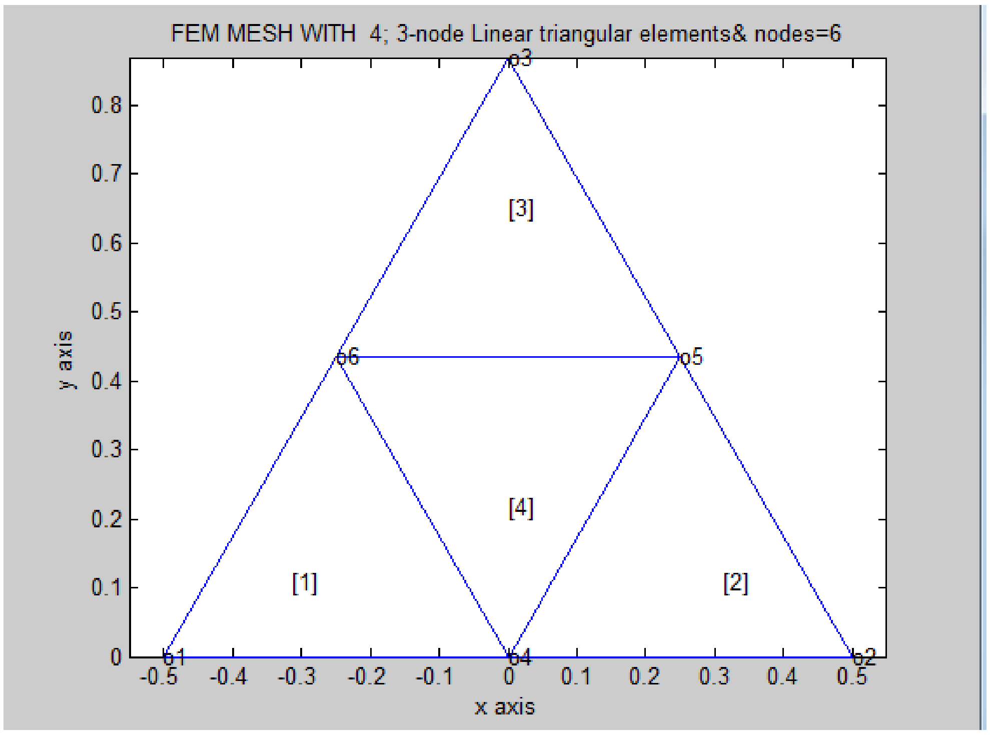


Fig 4(a)

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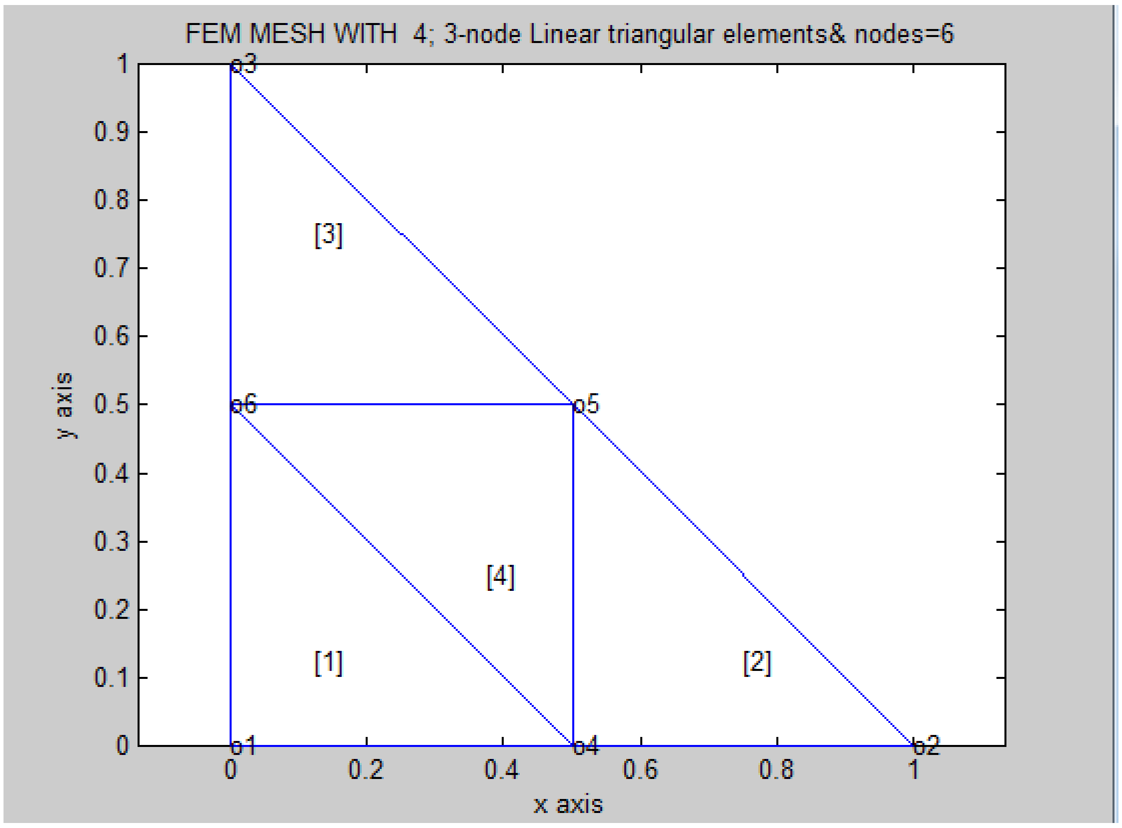


Fig.4(b)

Fig 4(a). Division of an arbitrary triangle into four triangles

Fig 4(b). Division of a standard triangle into four triangles

3.2 Four Divisions of Each side of an Arbitrary Triangle and Standard Triangle

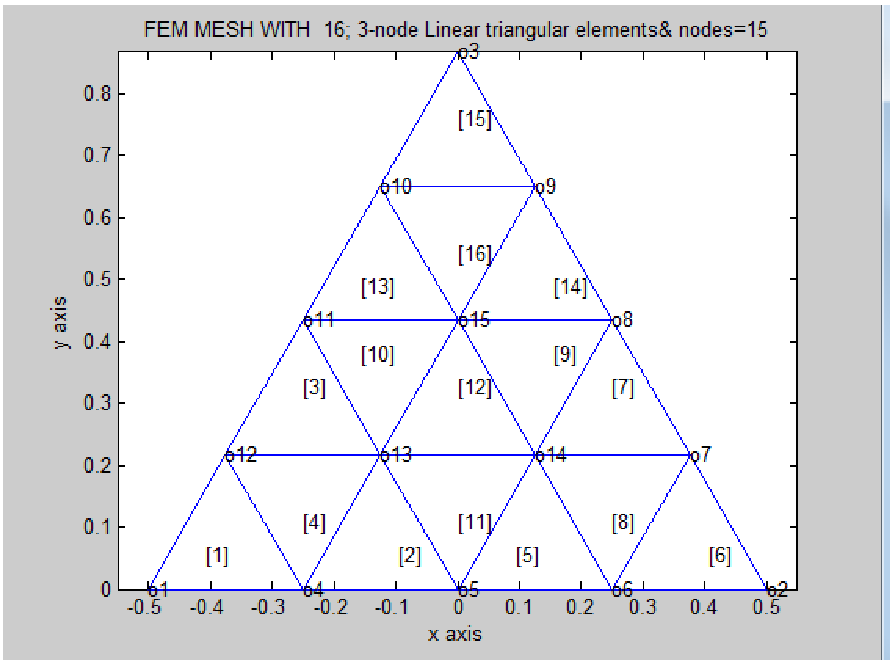

FEM MESH WITH 16; 3-node Linear triangular elements& nodes=15
x axis
y axis

Fig.5(a)

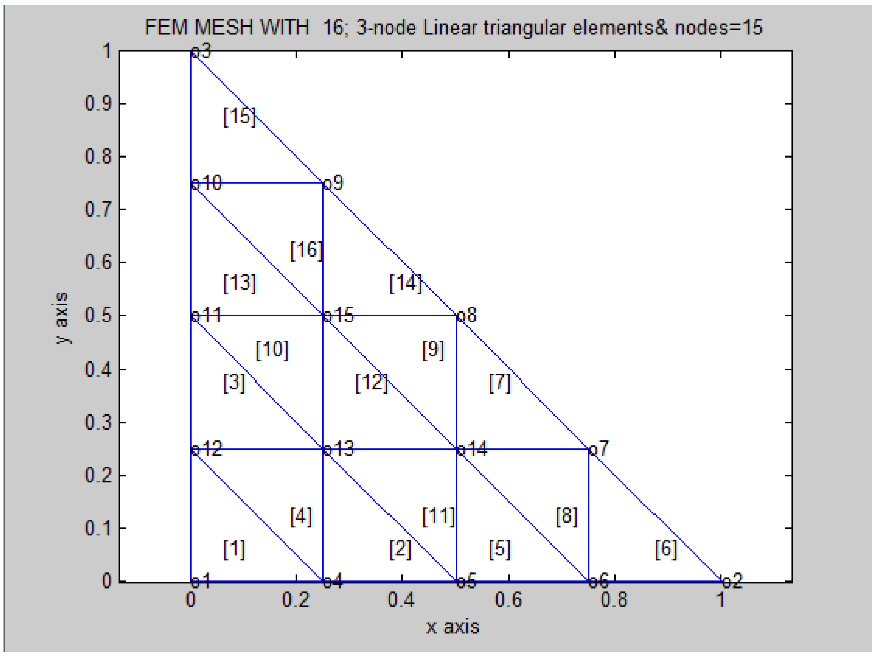


Fig.5(b)

Fig 5(a). Division of an arbitrary triangle into 4 six node arbitrary triangles which give rise to 16,three node aritrary triangles

Fig 5(b). Division of a standard triangle into 4 six node right isosceles triangles which give rise to 16,three node right isoscles triangles

In general, we note that to divide an arbitrary triangle into equal size six node triangle, we must divide each side of the triangle into an even number of divisions and locate points in the interior of triangle at equal spacing. We also do similar divisions and locations of interior points for the standard triangle. Thus n (even) divisions creates (n/2)^2 six node triangles in both the spaces. If the entries of the sub matrix $\underline{rr}\,(i\,;\,i+2,\ j\,;\,j+2)$ are nonzero then two six node triangles can be formed. If $\underline{rr}\,(i+1,\ \ j+2) = \underline{rr}\ \ (i+2,\ j+1;\ j+2) = 0$ then one six node triangle can be formed. If the sub matrices $\underline{rr}\,(i\,;\,i+2,\ j\,;\,j+2)$ is a (3×3) zero matrix , we cannot form the six node triangles. We now explain the creation of the six node triangles using the $\underline{rr}$ matrix_of eqn.(2). We can form six node triangles by using node points of three consecutive rows and columns of $\underline{rr}$ matrix. This procedure is depicted in Fig. 6a for three consecutive rows $i\,,i+1,\ i+2$ and three consecutive columns $j\,,\ j+1,\ j+2$ of the $\underline{rr}$ sub matrix Formation of six node triangle using sub matrix $\underline{rr}$

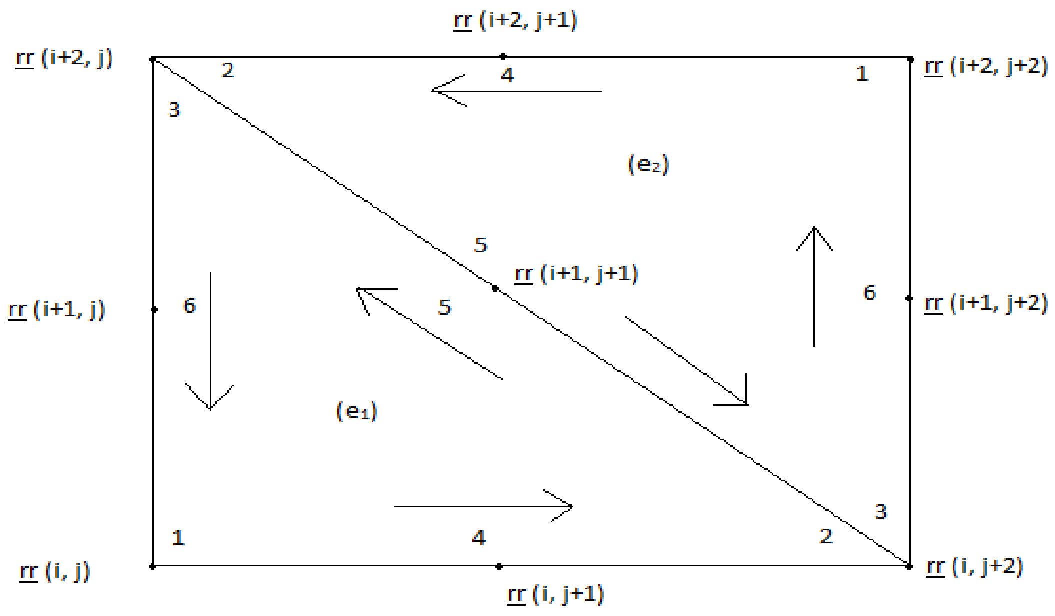


Fig. 6a Six node triangle formation for non zero sub matrix $\underline{rr}$

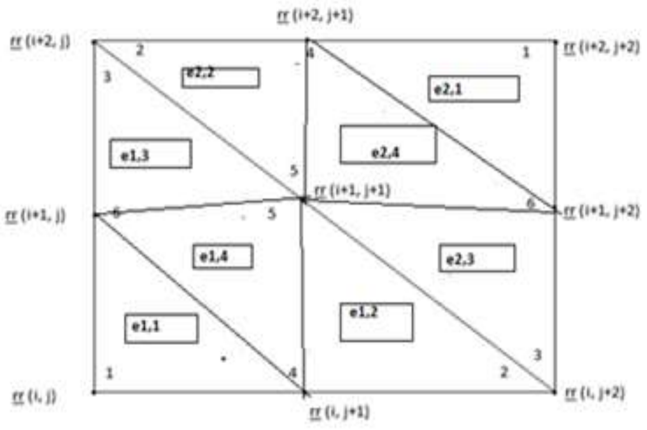


Fig. 6b Formation of 3-node triangles for non zero sub matrix $\underline{rr}$

If the sub matrix ($(\underline{rr}\ (k,l), k = i, i+1, i+2), l = j,\ j+1,\ j+2)$ is nonzero, then we can construct two six node triangles. The element nodal connectivity is then given by

(e₁) $< \underline{rr}\ \ (i,\ j), \underline{rr}\ (i, i+2), \underline{rr}\ (i+2,\ j), \underline{rr}\ (i, j+1), \underline{rr}\ (i+1, j+1), \underline{rr}\ (i+1, j) >$

(e₂) $< \underline{rr}\ \ (i+2, j+2), \underline{rr}\ (i+2, j), \underline{rr}\ (i, j+2), \underline{rr}\ (i+2, j+1), \underline{rr}\ (i+1, j+1), \underline{rr}$

$(i+1, j+2) >$ ------------------------------------(3)

If the elements of sub matrix ($(\underline{rr}\ (\ k,l), k = i, i+1, i+2),\ l = j,\ j+1,\ j+2)$ are nonzero, then as stated earlier, we can construct two six node triangles. We can create four 3-node tringles in each of these six node triangles (e_1) and (e_2). This procedure is depicted in Fig.6b, The nodal connectivity for the four 3-node triangles created in (e$_1$) and having element number n_1 are given as

$e1,1 = T_{4n_1-3} < \underline{rr}\ (i,\ j)\ , \underline{rr}\ (i,\ j+1),\ \underline{rr}\,(i+1,\ j)\ >$

$e1,2 = T_{4n_1-2} <\ \underline{rr}\ (i,\ j+2),\ \underline{rr}\,(i+1,\ j+1)\ , \underline{rr}\,(i, j+1) >$

$e1,3 = T_{4n_{1-1}} <\ \underline{rr}\ (i+2,\ j),\ \underline{rr}\,(i+1,\ j)\ , \underline{rr}\,(i+1, j+1) >$

$e1,4 = T_{4n_1} < \underline{rr}\ (i,\ j+1)\ , \underline{rr}\ (i+1,\ j+1),\ \underline{rr}\,(i+1,\ j)\ >$

----------------------(4)

and the nodal connectivity for the 4 triangles created in (e$_2$) are given as

$e2,1 = T_{4n_2-3} <\ \underline{rr}\ (i+2,\ j+2),\ \underline{rr}\,(i+2,\ j+1)\ , \underline{rr}\,(i+1,\ j+2) >$

$e2,2 = T_{4n_2-2} <\ \underline{rr}\ (i+2,\ j),\ \underline{rr}\,(i+1,\ j+1)\ , \underline{rr}\,(i+2,\ j+1) >$

$e2,3 = T_{4n_2-1} <\ \underline{rr}\ (i,\ j+2),\ \underline{rr}\,(i+1,\ j+2)\ , \underline{rr}\,(i+1,\ j+1) >$

$e2,4 = T_{4n_2} <\ \underline{rr}\ (i+2,\ j+1),\ \underline{rr}\,(i+1,\ j+1)\ , \underline{rr}\,(i+1, j+2) >$

--------------------------------(5)

4. Triangulation of the Polygonal Domain

We can generate polygonal meshes by piecing together of the triangles with straight sides. We designate them as subregions(called LOOPs). The user specifies the shape of these LOOPs by designating six coordinates of each LOOP

As an example, consider the geometry shown in Fig. 7(a). This is a a square region R which is simply chosen for illustration. We divide this region into four LOOPs as shown in Fig.7(d). These LOOPs 1,2,3 and 4 are triangles each with three sides. After the LOOPs are defined, the number of elements for each LOOP is selected to produce the mesh shown in Fig. 7(c).The complete mesh is shown in Fig.7(b)

Fig.7(a) Region R to be analyzed

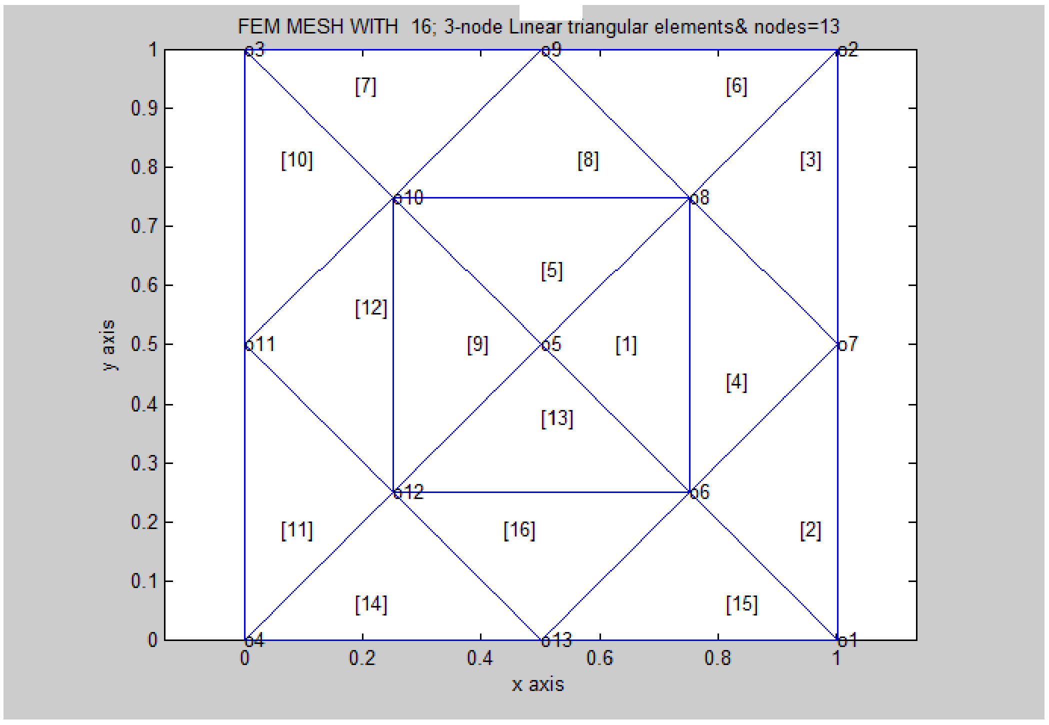
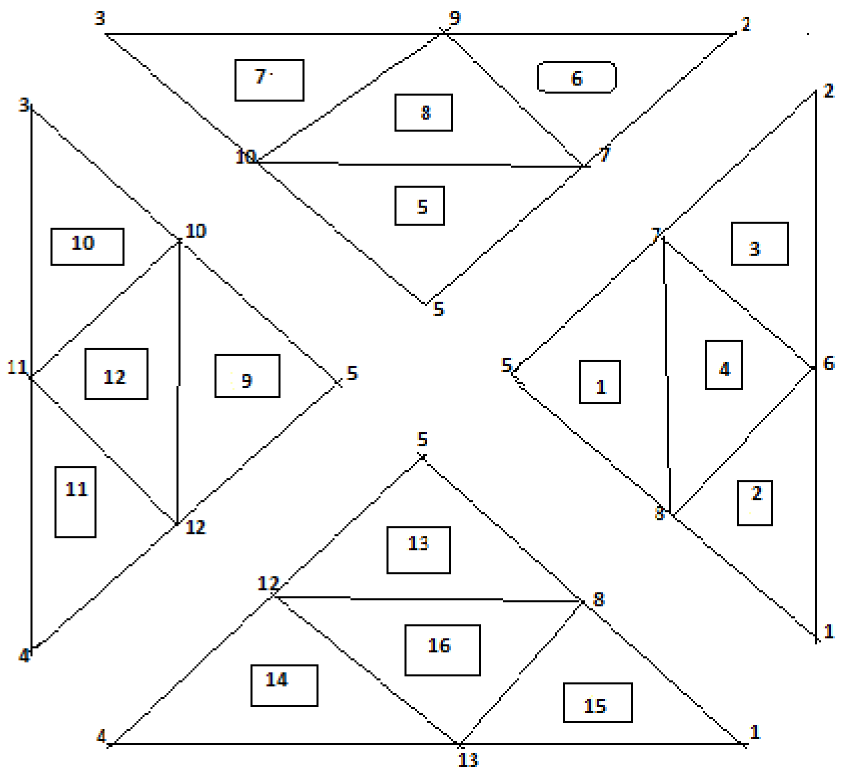
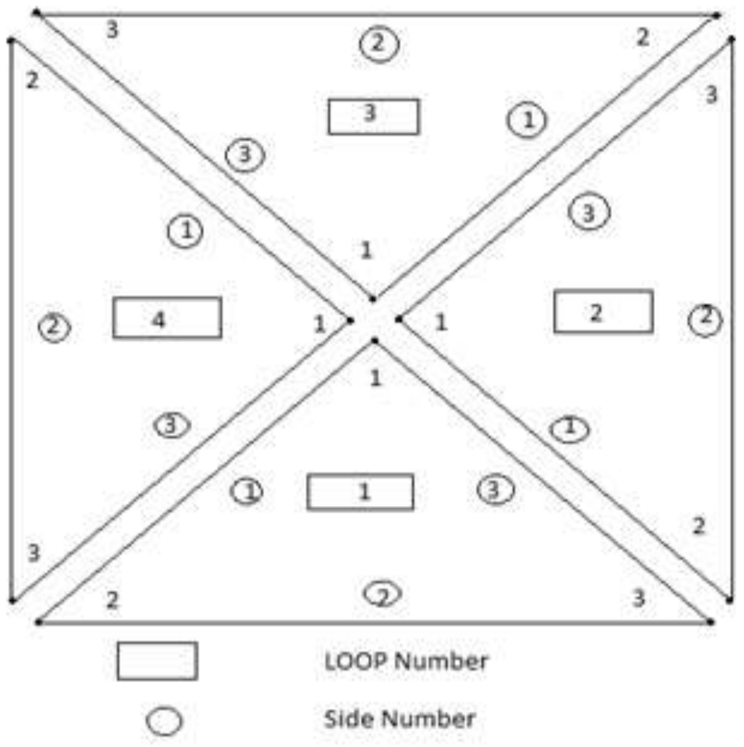


