PID-Controllers Tuning Optimization with PSO Algorithm for Nonlinear Gantry Crane System

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Abstract: In this paper, three PID controllers for anti-swing, rope length and position control of a gantry crane is designed based on the parameters tuning method by particle swarm optimization (PSO). The method searches the PID parameters that realizes the expected step response of the plant. The PID parameters are computed by PSO-based PID tuning method according to the obtained model. Simulation results have demonstrated satisfactory responses with the proposed controllers under conditions based on control system performances.

Keywords: Gantry Crane, PID Controller, Particle Swarm Optimization, Nonlinear Control.

1. Introduction

Gantry cranes are widely used in industry for transporting heavy loads and hazardous materials in shipping yards, construction sites, steel mills, nuclear power and waste storage facilities and many industrial sites require fast and safe transportation of payloads from one location to another. Increasing productivity of gantry cranes is indispensable where speed of operation and accuracy of control are much needed. The crane operation causes a swinging motion to the loads due to crane acceleration and deceleration during travel. This load swing could have many serious consequences such as damage to surrounding equipment or personnel and generation of excessive loads on the supporting structure of the crane [1]. Due to this, a lot of time is needed to unload until the payload stop from swaying. Without any precaution, it will cause efficiency drop, load damages and even accidents. In dealing with these issues, a control mechanism that account for position of the trolley and oscillation of the payload is required in order to move the trolley as fast as possible with low payload oscillation. For this reason, there has been increasing interest in the design of an anti-swing control scheme for crane system [2−7].

Nowadays, several control techniques have been proposed for controlling the gantry crane system. However, PID is seen good prospect and widely used in industries due to simple structure and robust performances in a wide range of operating conditions. Chang et al [2] combined PID and Fuzzy control to achieve a robust controller for an overhead crane. PID+Q controller has also been developed to reduce payload swing angle [3]. Nevertheless, they have some difficulties in tuning the PID parameters. Traditional tuning method such as trial and error is an easy way to tune the PID controller but it is not significant and satisfactory performances are not guaranteed. Another tuning method is Ziegler-Nichols that is still widely used due to their simplicity. Unfortunately, the way to find the parameters is very aggressive and leads to a large overshoot and oscillatory responses. Due to the some difficulties in finding the optimal value of PID parameters, many researchers have begun to use meta-heuristic methods in finding the most appropriate value parameters.

Recently, PID controller is developed with various tuning method based on optimization techniques. For instance, Genetic Algorithm (GA) has been applied to tune PID for automatic gantry crane [4], Ant Colony Algorithm (ACA) to optimize nonlinear PID controller [5]. Another optimization technique that can be utilized for finding optimal PID parameters is Particle Swarm Optimization (PSO) [6].

PSO was introduced in 1995 [7] and well known as simple optimization compared to the other of some optimization method. The method is an evolutionary algorithm which is inspired by the mechanism of biological swarm social behavior such as fish schooling and bird flocking.

This paper presents development of an optimal PID controller for control of a nonlinear gantry crane system. In this work, optimal PID parameters are obtained with the PSO algorithm based on a priority approach. A control structure with three PID controllers is proposed for position control of the trolley, control of hoist rope length and anti-sway of payload. The proposed PSO algorithm is used to find optimal parameters according to priority in time response. Simulation results have demonstrated satisfactory responses with the proposed controllers under various cases of conditions based on control system performances.

2. Dynamic Model of a Gantry Crane

In this section, a dynamical model of nonlinear gantry crane is formed in the case of simultaneous operation of both trolley
moving and payload lifting/lowering mechanisms. Assume the
dynamic model has the characteristic that the payload and the
trolley are connected by a massless, rigid link. The dynamic
model is depicted on Fig. 1.

![Dynamic model of gantry crane](image)

Figure 1: Dynamic model of gantry crane

The system includes two masses $M$ and $m$, those are the trolley
mass and payload mass, respectively. The dynamic system has
three degrees of freedom corresponding to three generalized
coordinates, $x(t), l(t)$ and $z(t)$, those are the displacement of
the trolley, the hoist rope length and the sway angle of the
payload, respectively. Furthermore, inner friction of rope was
considered as a damped element $c$. The friction of trolley
motion is characterized by coefficient $c$. The $F_s$ and $F_r$
individually indicate the forces of driving motors of trolley
moving mechanism and payload lifting mechanism.

