www.ijecs.in
International Journal Of Engineering And Computer Science ISSN: 2319-7242
Volume 5 Issue 5 May 2016, Page No. 16620-16638

Free Vibration Response Of Double-Bay Multi-Storey Building Frames With Stiffened Joints

E.C. Mbanusi*[1], D.A. Obodoh[2]
[1] Department Of Building, Nnamdi Azikiwe University, Awka, Nigeria
[2] Department Of Building, Nnamdi Azikiwe University Awka, Nigeria
E-Mail Address: [1]cyril.mbanusi@yahoo.com, [2]dominoanosike@yahoo.com.

ABSTRACT

This paper investigated self-excited vibration response of double-bay multi-storey building frames for the effect of joint stiffening on natural frequencies. One of the frames has normal rigid joints. Three others of the frames have stiffened joints of stiffened lengths: 250mm, 400mm and 750mm respectively. On account of limitations of shear frame models, the frames are modeled as frames with flexible horizontal members having multi degrees of freedom (MDOF). Classical displacement method of analysis is adopted using stiffness coefficients which are modified to include and embrace the contributions of joint stiffening. Results revealed: joint stiffening increased the natural frequencies of the frame; natural frequencies also increased with the length of stiffening. In addition, joint stiffening brought about enhanced values of other dynamic characteristics of the frame which are functions of natural frequencies.

Keywords: Free Vibration; Stiffening factor;
Natural frequencies; Restoring forces;
Multi-degree of freedom (MDOF);
Rotational and translational stiffness coefficients.

1. INTRODUCTION

Frames are the main load bearing components of structures of building and other skeletal formations such as ship, bicycle, bridge to mention but a few. They are made up of straight separate vertical, horizontal or inclined members connected together by means of welded, bolted, riveted or other types of joints.

Type and strength of the connecting joints are key elements to the overall stability of the frame. It is for this reason rigid types of connection are given preference over the hinged or pinned joints [1]. Furthermore, [2 and 3] have shown that stiffening the joints reduced the stress as well as enhanced the stability and the performance of the structure.

Frame can be a plane or space structure. Plane frames are two-dimensional and stable only in their own plane whereas space frames are three-dimensional and stable in all directions [4,6].

Forces acting on the frame structures are, very often, dynamic in nature. These time-dependent forces are characterized by variable intensity, frequency, and state, plane of acting and sense or direction, with respect to time. Most forms of loading have dynamic components, and some forms of structures especially where they are slender, are susceptible to the dynamic effects. Furthermore, the use of the structure, say as a laboratory housing sensitive instruments, may require that vibration be considered. Thus, there is an interrelation between the sources of dynamic excitation, the structural form and the purpose of the structure [5].

A structure [6] is said to be undergoing free vibration when it is disturbed from its position of static equilibrium and then allowed to vibrate without any external dynamic excitation. By free vibration, it is meant the motion of a structure without any dynamic excitation, external loads or forces or support motion. Free vibration is initiated by disturbing the structure from its equilibrium position by some initial displacements and/or by imparting some initial velocities [5, 6].

Sources of dynamic loads [5, 7-9] may include; (i) seismic disturbance (ii) wind (iii) industrial machinery (iv) human forces (v) moving vehicles (vi) blasting (vii) pile driving (viii) moving load on beam (ix) stationary vibrating loads (x) shock waves (xi) impact and sudden loading.

A dynamic load [5,6] is not only time-dependent, it is also essentially associated with the common characteristics of inducing or developing vibrations on the structure it acts. Such vibrations can often get excited at the same frequency as that of the forcing external dynamic load. Should the frequency of the external force coincide with any of the natural frequencies of the frame, a phenomenon known as resonance would ensue, leading to very large deflections and stresses beyond what the structure might have been designed for. At this point, the structure can even collapse [5,6,8]. Conventionally, evaluation of the natural frequencies [9], is a matter of priority interest to the engineer involved in the dynamic analysis of any structural system. This is for the reason the design would be better guided so as to avoid a situation where the structure ever plunged into the catastrophic regime of resonance amid the knowledge of such natural frequencies.

Shear frame model is a phenomenon of beam stiffness being sufficiently large relative to column stiffness, taken as rationale for assuming beams to be infinite for many practical situations [10]. Shear frame does not permit rotation of its joints but allows lateral vibration motion in its plane on account of the assumed infinite rigidity of its horizontal members [6].

Flaws exist when considering the frame as shear frame. Notable among the flaws are reality that even though shear frame approach can predict the natural frequencies with some degree of accuracy, it does not yield identical results with the actual deformation [11, 12]. Shear frame, as an analysis model [13], is not only insufficient but also deficient as a generalized dynamic model for multi-storey building structure.

Unlike the conventional analysis which assumes the shear frame model, this paper adopts an improved model for dynamic analysis of frames known as: frames with flexible horizontal members [6, 14]. In contrast to shear frames [6, 14], flexibility of beams as well as joint rotations are accounted for in computation of restoring forces which the elements of dynamic analysis are composed.

In numerical study using the improved model [14], it was apparent that, in comparison with the results of shear frame, flexibility of beams and associated joint rotations within a certain range have quite significant influence on model frequency of multi-storey building frames.

The double-bay four-storey simulated building frame, without any loss of generality, is considered as a multi-degrees of freedom (MDOF) system which permits only lateral translation or sway motion in its plane, fig, 4.

This paper presents the evaluation of the natural frequencies of an undamped vibrating double-bay four-storey frame with stiffened joints fig.3. This is with the aim to investigate the effect of stiffened joints on the natural frequencies using the dynamic displacement or stiffness method of analysis.

This paper seeks to achieve the aim through the following objectives.

i. To determine the bending moments due to unit translation at the four floor levels;

ii. To evaluate the shear forces due to unit translation at the four floor levels;
iii. To determine the restoring forces;
iv. To evaluate the natural frequencies of the simulated normal rigid frame and stiffened frames;

2. STRUCTURAL MODEL AND DYNAMIC FLOOR MASS

In this study, the building configuration adopted is simple and regular. It can essentially serve as an office or classroom block having a plan as shown in Fig. 1. A reinforced concrete frame of four-floors simulated model shown in Fig. 2 is to be used as the case study. Handling procedural arrangement is in four major phases, each comprising of four case studies as stated thus:

Phase 1 treats the same frame as a normal rigid frame structure under four separate cases, namely; unit translation at: (i) fourth floor level (ii) third floor level (iii) second floor level and (iv) first floor level respectively. Phase 2 deals with the simulated frame as a stiffened frame of 250mm stiffening length, for similar cases of translation as in phase 1. Phases 3 and 4 are similar to phase 2 but for the stiffening lengths are increased to 400mm and 750mm respectively. The floor thickness for each floor level is 150mm but for the topmost floor level, which is 125mm thick. A concrete grade of $30N/mm^2$ and of density $24KN/m^3$ is adopted. A provision of floor finishes of IKN/M^2 is made in respect of each of the floors. These culminated into the numerical value of dynamic floor mass, $M_1=M_2=$……… = Mn-1= 34404Kg and Mn=27523Kg: where n is taken to be number of floors.

