

PAPR Reduction Of OFDM Using An Exponential Companding Technique

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Abstract:

OFDM is a bandwidth efficient multicarrier modulation where the available spectrum is divided into subcarriers, with each subcarrier containing a low rate data stream. OFDM has gained a tremendous interest in recent years because of its robustness in the presence of severe multipath channel conditions with simple equalization, robustness against Inter-symbol Interference (ISI), multipath fading, in addition to its high spectral efficiency. However, the Peak-to-Average Power Ratio (PAPR) is a major drawback of multicarrier transmission system such as OFDM. High PAPR increases the complexity of analog-to-digital (A/D) and digital-to-analog (D/A) converters, and lowers the efficiency of power amplifiers. The aim of this project work is to investigate the OFDM scheme, and realize a fully functional system in software and analyzing how it reduces the Peak to Average Power Ratio (PAPR). Different PAPR Reduction techniques developed for OFDM are analyzed, based on the Computational Complexity, Bandwidth expansion and error performance. In this Project work, new Companding Transform Techniques is developed for PAPR reduction of OFDM system. This guarantees the improved performance in terms of BER while reducing PAPR effectively and efficiently.

INTRODUCTION:

Most first generations systems were introduced in the mid 1980's, and can be characterized by the use of analog transmission techniques and the use of simple multiple access techniques such as Frequency Division Multiple Access (FDMA). First generation telecommunications systems such as Advanced Mobile Phone Service (AMPS) only provided voice communications. They also suffered from a low user capacity, and security problems due to the simple radio interface used. Second generation systems were introduced in the early 1990's, and all use digital technology [1]. This provided an

increase in the user capacity of around three times. This was achieved by compressing the voice waveforms before transmission.

Third generation systems are an extension on the complexity of second-generation systems and are expected to be introduced after the year 2000. The system capacity is expected to be increased to over ten times original first generation systems. This is going to be achieved by using complex multiple access techniques such as Code Division Multiple Access (CDMA), or an extension of TDMA, and by improving flexibility of services available. The telecommunications industry faces the problem of providing telephone services to rural areas, where the customer base is

small, but the cost of installing a wired phone network is very high. One method of reducing the high infrastructure cost of a wired system is to use a fixed wireless radio network. The problem with this is that for rural and urban areas, large cell sizes are required to get sufficient coverage [2].

Multiple access schemes [8] are used to allow many simultaneous users to use the same fixed bandwidth radio spectrum. In any radio system, the bandwidth, which is allocated to it, is always limited. For mobile phone systems the total bandwidth is typically 50 MHz, which is split in half to provide the forward and reverse links of the system.

Sharing of the spectrum is required in order to increase the user capacity of any wireless network. FDMA, TDMA and CDMA are the three major methods of sharing the available bandwidth to multiple users in wireless system. There are many extensions, and hybrid techniques for these methods, such as OFDM, and hybrid TDMA and FDMA systems [3]. However, an understanding of the three major methods is required for understanding of any extensions to these methods.

OFDM PRINCIPLE:

Orthogonal frequency division multiplexing (OFDM) is a special form of Multi Carrier Modulation (MCM) [1] with densely spaced sub carriers with overlapping spectra, thus allowing for multiple-access. Multi Carrier Modulation is the principle of transmitting data by dividing the stream into several bit streams, each of which has a much lower bit rate, and by using these sub-streams to modulate several carriers.

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier transmission technique, which divides the available spectrum into many carriers, each one being modulated by a low rate data stream. OFDM is similar to FDMA in that the multiple user access is achieved by subdividing the available bandwidth into multiple channels that are then allocated to users. However, OFDM uses the spectrum much more efficiently by spacing the

channels much closer together. This is achieved by making all the carriers orthogonal to one another, preventing interference between the closely spaced carriers [4].

In geometry, orthogonal means, "involving right angles" (from Greek ortho, meaning right, and gon meaning angled). The term has been extended to general use, meaning the characteristic of being independent (relative to something else). It also can mean: non-redundant, non-overlapping, or irrelevant. Orthogonality is defined for both real and complex valued functions. The functions $\varphi_m(t)$ and $\varphi_n(t)$ are said to be orthogonal with respect to each other over the interval $a < t < b$ if they satisfy the condition:

$$\int_a^b \varphi_m(t) \varphi_n^*(t) dt = 0, \quad \text{Where } n \neq m \quad (1)$$

Let $X(0), X(1), \dots, X(N-1)$ represent the data sequence to be transmitted in an OFDM symbol with N subcarriers. The baseband representation of the OFDM symbol is given by:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X(n) e^{j2\pi n t / N} \quad 0 \leq t \leq T, \quad (2)$$