We use the Lagrangian approach to derive the equations of
motion. It follows from Fig. 1 that the cargo and trolley
position vectors are given by

$$r_p = \{x+l \sin \alpha, -l \cos \alpha\}$$

Then, the kinetic and potential energies of the whole system are
given by

$$K = \frac{1}{2} m x^2 + \frac{1}{2} M x^2$$

$$V = -mg \cdot l \cos \alpha$$

The expenditure energy of damping elements is of the form

$$\Phi = \frac{1}{2} c \dot{x} + \frac{1}{2} c \dot{\alpha}$$

Let the generalized forces corresponding to the generalized
displacements $q = \{x, l, \alpha\}$ be $F = \{F_s, F_r, F\}$. Constructing
the Lagrangian $L = K - V$ and using Lagrange’s equations

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} + \frac{\partial \Phi}{\partial \dot{q}_i} = F_i \quad (i = 1, 2, 3)$$

we obtain the following equations of motion:

$$(M + m) \ddot{x} + ml \sin \alpha \ddot{\alpha} + 2m \cos \alpha \ddot{l}$$

$$+ ml \sin \alpha \dot{\alpha}^2 + c \dot{x} = F_s$$

$$(ml + m) \sin \alpha \ddot{x} + ml \alpha \ddot{\alpha} - ml \cos \alpha + c \dot{l} = F_r$$

$$ml^2 \ddot{\alpha} + ml \cos \alpha \ddot{x} + 2ml \dot{\alpha} \dot{l} + mgl \sin \alpha = 0$$

In this gantry crane system, the object to be controlled are the
the trolley position $x(t)$, the rope length $l(t)$ and the payload
swing angle $\alpha(t)$, and control inputs are inputs $F_s$ and $F_r$, that
apply to each trolley and hoist. Besides, the linear forces is
originated from the torque of trolley motor and hoist motor as

$$T_s = r_s F_s = \frac{k_s}{R_s} u_s - \frac{k_s^2}{R_s} \omega_s$$

$$\dot{x} = r_s \omega_s$$

and

$$T_i = r_i F_i = \frac{k_i}{R_i} u_i - \frac{k_i^2}{R_i} \omega_i$$

$$\dot{l} = r_i \omega_i$$

where $r_s, r_i$ are radius of pulley of trolley motor and hoist
motor, respectively. $R_s, R_i$ are armature resistance of
trolley motor and hoist motor, respectively. $k_s, k_i$ are motor
torque constant of trolley motor and hoist motor, respectively.
$\omega_s, \omega_i$ are the angular velocity of trolley motor and hoist
motor, respectively. $u_s, u_i$ are the DC motor voltage of trolley
motor and hoist motor, respectively.

Furthermore, by combination of Eq. 6 and Eq. 9, and Eq. 7 and
Eq. 10, the nonlinear equation of the gantry crane can be
summarized as follows:

$$(M + m) \ddot{x} - ml \sin \alpha \ddot{\alpha} - ml \cos \alpha \ddot{l}$$

$$+ ml \sin \alpha \ddot{\alpha}^2 + c \dot{x} = F_s$$

$$(ml + m) \sin \alpha \ddot{x} - ml \cos \alpha + c \dot{l} = F_r$$

$$ml \ddot{\alpha} + ml \cos \alpha \ddot{x} + 2ml \dot{\alpha} \dot{l} + mgl \sin \alpha = 0$$

The system dynamics (11)÷(13) completely describes the
physical behaviors of the gantry crane system.

3. Control Design

3.1 Proposed Control Structure

For the successful sway suppression and hoist control of a
suspended load, it is important to know what part of the gantry
crane dynamics should be included in the control law design
process and what part can be neglected. For that reason, the
structure of the proposed controller for the gantry crane system
is shown in Fig. 2.

The proposed controller consists of PID controller for position
control of trolley, PI controller for length control of hoist rope
and PD for anti-swing control. The gantry crane model is
designed based on Fig. 1 with development of mathematical
modeling equation in Eq.11, Eq.12 and Eq.13. The gantry
system modelled with SIMULINK is shown in Fig. 3.
3.2 Particle Swarm Optimization

