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EVEN GRACEFUL LABELING OF SOME NEW GRAPHS

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Abstract

In this paper some new even graceful graphs are investigated. We prove that the graph obtained by joining two copies of even cycle C_n with path P_k and the splitting graph of $K_{1,n}$ admits even graceful labelling.

Keywords: Graceful labelling, graceful graph, even graceful labelling, Splitting graph

Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [2]. The symbols V(G) and E(G) will denote the vertex set and the edge set of a graph G. The cardinality of the edge set is called the size of the graph G. A graph with p vertices and q edges is called a (p,q) graph.

Definition 1.1

If the vertices are assigned values subject to certain conditions, then it is known as graceful labelling. The study of graceful graphs and graceful labelling methods was introduced by Rosa,[5].

A function f is called graceful labelling of the graph G if f: V→{0, 1, 2, …., q} is injective and the induced function f*:E → {1, 2, …. ,q} defined as f*(uv) = $|f(u) - f(v)|$ is bijective. A graph which admits graceful labelling is called graceful graph.

Definition 1.2

A graph G = (V (G), E (G)) is said to admit even graceful labelling if f: V(G) → {0, 1, 2, …..2q - 1} is injective and the induced function f *: E(G) →{2, 4, 6, ……2q-2} defined as f* (uv) = |f(u) – f(v)| is bijective. A graph which admits even graceful labelling is called an even graceful graph.

Definition 1.3

For a graph G, the splitting graph is obtained by adding to each vertex v, a new vertex v' so that v' is adjacent to every vertex that is adjacent to v in G.

Here we prove that the graph obtained by joining two copies of even cycles C_n and path P_k, and splitting graph of $K_{1.n}$ are even graceful graphs.

Main Results

Theorem 2.1: The graph obtained by joining two copies of cycle C_n of even order(n ≡ 2 mod 4) with the path P_k admits even graceful labelling.

Proof: Let $v_1, v_2, \dots, v_n$ be the vertices of the cycle C_n and $u_1, u_2, \dots, u_k$ be the vertices of the path P_k. Consider two copies of C_n of even order and n ≡ 2 mod 4. Let G be the graph obtained by connecting two copies of C_n with path P_k. Let $v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{[n+(k-2)]+n-1}, v_{2n+(k-2)}$ be the vertices of G and these vertices form a spanning path in G. In this spanning path the vertex v_n is the vertex common to the first copy of C_n and path P_k as well as the vertex $v_{[n+(k-2)]+1}$ is the vertex common to the second copy of C_n and the path P_k. Define f: V(G) →{, 1, 2, …..2q - 1} as follows:

For $1 \leq i \leq n-1$, $f(v_i)$ = { i-1, if i is odd and (4n+2k)-i if i is even}

For $n \leq i \leq \frac{3n}{2} + k$, $f(v_i)$ = { i if i is even and (4n+2k)-(i-3) if i is odd}

For $\frac{3n}{2} + k + 1 \leq i \leq 2n + k-2$, $f(v_i)$ = {(4n+2k)-(i+1) if i is odd and i if i is even }

In accordance with the above labelling pattern ,the graph under consideration admits even graceful labelling.

Illustration 2.2: Consider the graph obtained by attaching two copies of C_{10} by P_5. The labelling pattern is shown in Fig 1.

