

# Reliability Analysis of Failure Mode Screening Using Fuzzy Set Theory

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## **Abstract**

*In reliability analysis of a system using classical set theoretic approach or some statistical methods, we require exact knowledge about the functioning of the system. But in most of the cases, it is not possible to acquire the information to a high degree of exactness. This brings the importance of fuzzy set theory in reliability analysis. In this paper we shall study the failure mode screening methodology based upon fuzzy set theory.*

**Keywords:-** failure mode screening, fuzzy set theory

## **1. Introduction**

Most of the researches[1,4,5,6,11] in classical reliability theory are based on binary state assumption for states. In gracefully degradable systems, it is unrealistic to assume that the system possesses only two states that is, 'working' or 'failed'. Such systems may be considered working to certain degrees at different states of its degradation during its transition from fully working state to completely failed state. The degree may be any real number between 0 and 1. Degree 0 would represent the system in completely failed state while a fully working state would be represented by degree 1. The assignment of the degree may depend upon the limit of tolerance of the user about the adequate performance of the system. Zadeh[1] suggested a paradigm shift from the theory of total denial & affirmation to a theory of grading, to give new concept of sets called fuzzy sets. Fuzzy sets can express the gradual transition of the system from working state to failed state. The crisp set theory only dichotomizes the system in working state and failed state but fuzzy state theory can cover up all possible states between a fully working state and completely failed state. This

approach to the reliability theory is known as Profust reliability, wherein the binary state assumption is replaced by fuzzy state assumption.

This chapter presents an efficient methodology that is developed for the reliability prediction and the failure mode effects of the pacemaker using fuzzy logic. The reliability prediction is based on the general features and characteristics of the factors affecting the Heart pacemaker and a de-rating plan for the system design is developed in order to maintain low components' failure rates. These failure rates are used in the Failure mode effects, which uses fuzzy sets to represent the respective parameters. A fuzzy failure mode risk index is introduced that gives priority to the criticality of the components for the system operation, while a knowledge base is developed to identify the rules governing the fuzzy inputs and output. The work in this chapter also studies the functional behaviour of cardiac pacemaker. All failure-causing factors have been divided into four categories- catastrophic factors, critical factors, marginal factors and other factors. There are five failure causing factors namely voltage of the cell, refractory period,

impedance of the pacing circuit, Temperature, Sensitivity of pacemaker from other factors.

## 2. Classification and Fuzzification of Failure Causing

### Factor:-

#### (a) Classification of factors causing system failure:

The failure of a system may be caused by various factors. These factors may not play equal role in system failure. Rather they may have different importance. Thus these factors may be categorized as decisive, specific, momentous and related factors. These factors differ with each other in the sense that, their effects in the failure of the system are different. The factors that make the system immediately completely failed if they occur in their full strength, have

$$f_i(x) = \begin{cases} 0 & \text{for } x \leq a \text{ or } x \geq c \\ \frac{x-a}{b-a} & \text{for } a \leq x \leq b \\ 1 & \text{for } b \leq x \leq c \\ \frac{d-x}{d-c} & \text{for } c \leq x \leq d \end{cases} \text{-----(1)}$$

where  $x$  denote the numerical value of factor  $i$ . This factor  $i$  may be assigned any other fuzzy number i.e. by triangular/trapezoidal fuzzy number or fuzzy number of any other type. The fuzzification of these factors depend on the nature of occurrence during any experimental or functioning mode.

In our study we have classified these factors as (i) Decisive factors (ii) Specific factors In the present chapter these factors have been fuzzified on assigning them

$$f_v(x) = \begin{cases} 0 & \text{for } x < 0 \text{ or } x > 6 \\ 1 & \text{for } 0 \leq x \leq 1 \\ \frac{6-x}{5} & \text{for } 1 \leq x < 6 \end{cases} \text{-----(2)}$$

been put in the decisive category. The specific factors however have key role in making system failed, but they are less significant in comparison to decisive factors. The momentous factors include that variety of factors that have very significant role but they cannot cause system failed even in their full strength. Related factors are the factors involved in system failure although they do not play a major role in system failure.

