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Shape Analysis of the Quintic Trigonometric Bèzier curve with two shape parameter

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Abstract: A quintic trigonometric Bèzier curve with two shape parameters, is presented in this work. The shape of the curve can be adjusted as desired, by simply altering the value of shape parameter, without changing the control polygon. The quintic trigonometric Bèzier curve can be made close to the quintic Bèzier curve or closer to the given control polygon than the quintic Bèzier curve.

Keywords: Trigonometric Bèzier Basis Function, Shape Parameter, Open curves, Close curves.

1. Introduction

Trigonometric splines are an important class of splines, which were first discovered by schoenberg [8]. In recent years, In geometric modelling, for the development of the CAD/CAM software tools special attention were maid in the application of trigonometric splines. With the help of these observations, the problems of surface modelling can be handled more easily by trigonometric splines, specially problems relating to the data fitting on spherical objects. In recent years, trigonometric splines and polynomials plays a very important role in the Computer Aided Geometric Design (CAGD), specially in curve design: see [2], [3], [4], [5], [6], [10], [11].

The theory of the Bèzier curves is key feature in CAGD. These are considered as ideal geometric standard for the representation of piecewise polynomial curves. In recent years, trigonometric polynomial curves like Bèzier type are taken in discussion. A cubic trigonometric Bèzier curve with two shape parameters was discussed by Han et al [7]. It enjoyed all the geometric properties of the ordinary cubic Bèzier curve and was used for spur gear tooth design with S-shaped transition curve Abbas et al [1]. A study on class of TC- Bèzier curve with shape parameters was presented by Liu, et al [9].

The paper is organized as follows. In section 2, the basis functions of the quintic trigonometric Bèzier curve with two shape parameters are established and the properties of the basis function has been described. In section 3, quintic trigonometric Bèzier curves and their properties are discussed. In section 4, By using shape parameter, shape control of the curves is studied and open, closed quintic trigonometric Bèzier curves are presented.

2. Quintic Trigonometric Bèzier Basis Functions

In this section, definition and some properties of quintic trigonometric Bèzier basis functions with two shape parameters are given as follows:

Definition 2.1: For two arbitrarily real values of λ and μ, where $\lambda.\mu \in [-1,1]$ the following six functions of $t(t \in [0,1])$ are defined as quintic trigonometric Bèzier basis functions with two parameters λ and μ:

$$b_0(t) = \left(1 - \sin\frac{\pi}{2}t\right)^3 \left(1 - \lambda \sin\frac{\pi}{2}t\right)^2$$
$$b_1(t) = \sin\frac{\pi}{2}t\left(1 - \sin\frac{\pi}{2}t\right)^2 \left(1 - \lambda\, sin\frac{\pi}{2}t\right)\left(1 + \lambda - \lambda\, sin\frac{\pi}{2}t\right)$$
$$b_2(t) = \sin\frac{\pi}{2}t\left(1 - sin\frac{\pi}{2}t\right)\left(2 + \lambda - \lambda\, sin\frac{\pi}{2}t\right)$$
$$b_3(t) = \cos\frac{\pi}{2}t\left(1 - cos\frac{\pi}{2}t\right)\left(2 + \mu - \mu\, cos\frac{\pi}{2}t\right)$$
$$b_4(t) = \cos\frac{\pi}{2}t\left(1 - \cos\frac{\pi}{2}t\right)^2 \left(1 - \mu\, cos\frac{\pi}{2}t\right)\left(1 + \mu - \mu\, cos\frac{\pi}{2}t\right)$$
$$b_5(t) = \left(1 - \cos\frac{\pi}{2}t\right)^3 \left(1 - \mu\, cos\frac{\pi}{2}t\right)^2$$

For $\lambda = \mu = 0$, the basis functions are cubic trigonometric polynomials. For $\lambda, \mu \neq 0$, the basis functions are quintic trigonometric polynomials.

Theorem 2.1: The basis functions (2.1) have the following properties:

(a) **Nonnegativity**: $b_i(t) \geq 0, i = 0,1,2,3,4,5$.
(b) **Partition of Unity**: $\sum_{i=0}^{5} b_i(t) = 1$.
(c) **Monotonicity**: For a given parameter t, $b_0(t)$ and $b_5(t)$ are monotonically decreasing λ and μ respectively; $b_1(t)$ and $b_4(t)$ are monotonically increasing for the shape parameters λ and μ respectively.
(d) **Symmetry**: $b_i(t;\lambda,\mu) = b_{5-i}(1-t;\mu,\lambda)$, for i=0,1,2,3,4,5.

