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Noise Estimation for Images using Eigen Values and Frobenius norm

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***Abstract*—Performance of image denoising algorithm is critically dependent on the accuracy of noise level estimation. In the estimation process, restricting the influence of image content on the estimation dataset is a major challenge. In this paper, we propose a novel method which involves the application of Frobenius norm as the basis of content energy measurement. It involves the truncation of SVD (Singular Value Decomposition) components thereby effectively restricting the influence of image details. Application of linear regression for determining content parameter enhances the application scope of the proposed method. The experimental results demonstrates the effectiveness of the proposed approach.**

Keywords—Frobenius norm, noise estimation, singular value decomposition, linear regression, additive white gaussian noise

I. INTRODUCTION

Application of Image Denoising or Filtering algorithms is desirable for images corrupted with additive white gaussian noise. This step is applied before many processing algorithms such as image segmentation, restoration etc. Many of these filtering or denoising algorithms rely on the information about the incurred level of noise which is assumed to be known apriori. In real-time applications, this information is obtained by estimation techniques. Accuracy and reliability of estimation algorithms thus affects the filtering or denoising processes concomitantly.

More often the noise is assumed to be zero mean additive white gaussian in nature [3], whose distribution is characterized by following equation:

$$f(x) = \frac{1}{\sigma\sqrt{(2\pi)}} \; e^{\frac{(x-\mu)^2}{2\sigma^2}} \qquad (1)$$

where σ represents the standard deviation and μ is the mean which is assumed to be zero.

Pixel level intensity variations are expressed as:

$$H(i,j) = H_0(i,j) + N(i,j)) \qquad (2)$$

Where $P_0(i,j)$ represents intensity due to image content and $N(i,j)$ is the added noise.

One category of estimation methods is Filtering based. In filtering based techniques [3]- [4], image is processed using a filter to restrict or suppress noise. Then, noise variance is estimated by taking the difference between the original and the processed image. However, the method does not guarantee full suppression of noise or restriction of image content. This degrades the estimation accuracy. For enhancing the accuracy, processing is done multiple times and then variance is approximated by using the median of those estimated variances. In another category of block based estimation techniques [5], the image is divided into a number of smaller blocks. From this, homogeneous blocks are considered for estimation or blocks with very less variation of image content between them. Ascertaining that the content variation is due to image and not noise is a difficult task and makes the identification of homogeneous blocks a difficult task.

A class of non-linear estimation method involves the use of SVD (Singular Value Decomposition) [6]. In this method, estimation dataset is prepared by considering the tail of singular values [6]. In SVD domain, the later subspaces of singular values are dominated by noise. The influence of image details on the estimation dataset is restricted in this manner. However, for images of varying details the choice of appropriate number of singular values is a difficult task..

In this paper we propose a new approach for estimating additive white gaussian noise. The proposed method utilizes the tail of singular values from SVD subspace for preparation of dataset. The number of singular values considered for estimation is taken by using the Frobenius norm based technique. In this technique, a threshold value of image content rejection is chosen. Based upon the threshold, only those singular values are considered which are dominated by noise. This technique enhances the application scope of the proposed algorithm since the amount of singular values chosen for estimation purpose is of adaptive length. It depends on the information content of the image considered and ensures the noise dominated singular values are chosen for estimation purpose. Further, the relationship between the considered estimation dataset and noise level is established. The associated parameters are obtained by the use of Regression analysis.

This paper is organised as follows: Section II introduces the concept of Frobenius based Energy truncation and the method of Singular Value Decomposition (SVD). In section III we elaborate the proposed method of noise estimation. Section IV deals with the experimental results of the proposed method and Section V gives the concluding remarks.