#### (b) Fuzzification of failure causing factors:

These precipitating factors may be fuzzified by associating adequate fuzzy set to these factors. This facilitates us to quantify the contribution of a particular factor by a fuzzy number between 0 and 1

Let  $f_i$  be the factor causing the failure of a system. Then this factor may be fuzzified as below.

trapezoidal fuzzy numbers of distinct shapes. The shapes of these fuzzy numbers depend on the nature of the involved factor i.e. what sort of behaviour it imparts with the change in their numerical value. Fuzzification of the failure causing factors in this study has been done in the following manner.

(i) **Low voltage of the cell:** The voltage of the cell is considered lying in the range {0- 10 volt}. The fuzzification of this factor is done by the following example and also shown in fig. 7.4.1.

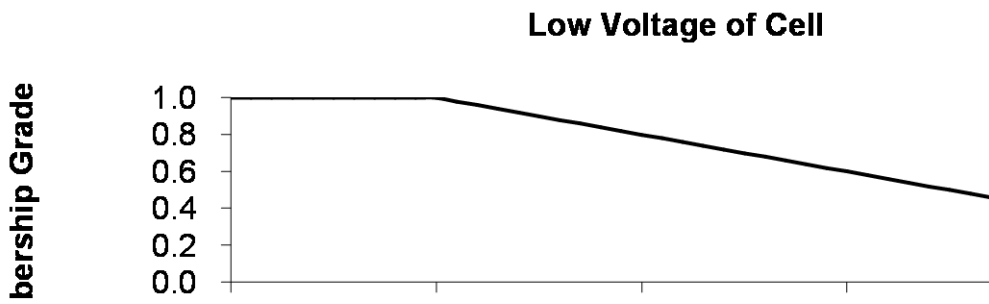


Fig.1

(ii) **Refractory index of the cell:** The refractory index of the lithium cell is fuzzified on the scale having the range {400-500 ms}.

$$f_{RI}(x) = \begin{cases} 0 & \text{for } x \leq 400 \text{ or } x \geq 500 \\ \frac{x-400}{30} & \text{for } 400 \leq x \leq 430 \\ 1 & \text{for } 430 \leq x \leq 470 \\ \frac{500-x}{30} & \text{for } 470 \leq x \leq 500 \end{cases} \quad \text{-----(3)}$$

Refractory Index of the cell



Fig.2

(iii) **Impedance of Pacing circuit:** The Impedance of the pacing circuit has the numerical values from 5000Ω to 20000Ω the expression for the fuzzification of this factor on the given range is given as

$$f_{RI}(x) = \begin{cases} 0 & \text{for } x \leq 4000 \text{ or } x \geq 16000 \\ \frac{x-5000}{5000} & \text{for } 4000 \leq x \leq 10000 \\ 1 & \text{for } 10000 \leq x \leq 15000 \\ \frac{20000-x}{5000} & \text{for } 15000 \leq x \leq 20000 \end{cases} \quad \text{-----(4)}$$

### Impedance of Pacing Circuit

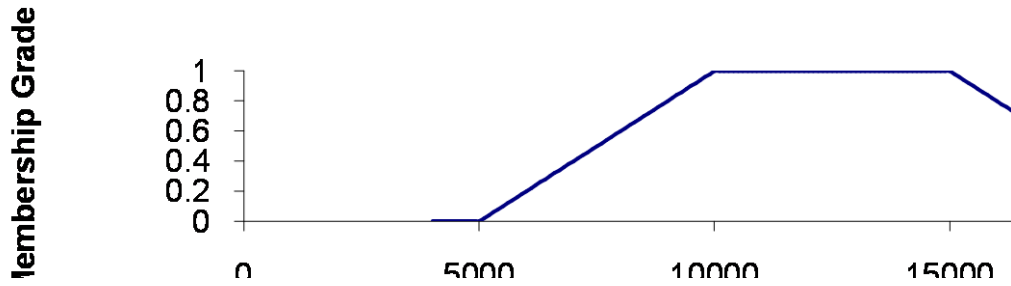


Fig. 3

**(iv) High Temperature of the working place:** The temperature of the place where the functioning of the pacemaker is going on also matters. In the present chapter this factor have been fuzzified by assigning the trapezoidal fuzzy set defined on the universal set  $\{0- 50^{\circ}\text{F}\}$  as the following expression and fig. 7.5 represents this factor.

$$f_{RI}(x) = \begin{cases} 0 & \text{for } x \leq 50 \\ \frac{.12x - .6}{3} & \text{for } 5 \leq x \leq 30 \\ 1 & \text{for } 30 \leq x \end{cases} \text{-----(5)}$$