Proof: (a) For $t \in [0,1]$ and $\lambda,\mu \in [-1,1]$, then $\left(1-\sin\frac{\pi}{2}t\right) \geq 0, \left(1-\lambda\sin\frac{\pi}{2}t\right) \geq 0, \left(1-\cos\frac{\pi}{2}t\right) \geq 0, \left(1-\mu\cos\frac{\pi}{2}t\right) \geq 0, \left(1+\lambda-\lambda\sin\frac{\pi}{2}t\right) \geq 0, \left(2+\lambda-\lambda\sin\frac{\pi}{2}t\right) \geq 0, \left(1+\mu-\mu\cos\frac{\pi}{2}t\right) \geq 0, \left(2+\mu-\mu\cos\frac{\pi}{2}t\right) \geq 0, \sin\frac{\pi}{2}t \geq 0, \cos\frac{\pi}{2}t \geq 0, \lambda \geq 0, \mu \geq 0$.
It is obvious that $b_i(t) \geq 0, i = 0,1,2,3,4,5$.

(b) $\sum_{i=0}^{5} b_i(t) = \left(1-\sin\frac{\pi}{2}t\right)^3\left(1-\sin\frac{\pi}{2}t\right)^2 + \sin\frac{\pi}{2}t\left(1-\sin\frac{\pi}{2}t\right)^2\left(1-\lambda\sin\frac{\pi}{2}t\right)\left(1+\lambda-\lambda\sin\frac{\pi}{2}t\right) + \sin\frac{\pi}{2}t\left(1-\sin\frac{\pi}{2}t\right)\left(2+\lambda-\lambda\sin\frac{\pi}{2}t\right) + \cos\frac{\pi}{2}t\left(1-\cos\frac{\pi}{2}t\right)\left(2+\mu-\mu\cos\frac{\pi}{2}t\right) + \cos\frac{\pi}{2}t\left(1-\cos\frac{\pi}{2}t\right)^2\left(1-\mu\cos\frac{\pi}{2}t\right)\left(1+\mu-\mu\cos\frac{\pi}{2}t\right) + \left(1-\cos\frac{\pi}{2}t\right)^3\left(1-\mu\cos\frac{\pi}{2}t\right)^2 = 1$.

The remaining cases follow obviously.

Fig. 1. shows the curves of the quintic trigonometric basis function for $\lambda = \mu = 1$ (red solid) and $\lambda = \mu = 1$ (blue dashed).

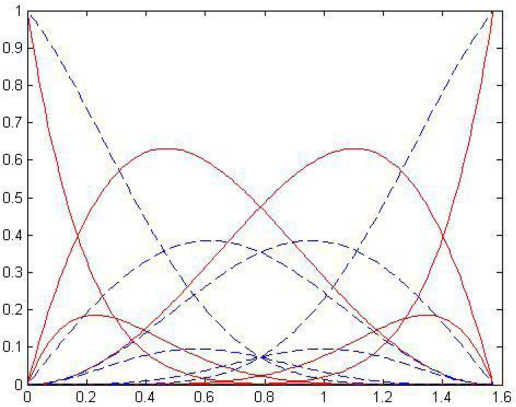

Figure 1: The quintic trigonometric basis function.

3. Qunitic trigonometric Bèzier curve

We constant the quintic trigonometric Bèzier curve with two shape parameters as follows:

Definition 3.1: Given the control points $P_i (i = 0,1,2,3,4,5)$ in R^2 or R^3, then

$$C(t) = \sum_{i=0}^{5} P_i b_i(t) \qquad (3.1)$$

$t \in [0,1], \lambda,\mu \in [-1,1]$ is called a quintic trigonometric Bèzier curve with two shape parameters.

The curve defined by (3.1) possesses some properties which can be obtained easily from the properties of the basis function.

Theorem 3.1: The Quintic trigonometric Bèzier curve (3.1) have the following properties.

(a) End point properties:

$C(0) = P_0, \quad C(1) = P_5$

$C'(0) = -\frac{\pi}{2}[(3+2\lambda)P_0 - (1+\lambda)P_1 - (2+\lambda)P_2]$

$C'(1) = \frac{\pi}{2}[(3+2\mu)P_5 - (1+\mu)P_4 - (2+\mu)P_3]$

(b) Symmetry: The control points P_i and $P_{5-i,}(i = 0,1,2,3,4,5)$ define the same curve in different parametrizations, that is

$C(t;\lambda,\mu;P_i) = C(1-t;\mu,\lambda;P_{5-i}), \quad t \in [0,1], \lambda,\mu \in [-1,1]$.