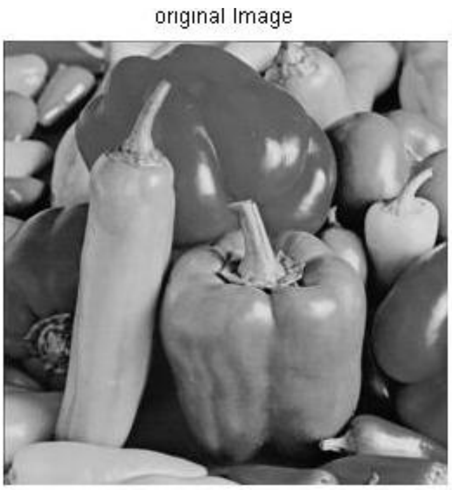
II. SINGULAR VALUE DECOMPOSITION AND FROBENIUS NORM

A. Singular Value Decomposition of Noisy Image

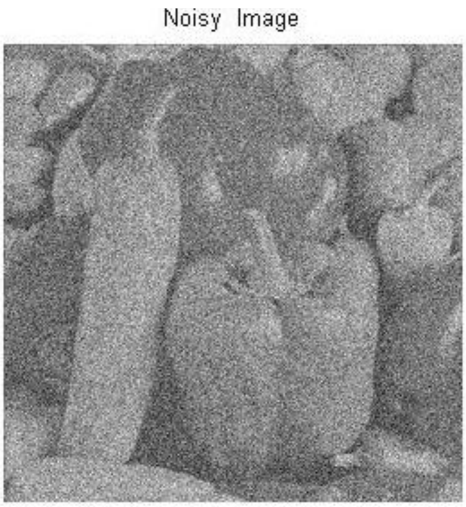
In general, any arbitrary real $m \times n$ matrix Z can be decomposed in the form as shown:

$$Z = U\Sigma V^T \qquad (3)$$

where U is the $m \times m$ orthogonal matrix V^T is transpose of $n \times n$ orthogonal matrix and Σ is a diagonal matrix containing singular values of Z in order of non-increasing magnitude. The columns of U and V are called the left and right singular vectors. This decomposition is called as Singular Value Decomposition (SVD).



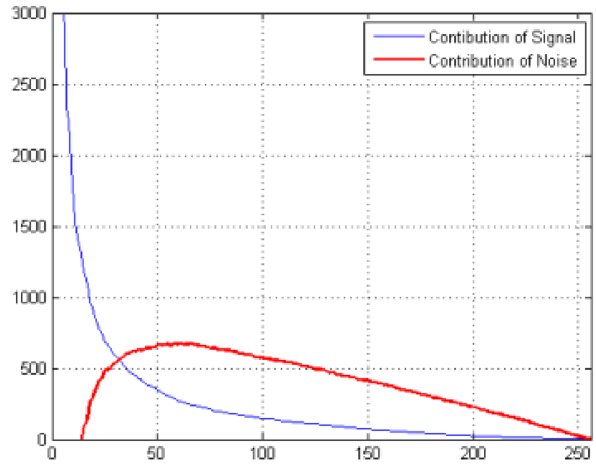
(a) Original Test Image



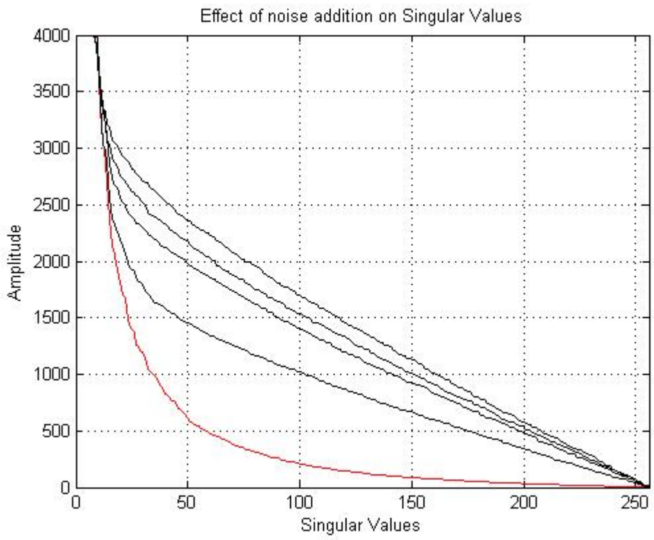
(a) Effect of Noise Addition

Fig 1. Original and Noisy Image

Figure 1(a) shows the test image which has been made noisy and SVD is applied on it. Fig 1(b) shows the noise corrupted test image. Influence of noise and image content on singular values is shown in Figure 2(a). It can be seen from Fig 2(a) that rear end of the SVD subspaces is mainly dominated by noise. Figure 2(b) shows the effect of noise addition on singular values. The red graph is for the original image without any noise. Subsquently, as the noise is added the later part of the singular values are more affected. Effect of the added noise is more pronounced in the rear part of the SVD subspaces.



(a) Singular Values



(b) Effect of Addition of Noise on Singular Values

Fig 2. Singular values of a noisy image

Taking the last M singular values into account and by calculating the average P_M at different noise levels (σ) it has been found that the relation between P_M and σ is of linear nature [6] as long as M $\in [\frac{r}{4}, \frac{4r}{5}]$ where r is the rank of singular matrix Σ. This linear relationship can be expressed as:

$$P_M = \alpha\sigma + \beta \tag{4}$$

Where $\alpha\sigma$ is the influence of noise and β is due to image content. Image details have little effect over α and mainly increases β [6]. By introducing different levels of noise σ and by calculating the corresponding P_M values we can estimate α and β by using Linear Regression.