2.1 DYNAMIC MODEL

Although every structural system is essentially system with distributed mass i.e. system is continuous and possesses infinite number of degrees of freedom (15, 16), the lumped mass system adopted in this work for the dynamic analysis provides satisfactorily approximate results (17).

Therefore, in the course of this work, each of the simulated double-bay, multi-storey frames is modeled as a structure having finite number of degrees of freedom, assuming lumped mass element concentration at the right corner of each floor level as shown in Fig. 4.

2.2 DYNAMIC ANALYSIS

The determination of the natural frequencies associated with the free vibration of structural system [9, 18] constitutes the basic principle in the dynamic analysis of structural system. As resonance at the lowest frequency would result in the maximum effects, [19], it is only the basic tone of vibration that is usually considered to be of paramount importance. [20] Indicated that with increasing number of floors, ten and above, the flexibility of building structures increases so as to necessitate inclusion of the higher modal frequencies determination as well, even if the buildings are regular. Although the number of floors for the building structure in this work did not fall within the stipulated in [20], the work did not deem it unnecessary to include determination of higher modal frequencies since resonance at the higher modal frequencies does not result in no dynamic effects.

2.3 EQUATION OF MOTION

Although the equations of static equilibrium differ from equations of dynamic equilibrium, D 'Alembert proposed a principle that makes it possible to use equation of static equilibrium in solving dynamic problems. This principle introduces terms that take into cognizance inertia forces expressed as products of masses by accelerations i.e. by the second derivatives of linear or angular displacements over time. The introduction of such terms (14, 1) automatically transforms equations of static equilibrium into equations of dynamic equilibrium; thus:

$$M\ddot{x}_{(t)} + C\dot{x}_{(t)} + Kx_{(t)} = P_{(t)} \ldots\ldots\ldots\ldots \quad (1)$$

Where: M, C, K are matrices of mass elements, damping elements and stiffness coefficients respectively:

$P_{(t)}$ is the column vector representing the external excitations.

$x_{(t)}, \dot{x}_{(t)}, \ddot{x}_{(t)}$ are vectors of displacements, velocities and accelerations of a system having finite number of degrees of freedom. Since this work focuses on the regime of free vibration only, there is complete absence of external excitation for this system. Therefore, the exciting force vector is assumed to be Zero. In this case, equation (1) becomes:

$$M\ddot{x}_{(t)} + C\dot{x}_{(t)} + Kx_{(t)} = O \quad \ldots\ldots (2)$$

Generally, to a certain degree, all vibrating systems are subject to damping effect for the reason energy is dissipated by friction and other resistances. Nonetheless, in some cases, damping is very small or the dynamic disturbance on the vibrating system, such as building, act for relatively short duration in such a manner the effect of damping becomes very, negligible and quite unimportant [21, 22, 23]. Hence, this work completely neglects the effect of damping so equation (2) reduces to:

$$M\ddot{x}_{(t)} + Kx_{(t)} = O \quad \ldots\ldots (3)$$

Adopting the stiffness matrix and displacement approach gives the impetus for evaluating the natural frequencies of the simulated frames in the way as solving the non-trivial equation thus:

$$\left| M^{-1} K_s - W^2 I \right| = O \quad \ldots\ldots (4)$$

Where:
$M^{-1} K_s$ = dynamic matrix
and
M^{-1} = inverse matrix of mass system
K_s = stiffness matrix
W = eigenvalues
I = identity matrix

3. METHODOLOGY

For frames with normal rigid joints, the traditional rotational and translational stiffness coefficients suitable for frames with normal rigid joints [6] are used for members and deemed to be fixed at both ends of the member where:

Case 1: Rotational Stiffness Coefficients

$$M_{(o)} = \frac{4EI}{L}, \quad M_{(L)} = \frac{-EI}{L}$$

$$\ldots\ldots 5$$

$$Q_{(o)} = \frac{-6EI}{L^2}, \quad Q_{(L)} = \frac{-6EI}{L^2}$$

Case II: Translational Stiffness Coefficients

$$M_{(o)} = \frac{6EI}{L^2}, \quad M_{(L)} = \frac{-6EI}{L^2}$$

$$Q_{(o)} = \frac{-12\,EI}{L^3}, \quad Q_{(L)} = \frac{-12\,EI}{L^3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

The stiffness coefficients for members of frames with stiffened joints are derived following ideas developed by [2], and making adaptation from the equation deduced.

Case I: Rotational Stiffness Coefficients

$$M_A = \frac{4EI}{L}\left[1 + 3\alpha - 3\alpha^2\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

$$\alpha = \frac{a}{L}$$

a = length of stiffened A end of the member

L = length of flexible portion of the member

$$Q_A = \frac{-6EI}{L^2}\left[1 + 2\alpha\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

$$M_B = \frac{-2EI}{L}\left[1 + 3(\alpha + \beta) + 6\alpha\beta\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

$$Q_B = \frac{-6EI}{L^2}\left[1 + 2\beta\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

Where:

$$\beta = \frac{b}{L}$$

b = length of stiffened B end of the member.

Note: In this work, a = b in each case. Therefore; $\alpha = \beta$

Case II: Translational Stiffness Coefficients

$$M_A = \frac{6EI}{L}\left[1 + 2\alpha\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

$$Q_A = Q_B = \frac{-12\,EI}{L^3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

$$M_B = \frac{-6EI}{L^2}\left[1 + 2\beta\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

i. The bending moment values for each of the member ends is computed as: $M = M_p + \sum_{i=1}^{n} M_i X_i$ ……………..……….(14)

ii. Where M_p is replaced with RiΔj since there is no external loading but unit translation, i.e. effects of applied unit translation at various floor levels, RíΔj, constitute the load vectors for the equilibrium equations.

iii. The final shear forces on each of the members of the frame is corrupted from the corresponding bending moments as:

$$H_i = \frac{M'e' - M'b'}{L} \quad \text{……………………………………..(15)}$$

Where:

M 'e' = value of moment at the far right end of the member length

M 'b' = value of the moment at the left end of the member length

L = length of member

iv. The restoring forces, kij, are systematically computed from appropriate shear force diagram as:

$$Kij = \sum_{k=i}^{n} Hq \quad \text{………………………………………………………. (16)}$$

Where: Hq = shear force value,
q = 1, 2, …., n

v. The natural frequencies of the simulated building frame with values of a = b= 0, are evaluated using the method of classical displacement with respect to frames with flexible horizontal members model [FHMM].

In addition, the natural frequencies of the simulated building frames with values of a = b = 250mm, a = b 400mm and a = b = 750mm, respectively, on separate notes, are evaluated using the same method of classical displacement with respect to frames with flexible horizontal members model as well.