Where T is the duration of the OFDM symbol. The input information symbols are assumed to be statistically independent and identically distributed. The amplitude, or modulus, of OFDM signal is given by

$$|x_t| = \sqrt{\text{Re}^2\{x_t\} + \text{Im}^2\{x_t\}} \quad (3)$$

The power of OFDM signal can be calculated as

$$|x_t|^2 = \frac{1}{N} \sum_{m=0}^{N-1} \sum_{k=0}^{N-1} X_m X_k^* \frac{\exp(j2\pi(m-k)t)}{N} \quad (4)$$

According to the central limit theorem, when N is large, both the real and imaginary parts of $x(t)$ become Gaussian distributed, each with

zero mean and a variance of $E[|x(t)|^2]/2$, and the amplitude of the OFDM symbol follows a Rayleigh distribution. Consequently it is possible that the maximum amplitude of OFDM signal may well exceed its average amplitude. Practical hardware (e.g. A/D and D/A converters, power amplifiers) has finite dynamic range; therefore the peak amplitude of OFDM signal must be limited. PAPR is mathematically defined as [5]:

$$PAPR = 10 \log_{10} \frac{\max[|x(t)|^2]}{1/T \int_0^T |x(t)|^2 dt} \text{ (dB)} \quad (5)$$

The peak power occurs when modulated symbols are added with the same phase. It is easy to see from above that PAPR reduction may be achieved by decreasing the numerator $\max[|x(t)|^2]$, increasing the denominator $1/T \int_0^T |x(t)|^2 dt$, or both. The effectiveness of a PAPR reduction technique is measured by the complementary cumulative distribution function (CCDF), which is the probability that PAPR exceeds some threshold, i.e.:

$$CCDF = \text{Probability}(PAPR > p_0), \quad (6)$$

Where p_0 is the threshold.

PAPR REDUCTION

Various methods are proposed for reduction of PAPR in OFDM systems and each of the method is explored to prove efficacy of proposed one. A vital factor in selection of any technique is its performance as regards to PAPR reduction. This means how much capable the technique is as regards to reduction of PAPR. This capability of any technique is algorithm dependent. For example Interleaver technique PAPR reduction capability is less than that of SLM [5]. However this PAPR reduction performance must be judged while giving a cautious thought to other detrimental effects which may result. Take an instance of clipping technique which has very high performance as

regards to PAPR reduction but the amount of resultant in-band distortion and out-of-band radiation is intolerable.

PROPOSED TECHNIQUE

A companding system compresses the signal at input and expands the signal at output in order to keep the signal level above the noise level during processing. In other words, companding amplifies small inputs so that the signal level is well above the Noise floor during processing. At the output, the original input signal is then restored by a simple attenuation. Companding increases the SNR when the input signal is low and therefore reduces the effect of a system's noise source.

Companding introduces minimum amount of OBI if the companding function $f(x)$ is infinitely differentiable. The functions that meet the above condition are the smooth functions. We now propose a new companding algorithm using a smooth function, namely the airy special function. The companding function is as follows [6]:

$$f(x) = \beta \cdot \text{sign}(x) \cdot [\text{airy}(0) - \text{airy}(\alpha \cdot |x|)] \quad (7)$$

where $\text{airy}(\cdot)$ is the airy function of the first kind. α is the parameter that controls the degree of companding (and ultimately PAPR). β is the factor adjusting the average output power of the compander to the same level as the average input power:

$$\beta = \sqrt{\frac{E[|x|^2]}{E\left[\left|\text{airy}(0) - \text{airy}(\alpha \cdot |x|)\right|^2\right]}} \quad (8)$$

where $E[\cdot]$ denotes the expectation.

The decompanding function is the inverse of companding can be written as

$$f^{-1}(x) = \frac{1}{\alpha} \cdot \text{sign}(x) \cdot \text{airy}^{-1}\left[\text{airy}(0) - \frac{|x|}{\beta}\right] \quad (9)$$