Fig.7(b) Example of completed mesh



7(c) Exploded view showing four loops



7(d) Example of a loop and side numbering scheme

4 Mesh Generation Scheme for Convex Polygonal Domains

How to define the LOOP geometry, specify the number of elements and piece together the LOOPs will now be explained

Joining LOOPs : A complete mesh is formed by piecing together LOOPs. This piecing is done sequentially thus, the first LOOP formed is the foundation LOOP, with subsequent LOOPs joined either to it or to other LOOPs that have already been defined. As each LOOP is defined, the user must specify for each of the three sides of the current LOOP.

In the present mesh generation code, we aim to create a convex polygon. This requires a simple procedure. We join side 3 0f LOOP 1 to side 1 of LOOP 2, side 3 of LOOP 2 will joined to side 1 of LOOP 3, side 3 of LOOP 3 will be joined to side 1 of LOOP 4. Finally side 3 of LOOP 4 will be joined to side 1 of LOOP 1.

When joining two LOOPs, it is essential that the two sides to be joined have the same number of divisions. Thus the number of divisions remains the same for all the LOOPs. We note that the sides of LOOP (i) and side of LOOP $(i+1)$ share the same node numbers. But we have to reverse the sequencing of node numbers of side 3 and assign them as node numbers for side 1 of LOOP $(i+1)$. This will be required for allowing the anticlockwise numbering for element connectivity

5. Application Examples

5.1 Mesh Generation Over an Arbitrary Triangle

In applications to boundary value problems due to symmetry considerations, we may have to discretize an arbitrary triangle. Our purpose is to have a code which automatically generates triangulations of the domain by assuming the input as coordinates of the vertices. We use the theory and procedure developed in section 2 and section 3 of this paper for this purpose. The following MATLAB codes are written for this purpose.

(1) triangulation4polygonal_domain_coordinates.m
(2) nodaladdresses_for4XnXn_LinearTriangles.m
(3) generate_area_coordinate_over_standard_triangle. m
(4) triangular_mesh4LinearTriangles_t3.m

We have included some meshes generated by using the above codes written in MATLAB. We illustrate the mesh generation for a standard triangle and an arbitrary triangle.Some sample commands to generate these meshes are included in comment lines. In all the codes sample input data is included for easy access.

5.2 Mesh Generation for a Convex Polygonal Domain

In several physical applications in science and engineering, the boundary value problem require meshes generated over convex polygons. Again our aim is to have a code which automatically generates a mesh of linear triangles for the complex domains such as those in [21,22]. We use the theory and procedure developed in sections 2, 3 and 4 for this purpose. The following MATLAB codes are written for this purpose.

(1) triangular_mesh4LinearTriangles_t3.m
(2) nodaladdresses_for4XnXn_LinearTriangles_trial.m
(3) triangulation4polygonal_domain_coordinates.m
(4) generate_area_coordinate_over_standard_triangle. m

We have included some meshes generated by using the above MATLAB codes. We further illustrate the application of above codes by generating meshes for a square, five sided and 20 sided regular convex polygons and cracked convex polygons. Some sample commands to generate the polygons stated above appear in the comment lines of programs. In all the cases the sample data is a part of the codes. The development of the MATLAB code is partly inspired by a procedure given in [23] and the literature available on the internet[24]

6 Conclusions

An automatic indirect triangular mesh generator which uses the splitting technique is presented for the two dimensional convex polygonal domains. This mesh generation is made fully automatic and allows the user to define the problem domain with minimum amount of input such as coordinates of boundary. Once this input is created, by selecting an appropriate interior point of the convex polygonal domain, we form the triangular subdomains. These subdomains are then triangulated to generate a fine mesh of six node triangular elements. We have then proposed an automatic 6-node triangular to 3-node triangularl conversion scheme in which each isolated 6-node triangle is split into four 3-node triangles according to the usual scheme, adding three vertices in the middle of the edges and joining them by straight lines of the 6-node triangular element. This task is made a bit simple since a fine mesh of six node triangles is first generated. Further, to preserve the mesh conformity a similar procedure is also applied to every triangle of the domain and this discretizes the given convex polygonal domain into all 3-node triangles, thus propogating a uniform refinement. This simple method generates high quality mesh whose elements confirm well to the requested shape by refining the problem domain. We have also appended MATLAB programs which provide the nodal coordinates, element nodal connectivity and graphic display of the generated all triangular mesh for the standard triangle, an arbitrary triangle, a square and an arbitrary convex polygonal domain. We believe that this work will be useful for various applications in science and engineering.

References

[1] Zienkiewicz. O. C, Philips. D. V, An automatic mesh generation scheme for plane and curved surface by isoparametric coordinates, Int. J.Numer. Meth.Eng, 3, 519-528 (1971)

[2] Gardan. W. J and Hall. C. A, Construction of curvilinear coordinates systems and application to mesh generation, Int. J.Numer. Meth. Eng 3, 461-477 (1973)

[3] Cavendish. J. C, Automatic triangulation of arbitrary planar domains for the finite element method, Int. J. Numer. Meth. Eng 8, 679-696 (1974)

[4] Moscardini. A. O , Lewis. B. A and Gross. M A G T H M – automatic generation of triangular and higher order meshes, Int. J. Numer. Meth. Eng, Vol 19, 1331-1353(1983)

[5] Lewis. R. W, Zheng. Y, and Usmam. A. S, Aspects of adaptive mesh generation based on domain decomposition and Delaunay triangulation, Finite Elements in Analysis and Design 20, 47-70 (1995)

[6] W. R. Buell and B. A. Bush, Mesh generation a survey, J. Eng. Industry. ASME Ser B. 95 332-338(1973)

[7] Rank. E, Schweingruber and Sommer. M, Adaptive mesh generation and transformation of triangular to quadrilateral meshes, Comn. Appl. Numer. Methods 9, 11 121-129(1993)

[8] Ho-Le. K, Finite element mesh generation methods, a review and classification, Computer Aided Design Vol.20, 21-38(1988)

[9] Lo. S. H, A new mesh generation scheme for arbitrary planar domains, Int. J. Numer. Meth. Eng. 21, 1403-1426(1985)

[10] Peraire. J. Vahdati. M, Morgan. K and Zienkiewicz. O. C, Adaptive remeshing for compressible flow computations, J. Comp. Phys. 72, 449-466(1987)

[11] George. P.L, Automatic mesh generation, Application to finite elements, New York , Wiley (1991)

[12] George. P. L, Seveno. E, The advancing-front mesh generation method revisited. Int. J.Numer. Meth. Eng 37, 3605-3619(1994)

[13] Pepper. D. W, Heinrich. J. C, The finite element method, Basic concepts and applications, London, Taylor and Francis (1992)

[14] Zienkiewicz. O. C, Taylor. R. L and Zhu. J. Z, The finite element method, its basis and fundamentals, 6th Edn, Elsevier (2007)

[15] Masud. A, Khurram. R. A, A multiscale/stabilized finite element method for the advection- diffusion equation, Comput. Methods. Appl. Mech. Eng 193, 1997-2018(2004)
[16] Johnston. B.P, Sullivan. J. M and Kwasnik. A, Automatic conversion of triangular finite element meshes to quadrilateral elements, Int. J. Numer. Meth. Eng. 31, 67-84(1991)

[17] Lo. S. H, Generating quadrilateral elements on plane and over curved surfaces, Comput. Stuct. 31(3) 421-426(1989)

[18] Zhu. J, Zienkiewicz. O. C, Hinton. E, Wu. J, A new approach to development of automatic quadrilateral mesh generation, Int. J. Numer. Meth. Eng 32(4), 849-866(1991)

[19] Lau. T. S, Lo. S. H and Lee. C. S, Generation of quadrilateral mesh over analytical curved surfaces, Finite Elements in Analysis and Design, 27, 251-272(1997)

[20] Park. C, Noh. J. S, Jang. I. S and Kang. J. M, A new automated scheme of quadrilateral mesh generation for randomly distributed line constraints, Computer Aided Design 39, 258- 267(2007)
[21]Moin.P, Fundamentals of Engineering Numerical Analysis,second edition,Cambridge University Press(2010)
[22]Fausett.L.V, Applied Numerical Analysis Using MATLAB, second edition, Pearson Education.Inc(2008)
[23]Thompson.E.G, Introduction to the finite element method,John Wiley & Sons Inc.(2005)