This method requires a value of *'M'* to be chosen beforehand. It does not considers the amount of image content that is being restricted in the dataset. To avoid this we propose a approach which restricts the major amount of image information in the estiamtion dataset. Use of Frobenius based truncation achieves this. The method is explained below.

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B. Restriction of Content dominated Singular Values Using Frobenius Norm

The *norm* of a matrix A is a scalar quantity that gives a measure of magnitude of the matrix elements as a whole. For a matrix A with *n-elements* Norm-2, also known as Euclidean norm is defined as

$$Norm\ (A) = \sqrt{\sum_{i=1}^{n}(A_i)^2} \quad (5)$$

Euclidean norm is the square of sum of squares of all the matrix elements. It represents the largest singular value that can be obtained from SVD components of matrix A.

Similarly, the Frobenius norm of the matrix A can be defined as:

$$||A|| = \sqrt{\sum_{i=1}^{n}(\sigma_i)^2} \quad (6)$$

For restriction of the image content dominant singular values we define a parameter E_k as follows:

$$E_K = \frac{||A_K||_F}{||A||_F} \quad (7)$$

A_K refers to the truncated image at rank 'k'.i.e; only the singular values upto 'k' are considered for calculation of Frobenius norm. Based on the corresponding value of E_K (amount of content to be restricted) we get the value of 'k'. This method does not uses a hard-threhold value of M (no. of singular values for estimation). Hence, it becomes adaptive to images of varying visual details.

C. Linear Regression Technique

A Linear Regression technique determines the best straight line that passes through a set of uncertain data points. It calculates the slope and intercept of the straight line.

Let us say we have set of *'k'* data points (P_{M1},σ_1), (P_{M2},σ_2), …, (P_{Mk},σ_k) that has been obtained from part A above. Since the relation between P_M and σ is of linear nature, we approximate it by the mathematical expression :

$$P_M = \alpha\sigma + \beta + \delta \quad (8)$$

where *'δ'* is the residual error.

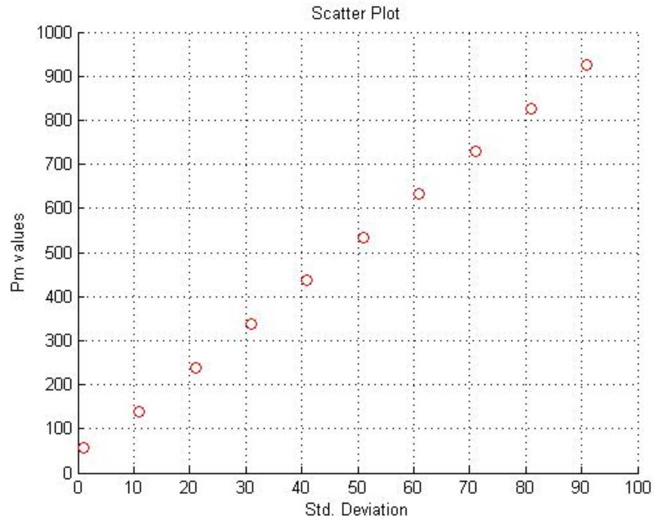

Fig 3. Scatter Plot showing the relation between P_M and σ

Fig 3. Shows the linear relationship between the averaged singular values upto length 'k' and standard deviation (σ).

Let us consider the sum of squares of the residual errors:

$$S_r = \sum_{i=1}^{k}(P_{Mi} - \beta - \alpha_i\sigma_i)^2 \quad (9)$$

Due to linear relation between P_M and σ we have:

$$\frac{\partial S_r}{\partial \beta} = -2\sum_{i=1}^{k}(P_{Mi} - \beta - \alpha\sigma_i) = 0 \quad (10)$$

$$\frac{\partial S_r}{\partial \alpha} = -2\sum_{i=1}^{k}\sigma_i(P_{Mi} - \beta - \alpha\sigma_i) = 0 \quad (11)$$

By solving equation (7) and (8) simultaneously, we get

$$\alpha = \frac{k\sum_{i=1}^{k}\sigma_i P_{Mi} - \sum_{i=1}^{k}\sigma_i\sum_{i=1}^{k}P_{Mi}}{k\sum_{i=1}^{k}\sigma_i{}^2 - (\sum_{i=1}^{k}\sigma_i)^2} \quad (12)$$

$$\beta = \overline{P_M} - \alpha\overline{\sigma} \quad (13)$$

Where $\overline{P_M}$ and $\overline{\sigma}$ represents mean values of P_M and σ respectively.