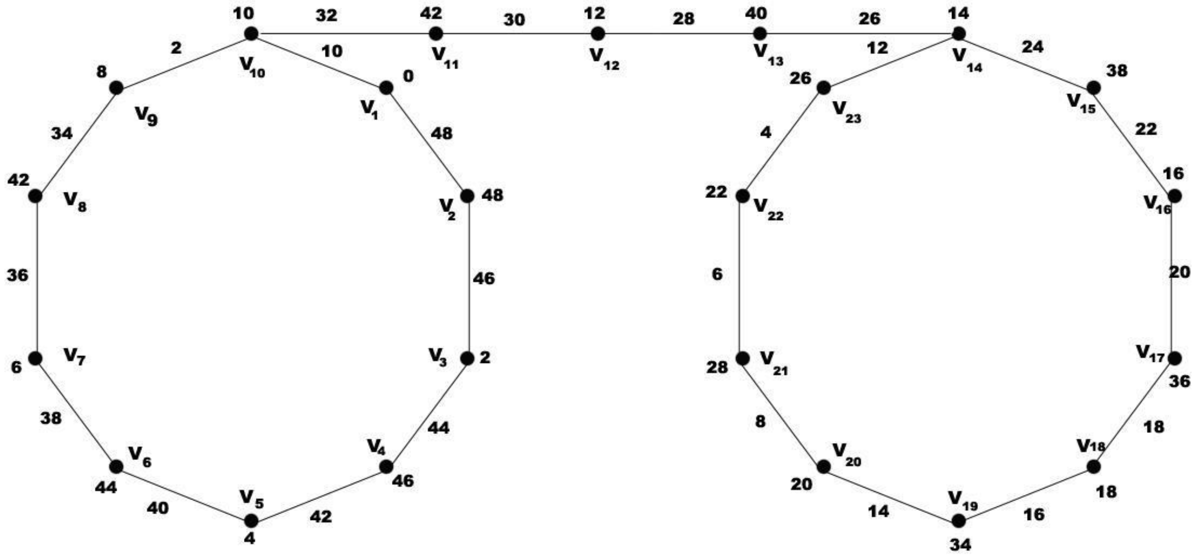

Fig 1

Theorem 2.3: Splitting graph of a star admits even graceful labelling.

Proof: Let v, v_1, v_2, ... , v_n be the vertices of star graph $K_{1,n}$ with v be the apex vertex. Let G be the splitting graph of $K_{1,n}$ and v', v_2', ... , v_n' be the newly added vertices to $K_{1,n}$ to form G.

Define f : V(G)⟶ {0,1, 2, …..2q - 1} by f(v) = 0; $f(v_i) = 6n - 4i + 4$ for $1 \leq i \leq n$,

$f(v') = 2$, and $f(v_i') = 2i$ for $1 \leq i \leq n$.

In view of the above defined labelling pattern, G admits even graceful labelling

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Illustration 2.4: Fig 2 shows the labelling pattern of the splitting graph of $K_{1,6}$.

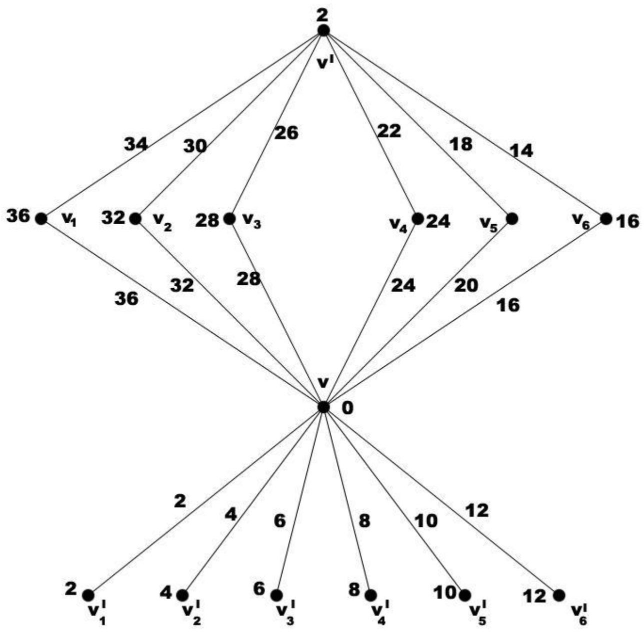

Fig 2

1. Conclusion Remarks

Gracefulness and even gracefulness of a graph are two entirely different concepts. A graph may possess one or both of these or neither. In the present work we investigate two families of even graceful graphs. To investigate similar results for other graph families and in the context of different labelling techniques is an open area of research.

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