### High Temperature

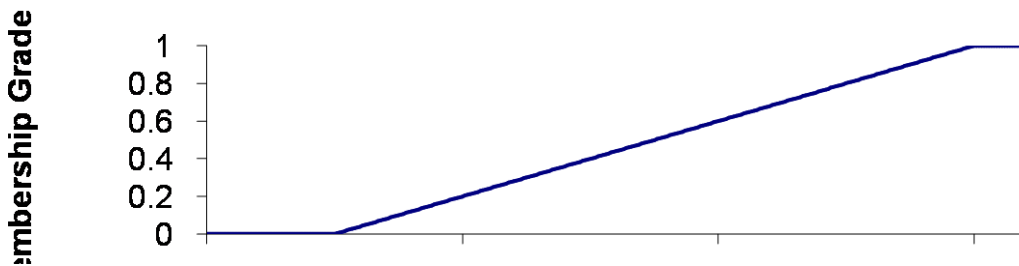


Fig.4

**(v) Close Distance:** This factor is fuzzified by assigning a fuzzy set to this factor defined on the universal set  $\{0- 20\text{cm}\}$  in the following manner and shown in the fig.

$$f_{RI}(x) = \begin{cases} 0 & \text{for } x \leq 50 \\ 1 & \text{for } 0 \leq x \leq 5 \\ \frac{20-x}{15} & \text{for } 5 \leq x \leq 20 \end{cases} \text{-----(6)}$$

## Close Distance

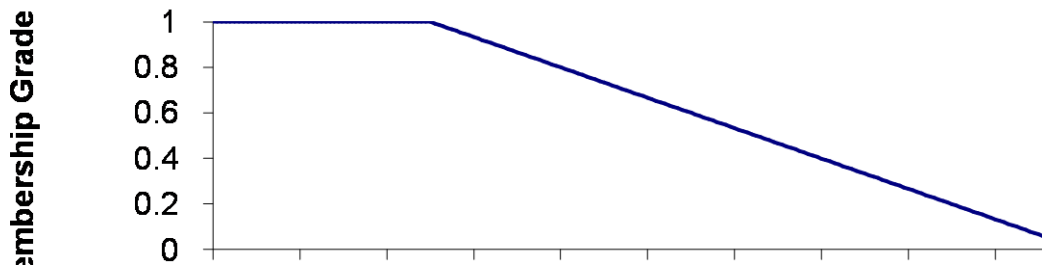


Fig.5

### 2. Measure of Failure Causing Factors:

Since the failure causing factors have a very important role in system analysis. Therefore the measure of these factors is a matter of serious concern for system behaviour (analysis). That is why these factors should be measured with the help of very effective tools.

However the best effective tools are usually used to measure these factors. But at the time of manufacturing of these tools, a major part of uncertainties are avoided for sake of simplicity. For example suppose the voltmeter gives the reading between 2.5 and 2.6, either we take the mean of these values or we adopt any one of these two values for computation. But during the analysis of micro systems, where such deviation of the estimated value from exact value may create a serious problem, we cannot dare to avoid this problem and we require some more effective tools to measure the involved factors. This problem can be sorted out by using fuzzy set theory. According to which a fuzzy number is assigned to the concerned variable in place of crisp numbers. For illustration let the needle of the voltmeter lies at any place between 2.5 and 2.6. Then we may assign a fuzzy number “about 2.5” or “almost 2.6” to the voltage.

In the present chapter we study the functioning of a pacemaker. The factors causing failure of a pacemaker are classified as below.

- (i) Decisive Factors (ii) Specific Factors

**(i) Decisive Factors:** As defined above this class includes the factors, occurrence of those makes the system failed immediately as they occur. Here voltage of lithium-iodine cell is taken as one of this type of factors. In a pacemaker the voltage produced by lithium iodide cell is 2.8V, however the pacemaker require the voltage of about 5V. Thus a voltage doubler circuit is placed to make it possible. The voltage produced by the cell and then doubled by the doubler is usually less than 5. This may be almost 4, about 3 and so on.