[24]ProgramMESHGEN:www.ce.memphis.edu/7111/notes/fem_code/MESHGEN_ tutorial.pdf
COMPUTER PROGRAMS IN MATLAB

```
%PROGRAM-(1) triangulation4polygonal_domain_coordinates.m
function[coord,gcoord,tnodes,nodetel,nnode,nel]=triangulation4polygonal_domain_coordinates(n1,n2,n3,nmax,numtri,n,mesh)
%n1=node number at(0,0)for  a choosen triangle
%n2=node number at(1,0)for  a choosen triangle
%n3=node number at(0,1)for  a choosen triangle
%eln=6-node triangles with centroid
%spqd=4-node special convex quadrilateral
%n must be even,i.e.n=2,4,6,.......i.e number of divisions
%nmax=one plus the number of segments of the polygon
%nmax=the number of segments of the polygon plus a node interior to the polygon
%numtri=number of T6 triangles in each segment i.e a triangle formed by
%joining the end poits of the segment to the interior point(e.g:the centroid) of the polygon
%[eln,spqd]=nodaladdresses_special_convex_quadrilaterals_trial(n1=1,n2=2,n3=3,nmax=3,n=2,4,6,...)
%[eln,spqd]=nodaladdresses_special_convex_quadrilaterals_trial([1;1;1;1],[2;3;4;5],[3;4;5;2],5,1,2)
%[eln,spqd]=nodaladdresses_special_convex_quadrilaterals_trial([1;1;1;1],[2;3;4;5],[3;4;5;2],5,4,4)
%[eln,spqd]=nodaladdresses_special_convex_quadrilaterals_trial([1;1;1;1],[2;3;4;5],[3;4;5;2],5,9,6)
%[eln,spqd]=nodaladdresses_special_convex_quadrilaterals_trial([1;1;1;1],[2;3;4;5],[3;4;5;2],5,16,8)
%PARVIZ MOIN EXAMPLE
syms U V W xi yi

switch mesh
case 1%for MOIN POLYGON
x=sym([1/2;1/2;1;  1;1/2;0;  0;0])%for MOIN EXAMPLE
y=sym([1/2;  0;0;1/2;  1;1;1/2;0])%for MOIN EXAMPLE



case 2%for a unit square: 0<=x,y<=1
x=sym([1/2;1/2;1;  1;  1;1/2;0;  0;0])%FOR UNIT SQUARE
y=sym([1/2;  0;0;1/2;  1;  1;1;1/2;0])%FOR UNIT SQUARE

case 3%for A POLYGON like MOIN OVER(-1/2)<=x,y<=(1/2)
   % 1   2   3  4    5  6   7  8
```

```
  x=sym([0;   0; 1/2;1/2;  0;-1/2;-1/2;-1/2])
  y=sym([0;-1/2;-1/2;  0;1/2; 1/2;   0;-1/2])

case 4%for a unit square: -0.5<=x,y<=0.5
  %       1    2   3   4   5   6    7   8
  x=sym([0;   0; 1/2;1/2;1/2;  0;-1/2;-1/2;-1/2])
  y=sym([0;-1/2;-1/2;  0;1/2;1/2; 1/2;   0;-1/2])
case 5%for a convexpolygonsixside
  %    1    2   3  4   5  6 7
x=sym([0.5;0.1;0.7;1;.75;.5;0])
y=sym([0.5;0;0.2;.5;.85;1;.25])
case 6%standard triangle
x=sym([0;1;0])
y=sym([0;0;1])
case 7%equilateral triangle

%x=sym([0;1;1/2])
%y=sym([0;0;sqrt(3)/2])
x=sym([-1/2;1/2;0])
y=sym([0;0;sqrt(3)/2])
case 8%for a convexpolygonsixside
  %    1    2   3  4   5  6  7  8
x=sym([0.5;0.1;0.7;1;.75;.5;.25;0])
y=sym([0.5;0;0.2;.5;.85;1;.625;.25])
case 9%for a convexpolygeightside
%   1  2   3  4   5    6  7     8   9
x=([.2;0.5;.8;1.0;.75;0.5;0.25;0.0;0.5])
y=([.2;.05;.2;0.5;.85;1.0;0.90;0.6;0.5])
case 10
%   1  2   3  4   5    6  7     8   9
x=([0.5;.2;0.5;.8;1.0;.75;0.5;0.25;0.0])
y=([0.5;.2;.05;.2;0.5;.85;1.0;0.90;0.6])
 case 11 %a square
 %  1 2 3 4  5
%x=([0;1;1;0;0.5])
%y=([0;0;1;1;0.5])
 %     1 2 3 4  5
x=sym([1;1;0;0;0.5])
y=sym([0;1;1;0;0.5])
case 12 %a 2-square [-1,1]x[-1,1]
 %       1    2   3   4   5   6    7   8    9
  x=sym([0;   0;  1;  1;  1;  0;  -1;  -1; -1])
  y=sym([0;  -1; -1;  0;  1;  1;   1;   0; -1])
case 13%for a unit square: 0<=x,y<=1
x=sym([1/2;1/2;1;  1;  1;1/2;0;  0;0])%FOR UNIT SQUARE
y=sym([1/2;  0;0;1/2;  1;  1;1;1/2;0])%FOR UNIT SQUARE
case 14%for a unit square: 0<=x,y<=1
x=sym([1/2;1/2;1;  1;  1;1/2;0;  0;0])%FOR UNIT SQUARE
y=sym([1/2;  0;0;1/2;  1;  1;1;1/2;0])%FOR UNIT SQUARE
case 15%for a unit square: 0<=x,y<=1
x=sym([1/2;1/2;1;  1;  1;1/2;0;  0;0])%FOR UNIT SQUARE
y=sym([1/2;  0;0;1/2;  1;  1;1;1/2;0])%FOR UNIT SQUAREpi

case 16 %a 2-square [-1,1]x[-1,1]
 %       1    2   3   4   5   6    7   8    9
  x=sym([0;   0;  1;  1;  1;  0;  -1;  -1; -1])
  y=sym([0;  -1; -1;  0;  1;  1;   1;   0; -1])

case 17%isosceles triangle(one triangle in a square-required for torsion analysis)
x=sym([0;sqrt(2)/2; -sqrt(2)/2])
y=sym([0;sqrt(2)/2;sqrt(2)/2])
case 18%isoscles triangle(torsion of an equilateral triangle,each side=2*sqrt(3))
```

```
 x=sym([-sqrt(3);sqrt(3); 0])
 y=sym([     -1;      -1; 2])
 case 19%triangle inscribed in a circle of radius=1(torsion of an equilateral triangle each side=sqrt(3))
 x=sym([-sqrt(3)/2;sqrt(3)/2; 0])
 y=sym([     -1/2;      -1/2; 1])
 case 20%square inscribed in a circle of radius=1,required for torsion analysis
%      1      2         3      4         5
x=sym([0;-sqrt(2)/2; sqrt(2)/2;sqrt(2)/2;-sqrt(2)/2])
y=sym([0;-sqrt(2)/2;-sqrt(2)/2;sqrt(2)/2; sqrt(2)/2])
case 21%regular pentagon inscribed in a circle of radius=1,required for torsion analysis
 %      1      2   3      4          5           6
x=sym([0;cos(pi/10);0;-cos(pi/10);-cos(3*pi/10); cos(3*pi/10)])
y=sym([0;sin(pi/10);1; sin(pi/10);-sin(3*pi/10);-sin(3*pi/10)])
case 22%regular hexagon inscribed in a circle of radius=1,required for torsion analysis
    %      1 2   3         4        5     6          7
    x=sym([0;1;cos(pi/3);-cos(pi/3);-1; -cos(pi/3); cos(pi/3)])
    y=sym([0;0;sin(pi/3); sin(pi/3); 0; -sin(pi/3);-sin(pi/3)])
    case 23%regular heptagon inscribed in a circle of radius=1,required for torsion analysis
 %     1     2           3             4           5       6       7        8
x=sym([0;-cos(5*pi/14); cos(5*pi/14); cos(pi/14);cos(3*pi/14);0;-cos(3*pi/14);-cos(pi/14)])
y=sym([0;-sin(5*pi/14);-sin(5*pi/14);-sin(pi/14);sin(3*pi/14);1; sin(3*pi/14);-sin(pi/14)])
    case 24 %regular octagon inscribed in a circle of radius=1,required for torsion analysis
 %     1 2     3     4     5       6       7     8     9
x=sym([0;1;cos(pi/4);0;-cos(pi/4);-1;-cos(pi/4); 0; cos(pi/4)])
y=sym([0;0;sin(pi/4);1; sin(pi/4); 0;-sin(pi/4);-1;-sin(pi/4)])
    case 25%(9-gon)nonagon
 %       1     2            3      4          5         6           7             8           9             10
  x=sym([0;cos(pi/18); cos(5*pi/18);0;-cos(5*pi/18);-cos(pi/18);-cos(3*pi/18);-cos(7*pi/18); cos(7*pi/18); cos(3*pi/18)])
  y=sym([0;sin(pi/18); sin(5*pi/18);1; sin(5*pi/18); sin(pi/18);-sin(3*pi/18);-sin(7*pi/18);-sin(7*pi/18);-sin(3*pi/18)])
    case 26%(10-gon)decagon
 %       1 2     3         4          5             6       7     8          9             10       11
  x=sym([0;1;cos(pi/5); cos(2*pi/5);-cos(2*pi/5);-cos(pi/5);-1;-cos(pi/5);-cos(2*pi/5); cos(2*pi/5); cos(pi/5)])
  y=sym([0;0;sin(pi/5); sin(2*pi/5); sin(2*pi/5); sin(pi/5); 0;-sin(pi/5);-sin(2*pi/5);-sin(2*pi/5);-sin(pi/5)])
    case 27%(11-gon)hendecagon
  %      1       2         3      4         5          6             7        8          9            10              11       12
  x=sym([0;cos(3*pi/22);cos(7*pi/22);0;-cos(7*pi/22);-cos(3*pi/22);-cos(pi/22);-cos(5*pi/22);-cos(9*pi/22); cos(9*pi/22);
cos(5*pi/22); cos(pi/22)])
  y=sym([0;sin(3*pi/22);sin(7*pi/22);1; sin(7*pi/22); sin(3*pi/22);-sin(pi/22);-sin(5*pi/22);-sin(9*pi/22);-sin(9*pi/22);-
sin(5*pi/22);-sin(pi/22)])
  case 28%(12-gon)dodecagon
  %      1 2   3           4     5       6          7       8       9        10     11      12      13
  x=sym([0;1;cos(pi/6);cos(pi/3);0;-cos(pi/3);-cos(pi/6);-1;-cos(pi/6);-cos(pi/3); 0; cos(pi/3); cos(pi/6)])
  y=sym([0;0;sin(pi/6);sin(pi/3);1; sin(pi/3); sin(pi/6); 0;-sin(pi/6);-sin(pi/3);-1;-sin(pi/3);-sin(pi/6)])
  case 29%(13-gon)tridecagon
  %      1       2         3          4       5     6               7           8       9              10           11            12          13           14
  x=sym([0;cos(pi/26);cos(5*pi/26);cos(9*pi/26);0;-cos(9*pi/26);-cos(5*pi/26);-cos(pi/26);-cos(3*pi/26);-cos(7*pi/26);-
cos(11*pi/26); cos(11*pi/26); cos(7*pi/26); cos(3*pi/26)])
  y=sym([0;sin(pi/26);sin(5*pi/26);sin(9*pi/26);1; sin(9*pi/26); sin(5*pi/26); sin(pi/26);-sin(3*pi/26);-sin(7*pi/26);-
sin(11*pi/26);-sin(11*pi/26);-sin(7*pi/26);-sin(3*pi/26)])
    case 30%(14-gon)tetradecagon
  %      1 2        3          4            5         6               7           8       9       10          11             12          13           14            15
  x=sym([0;1;cos(2*pi/14);cos(4*pi/14);cos(6*pi/14);-cos(6*pi/14);-cos(4*pi/14);-cos(2*pi/14);-1;-cos(2*pi/14);-cos(4*pi/14);-
cos(6*pi/14); cos(6*pi/14); cos(4*pi/14); cos(2*pi/14)])
  y=sym([0;0;sin(2*pi/14);sin(4*pi/14);sin(6*pi/14); sin(6*pi/14); sin(4*pi/14); sin(2*pi/14); 0;-sin(2*pi/14);-sin(4*pi/14);-
sin(6*pi/14);-sin(6*pi/14);-sin(4*pi/14);-sin(2*pi/14)])
    case 31%(15-gon)pentadecagon
  %      1       2         3            4            5       6    7                8             9       10          11            12         13           14            15
16
   x=sym([0; cos(pi/30);cos(3*pi/30);cos(7*pi/30);cos(11*pi/30);0;-cos(11*pi/30);-cos(7*pi/30);-cos(3*pi/30);-cos(pi/30);-
cos(5*pi/30);-cos(9*pi/30);-cos(13*pi/30); cos(13*pi/30); cos(9*pi/30); cos(5*pi/30)])
   y=sym([0;-sin(pi/30);sin(3*pi/30);sin(7*pi/30);sin(11*pi/30);1; sin(11*pi/30); sin(7*pi/30); sin(3*pi/30);-sin(pi/30);-
sin(5*pi/30);-sin(9*pi/30);-sin(13*pi/30);-sin(13*pi/30);-sin(9*pi/30);-sin(5*pi/30)])
    case 32%(16-gon)hexadecagon
  % 1 2    3      4          5       6      7             8        9     10      11         12         13    14       15            16         17
  x=sym([0;1;cos(pi/8);cos(pi/4);cos(3*pi/8);0;-cos(3*pi/8);-cos(pi/4);-cos(pi/8);-1;-cos(pi/8);-cos(pi/4);-cos(3*pi/8); 0;
cos(3*pi/8); cos(pi/4); cos(pi/8)]);
  y=sym([0;0;sin(pi/8);sin(pi/4);sin(3*pi/8);1; sin(3*pi/8); sin(pi/4); sin(pi/8); 0;-sin(pi/8);-sin(pi/4);-sin(3*pi/8);-1;-sin(3*pi/8);-
sin(pi/4);-sin(pi/8)]);
   case 33%(17-gon)heptadecogon
  %  1         2          3      4             5       6         7             8          9           10          11          12              13              14
15            16              17       18
  x=sym([0;cos(pi/34);cos(5*pi/34);cos(9*pi/34);cos(13*pi/34);0;-cos(13*pi/34);-cos(9*pi/34);-cos(5*pi/34);-cos(pi/34);-
cos(3*pi/34);-cos(7*pi/34);-cos(11*pi/34);-cos(15*pi/34); cos(15*pi/34); cos(11*pi/34); cos(7*pi/34); cos(3*pi/34)]);
  y=sym([0;sin(pi/34);sin(5*pi/34);sin(9*pi/34);sin(13*pi/34);1; sin(13*pi/34); sin(9*pi/34); sin(5*pi/34); sin(pi/34);-
sin(3*pi/34);-sin(7*pi/34);-sin(11*pi/34);-sin(15*pi/34);-sin(15*pi/34);-sin(11*pi/34);-sin(7*pi/34);-sin(3*pi/34)]);
 case 34%(18-gon)octadecogon
 %  1 2          3      4             5            6          7            8            9            10        11          12           13              14           15
16              17       18              19
 x=sym([0;1;cos(4*pi/36);cos(8*pi/36);cos(12*pi/36);cos(16*pi/36);-cos(16*pi/36);-cos(12*pi/36);-cos(8*pi/36);-cos(4*pi/36);-
1;-cos(4*pi/36);-cos(8*pi/36);-cos(12*pi/36);-cos(16*pi/36); cos(16*pi/36); cos(12*pi/36); cos(8*pi/36); cos(4*pi/36)]);
 y=sym([0;0;sin(4*pi/36);sin(8*pi/36);sin(12*pi/36);sin(16*pi/36); sin(16*pi/36); sin(12*pi/36); sin(8*pi/36); sin(4*pi/36); 0;-
sin(4*pi/36);-sin(8*pi/36);-sin(12*pi/36);-sin(16*pi/36);-sin(16*pi/36);-sin(12*pi/36);-sin(8*pi/36);-sin(4*pi/36)]);
case 35%(19-gon)enneadecogon
 %  1        2           3          4            5       6      7            8            9            10           11         12            13              14            15
16                  17      18              19            20
 x=sym([0;cos(3*pi/38);cos(7*pi/38);cos(11*pi/38);cos(15*pi/38);0;-cos(15*pi/38);-cos(11*pi/38);-cos(7*pi/38);-cos(3*pi/38);-
cos(pi/38);-cos(5*pi/38);-cos(9*pi/38);-cos(13*pi/38);-cos(17*pi/38); cos(17*pi/38); cos(13*pi/38); cos(9*pi/38); cos(5*pi/38);
cos(pi/38)]);
 y=sym([0;sin(3*pi/38);sin(7*pi/38);sin(11*pi/38);sin(15*pi/38);1; sin(15*pi/38); sin(11*pi/38); sin(7*pi/38); sin(3*pi/38);-
sin(pi/38);-sin(5*pi/38);-sin(9*pi/38);-sin(13*pi/38);-sin(17*pi/38);-sin(17*pi/38);-sin(13*pi/38);-sin(9*pi/38);-sin(5*pi/38);-
sin(pi/38)]);
  case 36%(20-gon)icosagon
 % 1 2     3          4           5          6      7          8            9            10          11   12      13           14            15            16      17
18              19             20      21
x=sym([0;1;cos(pi/10);cos(2*pi/10);cos(3*pi/10);cos(4*pi/10);0;-cos(4*pi/10);-cos(3*pi/10);-cos(2*pi/10);-cos(pi/10);-1;-
cos(pi/10);-cos(2*pi/10);-cos(3*pi/10);-cos(4*pi/10); 0; cos(4*pi/10); cos(3*pi/10); cos(2*pi/10); cos(pi/10)]);
y=sym([0;0;sin(pi/10);sin(2*pi/10);sin(3*pi/10);sin(4*pi/10);1; sin(4*pi/10); sin(3*pi/10); sin(2*pi/10); sin(pi/10); 0;-sin(pi/10);-
sin(2*pi/10);-sin(3*pi/10);-sin(4*pi/10);-1;-sin(4*pi/10);-sin(3*pi/10);-sin(2*pi/10);-sin(pi/10)]);

case 50%isoscles triangle(torsion of an equilateral triangle,each side=1 unit)
 x=sym([-1/2;1/2;       0])
 y=sym([   0;  0;sqrt(3)/2])

 case 51%one special quadrilateral with centroid in a triangle(torsion of an equilateral triangle,each side=1 unit)
%               1         2         3         4        5
 x=sym([          0;        0;      1/4;        0;     -1/4])
 y=sym([7*sqrt(3)/24;sqrt(3)/6;sqrt(3)/4;sqrt(3)/2;sqrt(3)/4])
 case 52
 x=sym([0;1;1])
 y=sym([0;0;1])
  case 53
 x=sym([0;1;0])
 y=sym([0;1;1])
 case 54
 x=sym([0;1;1;0;-1; 0])
 y=sym([0;0;1;1; 0;-1])
  case 55
 %      1  2  3 4 5  6
 x=sym([0;-1; 1;2;0;-2])
 y=sym([0;-1;-1;0;1; 0])

case 56
 x=sym([0;1;1;0;-1])
 y=sym([0;0;1;1;-1])
```

```
end
if (nmax>3)
%[eln,trielm,rrr,tnodes,nodetel]=nodaladdresses_for4XnXn_LinearTriangles_trial(n1,n2,n3,nmax,numtri,n);
[eln,trielm,rrr,tnodes,nodetel]=nodaladdresses_for4XnXn_LinearTriangles_trial(n1,n2,n3,nmax,numtri,n);
end
if (nmax==3)
    [eln,trielm,rrr,tnodes,nodetel]=nodaladdresses_for4XnXn_LinearTriangles(n);
end
[U,V,W]=generate_area_coordinate_over_the_standard_triangle(n);
ss1='number of 6-node triangles =';
[p1,q1]=size(eln);
disp([ss1 num2str(p1)])
%
eln
%
ss2='number of triangular elements&nodes per element =';
[nel,nnel]=size(trielm);
disp([ss2 num2str(nel) ','  num2str(nnel)])
%
trielm
%
nnode=max(max(trielm));
ss3='number of nodes of the triangular domain& number of triangular elements=';
disp([ss3 num2str(nnode) ',' num2str(nel)])
  nitri=nmax-1;

if (nmax==3)
    nitri=1;
end
for itri=1:nitri
disp('vertex nodes of the itri triangle')
[n1(itri,1) n2(itri,1) n3(itri,1)]
x1=x(n1(itri,1),1)
x2=x(n2(itri,1),1)
x3=x(n3(itri,1),1)
%
y1=y(n1(itri,1),1)
y2=y(n2(itri,1),1)
y3=y(n3(itri,1),1)
rrr(:,:,itri)
U'
V'
W'
kk=0;
for ii=1:n+1
    for jj=1:(n+1)-(ii-1)
        kk=kk+1;
        mm=rrr(ii,jj,itri);
        uu=U(kk,1);vv=V(kk,1);ww=W(kk,1);
        xi(mm,1)=x1*ww+x2*uu+x3*vv;
        yi(mm,1)=y1*ww+y2*uu+y3*vv;
    end
end
[xi yi]
%add coordinates of centroid
 ne=(n/2)^2;
% stdnode=kk;
 %for iii=1+(itri-1)*ne:ne*itri
     %kk=kk+1;
  %   node1=eln(iii,1)
   %  node2=eln(iii,2)
    % node3=eln(iii,3)
    % mm=eln(iii,7)
    % xi(mm,1)=(xi(node1,1)+xi(node2,1)+xi(node3,1))/3;
     %yi(mm,1)=(yi(node1,1)+yi(node2,1)+yi(node3,1))/3;
 %end

end%for itri
N=(1:nnode)'
[N xi yi]
%
coord(:,1)=(xi(:,1));
coord(:,2)=(yi(:,1));
gcoord(:,1)=double(xi(:,1));
gcoord(:,2)=double(yi(:,1));
%disp(gcoord)

%=====================================================================
%PROGRAM-(2) nodaladdresses_for4XnXn_LinearTriangles.m

function[eln,trielm,rrr,tnodes,nodetel]=nodaladdresses_for4XnXn_LinearTriangles(n)
%we first generate 6-node triangles
%hence it is necessary that n is an even number,i.e: n=2,4,6,8......etc
%this generates (n/2)^2 triangles with 6-nodes each
%in each six node triangle we can make 4-Linear Triangles
%standard triangle is divided into n^2 right isoscles
%triangles each of side length (1/n) units
%computes nodal connections of these right isiscles triangles
%assumes nodal addresses for the standard triangle has local nodes
%as {1,2,3} which correspond to global nodes {1,(n+1),(n+1)*(n+2)/2}
%respectively and then generates  nodal addresses for
%six node triangles and special convex quadrilaterals
%eln=6-node triangles
%triel=3-node linear triangular elements created in 6-node triangle
%n=number of divisions of a side and n must be even,i.e.n=2,4,6,.......
%syms mst_tri x
%disp('vertex nodes of triangle')
elm(1,1)=1;
elm(n+1,1)=2;
elm((n+1)*(n+2)/2,1)=3;
%disp('vertex nodes of triangle')
%base edge
kk=3;
for k=2:n
    kk=kk+1;
    elm(k,1)=kk;
end
%disp('left edge nodes')
nni=1;
for i=0:(n-2)
    nni=nni+(n-i)+1;
    elm(nni,1)=3*n-i;
end
%disp('right edge nodes')
nni=n+1;
for i=0:(n-2)
    nni=nni+(n-i);
    elm(nni,1)=(n+3)+i;
end

%disp('interior nodes')
nni=1;jj=0;
for i=0:(n-3)
    nni=nni+(n-i)+1;
    for j=1:(n-2-i)
        jj=jj+1;
        nnj=nni+j;
        elm(nnj,1)=3*n+jj;
    end
end
%disp(elm)
%disp(length(elm))

jj=0;kk=0;
for j=0:n-1
    jj=j+1;
for k=1:(n+1)-j
    kk=kk+1;
    row_nodes(jj,k)=elm(kk,1);
end
end
row_nodes(n+1,1)=3;
%for jj=(n+1):-1:1
%    (row_nodes(jj,:));
%end
%[row_nodes]
rr=row_nodes;




rr=row_nodes;
rr
rrr(:,:,1)=rr;

%rr
%disp('element computations')
if rem(n,2)==0
ne=0;N=n+1;

for k=1:2:n-1
N=N-2;
i=k;
for j=1:2:N
    ne=ne+1;
eln(ne,1)=rr(i,j);
eln(ne,2)=rr(i,j+2);
eln(ne,3)=rr(i+2,j);
eln(ne,4)=rr(i,j+1);
eln(ne,5)=rr(i+1,j+1);
eln(ne,6)=rr(i+1,j);
end%i
%me=ne
%N-2
if (N-2)>0
for jj=1:2:N-2
ne=ne+1;
eln(ne,1)=rr(i+2,jj+2);
eln(ne,2)=rr(i+2,jj);
eln(ne,3)=rr(i,jj+2);
eln(ne,4)=rr(i+2,jj+1);;
eln(ne,5)=rr(i+1,jj+1);
eln(ne,6)=rr(i+1,jj+2);
end%jj
end
end%k
```

```
end
%
%ne=(n/2)^2,the number of six node triangles
%for kk=1:ne
%[eln(kk,1:6)]
%end

%nnd=(n+1)*(n+2)/2;
%for kkk=1:ne
 %   nnd=nnd+1;
  %  eln(kkk,7)=nnd;
%end
%
%to generate special quadrilaterals
%mm=0;

%for iel=1:ne
%    for jel=1:3
%    mm=mm+1;
%        switch jel
%         case 1
%          spqd(mm,1:4)=[eln(iel,7) eln(iel,6) eln(iel,1) eln(iel,4)];
%          nodes(mm,1:4)=spqd(mm,1:4);
%          nodetel(mm,1:3)=[eln(iel,2) eln(iel,3) eln(iel,1)];
 %         case 2
 %         spqd(mm,1:4)=[eln(iel,7) eln(iel,4) eln(iel,2) eln(iel,5)];
 %         nodes(mm,1:4)=spqd(mm,1:4);
 %         nodetel(mm,1:3)=[eln(iel,3) eln(iel,1) eln(iel,2)];
 %         case 3
 %         spqd(mm,1:4)=[eln(iel,7) eln(iel,5) eln(iel,3) eln(iel,6)];
 %         nodes(mm,1:4)=spqd(mm,1:4);
 %         nodetel(mm,1:3)=[eln(iel,1) eln(iel,2) eln(iel,3)];
 %        end%switch
 %    end
 %  end

%for mmm=1:mm
   %spqd(:,1:4)
 %end
%
%ss1='number of 6-node triangles with centroid=';
%[p1,q1]=size(eln);
%disp([ss1 num2str(p1)])
%
%eln
%
%ss2='number of 4-node special convex quadrilaterals =';
%[p2,q2]=size(spqd);
%disp([ss2 num2str(p2)])
%
%spqd
%generate 4-Linear triangles in a 6-node triangles,call these as stel
mm=0;mmm=0;
for iel=1:ne
   for jel=1:4
   %mm=mm+1;mmm=mmm+1;
      switch jel
      case 1
          mm=mm+1;mmm=mmm+1;
         trielm(mmm,1:3)=[eln(iel,1) eln(iel,4) eln(iel,6)];
         tnodes(mmm,1:3)=trielm(mmm,1:3);
         nodetel(mm,1:3)=[eln(iel,1) eln(iel,2) eln(iel,3)];
```

```
        case 2
        mm=mm+1;mmm=mmm+1;
        trielm(mmm,1:3)=[eln(iel,2) eln(iel,5) eln(iel,4)];
        tnodes(mmm,1:3)=trielm(mmm,1:3);
        nodetel(mm,1:3)=[eln(iel,2) eln(iel,3) eln(iel,1)];
        case 3
        mm=mm+1;mmm=mmm+1;
        trielm(mmm,1:3)=[eln(iel,3) eln(iel,6) eln(iel,5)];
        tnodes(mmm,1:3)=trielm(mmm,1:3);
        nodetel(mm,1:3)=[eln(iel,3) eln(iel,1) eln(iel,2)];
         case 4
         mmm=mmm+1;
         trielm(mmm,1:3)=[eln(iel,4) eln(iel,5) eln(iel,6)];
         tnodes(mmm,1:3)=trielm(mmm,1:3);


      end%switch
    end
  end

eln
trielm
tnodes
nodetel
==================================================================
```

```
%PROGRAM-(3) generate_area_coordinate_over_the_standard_triangle.m
function[U,V,W]=generate_area_coordinate_over_the_standard_triangle(n)
syms ui vi wi U V W
kk=0;
for j=1:n+1
   for i=1:(n+1)-(j-1)
       kk=kk+1;
       ui(i,j)=(i-1)/n;
       vi(i,j)=(j-1)/n;
       wi(i,j)=1-ui(i,j)-vi(i,j);
       U(kk,1)=ui(i,j);
       V(kk,1)=vi(i,j);
       W(kk,1)=wi(i,j);
   end
end
%  ui
 % vi
  %wi
  U'
  V'
  W'
```

```
%===============================================================================================================
%PROGRAM 4: nodaladdresses_for4XnXn_LinearTriangles_trial.m
function[eln,trielm,rrr,tnodes,nodetel]=nodaladdresses_for4XnXn_LinearTriangles_trial(n1,n2,n3,nmax,numtri,n)
%function[eln,spqd,rrr,nodes,nodetel]=nodaladdresses_special_convex_quadrilaterals_trial(n1,n2,n3,nmax,numtri,n)
%n1=node number at(0,0)for  a choosen triangle
%n2=node number at(1,0)for  a choosen triangle
%n3=node number at(0,1)for  a choosen triangle
%eln=6-node triangles with centroid
%spqd=4-node special convex quadrilateral
%n must be even,i.e.n=2,4,6,.......i.e number of divisions
%nmax=one plus the number of segments of the polygon
%nmax=the number of segments of the polygon plus a node interior to the polygon
%numtri=number of T6 triangles in each segment i.e a triangle formed by
%joining the end poits of the segment to the interior point(e.g:the centroid) of the polygon
%[eln,spqd]=nodaladdresses_special_convex_quadrilaterals_trial(n1=1,n2=2,n3=3,nmax=3,n=2,4,6,...)
%[eln,spqd,rrr,nodes,nodetel]=nodaladdresses_special_convex_quadrilaterals_trial([1;1;1;1],[2;3;4;5],[3;4;5;2],5,1,2)
%[eln,spqd,rrr,nodes,nodetel]=nodaladdresses_special_convex_quadrilaterals_trial([1;1;1;1],[2;3;4;5],[3;4;5;2],5,4,4)
%[eln,spqd,rrr,nodes,nodetel]=nodaladdresses_special_convex_quadrilaterals_trial([1;1;1;1],[2;3;4;5],[3;4;5;2],5,9,6)
```

```
%[eln,spqd,rrr,nodes,nodetel]=nodaladdresses_special_convex_quadrilaterals_trial([1;1;1;1],[2;3;4;5],[3;4;5;2],5,16,8)
%PARVIZ MOIN EXAMPLE
%[eln,spqd,rrr,nodes,nodetel]=nodaladdresses_special_convex_quadrilaterals_trial([1;1;1;1;1;1;1],[2;3;4;5;6;7;8],[3;4;5;6;7;8;2],8
,1,2)
%[eln,spqd,rrr,nodes,nodetel]=nodaladdresses_special_convex_quadrilaterals_trial([1;1;1;1;1;1;1],[2;3;4;5;6;7;8],[3;4;5;6;7;8;2],8
,4,4)
%syms mst_tri x
ne=0;

%nitri=nmax-1;
[nitri,cl]=size(n1)



for itri=1:nitri
    elm(1:(n+1)*(n+2)/2,1)=zeros((n+1)*(n+2)/2,1)
elm(1,1)=n1(itri,1)
elm(n+1,1)=n2(itri,1)
elm((n+1)*(n+2)/2,1)=n3(itri,1)
disp('vertex nodes of the itri triangle')
[n1(itri,1) n2(itri,1) n3(itri,1)]
if itri==1
kk=nmax;
for k=2:n
    kk=kk+1
    elm(k,1)=kk
end
disp('base nodes=')
%elm(2:n)
edgen1n2(1:n+1,itri)=elm(1:n+1,1)
end%itri==1
if itri>1
    elm(1:n+1,1)=edgen1n3(1:n+1,itri-1);
end%if itri>1
if itri==1
    lmax=nmax+3*(n-1);
end%if itri==1
if (itri>1)&(itri<nitri)
    lmax=nmax+2*(n-1);
end% if (itri>1)&(itri<nitri)
mmax=nmax;
if itri==1
    mmax=max(max(edgen1n2(1:n+1,1)))
end%f itri==1
disp('right edge nodes')
nni=n+1;hh=1;qq(1,1)=n2(itri,1);
for i=0:(n-2)
    hh=hh+1;
    nni=nni+(n-i);
    elm(nni,1)=(mmax+1)+i;
    qq(hh,1)=(mmax+1)+i;

end
qq(n+1,1)=n3(itri,1);
edgen2n3(1:n+1,itri)=qq;

if itri<nitri
disp('left edge nodes')
nni=1;gg=1;pp(1,1)=n1(itri,1);
for i=0:(n-2)
    gg=gg+1;
    nni=nni+(n-i)+1;
    elm(nni,1)=lmax-i;
    pp(gg,1)=lmax-i;
end
pp(n+1,1)=n3(itri,1);
edgen1n3(1:n+1,itri)=pp
end%if itri<nitri



%if itri==n
% elm(1:n+1,1)=edgen1n2(1:n+1,1)
%end

if itri==nitri
disp('left edge nodes')
nni=1;gg=1;
for i=0:(n-2)
    gg=gg+1;
    nni=nni+(n-i)+1;
    elm(nni,1)=edgen1n2(gg,1);
end
%pp(n+1,1)=n3(itri,1);
%edgen1n3(1:n+1,itri)=pp
end%if itri==nitri
if itri==nitri
lmax=max(max(edgen2n3(1:n+1,itri)));
end%if itri==nitri

%elm
disp('interior nodes')
nni=1;jj=0;
for i=0:(n-3)
    nni=nni+(n-i)+1;
    for j=1:(n-2-i)
        jj=jj+1;
        nnj=nni+j;
        elm(nnj,1)=lmax+jj;
        [nnj lmax+jj];
    end
end
%disp(elm);
%disp(length(elm));

jj=0;kk=0;
for j=0:n-1
    jj=j+1;
for k=1:(n+1)-j
    kk=kk+1;
    row_nodes(jj,k)=elm(kk,1);
end
end
row_nodes(n+1,1)=n3(itri,1);
%for jj=(n+1):-1:1
%    (row_nodes(jj,:));
%end
%[row_nodes]
rr=row_nodes;
rr
rrr(:,:,itri)=rr;
disp('element computations')
if rem(n,2)==0
N=n+1;
```

```
for k=1:2:n-1
N=N-2;
i=k;
for j=1:2:N
    ne=ne+1
eln(ne,1)=rr(i,j);
eln(ne,2)=rr(i,j+2);
eln(ne,3)=rr(i+2,j);
eln(ne,4)=rr(i,j+1);
eln(ne,5)=rr(i+1,j+1);
eln(ne,6)=rr(i+1,j);
end%j
%me=ne
%N-2
if (N-2)>0
for jj=1:2:N-2
ne=ne+1
eln(ne,1)=rr(i+2,jj+2);
eln(ne,2)=rr(i+2,jj);
eln(ne,3)=rr(i,jj+2);
eln(ne,4)=rr(i+2,jj+1);;
eln(ne,5)=rr(i+1,jj+1);
eln(ne,6)=rr(i+1,jj+2);
end%jj
end%if(N-2)>0
end%k

end% if rem(n,2)==0
ne
%for kk=1:ne
%[eln(kk,1:6)]
%end
%add node numbers for element centroids

nnd=max(max(eln))
%if (n>3)
%for kkk=1+(itri-1)*numtri:ne
 %   nnd=nnd+1;
  %  eln(kkk,7)=nnd;
%end
%end
%if n==2
 %for kkk=itri:ne
  %  nnd=nnd+1;
   % eln(kkk,7)=nnd;
 %end
%end
%for kk=1:ne
%[eln(kk,1:7)]
%end
%to generate special quadrilaterals
%mm=0;

%for iel=1:ne
 %   for jel=1:3
  %  mm=mm+1;
   %     switch jel
   %     case 1
   %       spqd(mm,1:4)=[eln(iel,7) eln(iel,6) eln(iel,1) eln(iel,4)];
    %      nodes(mm,1:4)=spqd(mm,1:4);
     %     nodetel(mm,1:3)=[eln(iel,2) eln(iel,3) eln(iel,1)];
     %    case 2
     %    spqd(mm,1:4)=[eln(iel,7) eln(iel,4) eln(iel,2) eln(iel,5)];
      %   nodes(mm,1:4)=spqd(mm,1:4);
       %  nodetel(mm,1:3)=[eln(iel,3) eln(iel,1) eln(iel,2)];
       %  case 3
      %   spqd(mm,1:4)=[eln(iel,7) eln(iel,5) eln(iel,3) eln(iel,6)];
      %   nodes(mm,1:4)=spqd(mm,1:4);
      %   nodetel(mm,1:3)=[eln(iel,1) eln(iel,2) eln(iel,3)];
      % end%switch
      %end
      %end
nmax=max(max(eln));
%nel=mm;
%
%ne
%spqd


end%itri

%generate 4-Linear triangles in a 6-node triangles,call these as stel
mm=0;mmm=0;
for iel=1:ne
    for jel=1:4
    %mm=mm+1;mmm=mmm+1;
       switch jel
       case 1
           mm=mm+1;mmm=mmm+1;
          trielm(mmm,1:3)=[eln(iel,1) eln(iel,4) eln(iel,6)];
          tnodes(mmm,1:3)=trielm(mmm,1:3);
          nodetel(mm,1:3)=[eln(iel,1) eln(iel,2) eln(iel,3)];
          case 2
          mm=mm+1;mmm=mmm+1;
          trielm(mmm,1:3)=[eln(iel,2) eln(iel,5) eln(iel,4)];
          tnodes(mmm,1:3)=trielm(mmm,1:3);
          nodetel(mm,1:3)=[eln(iel,2) eln(iel,3) eln(iel,1)];
          case 3
          mm=mm+1;mmm=mmm+1;
          trielm(mmm,1:3)=[eln(iel,3) eln(iel,6) eln(iel,5)];
          tnodes(mmm,1:3)=trielm(mmm,1:3);
          nodetel(mm,1:3)=[eln(iel,3) eln(iel,1) eln(iel,2)];
           case 4
           mmm=mmm+1;
           trielm(mmm,1:3)=[eln(iel,4) eln(iel,5) eln(iel,6)];
           tnodes(mmm,1:3)=trielm(mmm,1:3);


       end%switch
     end
   end

eln
trielm
tnodes
nodetel

 %to generate special quadrilaterals
%mm=0;

%for iel=1:ne
%    for jel=1:3
 %   mm=mm+1;
 %      switch jel
 %       case 1
 %        spqd(mm,1:4)=[eln(iel,7) eln(iel,6) eln(iel,1) eln(iel,4)];
 %        nodes(mm,1:4)=spqd(mm,1:4);
 %        nodetel(mm,1:3)=[eln(iel,2) eln(iel,3) eln(iel,1)];
 %        case 2
 %        spqd(mm,1:4)=[eln(iel,7) eln(iel,4) eln(iel,2) eln(iel,5)];
 %        nodes(mm,1:4)=spqd(mm,1:4);
 %        nodetel(mm,1:3)=[eln(iel,3) eln(iel,1) eln(iel,2)];
 %        case 3
 %        spqd(mm,1:4)=[eln(iel,7) eln(iel,5) eln(iel,3) eln(iel,6)];
 %        nodes(mm,1:4)=spqd(mm,1:4);
 %        nodetel(mm,1:3)=[eln(iel,1) eln(iel,2) eln(iel,3)];
 %      end%switch
 %   end
 %  end

%for mmm=1:mm
   %spqd(:,1:4)
 %end
%
ss1='number of 6-node triangles =';
[p1,q1]=size(eln);
disp([ss1 num2str(p1)])
%
eln
%
ss2='number of linear triangular elements&nodes per element =';
[nel,nnel]=size(trielm);
disp([ss2 num2str(nel) ','  num2str(nnel)])
%
nnode=max(max(trielm));
ss3='number of nodes of the triangular domain& number of linear triangles =';
disp([ss3 num2str(nnode) ',' num2str(nel)])


%======================================================================================================
%PROGRAM5 triangular_mesh4LinearTriangles_t3.m
function[]=triangular_mesh4LinearTriangles_t3(n1,n2,n3,nmax,numtri,ndiv,mesh,xlength,ylength,ndelete)
%maximumnumber of nodes on the actual domain
%
%clf
%(1)=generate 2-D quadrilateral mesh
%for a rectangular shape of domain
%quadrilateral_mesh_q4(xlength,ylength)
%xnode=number of nodes along x-axis
%ynode=number of nodes along y-axis
%xzero=x-coord of bottom left corner
%yzero=y-coord of bottom left corner
%xlength=size of domain alog x-axis
%ylength=size of domain alog y-axis
%type=0 for closed polygonal domain
%type=1 for open polygonal domain
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1],[2;3;4;5;6;7;8],[3;4;5;6;7;8;2],8,1,2,1,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1],[2;3;4;5;6;7;8],[3;4;5;6;7;8;2],8,4,4,1,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1],[2;3;4;5;6;7;8],[3;4;5;6;7;8;2],8,9,6,1,1,1)
%triangular_mesh4LinearTriangles_t3([1;1;1;1;1;1;1],[2;3;4;5;6;7;8],[3;4;5;6;7;8;2],8,1,2,1,1,1)
%triangular_mesh4LinearTriangles_t3([1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9],[3;4;5;6;7;8;9;2],9,1,2,2,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9],[3;4;5;6;7;8;9;2],9,1,2,2,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9],[3;4;5;6;7;8;9;2],9,4,4,2,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9],[3;4;5;6;7;8;9;2],9,9,6,2,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9],[3;4;5;6;7;8;9;2],9,1,2,4,1,1)
%quadrilateral_mesh4MOINEX_q4([9;9;9;9;9;9;9;9],[1;2;3;4;5;6;7;8],[2;3;4;5;6;7;8;1],9,1,2,9,1,1)
```

```
%quadrilateral_mesh4MOINEX_q4([9;9;9;9;9;9;9;9],[1;2;3;4;5;6;7;8],[2;3;4;5;6;7;8;1],9,4,4,9,1,1)
%quadrilateral_mesh4MOINEX_q4([9;9;9;9;9;9;9;9],[1;2;3;4;5;6;7;8],[2;3;4;5;6;7;8;1],9,9,6,9,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9],[3;4;5;6;7;8;9;2],9,1,2,10,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9],[3;4;5;6;7;8;9;2],9,4,4,10,1,1)
%quadrilateral_mesh4MOINEX_q4([1],[2],[3],3,1,2,6,1,1)
%triangular_mesh4LinearTriangles_t3([1],[2],[3],3,1,2,6,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9],[3;4;5;6;7;8;9;2],9,25,10,2,1,1)
%%quadrilateral_mesh4MOINEX_q4([1],[2],[3],3,1,2,17,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1],[2;3;4;5;6],[3;4;5;6;2],6,9,6,21,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1],[2;3;4;5;6;7],[3;4;5;6;7;2],7,9,6,22,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1],[2;3;4;5;6;7;8],[3;4;5;6;7;8;2],8,1,2,23,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9],[3;4;5;6;7;8;9;2],9,25,10,24,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9;10],[3;4;5;6;7;8;9;10;2],10,1,2,25,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9;10;11],[3;4;5;6;7;8;9;10;11;2],11,1,2,26,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9;10;11;12],[3;4;5;6;7;8;9;10;11;12;2],12,9,6,27,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9;10;11;12;13],[3;4;5;6;7;8;9;10;11;12;13;2],13,9,6,2
8,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9;10;11;12;13;14],[3;4;5;6;7;8;9;10;11;12;13;14;2],
14,1,2,29,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9;10;11;12;13;14;15],[3;4;5;6;7;8;9;10;11;12;13;1
4;15;2],15,1,2,30,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9;10;11;12;13;14;15;16],[3;4;5;6;7;8;9;10;11;12
;13;14;15;16;2],16,1,2,31,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9;10;11;12;13;14;15;16;17],[3;4;5;6;7;8;9;10;
11;12;13;14;15;16;17;2],17,1,2,32,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9;10;11;12;13;14;15;16;17;18],[3;4;5;6;7;8;
9;10;11;12;13;14;15;16;17;18;2],18,1,2,33,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9;10;11;12;13;14;15;16;17;18],[3;4;5;6;7;8;
9;10;11;12;13;14;15;16;17;18;2],18,1,2,33,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9;10;11;12;13;14;15;16;17;18;19],[3;4;5;6
;7;8;9;10;11;12;13;14;15;16;17;18;19;2],19,1,2,34,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9;10;11;12;13;14;15;16;17;18;19;20],[3;
4;5;6;7;8;9;10;11;12;13;14;15;16;17;18;19;20;2],20,1,2,35,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9;10;11;12;13;14;15;16;17;18;19;20;2
1],[3;4;5;6;7;8;9;10;11;12;13;14;15;16;17;18;19;20;21;2],21,1,2,36,1,1)
%quadrilateral_mesh4MOINEX_q4([1;1;1;1],[2;3;4;5],[3;4;5;2],5,1,2,51,1,1)
%triangular_mesh4LinearTriangles_t3([1;1;1;1;1;1;1],[2;3;4;5;6;7;8],[3;4;5;6;7;8;2],8,1,2,1,1,1)
%triangular_mesh4LinearTriangles_t3([1;1;1;1;1],[2;3;4;5;6],[3;4;5;6;2],6,1,2,54,2,2)
%triangular_mesh4LinearTriangles_t3([1;1;1;1;1],[2;3;4;5;6],[3;4;5;6;2],6,1,2,55,4,2)
%triangular_mesh4LinearTriangles_t3([1;1;1;1],[2;3;4;5],[3;4;5;2],5,1,2,56,2,2,0)
%triangular_mesh4LinearTriangles_t3([1;1;1;1],[2;3;4;5],[3;4;5;2],5,1,2,56,2,2,2)
%triangular_mesh4LinearTriangles_t3([5;5;5;5],[1;2;3;4],[2;3;4;1],5,1,2,11,1,1,0)
%triangular_mesh4LinearTriangles_t3([1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1],[2;3;4;5;6;7;8;9;10;11;12;13;14;15;16;17;18;19;2
0;21],[3;4;5;6;7;8;9;10;11;12;13;14;15;16;17;18;19;20;21;2],21,1,2,36,1,1,0)
[nitri,cl]=size(n1)
if(ndelete==0)
  nmax=nitri+1
end

[coord,gcoord,tnodes,nodetel,nnode,nel]=triangulation4polygonal_domain_coordinates(n1,n2,n3,nmax,numtri,ndiv,mesh)
[nel,nnel]=size(tnodes);

disp([xlength,ylength,nnode,nel,nnel])
%gcoord(i,j),where i->node no. and j->x or y
%_____________________________________________________________
%
%plot the mesh for the generated data
%x and y coordinates
xcoord(:,1)=gcoord(:,1);
ycoord(:,1)=gcoord(:,2);
%extract coordinates for each element
%clf
```

```
figure(ndiv/2)
NDEL=ndelete*4*numtri
NEL=nel-NDEL;
NNODE=max(max(tnodes(1:NEL,1:3)));
NNNODE=NNODE
if (ndelete>1)
NNNODE=NNODE-ndelete+1
end

for i=1:NEL
for j=1:nnel
x(1,j)=xcoord(tnodes(i,j),1);
y(1,j)=ycoord(tnodes(i,j),1);
end;%j loop
xvec(1,1:4)=[x(1,1),x(1,2),x(1,3),x(1,1)];
yvec(1,1:4)=[y(1,1),y(1,2),y(1,3),y(1,1)];
axis equal
%axis tight
rtext=0;
switch mesh
case 1
axis([0 xlength 0 ylength])
case 2
axis([0 xlength 0 ylength])
case 3
 xl=xlength/2;yl=ylength/2;
axis([-xl xl -yl yl])
case 4
xl=xlength/2;yl=ylength/2;
axis([-xl xl -yl yl])
case 12
    axis([-xlength xlength -ylength ylength])

case 17
    axis([-xlength xlength 0 ylength])
     rpoly=' one isosceles triangle in a  square';
    rtext=1;
case 18
    axis([-2*xlength 2*xlength -ylength 2*ylength])
    rpoly='   equilateral triangle';
    rtext=1;
case 19
    axis([-xlength xlength -ylength/2 ylength])
     rpoly='   equilateral triangle';
    rtext=1;

 case 20
    axis([-xlength xlength -ylength ylength])
     rpoly='    square';
    rtext=1;
 case 21
    axis([-xlength xlength -ylength ylength])
     rpoly='     pentagon';
    rtext=1;
 case 22
    axis([-xlength xlength -ylength ylength])
     rpoly='     hexagon';
    rtext=1;
  case 23
    axis([-xlength xlength -ylength ylength])
     rpoly='     heptagon';
    rtext=1;
    case 24
    axis([-xlength xlength -ylength ylength])
     rpoly='     octagon';
    rtext=1;
     case 25
    axis([-xlength xlength -ylength ylength])
     rpoly='     nonadecagon';
    rtext=1;
     case 26
    axis([-xlength xlength -ylength ylength])
     rpoly='     decagon';
     rtext=1;
     case 27
    axis([-xlength xlength -ylength ylength])
     rpoly='     hendecagon';
    rtext=1;
     case 28
    axis([-xlength xlength -ylength ylength])
     rpoly='     dodecagon';
    rtext=1;
     case 29
    axis([-xlength xlength -ylength ylength])
     rpoly='     tridecagon';
    rtext=1;
     case 30
    axis([-xlength xlength -ylength ylength])
     rpoly='     tetradecagon';
    rtext=1;
     case 31
    axis([-xlength xlength -ylength ylength])
     rpoly='     pentadecagon';
    rtext=1;
      case 32
    axis([-xlength xlength -ylength ylength])
     rpoly='     hexadecagon';
    rtext=1;
      case 33
    axis([-xlength xlength -ylength ylength])
     rpoly='     heptadecagon';
    rtext=1;
     case 34
    axis([-xlength xlength -ylength ylength])
    rpoly='     octadecagon';
    rtext=1;
     case 35
    axis([-xlength xlength -ylength ylength])
    rpoly='     enneadecagon';
    rtext=1;
  case 36
    axis([-xlength xlength -ylength ylength])
    rpoly='     icosagon';
    rtext=1;

end
plot(xvec,yvec);%plot element
hold on;
%place element number
if ndiv<=4
midx=mean(xvec(1,1:4))
midy=mean(yvec(1,1:4))
text(midx,midy,['[',num2str(i),']']);
end
```

```
end;%i loop

xlabel('x axis')
ylabel('y axis')
st1='FEM MESH WITH  ';
st2=num2str(NEL);
st3='; 3-node Linear ';
st4='triangular';
st5=' elements'
st6='& nodes='
st7=num2str(NNNODE);
title([st1,st2,st3,st4,st5,st6,st7])
%put node numbers
if ndiv<=4
for jj=1:NNODE
 if (ndelete>1)&(jj<=(nmax-ndelete+1))
 text(gcoord(jj,1),gcoord(jj,2),['o',num2str(jj)]);
 GCOORD(jj,1:2)=gcoord(jj,1:2);
 end

 if (ndelete>1)&(jj>=(nmax+1))
 text(gcoord(jj,1),gcoord(jj,2),['o',num2str(jj-ndelete+1)]);
 GCOORD(jj-ndelete+1,1:2)=gcoord(jj,1:2);
 end
if(ndelete==0|(ndelete==1))
text(gcoord(jj,1),gcoord(jj,2),['o',num2str(jj)]);
GCOORD(jj,1:2)=gcoord(jj,1:2);
end
end
end
%
if ndiv>4
for jj=1:NNODE
 if (ndelete>1)&(jj<=(nmax-ndelete+1))
 %text(gcoord(jj,1),gcoord(jj,2),['o',num2str(jj)]);
 GCOORD(jj,1:2)=gcoord(jj,1:2);
 end

 if (ndelete>1)&(jj>(nmax-ndelete+1))
 %text(gcoord(jj,1),gcoord(jj,2),['o',num2str(jj-ndelete+1)]);
 GCOORD(jj-ndelete+1,1:2)=gcoord(jj,1:2);
 end
if((ndelete==0)|(ndelete==1))
%text(gcoord(jj,1),gcoord(jj,2),['o',num2str(jj)]);
GCOORD(jj,1:2)=gcoord(jj,1:2);
end
end
end



if rtext==1
  text(0.1,-1.2,rpoly)
end
%axis off
NEL
NNNODE
GCOORD
%element nodal connectivity matrix
if (ndelete>1)
nnmax=nmax-(ndelete-1)
```