4. RESULTS AND DISCUSSIONS

Results

Tables 1 through 8 compare the bending moment and shear force resistances produced from the simulated frame with normal rigid joints and simulated frames with stiffened joints.

i. The results revealed: (i) both the moment and shear force resistances from the stiffened frames were quite higher than the corresponding values from the normal rigid frame, no matter the lengths of stiffening.
ii. The values of bending moment and shear force resistances increased with the length of stiffening.
iii. It would be observed that in each phase, table 9, restoring force, kij, values satisfied Maxwell, Law of reciprocal.
iv. It is observed the natural frequencies produced in the frame with stiffened joints were greater than the corresponding values of natural frequencies for the normal rigid frame, table 10.
v. In addition, natural frequencies increased with the length of stiffening, table 10.
vi. Natural frequency increased with stiffening factor, fig. 5.

Discussion

The observed results could infer the following:-

i. The results were in consonance with earlier studies of Osadebe [2] and Aminu [3] regarding enhanced performance of frames with stiffened joints. This is so because, since bending moment and shear force resistances are significantly enhanced in the frames with stiffened joints, displacements associated with them such as joint rotation and joint rotation or side sway, flexural and shear cracks, would corresponding reduce thereby leading to increased stability, durability and enhanced structural integrity of the frame structure.

ii. The result suggest an optimal length of stiffening, say L_o, exists at which the maximum benefits of stiffening is utilized, i.e. at L_o, a balance would be hit between the desire for bending moment reduction and increased natural frequencies. Hence, it would be likely that above L_o, the gains of stiffening in terms of bending moment reduction and stability of frame would commence their depreciation, while below L_o, the framed structure exhibits traits of normal rigid frame in respect of low natural or modal frequencies.

iii. Stiffening of joints of framed structures increases their rigidity against dynamic forces since natural frequencies get enhanced through stiffening. Therefore, with stiffened joints vulnerability of the framed structure to resonance with any of the dynamic forces would be credibly reduced.

5. CONCLUSION AND RECOMMENDATION

CONCLUSION

Based on those results of this investigation, it is concluded that stiffening of connecting joints of framed structures increases the natural frequencies of the framed structure while at the same time satisfying the stability requirement.

Therefore, stiffening the joints of multi-storey buildings would safeguard these buildings against resonance and its associated repercussions and consequences.

An area to be investigated further would be the functional relationship between bending moment (m) natural or modal frequencies (w), and optimal length of stiffening, Lo.

RECOMMENDATION

This study resents the following recommendations drawn basically from this investigation:

i. Structural dynamics analysts and designers of frame structures should always include determination of not only values of the lowest modal frequencies but also those of the higher modal frequencies because resonance at the higher modal frequencies does not result in zero dynamic effects.

ii. Stiffening of joints of frame structures should be given credible encouragement as much as possible.

iii. Static and dynamic analysts and designers of frame structures should pursue further the exploitation of the gains of joints stiffening for enhanced structural integrity of such structures.

REFERENCES

1. Darkov, A., Structural Mechanics 4th ed., Mir Publishers, Moscow, 1989.

2. Osadebe, N. N., Stress Analysis of Frames with stiffened Joints. Proceedings on the 4th International Conference on Structural Engineering Analysis and Modeling, SEAM 4, Accra, 1997, pp. 134 -145.

3. Aminu, B. N., Matrix Analysis of Multi-storey Building Frame with stiffened Joints, M. Eng. Dissertation, Department of civil Engrg, Enugu State University of Science and Technology, Enugu, Nigeria, 1995.

4. Ymanskago, A. A (editor), Handbook for Designers II in Russia, Stroiizdat, Moscow, 1973

5. Smith, J. W., Vibration of Structures, Application in Civil Engineering Design, Chapman and Hall, New York, 1988.

6. Chopra, A. K., Dynamics of Structures. Theory and Application to Earthquake Engineering, Pearson, New Delhi, 2012.

7. Nelson, H. M. Bhatt, P., Marshal and Nelson's Structures, 3rd ed., Longman Publishers Ltd, Singapore, 1990.

8. Kolousek, V., Dynamics in Engineering Structures, Butterworth & Co. Ltd, London, 1973.

9. Biggs, J. M., Introduction to Structural Dynamics, McGraw-Hill Book Company, New York, 1964.

10. Hurty, W. C., and Robinstein, M. F., Dynamics of Structures, Prentice-Hall, Inc., New Jersey, 1964.

11. Clough, R. W., and Penzien, J., Dynamics of Structures, McGraw-Hill int. Student Edition, Tokyo, 1975.

12. Masur, E. F., Discussion on "Effect of Joint Rotation on Dynamics of Structures" ASCE, mechanics Division, Vol. 88, 1962, pp. 83-85, 1962

13. David, J., Advanced Structural mechanics, Thomas Telford, London, 2000.

14. Osadebe, N. N., Free Vibrations of Multi-storey Frames with Flexible Horizontal Members, Translations of Nigeria Society of Engineers, Vol. 33, 1998, No. 30-41, 1998.

15. Clough, R. W., and Penzien, J., Dynamics of Structures, McGraw-Hill, New York, 1993.

16. Polyakov, S.V., Design of Earthquake Resistant Structures, Mir Publishers, Moscow, 1985.

17. Feodosyev, V., Strength of Materials, 2nd ed., Mir Publishers, Moscow, 1973.

18. Rao, S. S., Mechanical Vibrations, Pearson Education Inc., and Dorling Kindersley Publishing Inc. Law price Ed., 2006.

19. Tauchert T. R., Energy Principles in Structural Mechanics, McGraw-Hill Book Company, New York, 1974.

20. Hemant, B. K., Durgesh, C. R., and Sudhir, K. J., A case for use of Dynamic Analysis in Designing for Earthquake Forces, Scientific corresponding www.iitk.ac.in/nicee/RP/2006, Dynamic Analysis Current Science pdf, 2006.

21. Humar, J. L., Dynamics of Structures, 2nd ed., A. A. Balkema Publishers, Lisses, the Netherlands, 2002.

DOI: 10.18535/ijecs/v5i5.49

22. Soong, T. T., and Dargush, G. F., Passive Energy Dissipation Systems in Structural Engineering Wiley, Chichester, U. K., 1997.

