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Perfect Difference Number System

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Abstract

A new number system, Perfect Difference Number System (PDNS), based on the mathematical notion of Perfect Difference Sets (PDS) is proposed. It is expected that PDNS will take its place in the family of other number systems and will benefit computer theory and applications.

1. Introduction

The possibility of using projective geometry for the creation of an abstract number system was considered as early as 1910.[i] In the present paper we will discuss a specific number system, Perfect Difference Number System (PDNS), based on one of the derivatives of projective geometry - Perfect Difference Sets (PDS). This number system is oriented for the use in computers and communication systems.

2. Perfect Different Sets

Perfect Difference Sets find numerous practical applications in different areas of science and technology, specifically in networking and coding theory.[ii,iii,iv,v,vi] Perfect Difference Sets are based on the properties of augmented Galois Fields (*GF(n)* Fields) and exist for every order of n such that:

$n = p^r$

where p is a prime number and r is an integer.

Perfect Difference Sets were first discussed by James Singer in 1938.[vii] His formulation was in terms of points and lines in a finite projective plane.

Starting with a Perfect Difference Set of the form:

$\{d_0, d_1, \ldots, d_i, \ldots, d_{n-1}, d_n\}$

one can form a matrix of differences:

$d_0 - d_0$	$d_1 - d_0$	…	$d_{n-1} - d_0$	$d_n - d_0$
$d_0 - d_1$	$d_1 - d_1$	…	$d_{n-1} - d_1$	$d_n - d_1$
…	…	…	…	…
$d_0 - d_{n-1}$	$d_1 - d_{n-1}$	…	$d_{n-1} - d_{n-1}$	$d_n - d_{n-1}$
$d_0 - d_n$	$d_1 - d_n$	…	$d_{n-1} - d_n$	$d_n - d_n$

Differences of the type $d_u - d_v$ (mod N), where $u \neq v$ and $N = n^2 + n+1$, are not equal to one another and will take all values from the sequence 1, 2, ⋯ N − 1 once and only once. These differences are congruent to the integers 1, 2, ⋯ N − 1.

The structure of the matrix of differences is rather obvious. There are zeroes on the main diagonal of the matrix; considering that $d_0 = 0$, its upper row repeats the original PDS (OPDS). Following rows are formed as OPDS – d_1, OPDS – d_2 , down to the lowest row OPDS − d_n. Differences above the main diagonal ($d_u > d_v$) are calculated according to the expression $d_u - d_v$; differences below the main diagonal ($d_u < d_v$)are calculated according to the expression $N+d_u - d_v$.

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3. Examples of Perfect Difference Sets

PDS {0, 1, 3}; $n = 2$, $N = 7$

PDS {0, 1, 3, 9}; $n = 3$, $N= 13$

PDS {0, 1, 3, 7, 15, 31, 36, 54, 63}; $n = 8$, $N = 73$

Differences (mod 7) for the first of these PDSs are:
1 = 1−0
2 = 3−1
3 = 3−0
4 = 0−3
5 = 1−3
6 = 0−1

Differences (mod 13) for the second of these PDS are:
1 = 1−0
2 = 3−1
3 = 3−0
4 = 0−9
5 = 1−9
6 = 9−3
7 = 3−9
8 = 9−1
9 = 9−0
10 = 0−3
11 = 1−3
12 = 0−1

Differences for the third PDS are listed in the Appendix.

4. Discussion

The most essential property of PDS is the ability to make a big sequence of consecutive numbers from 0 to N out of the small number n of its elements, where n is (approximately) the square root of N. This property was instrumental in creating optimal codes and the highly efficient interconnection networks mentioned above, as well as other practical structures and systems.

Considering the fact that differences, modulo N, of PDS elements are congruent to the positive integers from 0 to N, all these differences (or even differences of their indexes) can be used as a unique representation of corresponding numbers. This opens the way to use the PDS apparatus as the foundation for a new number system proposed here - Perfect Difference Number System (PDNS). The potential benefits, challenges and presumable properties of this system are briefly outlined in this short memorandum, together with possible structures of the following algorithms.

A PDNS allows us to express a large set of integers $0 - N$ via a set of much smaller size $0 - n$, in a simple and highly regular fashion. The potential result of this for computing is that instead of processing the massive of N numbers, one would be able to achieve same results by processing the essentially smaller massive n. Possibly, processing could be reduced simply to the operating elements of PDS (or even their indexes) stored in memory. Utmost economy in the representation of numbers in PDNS may result in the possibility of using approaches that are prohibitively complex using conventional number systems.

5. Conclusion

To put this general concept to practical use it is necessary first to develop basic operations in this new number system similar to operations in existing number systems.[viii,ix] One may expect that these operations will have properties characteristic for modular systems. In addition to the basic operations of addition and multiplication, it will be necessary also to develop the operation of conversion from and to conventional number systems, as well as hybrids of both systems. In these hybrids, blocks using the PDNS approach may serve as elements of a new position system having redundancy. For example, PDNS with n = 8, N = 73 may be used as a replacement of the binary system with 64 states; system with n = 32, N = 1057 may substitute one with 1024 states, and system with n = 131, N = 17,293 - one with 16,384 states. We may expect that the new number system will positively influence computer algorithms and architecture.[x]

Obviously this will require substantial investment of resources and time, but the potential benefits most probably will justify these efforts. We have reason to believe that the Perfect Difference Number System will take its place in the family of other number systems and will benefit computer theory and practical applications. Further expanding the potential of this new number system may be achieved by exploiting multidimensional and functional approaches similar to those used in PDS networks.[xi,xii,xiii,xiv,xv]

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APPENDIX: Differences of Indexes for Third PDS

PDS {00, 01, 03, 07, 15, 31, 36, 54, 63}.

01 = (1,0)
02 = (2,1)
03 = (2,0)
04 = (3,2)
05 = (6,5)
06 = (3,1)
07 = (3,0)
08 = (4,3)
09 = (8,7)
10 = (0,8)
11 = (1,8)
12 = (4,2)
13 = (2,8)
14 = (4,1)
15 = (4,0)
16 = (5,4)
17 = (3,8)
18 = (7,6)
19 = (0,7)
20 = (1,7)
21 = (6,4)
22 = (2,7)
23 = (7,5)
24 = (5,3)
25 = (4,8)
26 = (3,7)
27 = (8,6)
28 = (5,2)
29 = (6,3)
30 = (5,1)
31 = (5,0)
32 = (8,5)
33 = (6,2)
34 = (4,7)
35 = (6,1)
36 = (6,0)
37 = (0,6)
38 = (1,6)
39 = (7,4)
40 = (2,6)
41 = (5,8)
42 = (0,5)
43 = (1,5)
44 = (3,6)
45 = (2,5)
46 = (6,8)
47 = (7,3)
48 = (8,4)
49 = (3,5)
50 = (5,7)
51 = (7,2)
52 = (4,6)
53 = (7,1)
54 = (7,0)
55 = (6,7)
56 = (8,3)
57 = (4,5)
58 = (0,4)
59 = (1,4)
60 = (8,2)
61 = (2,4)
62 = (8,1)
63 = (8,0)
64 = (7,2)
65 = (3,4)
66 = (0,3)
67 = (1,3)
68 = (5,6)
69 = (2,3)
70 = (0,2)
71 = (1,2)
72 = (0,1)

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