(c) Geometric invariance: The shape of the curve (3.1) is independent of the choice of coordinates, i.e., for $(i = 0,1,2,3,4,5)$, it satisfies the following two equation:

$C(t;\lambda,\mu;P_i + q) = C(t;\lambda,\mu;P_i) + q$

$C(t;\lambda,\mu;P_i * T) = C(t;\lambda,\mu;P_i) * T$

where q is an arbitrary vector in R^2 or R^3 and T is an arbitrary $d \times d$ matrix, $d = 2$ *or* 3.

(d) Convex hull property: From the non-negativity and partition of unity of basis functions, it follows that the whole curve is located in the convex hull generated by its control points P_i.

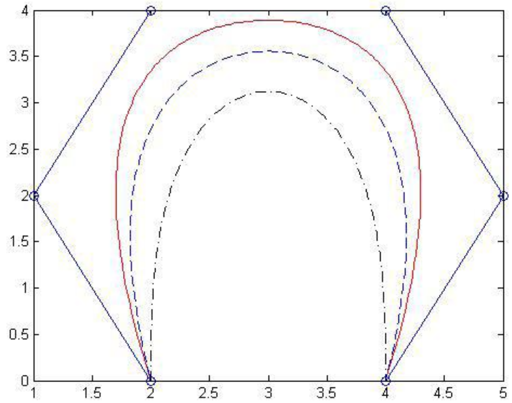

Figure 2: The quintic trigonometric Bèzier curves with two shape parameters

4. Shape control of the quintic Trigonometric Bèzier curve

The parameters λ and μ controls the shape of the curve (3.1). In figures 2, The quintic trigonometric Bèzier curve *C(t)* gets closer to the control polygon as the values of the parameters λ and μ increases. In figures 2, the curves are generated by setting the values of λ, μ as $\lambda = \mu = -1$ (black dash-dotted lines), $\lambda = \mu = 0$ (blue dashed lines), $\lambda = \mu = 1$(red solid lines).

In figures 3(a), the curves are generated by changing λ to $\lambda = -1$ (black dash-dotted lines), $\lambda = 0$ (blue dashed lines), $\lambda = 0$ (red solid lines), and setting $\mu = 1$. In figure 3(b), the curves are generated by changing μ to $\mu = -1$ (black dash-dotted lines), $\mu = 0$ (blue dashed lines), $\mu = 0$ (red solid lines), and setting $\lambda = 1$.

In figures 4, the curves are generated by changing λ,μ to $\lambda = \mu = -1$ (black dash-dotted lines), $\lambda = \mu = 0$ (blue dashed lines), $\lambda = \mu = 0$ (red solid lines).

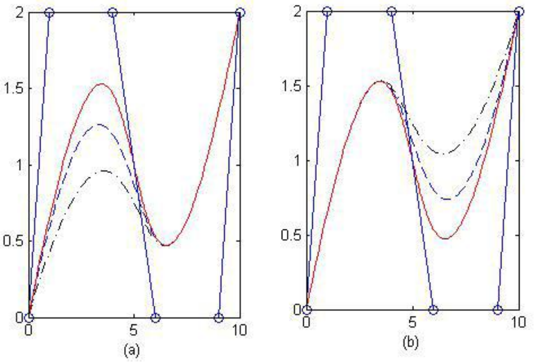

Figure 3: The effect on the shape of quintic trigonometric Bèzier curves with altering the values of λ and μ.

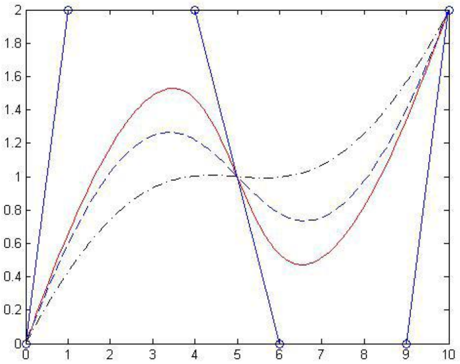

Figure 4: The effect on the shape of quintic trigonometric Bèzier curves with altering the values of λ and μ simultaneously.

In order to construct a closed quintic trigonometric Bèzier curves, we can set $P_n = P_0$. In figure 5(a), The closed quintic trigonometric Bèzier curves of altering the values of the shape parameters λ and μ at the same time. The quintic trigonometric Bèzier curves are generated by setting $\lambda = -1, \mu = -1$ (black solid lines), $\lambda = 0, \mu = 0$ (blue dashed lines) and $\lambda = 1, \mu = 1$ (red solid lines).