23. Coates, R. C., Coutie, M. G. Kong, F. K., Structural Analysis 2nd ed. ELBS, 1980

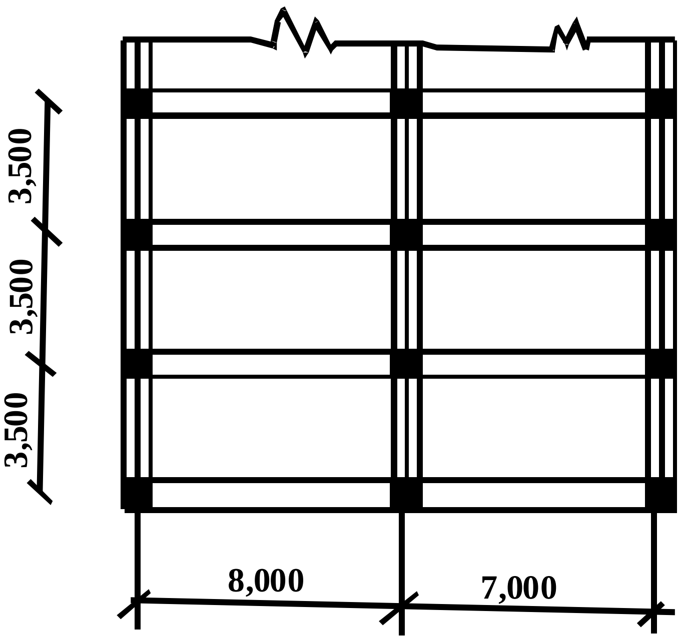

Fig. 1: Plan of the simulated double-bay Multi-Storey Framed Building Structure

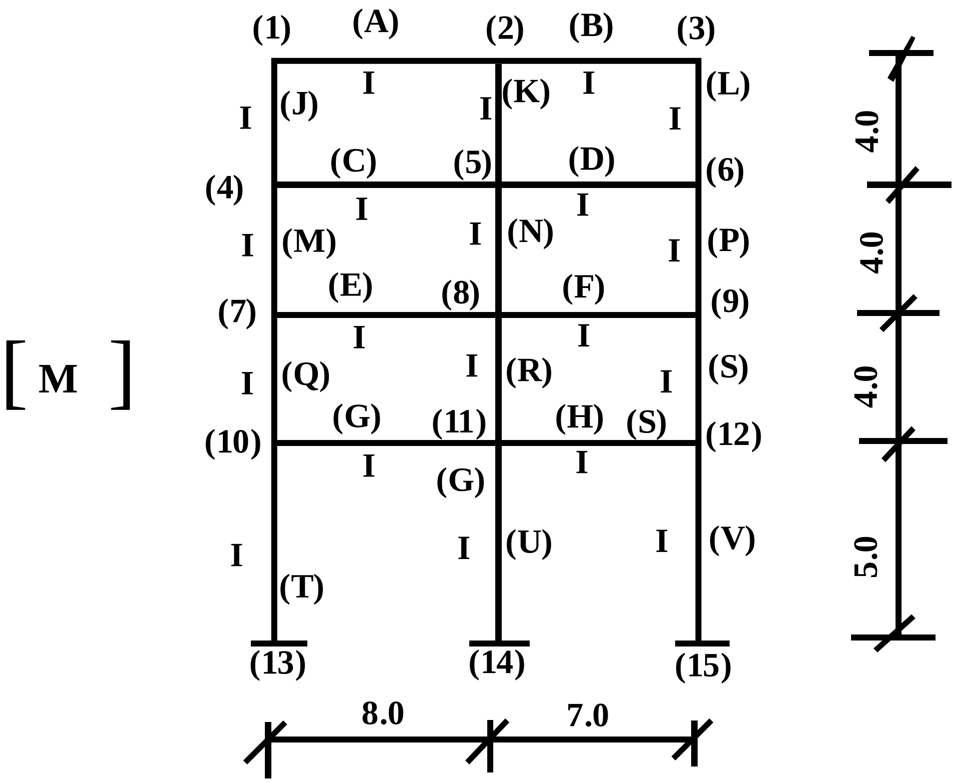

Fig. 2: A Double-bay Four-Storey simulated Normal Rigid Building Frame.

DOI: 10.18535/ijecs/v5i5.49

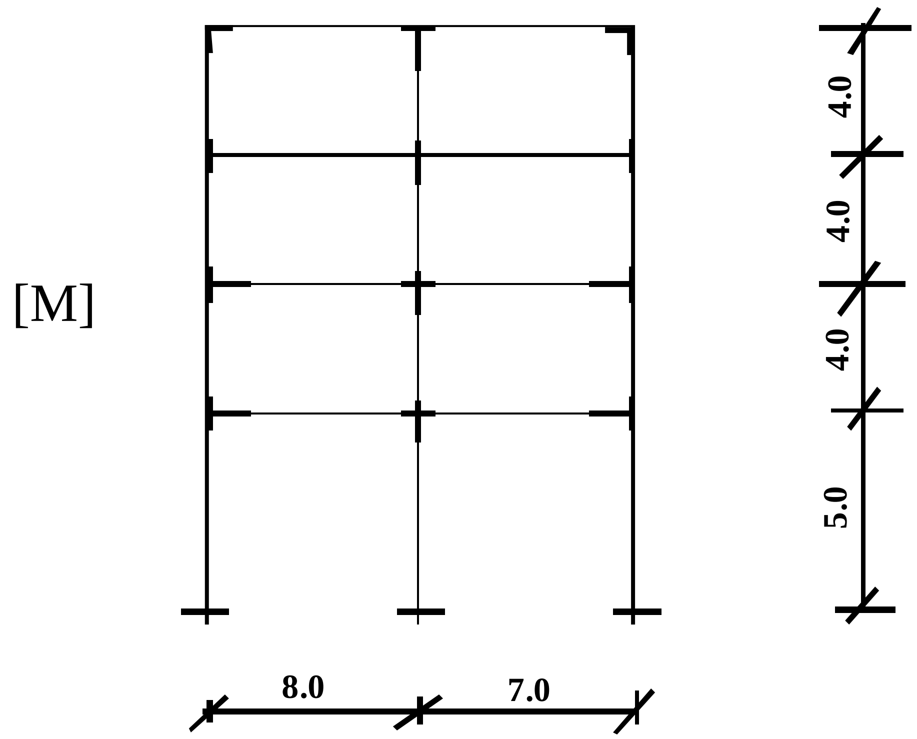

Fig. 3: Double-bay Four-storey simulated Stiffened Building Frame

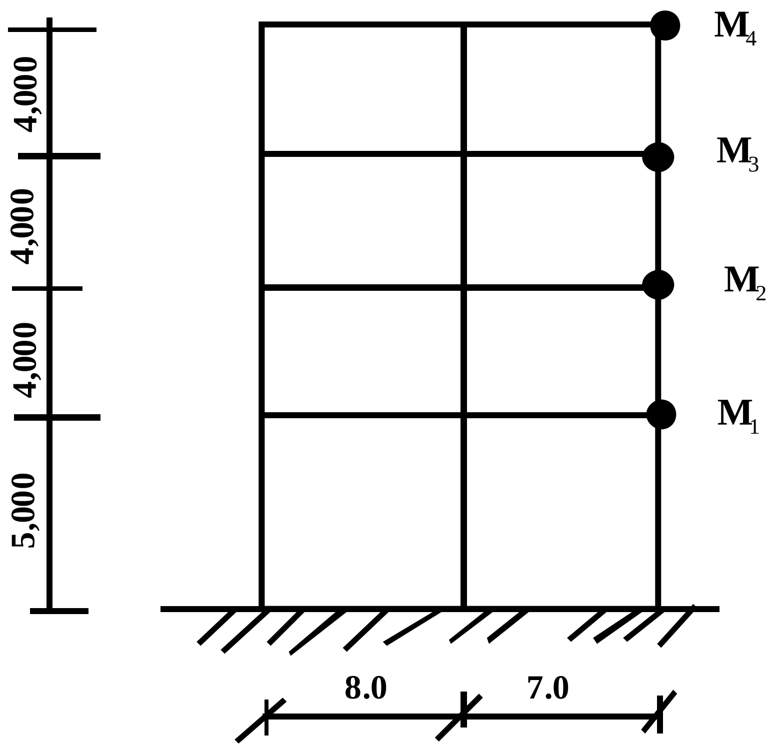

Fig. 4: Dynamic model for the Double-bay four-storey simulated Building Frame

DOI: 10.18535/ijecs/v5i5.49

Table 1: Values of bending Moment due to unit translation at 4th floor level only.