Suppose during any operation the voltage have three values between 0 and 5. Let “about 2”, “about 3” and “about 4” are the fuzzy numbers for the values attained by the voltage. These fuzzy numbers “about 2”, “about 3” and “about 4” assigned to voltage are defined as below and shown in fig. 2

$$f_V'(x) = \begin{cases} \frac{x}{2} & \text{for } 0 \leq x < 2 \\ \frac{3.5-x}{1.5} & \text{for } 2 \leq x < 3.5 \end{cases} \text{-----(7)}$$

$$f_V''(x) = \begin{cases} \frac{x-1}{3} & \text{for } 1 \leq x < 3 \\ 4-x & \text{for } 3 \leq x < 4 \end{cases} \text{-----(8)}$$

$$f_V'''(x) = \begin{cases} \frac{x-2.5}{1.5} & \text{for } 2.5 \leq x < 4 \\ 3.5-x & \text{for } 4 \leq x < 5 \end{cases} \quad \text{-----(9)}$$

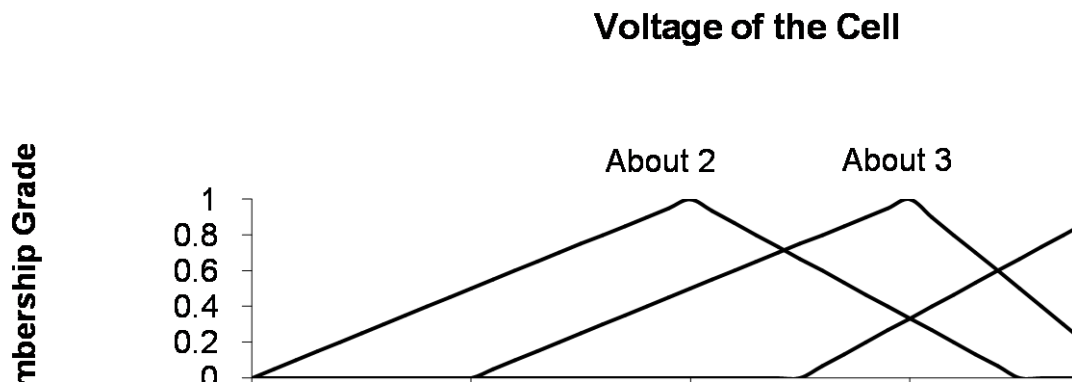


Fig. 6

Now it is not easy to take any of the above fuzzy numbers for our purpose. Rather we have to find out a fuzzy number that suits with all these fuzzy numbers. That can be obtained by using the technique mentioned above for getting

best-approximated fuzzy number. Hence using the technique mentioned above we get the best-approximated fuzzy number for our purpose.

$$f_V(x) = \begin{cases} \frac{x-1.17}{1.83} & \text{for } 1.17 \leq x < 3 \\ \frac{4.17-x}{1.17} & \text{for } 3 \leq x < 4.17 \end{cases} \quad \text{-----(10)}$$

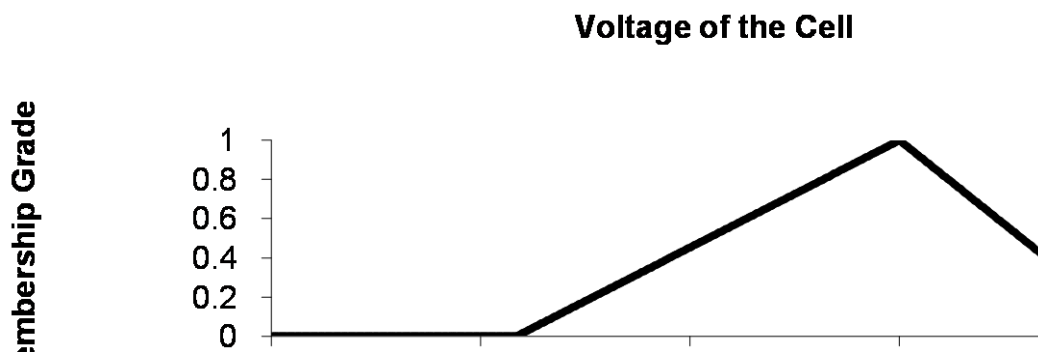


Fig.7

Another factor that has been put in the family of decisive factors is refractory index. For a commercial pacemaker, the value of refractory index ranges from 400-500 ms. Here during our operation three fuzzy numbers

“about 300”, “about 400” and “about 450” are assigned to this factor refractory index. Suppose these three fuzzy numbers can be defined by the following expression and fig. 4 gives the pictorial form of these fuzzy numbers.