Where the superscript-1 represents the inverse operation. Notice that the input to the

decompander is a quantized signal with finite set of values. We can therefore numerically pre-compute $f^{-1}(x)$ and use table look-up to perform the decompanding in practice. Next we examine the BER performance of the algorithm. Let $y(t)$ denote the output signal of the compander, $w(t)$ the white Gaussian noise. The received signal can be expressed as:

$$Z(t) = y(t) + w(t) \quad (10)$$

The decompanded signal $\bar{x}(t)$ simply is:

$$\bar{x}(t) = f^{-1}[Z(t)] = f^{-1}[y(t) + w(t)] \quad (11)$$

EXPONENTIAL COMPANDING TECHNIQUE

We propose in this section a new nonlinear companding technique, namely “exponential companding”, [5] that can effectively reduce the PAPR of transmitted (companded) OFDM signals by transforming the statistics of the amplitudes of these signals into uniform distribution. The new scheme also has the advantage of maintaining a constant average power level in the nonlinear companding operation. The strict linearity requirements on HPA can then be partially relieved. Let $|t_n|^d$ the d^{th} power of the amplitude of companded signal t_n , have a uniform distribution in the interval $[0, \alpha]$. The exponent is called the degree of a specific exponential companding scheme. The CDF of

$$t_n^d \text{ is simply } F_{|t_n|^d}(x) = \frac{x}{\alpha}, \quad 0 \leq x \leq \alpha \quad (12)$$

The amplitude t_n^d of companded signal has the following CDF

$$\begin{aligned} F_{|t_n|^d}(x) &= \text{prob}\{|t_n|^d \leq x\} \\ &= \text{prob}\{t_n^d \leq x^d\} \end{aligned}$$

$$= \frac{x^d}{\alpha}, \quad 0 \leq x \leq \sqrt[d]{\alpha} \quad (13)$$

The inverse function of $F_{|t_n|^d}(x)$ is therefore

$$F_{|t_n|^d}^{-1}(x) = \sqrt[d]{\alpha x}, \quad 0 \leq x \leq 1$$

$$F_{|s_n|}(x) = \text{prob}\{|s_n| \leq x\}$$

$$= F_{|t_n|^d}(h(x)), \quad 0 \leq x \leq h^{-1}(\sqrt[d]{\alpha}) \quad (14)$$

Considering the phase of input signals, the companding function $h(x)$

$$\begin{aligned} h(x) &= \text{sgn}(x) F_{|t_n|^d}^{-1}(F_{|s_n|}(x)) \\ &= \text{sgn}(x) \sqrt[d]{\alpha [1 - \exp(-x^2/\sigma^2)]} \quad (15) \end{aligned}$$

Here, $\text{sgn}(x)$ is the sign function. The positive constant α determines the average power of output signals. In order to keep the input and output signals at the same average power level. Let at the receiver side, the inverse function of $h(x)$ is used in the de-companding operation [7], i.e.,

$$\begin{aligned} \alpha &= \left[\frac{E[|s_n|^2]}{E[\sqrt[d]{1 - \exp(-s_n^2/\sigma^2)}]^2} \right]^{d/2} \\ h^{-1}(x) &= \text{sgn}(x) \sqrt{-\alpha^2 \log_e(1 - \frac{x^d}{\alpha})} \quad (16) \end{aligned}$$

With degree as a parameter, the companded signals have uniformly distributed amplitudes and powers, respectively for the cases $d=1$ and $d=2$. When, $d \geq 2$ the proposed function $h(x)$ can compress large input signals and expand small signals simultaneously. While the μ -law companding scheme can only enlarge small signals and does not change the signal peaks, which leads to a higher average power level of output signals. As seen, the differences between exponential companding functions are ignorable when $d \geq 8$.