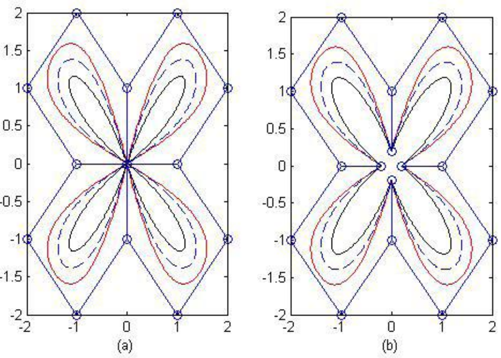

Figure 5: The close and open quintic trigonometric Bèzier curves with different values of shape parameter λ and μ (flower).

In order to construct a open quintic trigonometric Bèzier curves, we can set $P_n \neq P_0$. In figure 5(b), The open quintic trigonometric Bèzier curves of altering the values of the shape parameters λ and μ at the same time. The quintic trigonometric Bèzier curves are generated by setting $\lambda = -1, \mu = -1$(black solid lines), $\lambda = 0, \mu = 0$ (blue dashed lines) and $\lambda = 1, \mu = 1$ (red solid lines).

In figure 6, Now we show some relation of the quintic trigonometric Bèzier curves and quintic Bèzier curves corresponding to their control polygons. From figure 6(a) and 6(b), we can see that the quintic trigonometric Bèzier curve is closer to the given control polygon than the quintic Bèzier curve for some λ and μ.

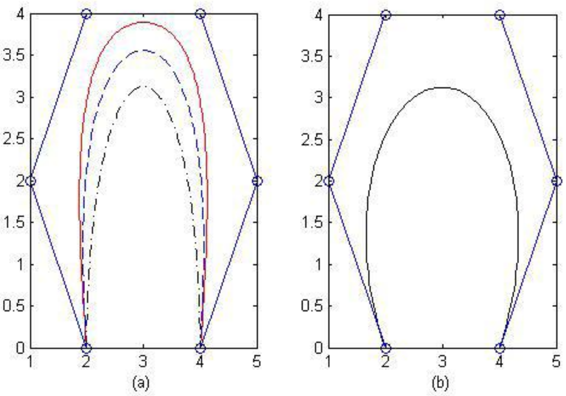

Figure 6: The relationship between the quintic trigonometric Bèzier curve and quintic Bèzier curve.

5. Conclusion

In this paper, we have presented the quintic trigonometric Bèzier curve with two shape parameters. Each section of the curve only refers to the six control points. We can design different shape curves by changing parameters. The proposed curve can be used to generate open and close curves. In future, it can be extended to tensor surfaces.

References

[1] Abbas M, Yahaya SH, Majid AA, Ali JM, Spur gear tooth design with shaped transition curve using T-B´ezier function. Procedia Eng 50, 2012, 211-21.

[2] Bashir, U., Abbas,M. Awang, M. N. H., and Ali. J., The quadratic Trigonometric B´ezier curve with single shape parameter. Journal of Basis and Applied Science Research, 2012, 2(3):2541-2546.

[3] Dube, M.,sharma, R., Quartic trigonometric Bezier curve with a shape parameter. International Journal of Mathematics and Computer Applications Research, 2013, vol.3, Issue3,(89-96):2249-6955.

[4] Dube, M.,sharma, R.,Quadratic NUAT B-spline curves with multiple shape parameters. International Journal of Machine Intelligence, 2011, (3):18-24.

[5] Han, X., Piecewise quadratic trigonometric polynomial curves. Mathematics of Computation. 2003, 72(243):1369-1378.

[6] Han, X., Cubic trigonometric polynomial curves with a shape parameter. Computer Aided Geometric Design, 2004, 21(6):535-548.

[7] Han, X.A., Y.C. Ma, and X.L. Huang, The cubic trigonometric B´ezier curves with two shape parameters. Applied Mathematics Letters, 2009, 22(2):226-231.

[8] I. J. Schoenberg, On trigonometric spline interpolation. Journal Math Mech, 1964, 13:795-825.

[9] Liu, H., L. Li, and D. Zhang, Study on a Class of TC-B´ezier curves with shape parameters. Journal of Information and Computational Science, 2011, 8(7): 1217-1223.
[10] Wei X. Xu, L. Qiang Wang, Xu Min Liu, Quadratic TC-B´ezier curves with shape parameter.Advanced Materials Research, 2011, (179- 180):1187-1192.
[11] Yang L,Li J,Chen G, A class of quasi-quartic trigonometric B´ezier curve and surfaces. Journal of Information and Computational Science, 2012, 7,72-80.