	Normal Rigid Frame a = b = 0	Stiffened Frame a = b = 250mm (EI)	Stiffened Frame a =b = 400mm (EI)	Stiffened Frame a= b= 750mm (EI)
M_{1-2}	0.12574	0.16790	0.20424	0.35143
M_{1-4}	0.12574	0.16790	0.20424	0.35143
M_{2-1}	-0.10436	-0.14337	-0.17719	-0.31205
M_{2-3}	0.11622	0.16368	0.20548	0.37621
M_{2-5}	-0.22058	-0.30705	-0.38268	-0.68827
M_{3-2}	-0.13759	-0.18785	-0.23177	-0.41246
M_{3-6}	-0.13759	-0.18785	-0.23177	-0.41246
M_{4-1}	-0.17070	-0.22532	-.0.26624	0.33908
M_{4-5}	0.07500	0.09648	0.11445	0.18634
M_{4-7}	-0.09570	-0.12884	-0.15179	-0.15273
M_{5-4}	-0.07033	-0.09290	-0.11200	-0.19219
M_{5-2}	0.23213	0.32174	0.39796	0.68711
M_{5-6}	0.07971	0.10837	0.13313	0.24013
M_{5-8}	0.08210	0.12045	0.15283	0.25477
M_{6-3}	0.17838	0.23928	0.28686	0.40169
M_{6-5}	-0.08438	-0.11185	-0.13540	-0.23389
M_{6-9}	0.09400	0.12743	0.15148	0.16779
M_{7-4}	0.03206	0.05397	0.07523	0.16539
M_{7-8}	-0.01324	-0.02169	-0.03038	-0.08394
M_{7-10}	0.01882	0.03226	0.04485	0.08145
M_{8-7}	0.01070	0.01846	0.02661	0.07978
M_{8-5}	-0.03289	-0.05951	-0.8806	-0.26784
M_{8-9}	-0.01187	-0.02104	-0.03087	-0.09646
M_{8-11}	-0.01033	-0.02002	-0.03056	-0.09159
M_{9-6}	-0.03217	-0.05467	-0.07698	-0.18015
M_{9-8}	0.01441	0.02422	0.03449	0.09959
M_{9-12}	-0.01776	-003046	-0.04247	-0.08057
M_{10-7}	-0.00607	-0.00758	-0.02121	-0.04945
M_{10-11}	0.00250	0.00530	0.00878	0.03873
M_{10-13}	-0.00356	-0.00758	-0.01241	-0.04945
M_{13-10}	0.00178	0.00382	0.00893	0.02633
M_{11-10}	-0.00167	-0.00384	-0.00668	-0.03369
M_{11-8}	0.00433	0.01024	0.01807	0.09606
M_{11-12}	0.00178	0.00422	0.00748	0.03926
M_{11-14}	0.00088	0.00218	0.00393	0.02310
M_{14-11}	-0.00044	-0.00110	-0.00283	-0.01230
M_{12-11}	-0.00262	-0.00566	-0.00952	-0.04347
M_{12-9}	0.00585	0.01247	0.02061	0.08667
M_{12-15}	0.00323	0.00681	0.01110	0.04322
M_{15-12}	-0.00161	-0.00343	-0.00798	-0.02302

DOI: 10.18535/ijecs/v5i5.49

Table 2: **Values of bending Moment due to unit translation at 3rd floor level only.**

	Normal Rigid Frame a = b = 0	Stiffened Frame a = b = 250mm (EI)	Stiffened Frame a =b = 400mm (EI)	Stiffened Frame a= b= 750mm (EI)
M_{1-2}	-0.14847	-0.20655	-0.25976	-0.52565
M_{1-4}	-0.14847	-0.20655	-0.25976	-0.52565
M_{2-1}	0.12004	0.17146	0.21867	0.43660
M_{2-3}	-0.13313	-0.19467	-0.25186	-0.51706
M_{2-5}	0.25316	0.36614	0.47051	0.95368
M_{3-2}	0.16156	0.22926	0.29184	0.60009
M_{3-6}	0.16156	0.22926	0.29184	0.60009
M_{4-1}	0.27573	0.36947	0.43773	0.48297
M_{4-5}	0.00949	0.01695	0.02513	0.09028
M_{4-7}	0.28522	0.38643	0.46287	0.57324
M_{5-4}	-0.00498	-0.00964	-0.01486	-0.04478
M_{5-2}	-0.31455	-0.44262	-0.55100	-0.94431
M_{5-6}	0.00504	0.00996	0.01550	0.04439
M_{5-8}	-0.32456	-0.46221	-0.58134	-1.03348
M_{6-3}	-0.28059	-0.37998	-0.45524	-0.56203
M_{6-5}	-0.00955	-0.01720	-0.02557	-0.08802
M_{6-9}	-0.29014	-0.39718	-0.48082	-0.65005
M_{7-4}	-0.22344	-0.31539	-0.39222	-0.57887
M_{7-8}	0.09602	0.13139	0.16344	0.30406
M_{7-10}	-0.12742	-0.18399	-0.22877	-0.27481
M_{8-7}	-0.08538	-0.11948	-0.15073	-0.30395
M_{8-5}	0.27505	0.40055	0.51530	1.05038
M_{8-9}	0.09605	0.13799	0.17703	0.37445
M_{8-11}	0.09362	0.14307	0.18754	0.37188
M_{9-6}	0.22989	0.32763	0.41122	0.65651
M_{9-8}	-0.10670	-0.14968	-0.18925	-0.37241
M_{9-12}	0.12319	0.17797	0.22196	0.28412
M_{10-7}	0.04150	0.07411	0.10901	0.29675
M_{10-11}	-0.01781	-0.03164	-0.04669	-0.13821
M_{10-13}	0.02369	0.04247	0.06231	0.15853
M_{13-10}	-0.01184	-0.02139	-0.04480	-0.08431
M_{11-10}	0.01342	0.02498	0.05412	0.12871
M_{11-8}	-0.03778	-0.07103	-0.10846	-0.39145
M_{11-12}	-0.01472	-0.02805	-0.04330	-0.15389
M_{11-14}	-0.00964	-0.01799	-0.02719	-0.10886
M_{14-11}	0.00482	0.00906	0.01955	0.05797
M_{12-11}	0.01911	0.03460	0.05175	0.16117
M_{12-9}	-0.04104	-0.07356	-0.10862	-0.30487
M_{12-15}	-0.02193	-0.03896	-0.05687	-0.14369
M_{15-12}	0.01097	0.01962	0.04089	0.07652

Table 3: Values of bending Moment due to unit translation at 2nd floor level only.