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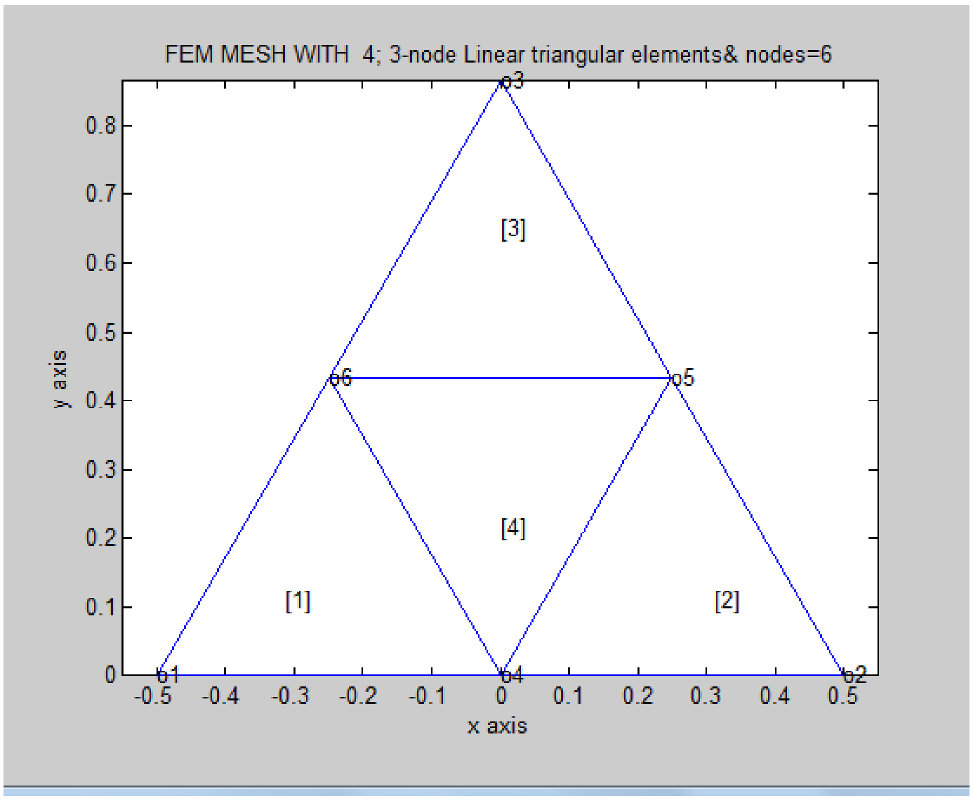
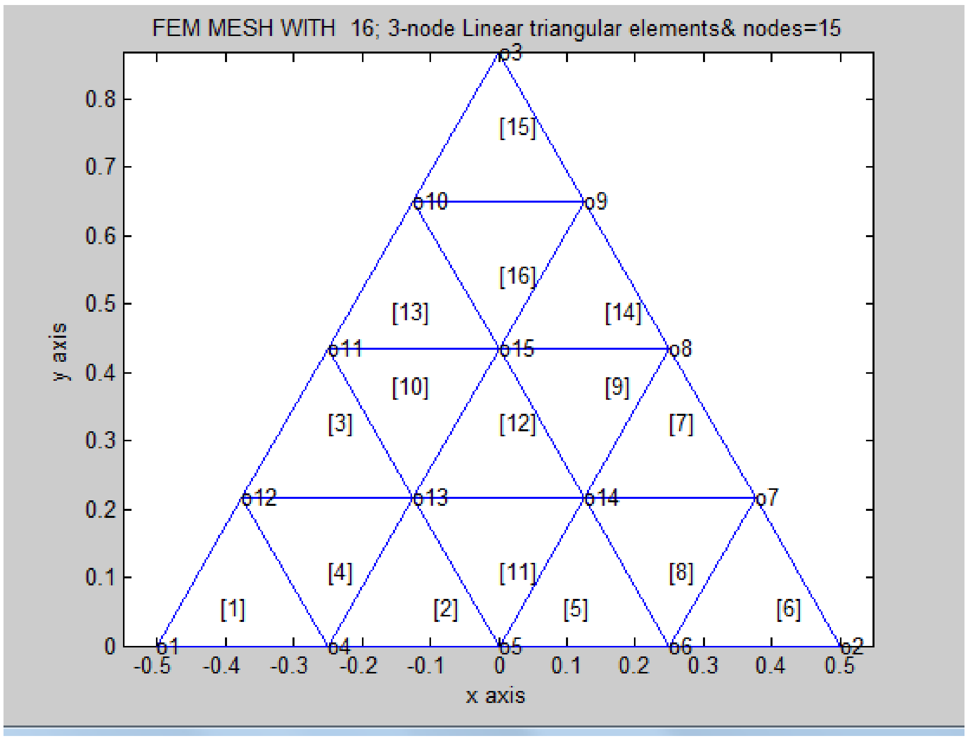
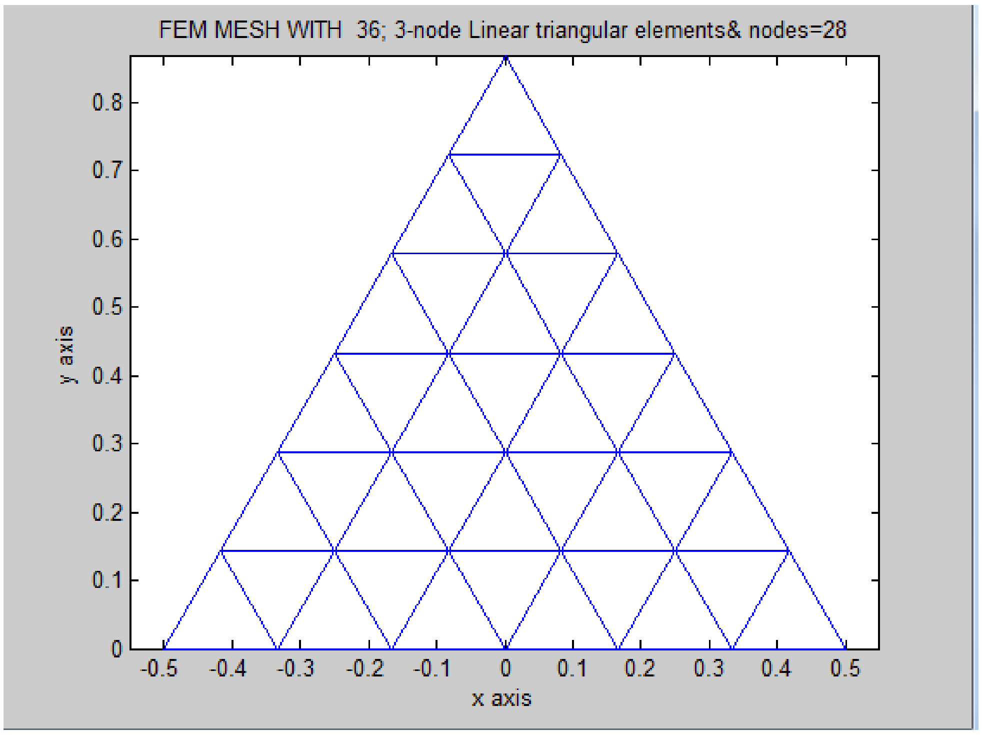
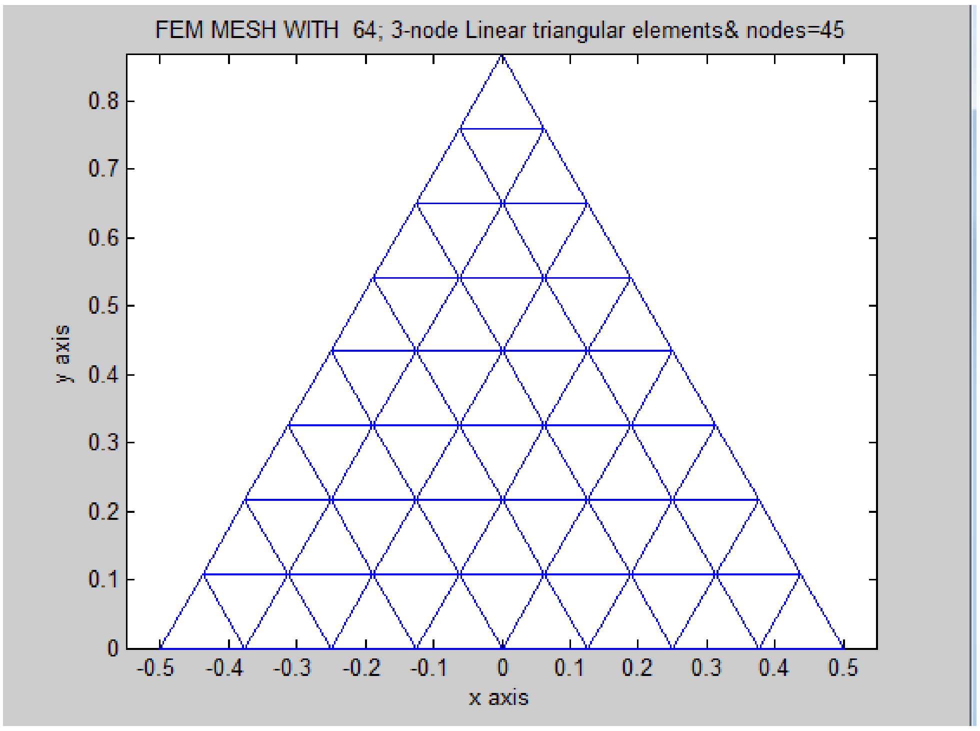
```
for iel=1:NEL
    for jel=1:3
      if(tnodes(iel,jel)>nnmax)
          tnodes(iel,jel)=tnodes(iel,jel)-(ndelete-1);
      end
    end
end
end
tnodes


%=================================================================
```