SIMULATION RESULTS

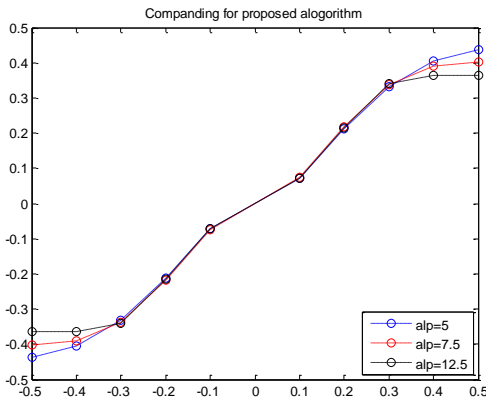


Fig: Transfer curve of proposed algorithm

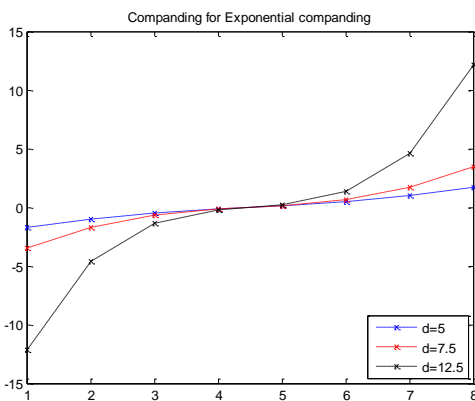


Fig: Transfer curve of Exponential algorithm

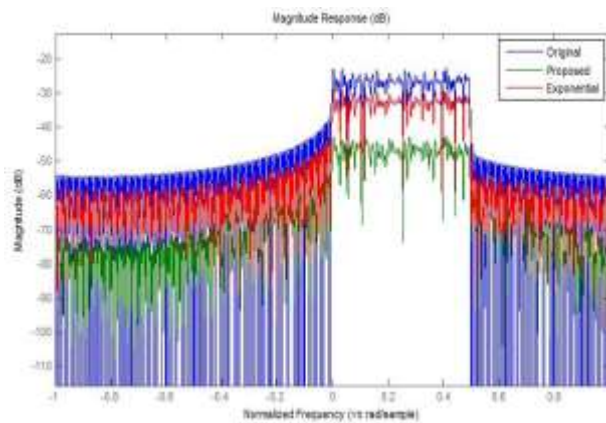


Fig: Frequency Response of proposed and Exponential

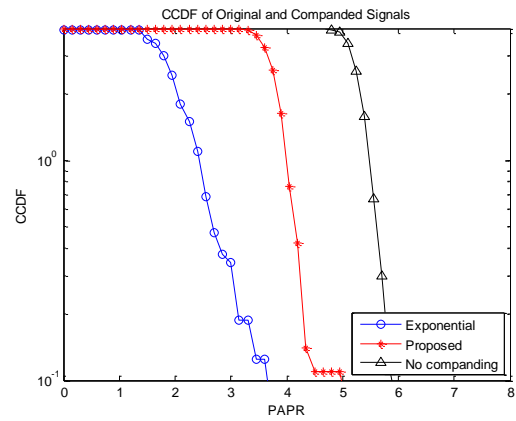


Fig: PAPR Reduction comparison of companding methods

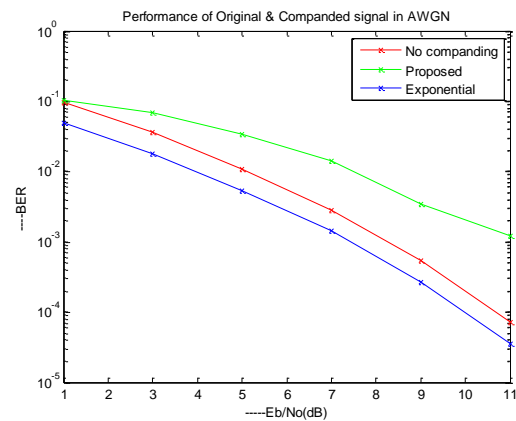


Fig: BER comparison of companding methods

CONCLUSION

In this paper, reduction of PAPR of OFDM by using proposed methods i.e., companding transform and Exponential Technique. PAPR is effectively reduced by Exponential Transform Technique. And also Bit Error Rate reduced. Exponential companding Transform gives better results than proposed technique. Those results are shown in simulation results by using MATLAB simulations.

REFERENCES

- [1] R.van Nee and R.Prasad, "OFDM for Wireless Multimedia Communications. Boston, MA: Artech House, 2000.
- [2] S.H.Han and J. H. Lee, "An Overview of peak-to average power ratio reduction techniques for multicarrier transmission," *IEEE Wireless Commun.*, vol. 12, pp. 56-65, Apr. 2005.

- [3] R.W.Bauml, R.F.H.Fischer, and J.B. Huber, "Reducing the peak-to-average power ratio of multi carrier modulation by selective mapping," IEE Electron. Lett., vol.32, pp. 2056-2057, Oct. 1996.
- [4] S.H Muller and J.B.Huber, "OFDM with reduced peak-to average power ratio by optimum combination of partial transmit sequences," IEE Electron. Lett., vol .33,pp. 368-369, Feb.1997.
- [5] X. Wang, T. T. Tjhung, and C. S. Ng, "Reduction of peak-to-average power ratio of OFDM system using a companding technique," *IEEE Trans.Broadcast.*, vol. 45, no. 3, pp. 303-307, Sept.1999.
- [6] T. Jiang and G. Zhu, "Nonlinear companding transform for reducing peakto- average power ratio of OFDM signals," *IEEE Trans. Broadcast.* vol. 50, no. 3, pp. 342-346, Sept. 2004.
- [7] T. Jiang, Y. Yang, and Y. Song, "Exponential companding technique for PAPR reduction in OFDM systems," *IEEE Trans. Broadcast.*, vol. 51, no. 2, pp. 244-248, June 2005.