	Normal Rigid Frame a = b = 0	Stiffened Frame a = b = 250mm (EI)	Stiffened Frame a =b = 400mm (EI)	Stiffened Frame a= b= 750mm (EI)
M_{1-2}	0.02694	0.04770	0.07082	0.26946
M_{1-4}	0.02694	0.04770	0.07082	0.26946
M_{2-1}	-0.01797	-0.03345	-0.05089	-0.17105
M_{2-3}	0.01925	0.03659	0.05631	0.18666
M_{2-5}	-0.03722	-0.07005	-0.10721	-0.35770
M_{3-2}	-0.02823	-0.05067	-0.07577	-0.28018
M_{3-6}	-0.02823	-0.05067	-0.07577	-0.28018
M_{4-1}	-0.12571	-0.18024	-0.22225	-0.21830
M_{4-5}	-0.09944	-0.13888	-0.17648	-0.41707
M_{4-7}	-0.22515	-0.31914	-0.39874	-0.63538
M_{5-4}	0.08663	0.12252	0.15621	0.32955
M_{5-2}	0.09243	0.14018	0.18233	0.35256
M_{5-6}	-0.09718	-0.14075	-0.18197	-0.38753
M_{5-8}	0.27624	0.40345	0.52051	1.06964
M_{6-3}	0.12155	0.17437	0.21576	0.23506
M_{6-5}	0.10998	0.15685	0.20166	0.47063
M_{6-9}	0.23153	0.33123	0.41742	0.70559
M_{7-4}	0.29978	0.41283	0.50107	0.60058
M_{7-8}	-0.00036	-0.00089	-0.00100	-0.04097
M_{7-10}	0.29942	0.41194	0.50005	0.64150
M_{8-7}	0.00043	0.00117	0.00193	0.00243
M_{8-5}	-0.32512	-0.46320	-0.58259	-1.06722
M_{8-9}	-0.00050	-0.00142	-0.00248	-0.01162
M_{8-11}	-0.32418	-0.46060	-0.57818	-1.05315
M_{9-6}	-0.30295	-0.42008	-0.51367	-0.67472
M_{9-8}	0.00043	0.00113	0.00154	0.03099
M_{9-12}	-0.30252	-0.41895	-0.51212	-0.70571
M_{10-7}	0.22326	-0.31600	-0.39802	-0.65050
M_{10-11}	0.10171	0.14342	0.18137	0.32764
M_{10-13}	-0.12155	-0.17257	-0.21665	-0.32286
M_{13-10}	0.06077	0.08691	0.15579	0.17193
M_{11-10}	-0.08947	-0.12854	-0.16457	-0.33017
M_{11-8}	0.27236	0.39441	0.50627	1.07352
M_{11-12}	0.10051	0.14807	0.19264	0.40725
M_{11-14}	0.08238	0.11781	0.14904	0.33612
M_{14-11}	-0.04119	-0.05933	-0.10717	-0.17899
M_{12-11}	-0.11275	-0.16269	-0.20886	-0.40241
M_{12-9}	0.22940	0.32733	0.41489	0.71534
M_{12-15}	0.11665	0.16464	0.20602	0.31293
M_{15-12}	0.05833	-0.08291	-0.14815	-0.16664

Table 4: Values of bending Moment due to unit translation at 1st floor level only

	Normal Rigid Frame a = b = 0	Stiffened Frame a = b = 250mm (EI)	Stiffened Frame a =b = 400mm (EI)	Stiffened Frame a= b= 750mm (EI)
M_{1-2}	-0.00489	-0.01075	-0.01848	-0.12792
M_{1-4}	-0.00489	-0.01075	-0.01848	-0.12792
M_{2-1}	0.00254	0.00604	0.01075	0.05390
M_{2-3}	-0.00257	-0.00622	-0.01115	-0.04957
M_{2-5}	0.00511	0.01226	0.02189	0.10347
M_{3-2}	0.00492	0.01088	0.01874	0.12097
M_{3-6}	0.00492	0.01088	0.01874	0.12097
M_{4-1}	0.02427	0.04346	0.06232	0.09824
M_{4-5}	0.01723	0.02993	0.04407	0.18448
M_{4-7}	0.04151	0.07339	0.10638	0.28272
M_{5-4}	-0.01265	-0.02268	-0.03372	-0.10984
M_{5-2}	-0.01061	-0.02060	-0.03141	-0.11058
M_{5-6}	0.01379	0.02525	0.03795	0.11828
M_{5-8}	-0.03705	-0.06853	-0.10308	-0.33869
M_{6-3}	-0.02256	-0.04019	-0.05754	-0.09574
M_{6-5}	-0.01839	-0.03240	-0.04803	-0.18983
M_{6-9}	-0.04095	-0.07259	-0.10557	-0.28548
M_{7-4}	-012667	-0.18129	-0.22457	-0.24392
M_{7-8}	-0.09450	-0.12711	-0.15644	-0.33424
M_{7-10}	-0.22117	-0.30838	-0.38102	-0.57808
M_{8-7}	-0.09332	-0.13089	-0.16456	-0.31517
M_{8-5}	0.26661	0.37876	0.47719	0.91701
M_{8-9}	0.09019	0.13422	0.17189	0.33504
M_{8-11}	0.08309	0.11366	0.14077	0.26680
M_{9-6}	0.12211	0.17445	0.21626	0.25108
M_{9-8}	0.10474	0.14412	0.17976	0.37936
M_{9-12}	0.22685	0.31858	0.39602	0.63044
M_{10-7}	0.27918	0.37118	0.44331	0.55637
M_{10-11}	-0.01902	-0.03730	-0.05387	-0.08830
M_{10-13}	0.26016	0.33388	0.38944	0.46813
M_{13-10}	-0.25010	-0.31405	-0.37355	-0.45937
M_{11-10}	0.01914	0.03720	0.05412	0.12129
M_{11-8}	-0.30155	-0.41434	-0.50924	-0.92016
M_{11-12}	-0.02188	-0.04365	-0.06467	-0.15820
M_{11-14}	-0.26053	-0.33350	-0.39046	-0.64067
M_{14-11}	0.25027	0.31386	0.36219	0.55127
M_{12-11}	0.02177	0.04373	0.06436	0.12585
M_{12-9}	-0.28198	-0.37749	-0.45376	-0.61172
M_{12-15}	-0.26021	-0.33376	-0.38938	-0.48587
M_{15-12}	0.25011	0.31398	0.36165	0.46883