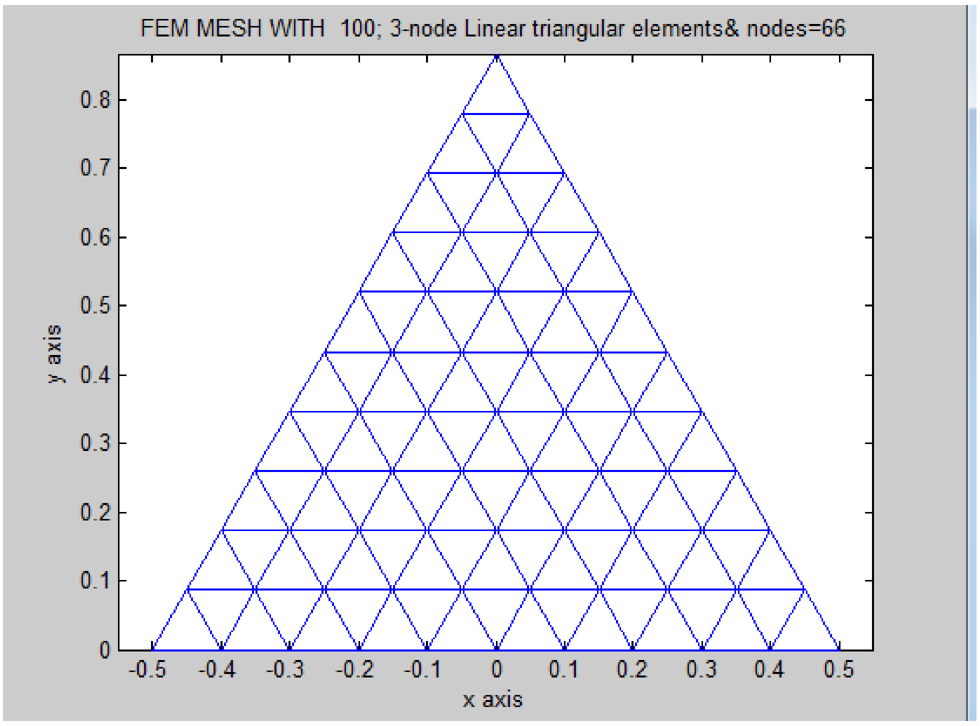
FIGURES



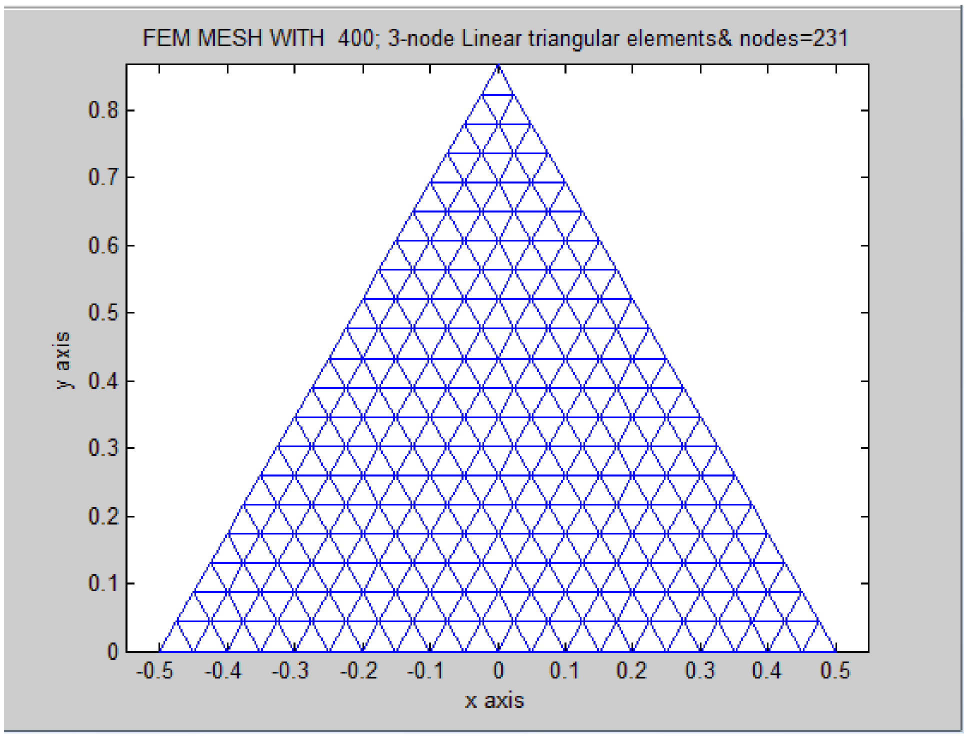

FEM MESH WITH 16; 3-node Linear triangular elements& nodes=15
y axis
x axis
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
-0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4 0.5
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
[1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13] [14] [15] [16]

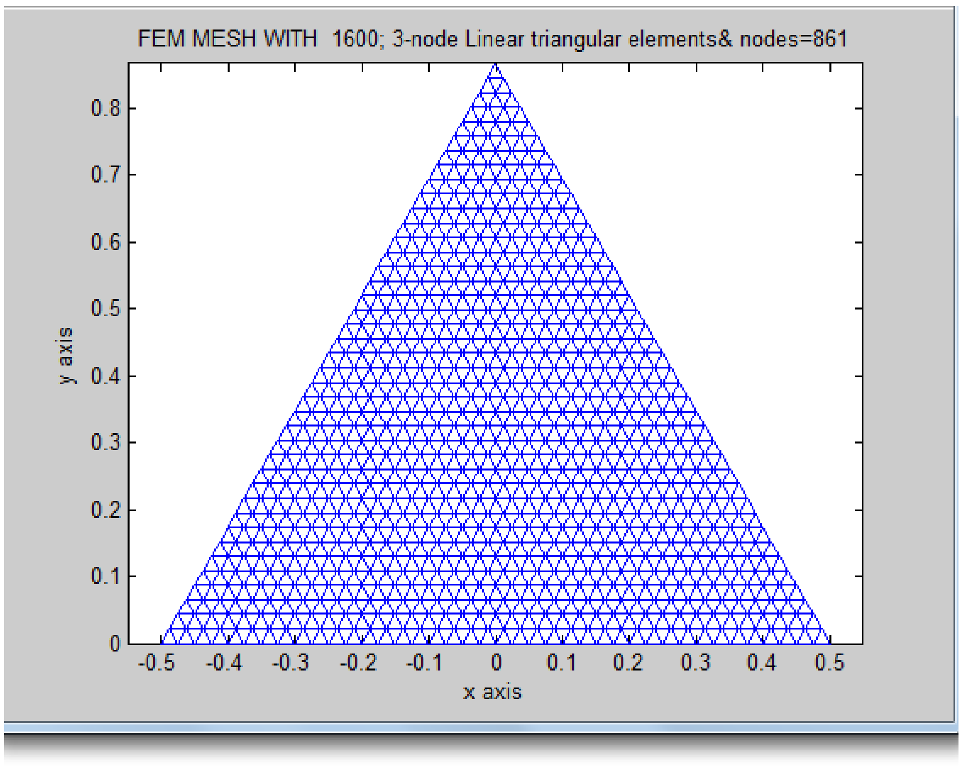

FEM MESH WITH 36; 3-node Linear triangular elements& nodes=28
y axis
x axis
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
-0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4 0.5


FEM MESH WITH 64; 3-node Linear triangular elements& nodes=45
y axis
x axis
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5


FEM MESH WITH 100; 3-node Linear triangular elements& nodes=66
y axis
x axis
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5