Algorithm PSO is an optimization algorithm based on evolutionary computation technique. The basic PSO is developed from research on swarm such as fish schooling and bird flocking. After it was firstly introduced in 1995 [7], a modified PSO was then introduced in 1998 to improve the performance of the original PSO. A new parameter called inertia weight is added [8]. This is a commonly used PSO where inertia weight is linearly decreasing during iteration in addition to another common type of PSO which is reported by Clerc [9]. The latter is the one used in this paper. In PSO, instead of using genetic operators, individuals called as particles are “evolved” by cooperation and competition among themselves through generations. A particle represents a potential solution to a problem. Each particle adjusts its flying according to its own flying experience and its companion flying experience. Each particle is treated as a point in a n-dimensional space. The ith particle is represented as \( X_i = (x_{i1}, x_{i2}, \ldots, x_{in}) \). The best previous position (giving the minimum fitness value) of any particle is recorded and represented as \( P_i = (p_{i1}, p_{i2}, \ldots, p_{in}) \), this is called pbest. The index of the best particle among all particles in the population is represented by the symbol g, called as gbest. The velocity for the particle i is represented as \( V_i = (v_{i1}, v_{i2}, \ldots, v_{in}) \). The particles are updated according to the following equations:

\[
\begin{align*}
    v_{id}^{k+1} &= w \cdot v_{id}^{k} + c_1 \cdot \text{rand()} \cdot (p_{id}^{k} - x_{id}^{k}) + c_2 \cdot \text{rand()} \cdot (p_{gd}^{k} - x_{id}^{k}) \\
    x_{id}^{k+1} &= x_{id}^{k} + v_{id}^{k+1}
\end{align*}
\]

(14)
(15)

where \( c_1 \) and \( c_2 \) are two positive constants. As recommended in Clerc’s PSO, the constants are \( c_1 = c_2 = 1.494 \). While \( \text{rand()} \) is random function between 0 and 1, and \( k \) represents iteration. Eq.14 is used to calculate particle’s new velocity according to its previous velocity and the distances of its current position from its own best experience (position) and the group’s best experience. Then the particle flies toward a new position according to Eq.15. The performance of each particle is measured according to a predefined fitness function (performance index), which is related to the problem to be solved. Inertia weight, \( w \) is brought into the equation to balance between the global search and local search capability. It can be a positive constant or even positive linear or nonlinear function of time. A guaranteed convergence of PSO proposed by Clerc set \( w=0.729 \). It has been also shown that PSO with different number of particles (swarm size) has reasonably similar performance [10]. Swarm size of 10-50 is usually selected.

3.3 Implementation of PSO-Based PID Tuning

For this proposed control structure, the particle position in PSO can be modelled as Eq.16.

\[
    X = [K_p, K_i, K_d, K_p, K_i, K_d]
\]

(16)

where \( X \) is the particle position, \( K_p, K_i, K_d \) are the proportional, integral, and derivative values of PID controller to control position of the trolley. \( K_{pl} \) and \( K_{dl} \) are the proportional, and integral values of PI controller to control length of the rope. While \( K_{ps} \) and \( K_{ds} \) are the proportional, and derivative values of PD controller to control oscillation of the gantry crane.

It is initialized and started with a number of random particles. Initialization of particles is performed using Eq.17.

\[
    X' = x_{\text{min}} + \text{rand} \left( x_{\text{max}} - x_{\text{min}} \right)
\]

(17)

where \( x_{\text{max}} \) and \( x_{\text{min}} \) are the maximum and minimum values in the search space boundary. Then, the particles find for the local best, pbest and subsequently global best, gbest in every iteration in order to search for optimal solution. Each particle is assessed by fitness function. Thus, all particles try to replicate their historical success and in the same time try to follow the success of the best agent. It means that the pbest and gbest are updated if the particle has a minimum fitness value compared to the current pbest and gbest value. Nevertheless, only particles that within the range of the system’s constraint is accepted.

Furthermore, performance index is defined as a quantitative measure to depict the system performance of the designed PID, PI and PD controller. Using this technique an ‘optimum system’ can often be designed and a set of PID, PI and PD parameters in the system can be adjusted to meet the required specification. For proposed control structure, the system performance can be used ISE index. It is defined as follows:

\[
    \text{ISE} = \int_{0}^{\infty} \left( e^2(t) + \int_{0}^{t} e^2(t) \, dt + \int_{0}^{t} \dot{e}^2(t) \, dt \right) \, dt
\]

(18)

where \( e \), \( e \) and \( \dot{e} \) are tracking errors of the trolley position, hoisting rope length and sway angle, respectively. They are defined by

\[
    e = x(t) - x_d, \quad e = l(t) - l_d, \quad \dot{e} = a(t)
\]

(19)

with \( x_d \) and \( l_d \) are the desired trolley position and hoisting rope length, respectively.