Table 5: Values of Shear Force due to unit translation at 4th floor level only

	Normal Rigid Frame a = b = 0	Stiffened Frame a = b = 250mm (EI)	Stiffened Frame a =b = 400mm (EI)	Stiffened Frame a= b= 750mm (EI)
Q_{1-2}	-0.02876	-0.03891	-0.04768	-0.08294
Q_{2-3}	-0.03626	-0.05022	-0.06246	-0.11267
Q_{4-1}	0.07411	0.09831	0.11762	0.17263
Q_{2-5}	0.11318	0.15720	0.19516	0.34385
Q_{3-6}	0.07899	0.10678	0.12966	0.20354
Q_{4-5}	-0.01817	-0.02367	-0.02831	-0.04732
Q_{5-6}	-0.02344	-0.03146	-0.03836	-0.06772
Q_{7-4}	-0.03194	-0.04570	-0.05676	-0.07953
Q_{5-8}	-0.02875	-0.04499	-0.06022	-0.13065
Q_{6-9}	-0.03154	-0.04553	-0.05712	-0.08699
Q_{7-8}	0.00299	0.00502	0.00712	0.02047
Q_{8-9}	0.00375	0.00647	0.00934	0.02801
Q_{10-7}	0.00622	0.01129	0.01652	0.04241
Q_{8-11}	0.00367	0.00757	0.01216	0.04691
Q_{9-12}	0.00590	0.01073	0.01577	0.04181
Q_{10-11}	-0.00052	-0.00114	-0.00193	-0.01516
Q_{11-12}	-0.00063	-0.00141	-0.00243	-0.01182
Q_{13-10}	-0.00107	-0.00228	-0.00427	-0.01516
Q_{11-14}	-0.00026	-0.00066	-0.00135	-0.00708
Q_{12-15}	-0.00097	-0.00205	-0.00382	-0.01325

Table 6: Values of Shear Force due to unit translation at 3rd floor level only

	Normal Rigid Frame a = b = 0	Stiffened Frame a = b = 250mm (EI)	Stiffened Frame a =b = 400mm (EI)	Stiffened Frame a= b= 750mm (EI)
Q_{1-2}	0.03356	0.04725	0.05980	0.12028
Q_{2-3}	0.04210	0.06056	0.07769	0.15959
Q_{4-1}	-0.10605	-0.14401	-0.17437	-0.25216
Q_{2-5}	-0.14193	-0.20219	-0.25538	-0.47450
Q_{3-6}	-0.11054	-0.15231	-0.18677	-0.29053
Q_{4-5}	-0.00181	-0.00332	-0.00500	-0.01688

Q_{5-6}	-0.00208	-0.00388	-0.00587	-0.01892
Q_{7-4}	0.12717	0.17546	0.21377	0.28803
Q_{5-8}	0.14990	0.21569	0.27416	0.52097
Q_{6-9}	0.13001	0.18120	0.22301	0.32664
Q_{7-8}	-0.02268	-0.03136	-0.03927	-0.09297
Q_{8-9}	-0.02896	-0.04110	-0.05233	-0.10669
Q_{10-7}	-0.04223	-0.06453	-0.08445	-0.14289
Q_{8-11}	-0.03285	-0.05353	-0.07400	-0.09297
Q_{9-12}	-0.04106	-0.04106	-0.08265	-0.14725
Q_{10-11}	0.00390	0.00708	0.01260	0.03337
Q_{11-12}	0.00483	0.00895	0.01358	0.04501
Q_{13-10}	0.00711	0.01277	0.02142	0.04857
Q_{11-14}	0.00289	0.00541	0.00935	0.03337
Q_{12-15}	0.00658	0.01172	0.01955	0.04404

Table 7: Values of Shear Force due to unit translation at 2nd floor level only.

	Normal Rigid Frame a = b = 0	Stiffened Frame a = b = 250mm (EI)	Stiffened Frame a =b = 400mm (EI)	Stiffened Frame a= b= 750mm (EI)
Q_{1-2}	-0.00561	-0.01014	-0.01521	-0.05506
Q_{2-3}	-0.00678	-0.01247	-0.01887	-0.06669
Q_{4-1}	0.03816	0.05699	0.07327	0.12194
Q_{2-5}	0.03241	0.05256	0.07239	0.17757
Q_{3-6}	0.03745	0.05626	0.07239	0.12881
Q_{4-5}	0.02326	0.03268	0.04159	0.09333
Q_{5-6}	0.02959	0.04251	0.05480	0.12259
Q_{7-4}	-0.13123	-0.18299	-0.22495	-0.30899
Q_{5-8}	-0.15034	-0.21666	-0.27578	-0.53422
Q_{6-9}	-0.13362	-0.18783	-0.23277	-0.34508
Q_{7-8}	0.00010	0.00026	0.00037	0.000543
Q_{8-9}	0.00013	0.00036	0.00057	0.00609
Q_{10-7}	0.13067	0.18199	0.22452	0.32300
Q_{8-11}	0.14914	0.21375	0.27111	0.53167
Q_{9-12}	0.13298	0.18657	0.23175	0.35526
Q_{10-11}	-0.02390	-0.03400	-0.04324	-0.08223

Q_{11-12}	-0.03047	-0.04439	-0.05736	-0.11567
Q_{13-10}	-0.03646	-0.05190	-0.07449	-0.09896
Q_{11-14}	-0.10216	-0.03543	-0.05124	-0.10302
Q_{12-15}	-0.10206	-0.04951	-0.07083	-0.09591

Table 8: Values of shear force due to Unit Translation at 1st Floor Level only

	Normal Rigid Frame a = b = 0	Stiffened Frame a = b = 250mm (EI)	Stiffened Frame a =b = 400mm (EI)	Stiffened Frame a= b= 750mm (EI)
Q_{1-2}	0.00093	0.00210	0.00365	0.02273
Q_{2-3}	0.00107	0.00244	0.00427	0.02436
Q_{4-1}	-0.00729	-0.01355	-0.02020	-0.05654
Q_{2-5}	-0.00393	-0.00822	-0.01333	-0.05351
Q_{3-6}	-0.00687	-0.01277	-0.01907	-0.05418
Q_{4-5}	0.00374	-0.00658	-0.00972	-0.03679
Q_{5-6}	-0.00460	-0.00824	-0.01228	-0.04402
Q_{7-4}	0.04205	0.06367	0.08274	0.13166
Q_{5-8}	0.03181	0.05069	0.06874	0.16843
Q_{6-9}	0.00408	0.06176	0.08046	0.13414
Q_{7-8}	0.02220	0.03010	0.03715	0.07513
Q_{8-9}	0.02829	0.03929	0.04919	0.09922
Q_{10-7}	-0.12509	-0.16989	-0.20608	-0.28361
Q_{8-11}	-0.14204	-0.19828	-0.24661	-0.45929
Q_{9-12}	-0.12721	-0.17402	-0.21245	-0.31054
Q_{10-11}	0.00477	0.00931	0.01350	0.02620
Q_{11-12}	0.00624	0.01248	0.01843	0.04058
Q_{13-10}	0.10205	0.12959	0.15260	0.18550
Q_{11-14}	0.10216	0.12947	0.15053	0.23839
Q_{12-15}	0.10206	0.12955	0.15021	0.19094