DOI: 10.18535/ijecs/v6i8.03





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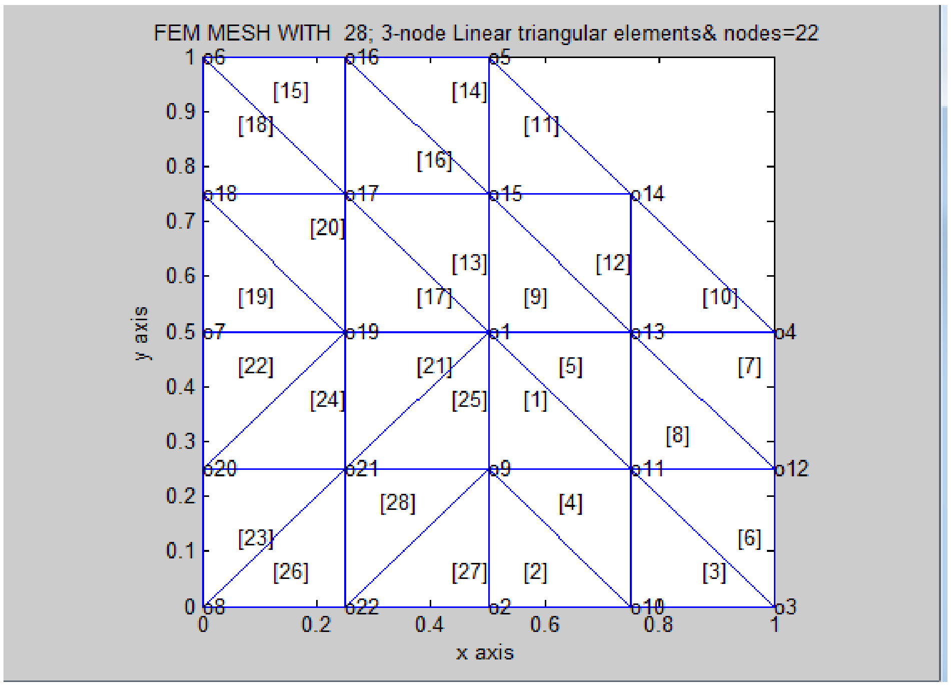
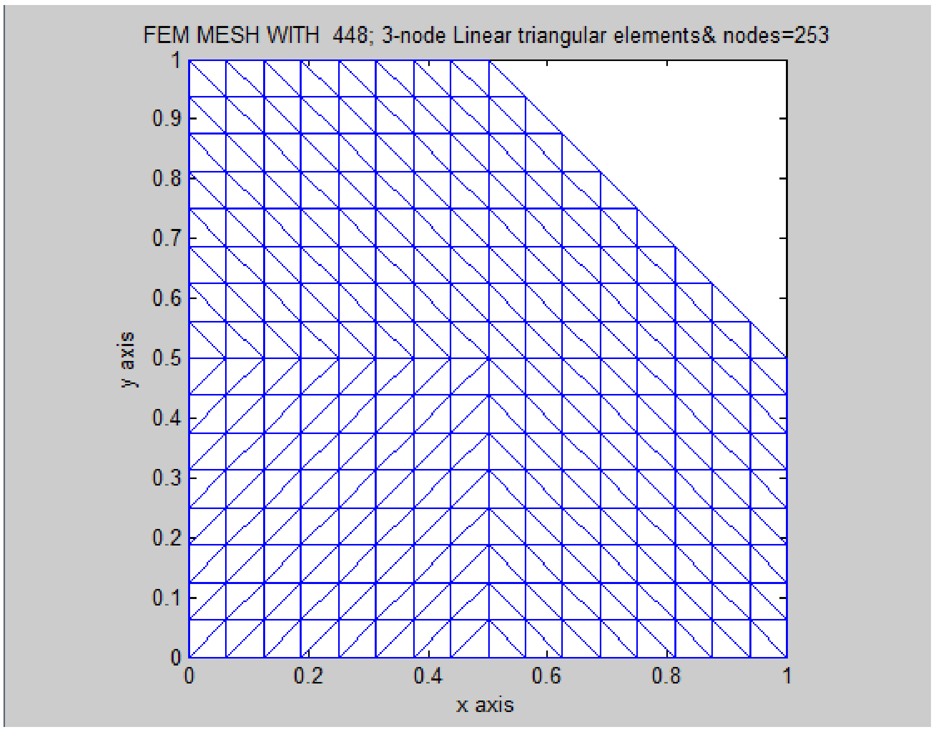
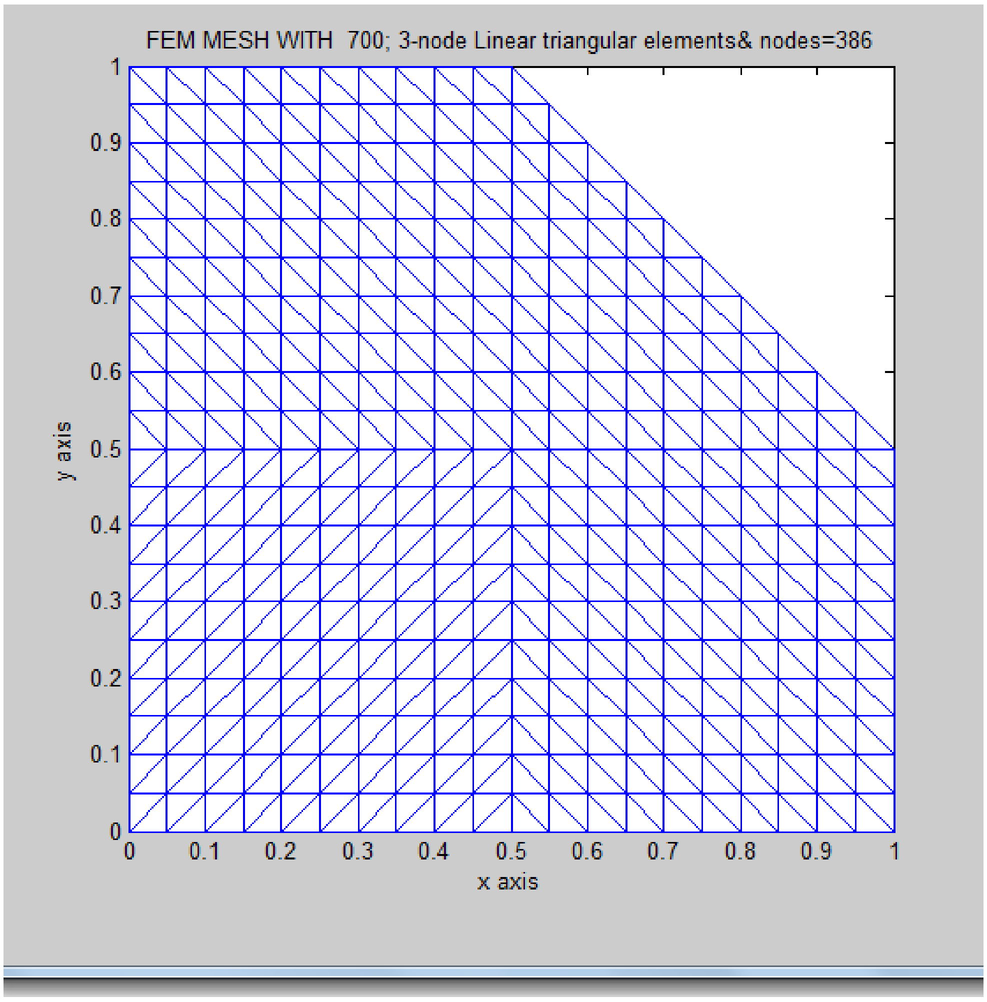
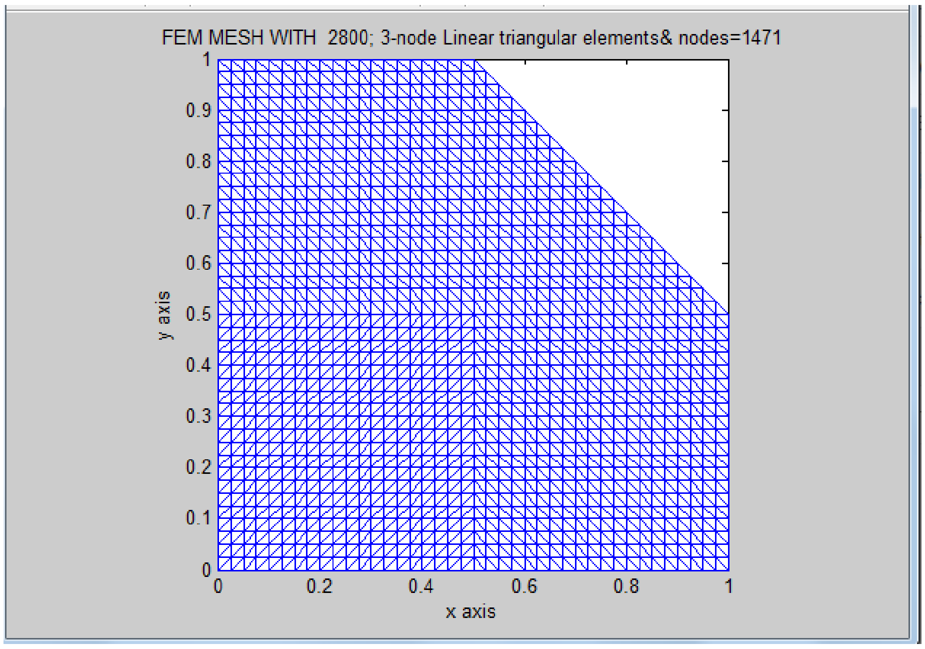
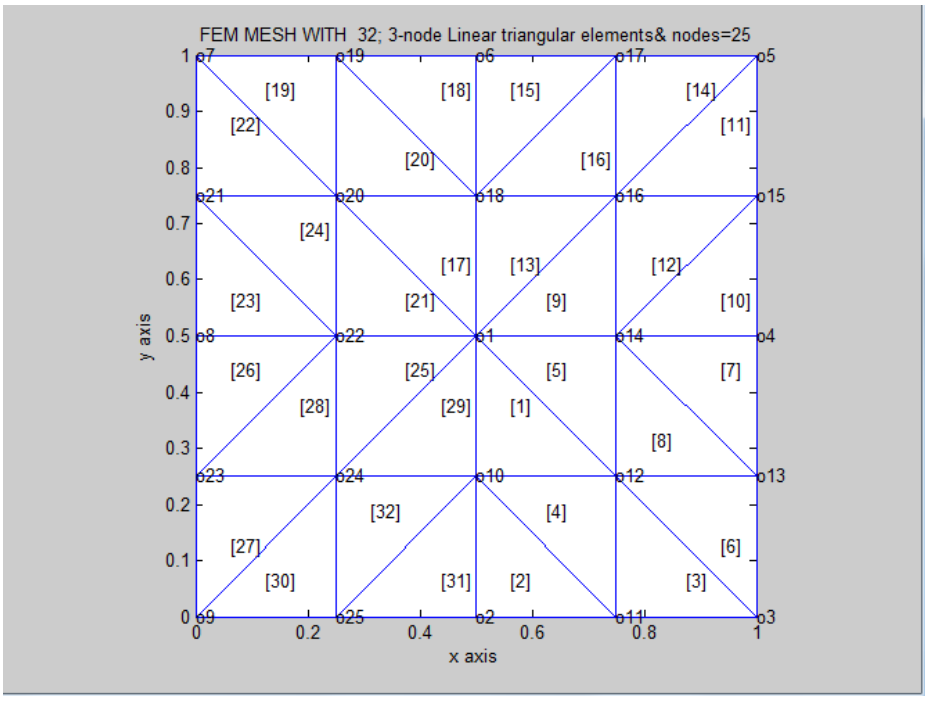

FEM MESH WITH 28; 3-node Linear triangular elements& nodes=22
x axis
y axis

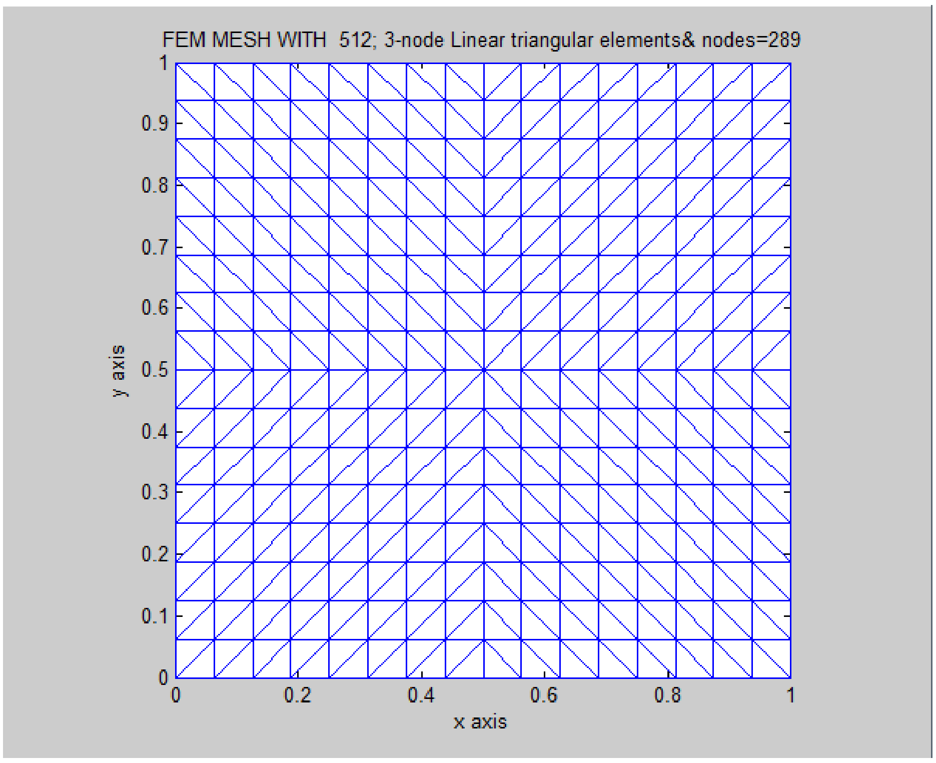

FEM MESH WITH 448; 3-node Linear triangular elements& nodes=253
x axis
y axis

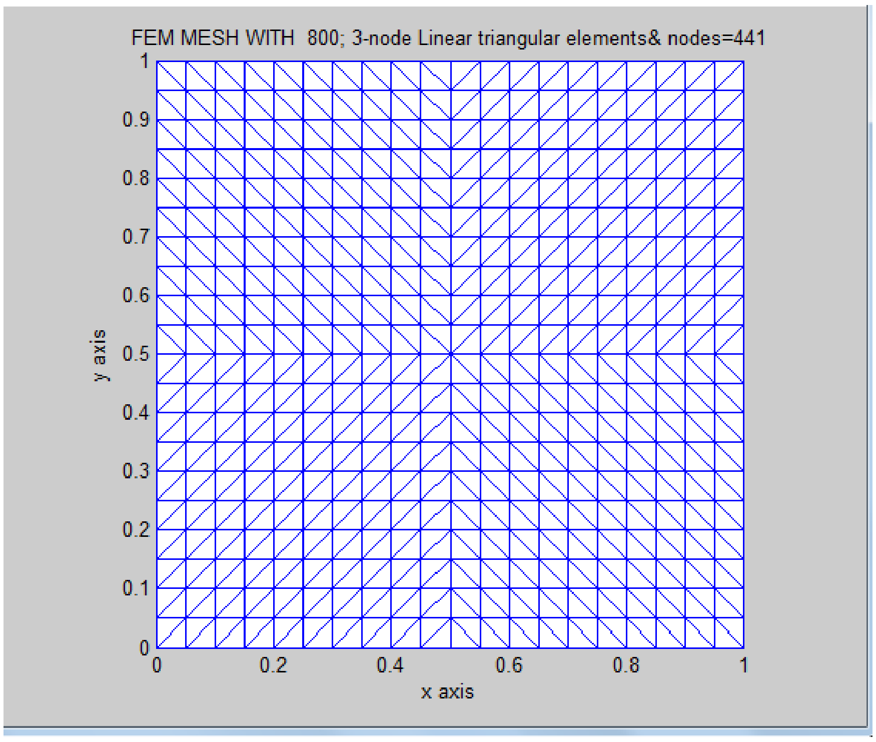

FEM MESH WITH 700; 3-node Linear triangular elements& nodes=386
y axis
x axis
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

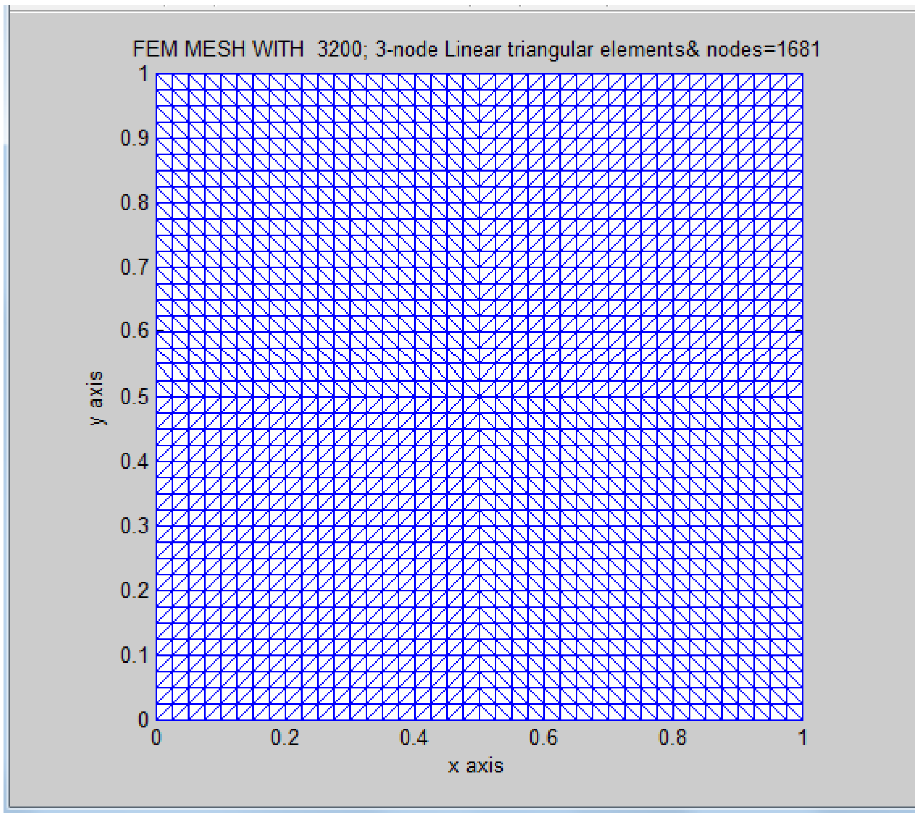

FEM MESH WITH 2800; 3-node Linear triangular elements& nodes=1471
y axis
x axis
0
0.2
0.4
0.6
0.8
1

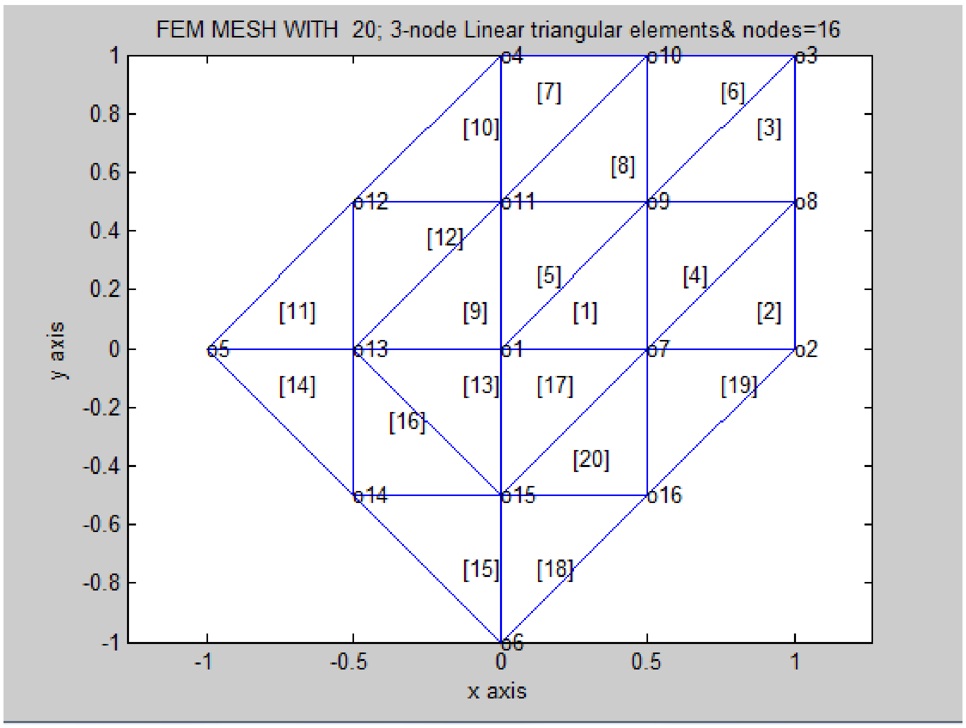
DOI: 10.18535/ijecs/v6i8.03


FEM MESH WITH 32; 3-node Linear triangular elements& nodes=25
x axis
y axis


FEM MESH WITH 512; 3-node Linear triangular elements& nodes=289
x axis
y axis

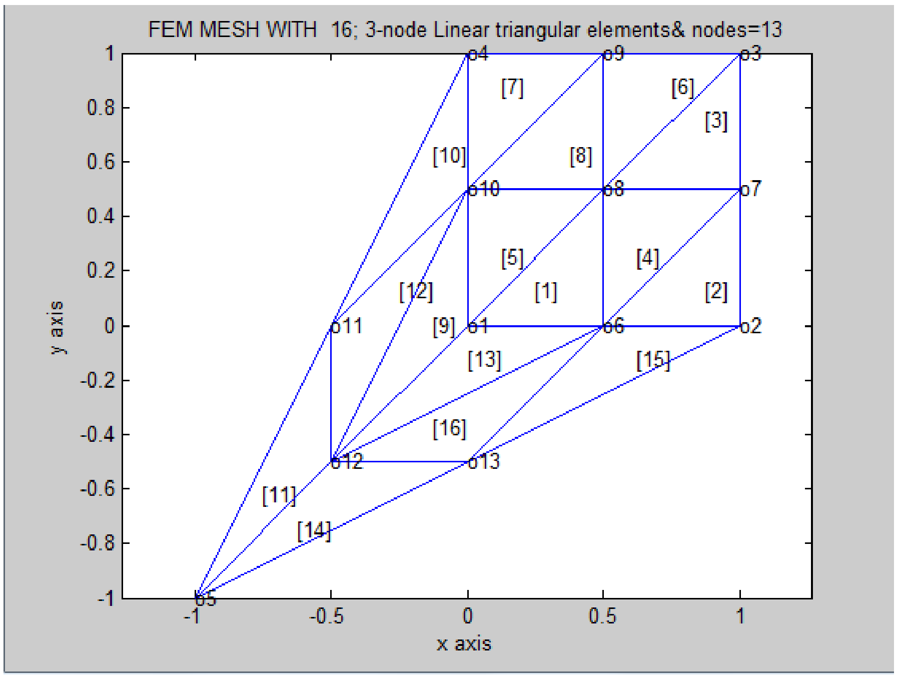

FEM MESH WITH 800; 3-node Linear triangular elements& nodes=441
y axis
x axis
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

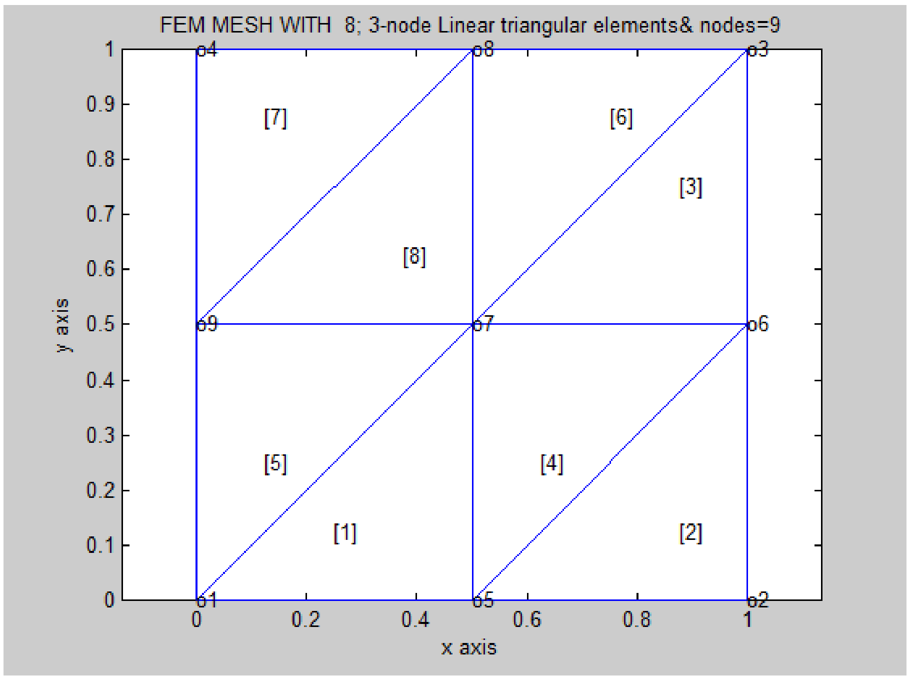

FEM MESH WITH 3200; 3-node Linear triangular elements& nodes=1681
y axis
x axis
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1