The conventional control system performance behaves poorly in characteristics and even it becomes unstable, when improper values of the controller tuning constants are used. The proposed PSO technique has the feature of tuning at every time, the particles are assumed new positions, they are ensured to follow the best particle by comparing the costs corresponding to these positions with the previously selected best particle cost [9].

The proposed PSO algorithm is used to tune and find seven optimal parameters of PID, PI and PD controllers. The flowchart shows the parameters selection using PSO, see Fig.3. In this study, 40 particles are considered with 50 iterations. The initial particles are bounded between \( 0 \) to \( 150 \). As default values, \( c_1 \) and \( c_2 \) are set as 1.494, \( w \) is set as 0.729.

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**Figure 3: The gantry crane system modelled with SIMULINK**
4. Simulation

In this paper we executed the computer simulation to verify the performance of the proposed control structure. Table 1 shows the specifications of gantry crane system we used.

**Table 1: System Parameters for gantry crane system Model**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trolley mass (M)</td>
<td>5 kg</td>
</tr>
<tr>
<td>Payload mass (m)</td>
<td>1 kg</td>
</tr>
<tr>
<td>Damping coefficient of trolley (c_x)</td>
<td>20 Ns/m</td>
</tr>
<tr>
<td>Damping coefficient of rope (c_r)</td>
<td>50 Ns/m</td>
</tr>
<tr>
<td>Gravitational (g)</td>
<td>9.81 m/s²</td>
</tr>
<tr>
<td>Radius of trolley pulley (r_T)</td>
<td>0.035 m</td>
</tr>
<tr>
<td>Resistance of trolley motor (R_T)</td>
<td>2.8 Ω</td>
</tr>
<tr>
<td>Torque constant of trolley motor</td>
<td>0.012 Nm/A</td>
</tr>
<tr>
<td>Radius of hoist pulley (r_H)</td>
<td>0.02 m</td>
</tr>
<tr>
<td>Resistance of hoist motor (R_H)</td>
<td>2.6 Ω</td>
</tr>
<tr>
<td>Torque constant of hoist motor (K_H)</td>
<td>0.007 Nm/A</td>
</tr>
<tr>
<td>Initial length of hoist rope (l_0)</td>
<td>0.5 m</td>
</tr>
</tbody>
</table>

Applying the method described in section 3 to find the parameters of PID controller, PI and PD as shown in Table 2.

**Table 2: Optimal PID, PI and PD parameters obtained using the improved PSO algorithm**

<table>
<thead>
<tr>
<th>PID, PI and PD Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>K_P</td>
<td>36.1842</td>
</tr>
<tr>
<td>K_I</td>
<td>0.49802</td>
</tr>
<tr>
<td>K_D</td>
<td>42.372</td>
</tr>
<tr>
<td>K_PL</td>
<td>121.32</td>
</tr>
<tr>
<td>K_IL</td>
<td>0.0478</td>
</tr>
<tr>
<td>K_DS</td>
<td>142.12</td>
</tr>
<tr>
<td>K_DS</td>
<td>0.0421</td>
</tr>
</tbody>
</table>

Subsequently, it is desirable to examine the controller’s performance under various loading conditions, desired positions and rope lengths. Fig. 5 shows the trolley displacement, payload oscillation and rope length responses respectively with payload of 1 kg and 5 kg, desired positions at 1 m and 1.5 m, desired rope lengths at 1 m, 1.5 m and 0.2 m.

**Figure 5: System Response**

(a) Trolley position, (b) Rope length and (c) Payload oscillation

It is noted for all conditions, quite a similar trolley position response is obtained. In all cases less steady state error, overshoot and settling time are obtained. However, slightly difference payload oscillation responses are observed with various payloads. Simulation results with a higher payload show less payload oscillation but required more a little time to settle down.
5. Conclusion

This paper has presented design of gantry crane system for controlling the trolley displacement, hoist rope length and payload oscillation. Nonlinear differential equations of the system including motion of trolley displacement, rope length and payload oscillation has been derived and used for verification of control algorithm. A control structure for the crane consists of PID controller for position control of trolley, PI controller for length control of hoist rope and PD for anti-swing control has proposed. Seven controller parameters of PID, PI and PD for the system have been obtained by using PSO algorithm. Simulation results have shown that the controllers are effective to move the trolley and length of the rope as fast as possible to the desired position and length with low payload oscillation.

References