$$f'_{RI}(x) = \begin{cases} \frac{x-220}{80} & \text{for } 220 \leq x < 300 \\ \frac{410-x}{110} & \text{for } 300 \leq x < 410 \end{cases} \text{-----(11)}$$

$$f''_{RI}(x) = \begin{cases} \frac{x-315}{85} & \text{for } 315 \leq x < 400 \\ \frac{495-x}{95} & \text{for } 400 \leq x < 495 \end{cases} \text{-----(12)}$$

$$f'''_{RI}(x) = \begin{cases} \frac{x-360}{90} & \text{for } 360 \leq x < 450 \\ \frac{530-x}{80} & \text{for } 450 \leq x < 530 \end{cases} \text{-----(13)}$$

### Refractory Index of Cell

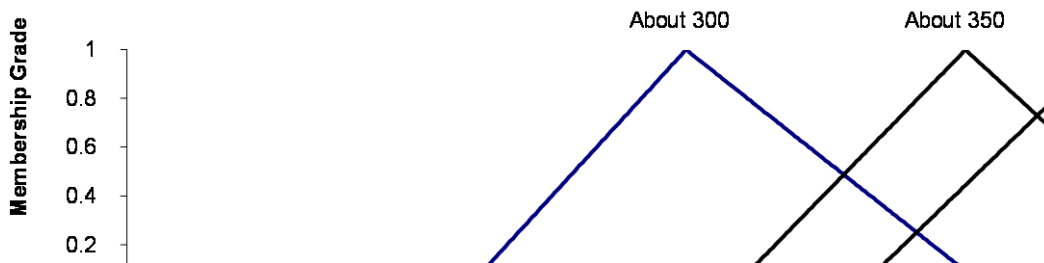
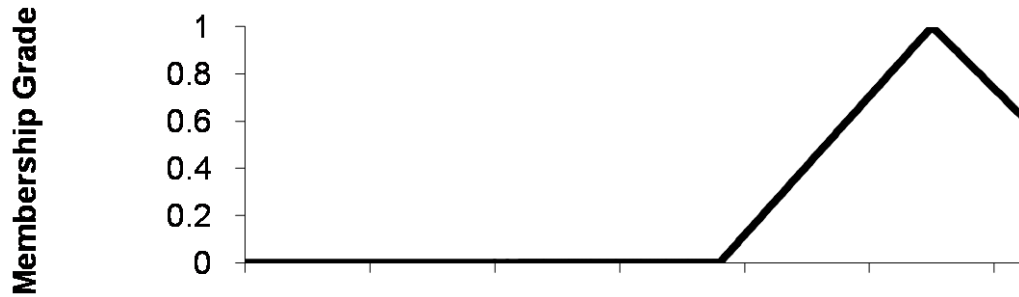


Fig.8

With the similar approach as mentioned above, we can find out a fuzzy number from these three fuzzy numbers. This fuzzy number thus obtained can be defined by the following expression and shown in fig.

$$f'_{RI}(x) = \begin{cases} \frac{x-290}{85} & \text{for } 290 \leq x < 375 \\ \frac{470-x}{95} & \text{for } 375 \leq x < 470 \end{cases} \text{-----(14)}$$

## Refractory Index of the Cell



**Fig.9**

**(ii) Specific Factors:** The impedance of the pacing circuit is the failure-causing factor that may be kept in the class of specific factors. The impedance of the pacing circuit lies between 5000 Ω to 20000 Ω. Whereas a demand pulse generator sensing amplifier is generally a high input impedance device the value of which may be 5 MΩ .

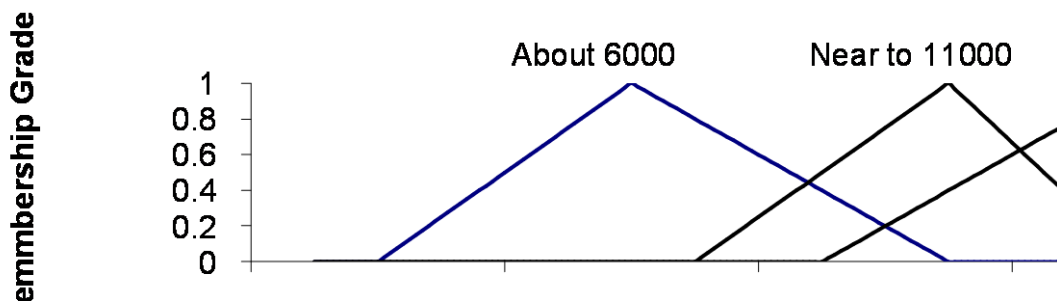
Here in our study we have assigned three fuzzy numbers “about 6000”, “near to 11000”, and “almost 14000” etc to the impedance of pacing circuit. Suppose these three fuzzy numbers can be defined as below.