FEM MESH WITH 20; 3-node Linear triangular elements& nodes=16
x axis
y axis

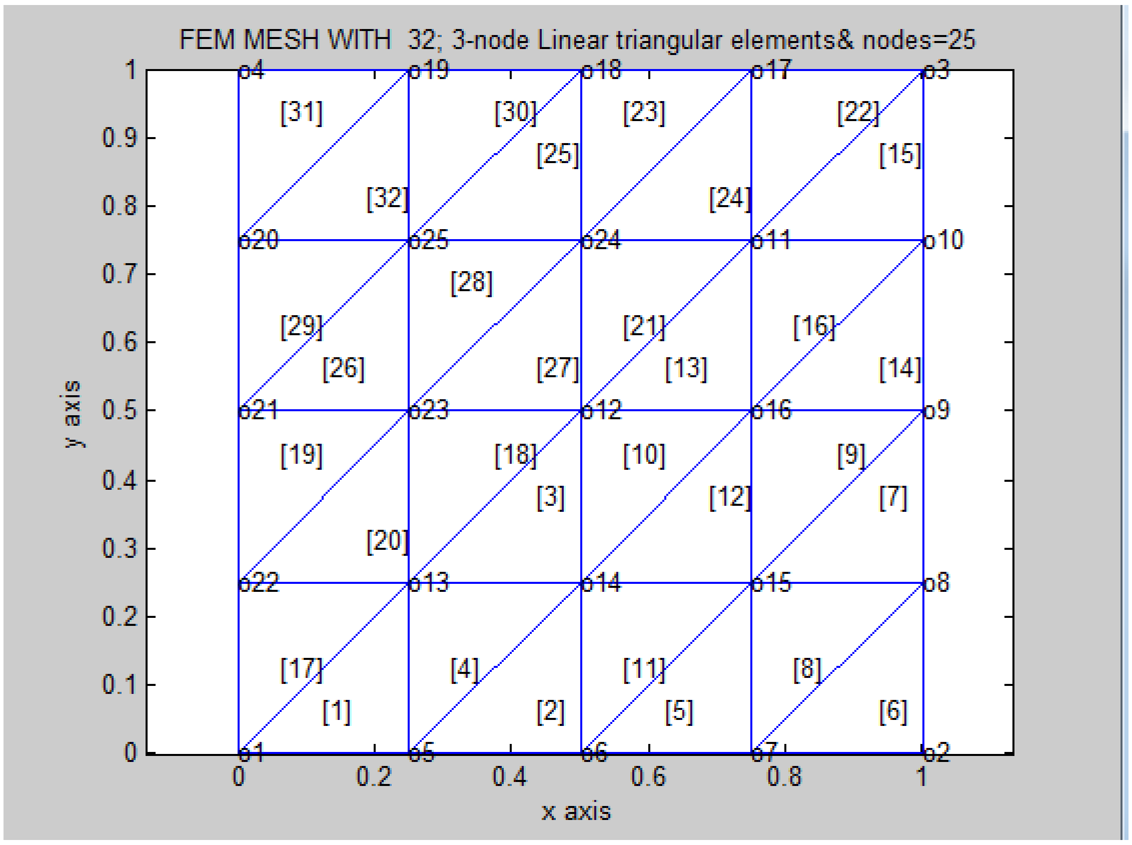
By setting ndelete=3,i.e by deleting 3-triangles in the above,we can get a 1-square .

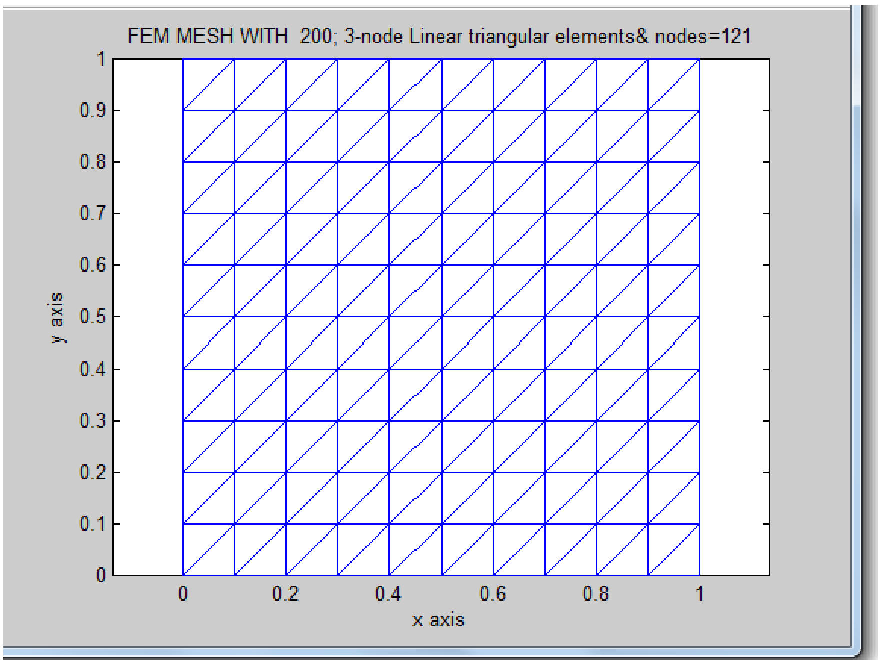
In a similar manner by deleting 2-triangles, we can also get the same 1-square from the following quadrilateral .

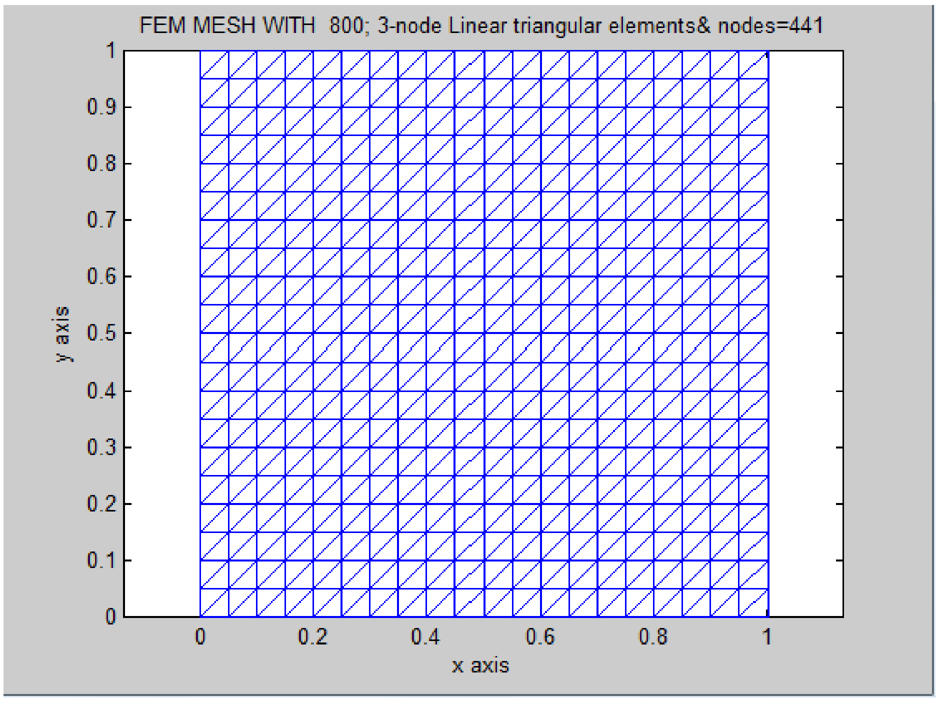

FEM MESH WITH 16; 3-node Linear triangular elements& nodes=13
x axis
y axis

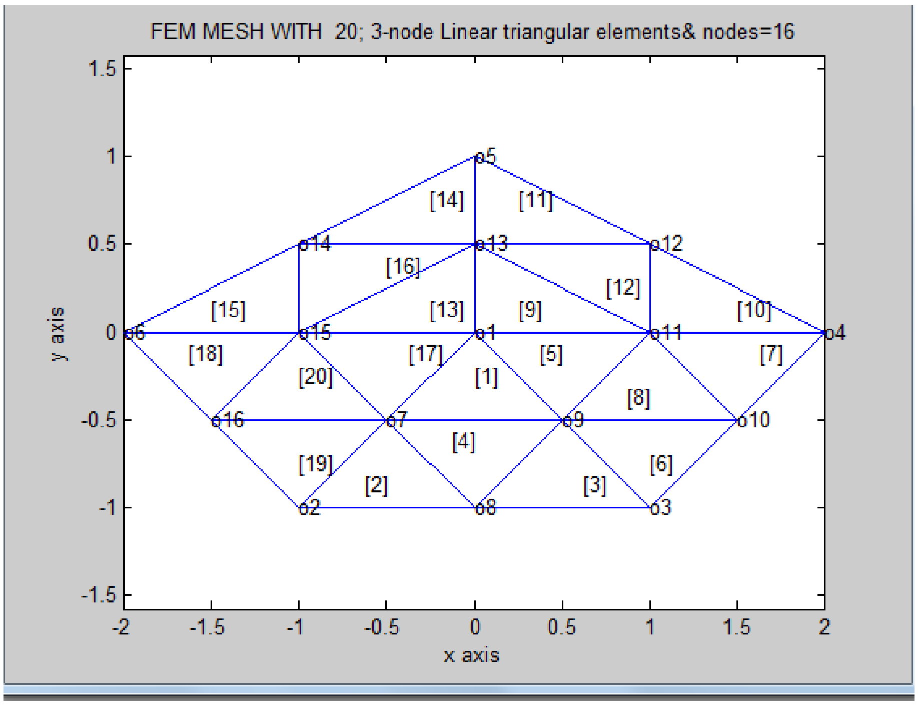


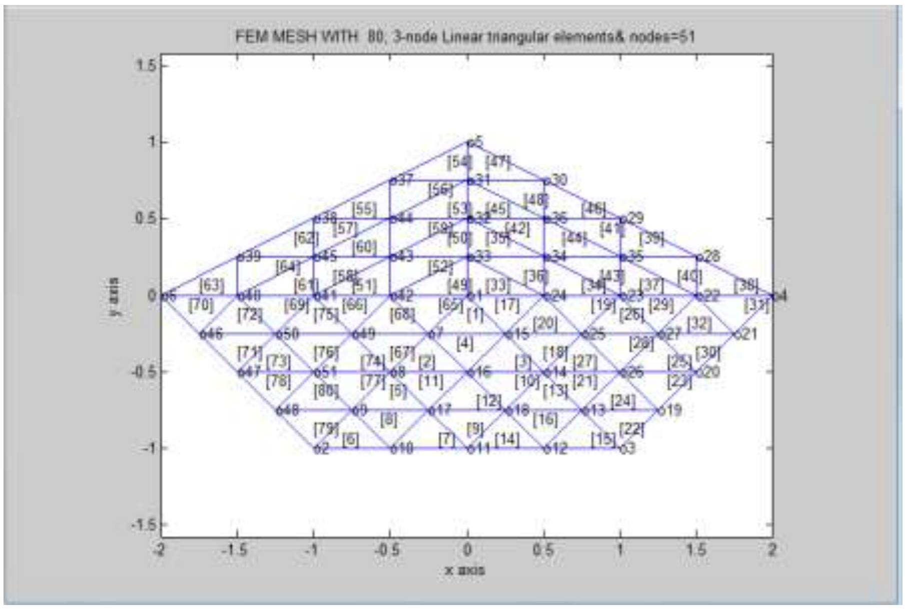
Using ndelete=3,i.e by deleting 3 triangular elements from the coarse mesh of 5- triangles in the above and ndelete=2 ,we have shown that it is possible to create meshes for 1-squares, a very useful concept for cracked polygons shown later here for icosagon in the end

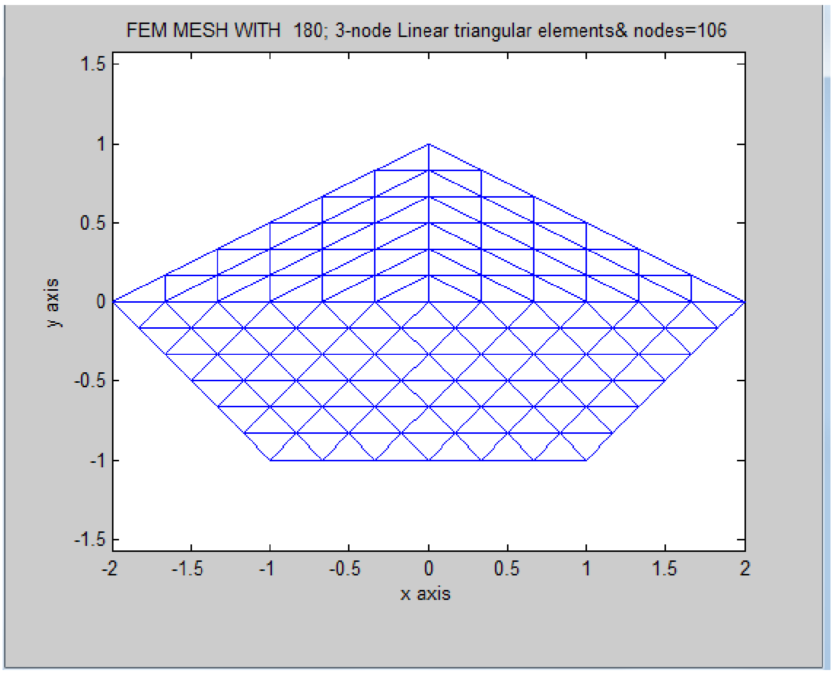

FEM MESH WITH 32; 3-node Linear triangular elements& nodes=25
y axis
x axis

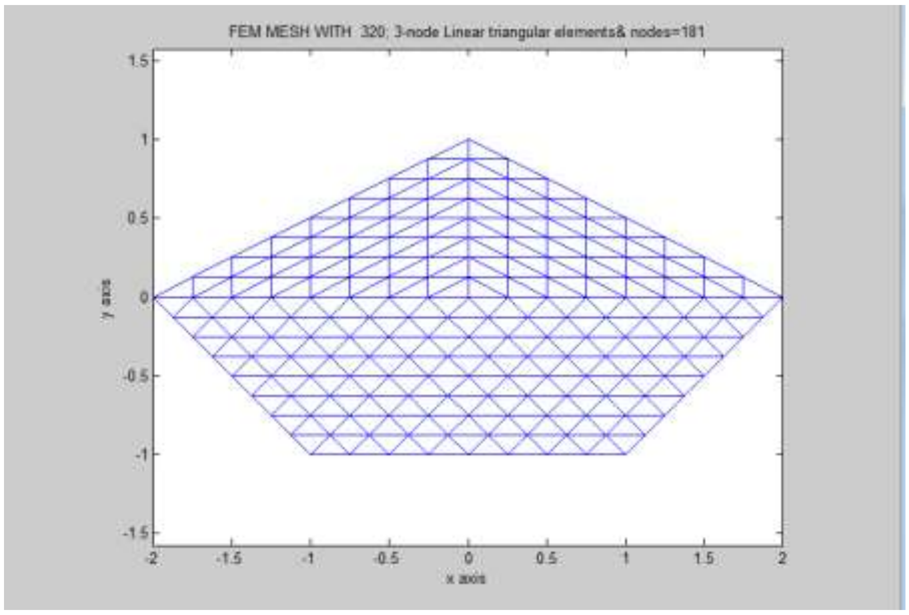

FEM MESH WITH 200; 3-node Linear triangular elements& nodes=121
y axis
x axis
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

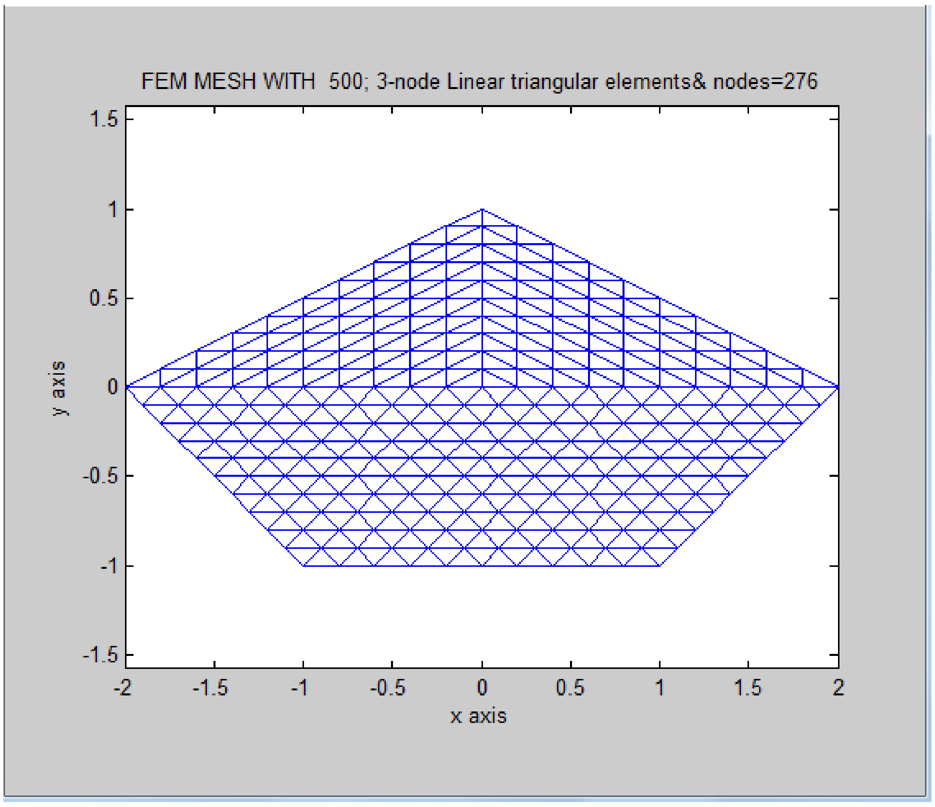

FEM MESH WITH 800; 3-node Linear triangular elements& nodes=441
y axis
x axis
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