Table 9: Values of Restoring Forces, Kij, for the four simulated frames

Kij	Normal Rigid Frame a = b = 0	Stiffened Frame a = b = 250mm (EI)	Stiffened Frame a =b = 400mm (EI)	Stiffened Frame a= b= 750mm (EI)
K_{11}	0.70061	0.93079	1.11845	1.66827
K_{21}	-0.50896	-0.72872	-0.91051	-1.49775
K_{31}	0.13271	0.21074	0.28797	0.60271
K_{41}	-0.01809	-0.03455	-0.05324	-0.16543
K_{12}	0.50896	-0.71872	-0.91051	-1.49775
K_{22}	0.82798	1.16979	1.46088	2.39821
K_{32}	-0.52321	-0.75329	-0.95204	-1.61660
K_{42}	0.10802	0.16580	0.21854	0.42831
K_{13}	0.13271	0.21074	0.28797	0.60271
K_{23}	-0.52321	-0.75329	-0.95204	-1.61660
K_{33}	0.76559	1.07086	1.32746	2.15282
K_{43}	-0.35851	-0.49851	-0.61653	-1.01719
K_{14}	-0.01809	-0.03455	-0.05324	-0.16543
K_{24}	0.10802	0.16580	0.21854	0.42831
K_{34}	-0.35851	-0.49851	-0.61653	-1.01719
K_{44}	0.26628	0.36228	0.44244	0.72002

Table 10: Values of Natural Frequencies, wi, for the four simulated Building Frames

Natural frequencies Wi	Normal Rigid Frame a = b = 0 $\left(\sqrt{E\,I}x10^{-3}\text{ rad/se}\right)$	Stiffened Frame a = b =250mm $\left(\sqrt{E\,I}x10^{-3}\text{ rad/se}\right)$	Stiffened Frame a =b = 400mm $\left(\sqrt{E\,I}x10^{-3}\text{ rad/se}\right)$	Stiffened Frame a= b= 750mm $\left(\sqrt{E\,I}x10^{-3}\text{ rad/se}\right)$
	$\alpha = \beta = 0$	$\alpha = \beta = 0.04$	$\alpha = \beta = 0.06$	$\alpha = \beta = 0.13$
W_1	0.65672	0.81697	0.93730	1.17533
W_2	2.13741	2.65992	3.12890	3.76203
W_3	3.89275	4.97313	6.08300	7.26325
W_4	5.58564	7.58335	9.75407	12.63431

DOI: 10.18535/ijecs/v5i5.49

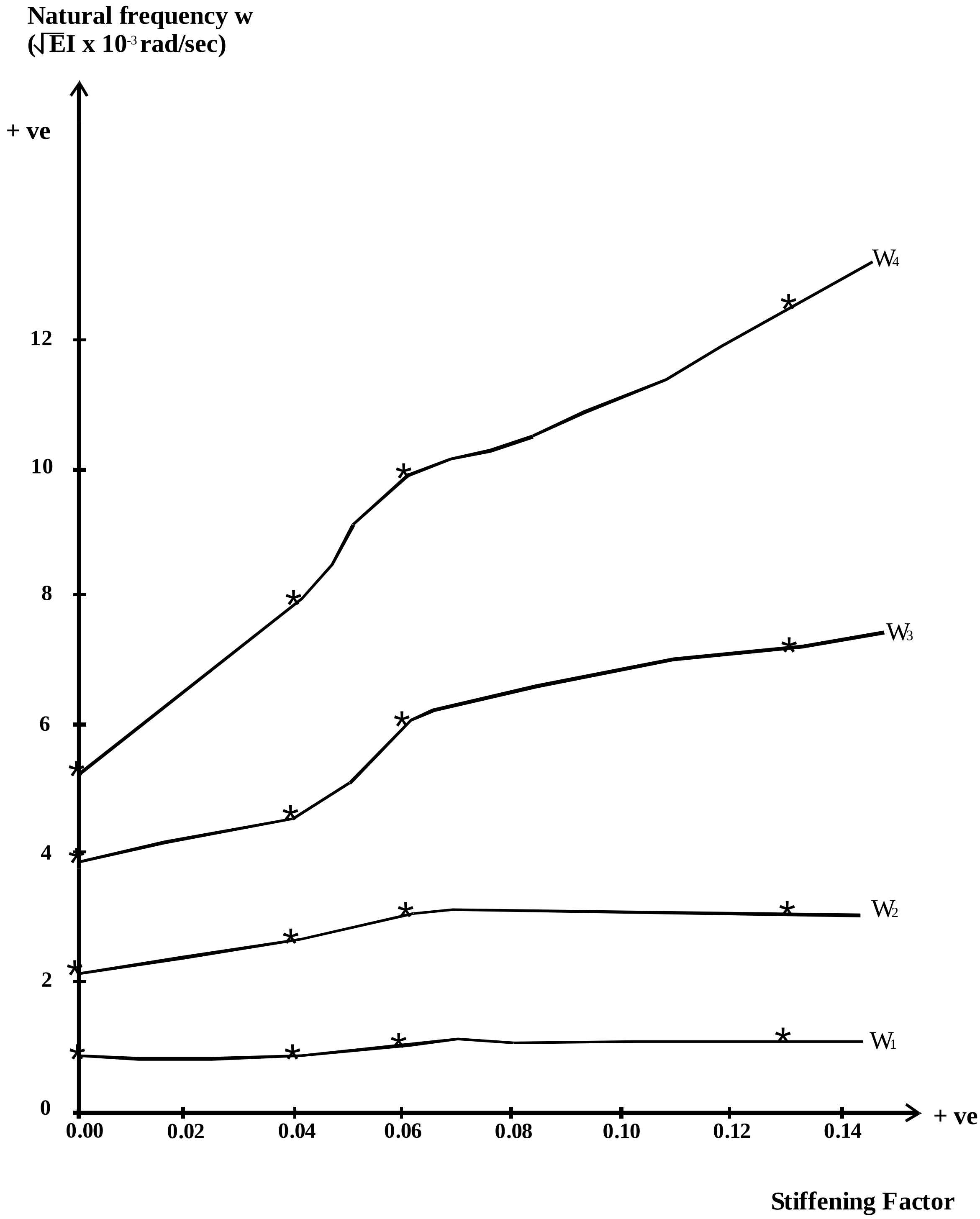

Fig 5: Graph of Natural Frequencies versus Stiffening Factors, α and β.