$$f_R'(x) = \begin{cases} \frac{x-2000}{4000} & \text{for } 2000 \leq x < 6000 \\ \frac{11000-x}{5000} & \text{for } 6000 \leq x < 11000 \end{cases} \quad \text{-----(15)}$$

$$f_R''(x) = \begin{cases} \frac{x-7000}{4000} & \text{for } 7000 \leq x < 11000 \\ \frac{14000-x}{3000} & \text{for } 11000 \leq x < 14000 \end{cases} \quad \text{-----(16)}$$

$$f_R'''(x) = \begin{cases} \frac{x-9000}{5000} & \text{for } 9000 \leq x < 14000 \\ \frac{18000-x}{4000} & \text{for } 14000 \leq x < 18000 \end{cases} \quad \text{-----(17)}$$

## Impedance of Pacing Circuit



**Fig.10**



In a similar manner as we have earlier discussed. We have to find out a fuzzy number out of these fuzzy numbers i.e a fuzzy number that is approximated with the help of all these three fuzzy numbers. This following expression is used to express this fuzzy number.

$$f_R(x) = \begin{cases} \frac{x-5670}{4330} & \text{for } 5670 \leq x < 10000 \\ \frac{14000-x}{4000} & \text{for } 10000 \leq x < 14000 \end{cases} \quad \text{-----(18)}$$

This fuzzy number can be shown as below.

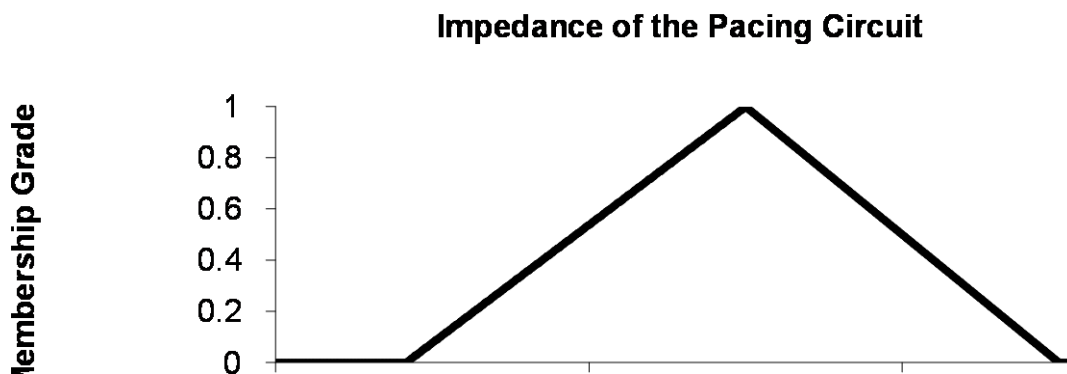


Fig.11

**3. Discussion and Interpretation: -**

This Chapter aimed at studying a failure mode screening methodology based upon the fuzzification of the effects of precipitating factors provoking the failure. The failure mode of a pacemaker is categorized into four classes: catastrophic, critical, marginal and related (minor) factors. On the basis of expert analysis it is observed that voltage of the cell and refractory period of the cell fall into the category of catastrophic factor while Pacing circuit impedance, Temperature and distance of pacemaker from cell phone respectively fall into the categories critical, marginal and minor.

**4. Conclusion:-**

This chapter thus applies an approximate reasoning algorithm, consisting of a fuzzy mathematical formulation to a pacemaker to enable the inference mechanism of a constructed expert system to identify the most likely equipment failure modes to precipitate. The process can be ignored if any of the catastrophic factor crosses its extreme idealized limits. In such case it passes the screening test without any calculations. This screening algorithm is useful in developed expert system only when all catastrophic factor values are within their extreme limits. In such case it enables the computer expert system to better comprehend the interdependencies of failure causing factors in the failure mode.

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