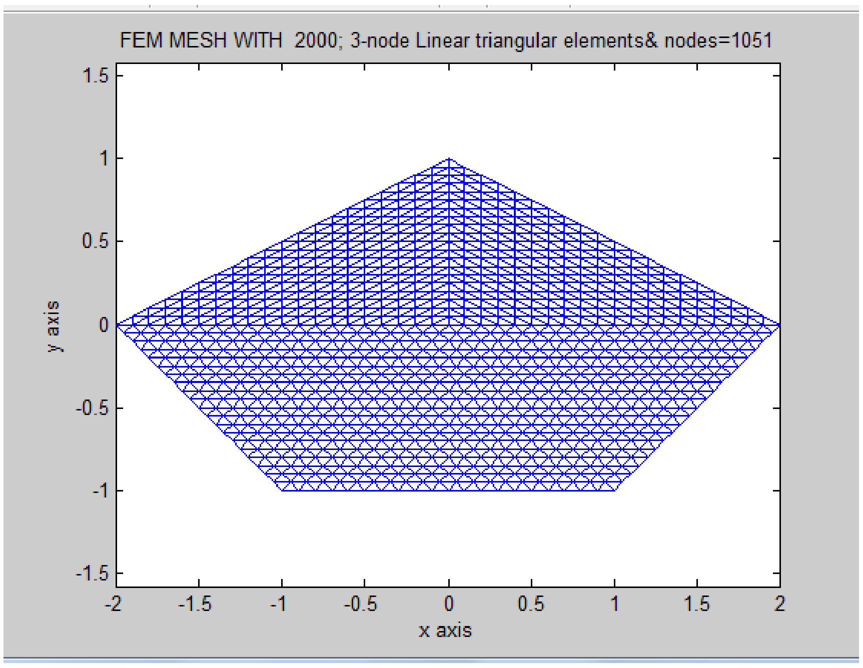

FEM MESH WITH 20; 3-node Linear triangular elements& nodes=16
y axis
x axis


FEM MESH WITH 80; 3-node Linear triangular elements& nodes=51
y axis
x axis

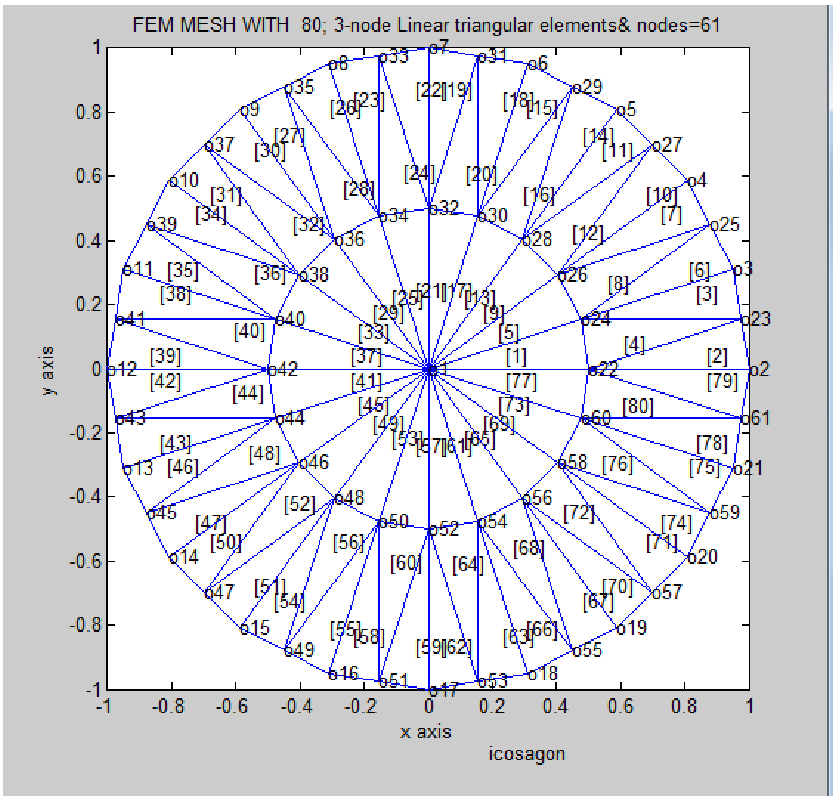

FEM MESH WITH 180; 3-node Linear triangular elements& nodes=106
y axis
x axis


FEM MESH WITH 320; 3-node Linear triangular elements& nodes=181
y axis
x axis

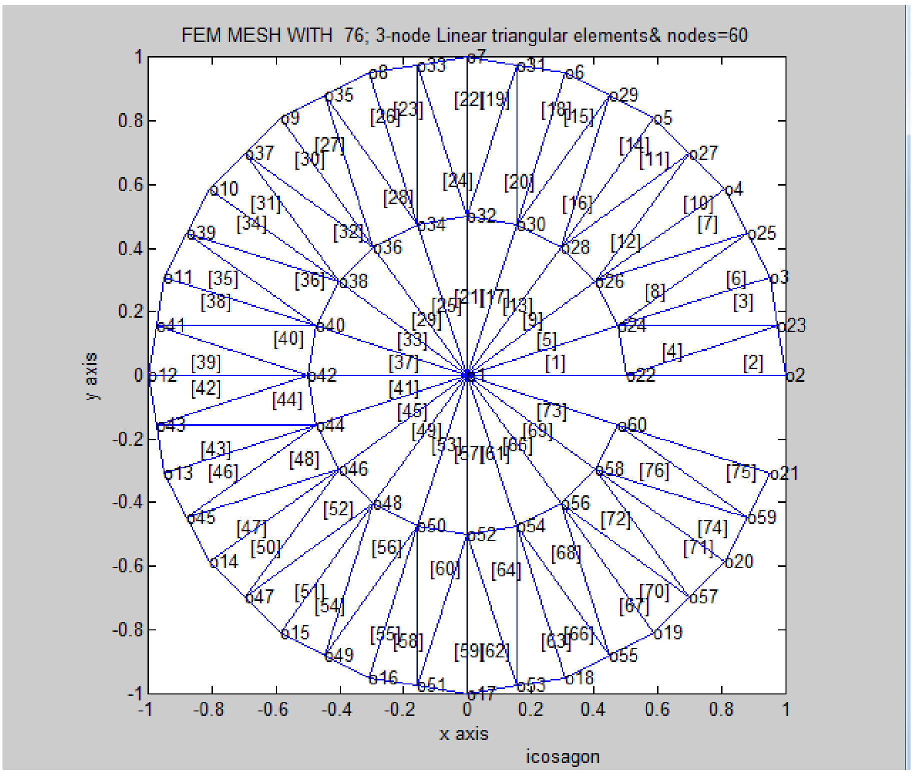

FEM MESH WITH 500; 3-node Linear triangular elements& nodes=276
y axis
x axis
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


FEM MESH WITH 2000; 3-node Linear triangular elements& nodes=1051
y axis
x axis
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

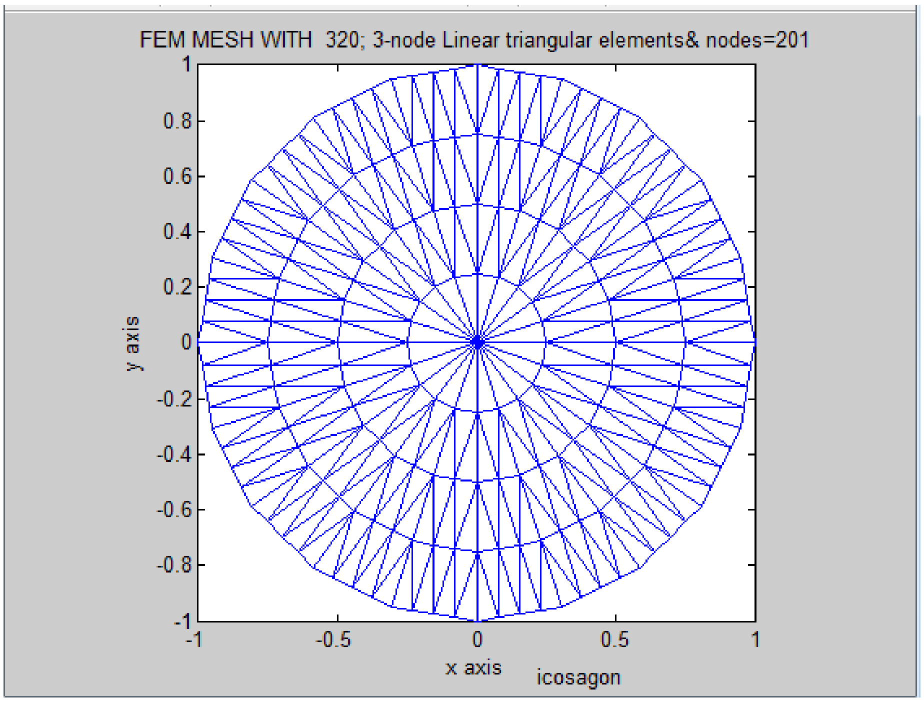
DOI: 10.18535/ijecs/v6i8.03

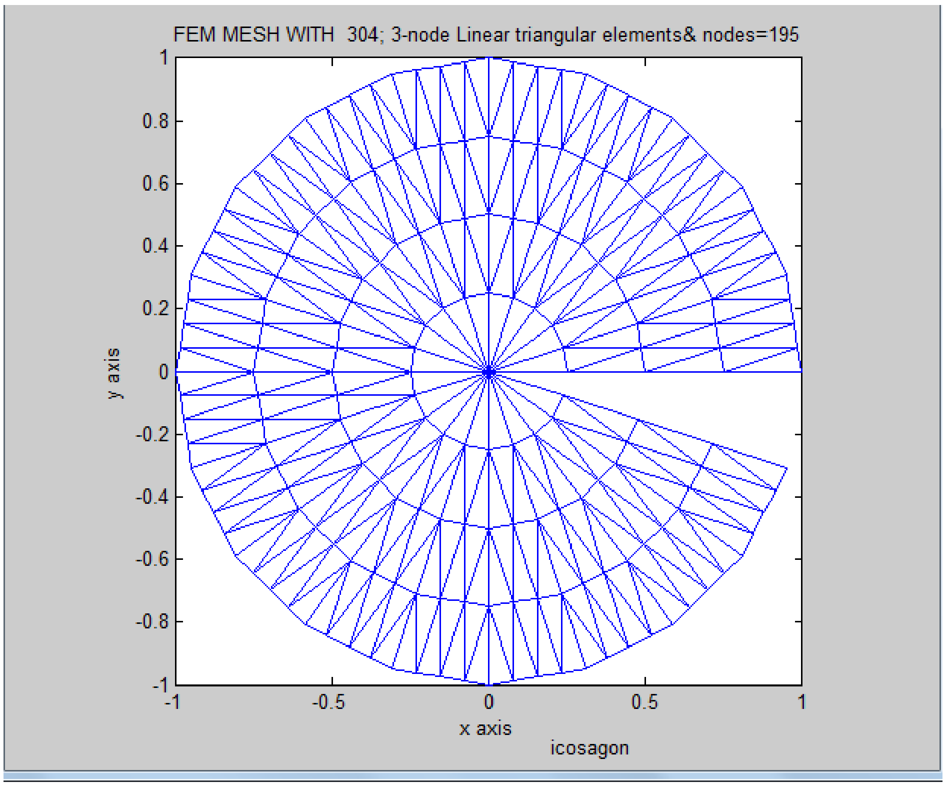


FEM MESH WITH 80; 3-node Linear triangular elements& nodes=61
x axis
y axis
icosagon

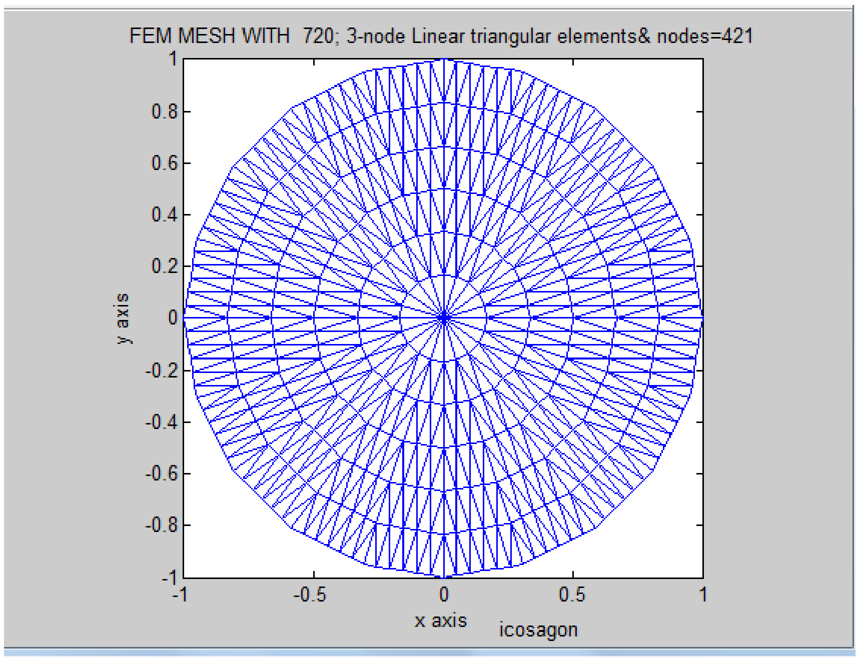


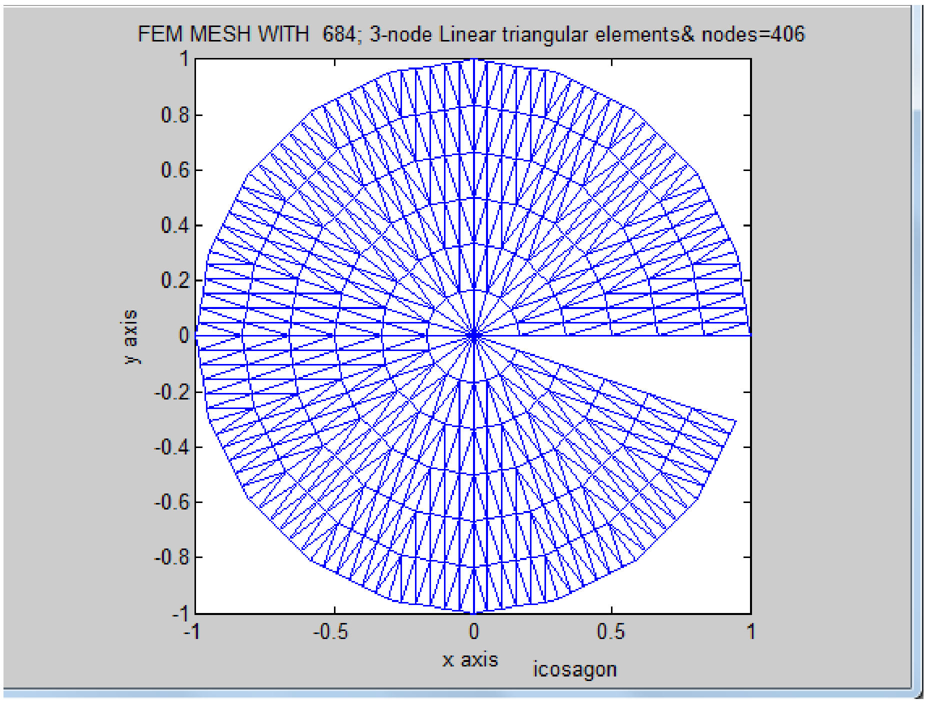
FEM MESH WITH 76; 3-node Linear triangular elements& nodes=60
x axis
y axis
icosagon

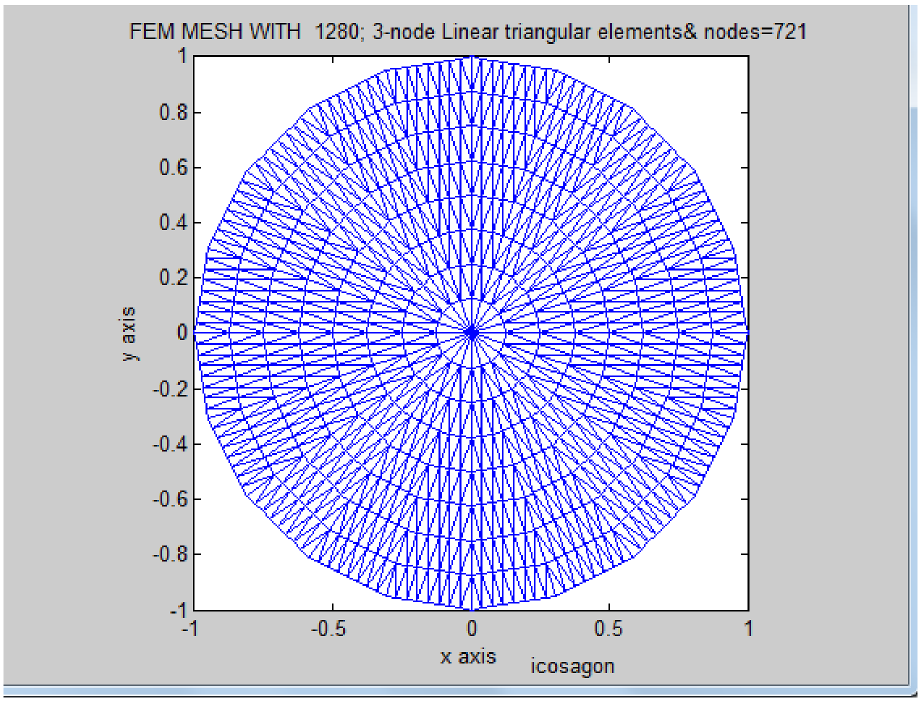

FEM MESH WITH 320; 3-node Linear triangular elements& nodes=201
y axis
x axis
icosagon
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.5
0.5

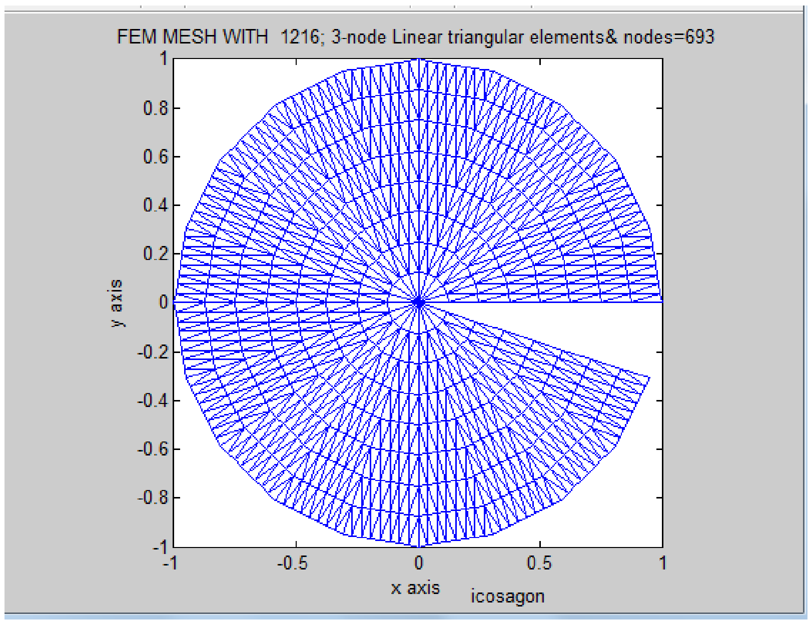

FEM MESH WITH 304; 3-node Linear triangular elements& nodes=195
y axis
x axis
icosagon
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.5
0.5

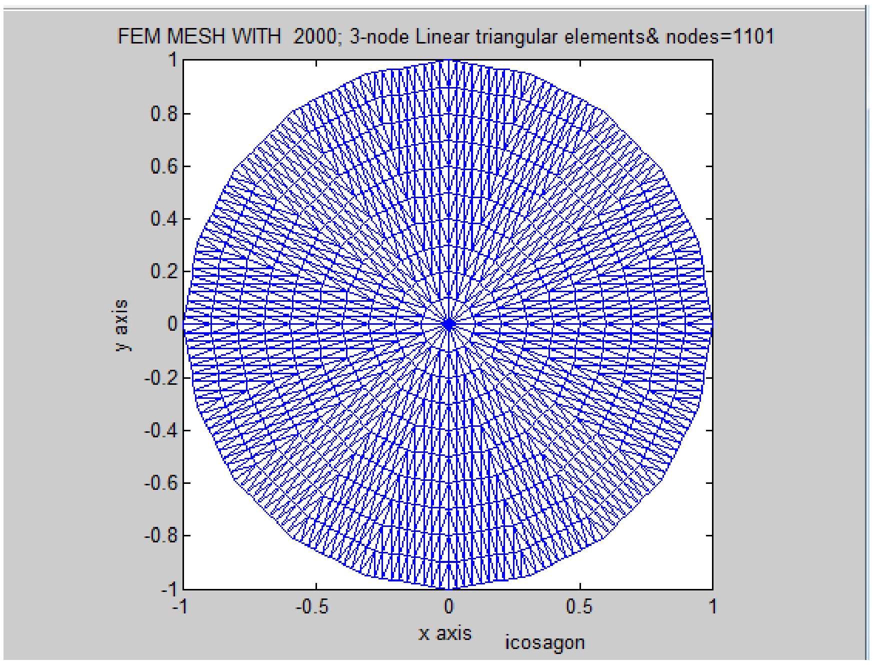
DOI: 10.18535/ijecs/v6i8.03

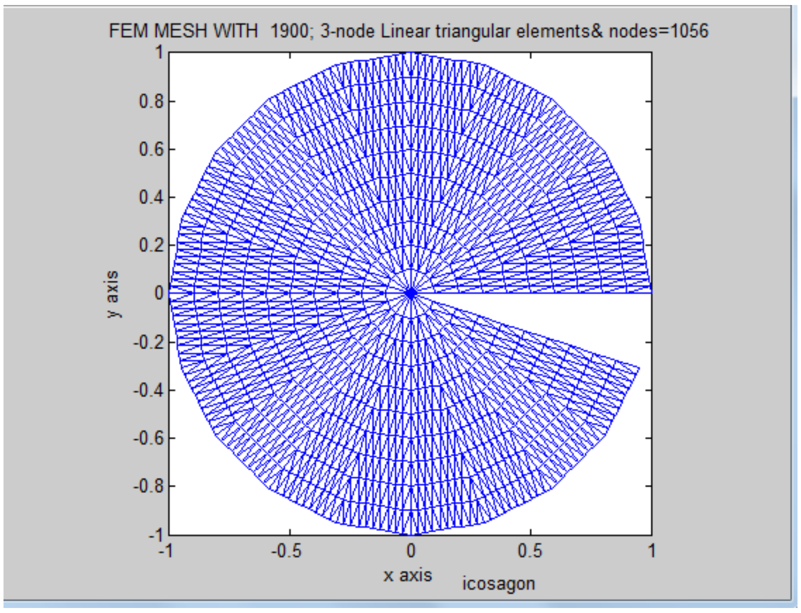

FEM MESH WITH 720; 3-node Linear triangular elements& nodes=421
y axis
x axis
icosagon


FEM MESH WITH 684; 3-node Linear triangular elements& nodes=406
y axis
x axis
icosagon


FEM MESH WITH 1280; 3-node Linear triangular elements& nodes=721
y axis
x axis
icosagon
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.5
0.5


FEM MESH WITH 1216; 3-node Linear triangular elements& nodes=693
y axis
x axis
icosagon
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.5
0.5


FEM MESH WITH 2000; 3-node Linear triangular elements& nodes=1101
y axis
x axis
icosagon
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1


FEM MESH WITH 1900; 3-node Linear triangular elements& nodes=1056
y axis
x axis
icosagon
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1