III. PROPOSED ESTIMATION METHOD

Based on the analysis of Section II, we propose the following steps in the estimation process:

1. Evaluate P_M by adding various noise levels σ for the given image.
2. Calculate α and β with the help of equations (12) and (13).

Once we have the knowledge of the parameters α and β we proceed towards the actual estimation part:

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3. Obtain the value of 'k' from Frobenius based truncation method for image details restriction upto 0.9995. (i.e; 99.95%)

4. Calculate the P_M for the input noisy image taking singular values upto 'k'

5. Use the values of α and β in the following equation to get the estimated value of noise standard deviation:

$$\widehat{\sigma} = \frac{P_M - \beta}{\alpha} \quad (14)$$

Where $\hat{\sigma}$ represents the estimated value.

IV. EXPERIMENTAL RESULTS AND DISCUSSIONS

The proposed method was tested on images (a), (b), (c) as shown in Fig. 4. These images are standard 256×256 grayscale images with varying visual information. Fig (c) has relatively less visual details whereas Fig. (a) is a complicated image and has more visual details.

Table I and Table II shows the computed results for these test images. In Table I, parameter values α (noise dependent) and β (image content dependent) are determined. For determination of these parameters we used the data points obtained by varying noise levels and their corresponding P_M values. Table II shows the estimation results obtained by the proposed regression technique.

(a) Barbara (b) Lena (c) Peppers

Fig 4. Test Images

From Table I, we observe that the value of α depends upon the noise level that corrupts the image. On the other hand, we see that the value of β depends on the image content. β value for Peppers image is the least and for Barbara image is the highest as it contains large visual details. The obtained values of α and β are used to determine the noise level as presented in table II. We observe from table II that the estimated noise level values $\widehat{\sigma}$ are close to the actual noise levels (σ). Each image has been tested for 10 times and mean value is taken in order to reduce chances of incorporating erroneous values.

TABLE I: CALCULATION OF α AND β FOR VARIOUS TEST IMAGES

	σ=1	σ=11	σ=21	σ=31	σ=41	σ=51	σ=61	σ=71	σ=81	σ=91	α	β
P_M for Barbara	82.01	150.39	240.51	336.61	434.98	533.07	630.01	725.88	822.79	921.82	9.51	50.62
P_M for Lena	73.27	147.87	245.10	341.94	440.23	534.19	634.16	730.73	827.85	926.08	9.89	49.12
P_M for Peppers	56.98	140.05	238.47	336.42	432.07	531.10	629.11	727.17	823.78	921.65	9.78	38.16

TABLE II: MEAN OF ESTIMATION RESULTS ($\hat{\sigma}$) FOR 10 TESTS WITH VARYING NOISE LEVELS (σ)

	$\sigma = 5$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 25$	$\sigma = 30$	$\sigma = 35$	$\sigma = 40$	$\sigma = 45$	$\sigma = 50$
Barbara	5.09	9.48	14.25	19.45	24.55	29.14	34.41	39.45	44.57	49.45
Lena	4.91	10.47	15.37	19.22	24.31	30.57	34.67	40.15	44.63	49.23
Peppers	4.78	9.16	15.52	20.14	24.22	30.26	34.85	39.75	44.54	50.16

TABLE III: Adaptive length of parameter 'k' for varying noise levels

	$\sigma = 5$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 25$	$\sigma = 30$	$\sigma = 35$	$\sigma = 40$	$\sigma = 45$	$\sigma = 50$
Barbara	105	136	157	171	182	187	193	195	197	198
Lena	109	133	152	166	181	189	194	196	198	201
Peppers	87	128	143	162	174	181	189	191	194	197

Table III gives the values of parameter 'k' which is used to restrict the content details on estimation dataset. This restriction is done using Frobenius norm. Value of E_K chose for the purpose is 0.9995. This ensures restriction of image details upto 99.95% on the estimation dataset thereby giving a reliable dataset.

V. CONCLUSION

In this paper, we have illustrated the use of SVD components for preparation of estimation dataset. Use of Frobenius based truncation of singular values restricts the effect of image content on the dataset to a large extent, which is difficult otherwise. The content related parameters are obtained by the use of Linear Regression.

The experimental results validates the effectiveness of the proposed approach. The adaptive length of singular values taken for estimation enlarges the application scope of the proposed method.

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