www.ijecs.in
International Journal Of Engineering And Computer Science ISSN:2319-7242
Volume 5 Issue 8 August 2016 Page No. 17690-17695

Collocation Method for Seventh Order Boundary Value Problems by Using Sextic B-splines

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Abstract: ***A finite element method involving collocation method with sextic B-splines as basis functions has been developed to solve seventh order boundary value problems. The seventh and six order derivatives for the dependent variable is approximated by the central differences. The basis functions are redefined into a new set of basis functions which in number match with the number of collocated points selected in the space variable domain. The proposed method is tested on three linear and two non-linear boundary value problems. The solution to a nonlinear problem has been obtained as the limit of a sequence of solutions of linear problems generated by the quasilinearization technique. Numerical results obtained by the present method are in good agreement with the exact solutions available in the literature.***

Keywords: Collocation method, Sextic B-spline, Seventh order boundary value problem, Absolute error.

1. Introduction

In this paper, we consider a general seventh order boundary value problem

$$\begin{aligned} &a_0(x)y^{(7)}(x)+a_1(x)y^{(6)}(x)+a_2(x)y^{(5)}(x)+a_3(x)y^{(4)}(x)\\ &+a_4(x)y'''(x)+a_5(x)y''(x)+a_6(x)y'(x)+a_7(x)y(x)\\ &=b(x), c<x<d \end{aligned} \quad (1)$$

subject to boundary conditions

$$\begin{aligned} &y(c)=A_0, y(d)=C_0, y'(c)=A_1, y'(d)=C_1,\\ &y''(c)=A_2, y''(d)=C_2, y'''(c)=A_3 \end{aligned} \quad (2)$$

where $A_0, A_1, A_2, A_3, C_0, C_1, C_2$ are finite real constants and $a_0(x), a_1(x), a_2(x), a_3(x), a_4(x), a_5(x), a_6(x), a_7(x)$ and $b(x)$ are all continuous functions defined on the interval $[c, d]$.

The seventh order boundary value problems generally arise in modelling induction motors with two rotor circuits. The induction motor behavior is represented by a fifth order differential equation model. This model contains two stator state variables, two rotor state variables and one shaft speed. Normally, two more variables must be added to account for the effects of a second rotor circuit representing deep bars, a starting cage or rotor distributed parameters. To avoid the computational burden of additional state variables when additional rotor circuits are required, model is often limited to the fifth order and rotor impedance is algebraically altered as function of rotor speed. This is done under the assumption that the frequency of rotor currents depends on rotor speed. This approach is efficient for the steady state response with sinusoidal voltage, but it does not hold up during the transient conditions, when rotor frequency is not a single value. The behavior of such models is shown as seventh order boundary value problems [1]. The existence and uniqueness of the solution for these types of problems have been discussed in Agarwal [2]. Finding the analytical solutions of such type of boundary value problems in general is not possible. Over the years, many researchers have worked on seventh order boundary value problems by using different methods for numerical solutions. Siddiqi et al. [3] developed the solution of special type of seventh order boundary value problems by using differential transformation method. Siddiqi et al. [4] presented the variational iteration principle to solve a special case of seventh order boundary value problems after transforming the given differential equation into a system of integral equations. Siddiqi and Iftikhar [5] presented the variational iteration technique for the solution of seventh order boundary value problems by using He’s polynomials. Siddiqi and Iftikhar [6] discussed Adomian decomposition method to solve the seventh order boundary value problems. Siddiqi and Iftikhar [7] discussed the numerical solution of higher order boundary value problems by using homotopy analysis method. Siddiqi and Iftikhar [8] dealt with variation of parameters method to solve a special case of seventh order boundary value problems. Siddiqi and Iftikhar [9] presented variational iteration homotopy perturbation method to solve the seventh order boundary value problems, where the variational iteration homotopy perturbation method is formulated by coupling of variational iteration method and homotopy perturbationmethod. Mustafa and Ali [10], Ghazala and Rehman [11] got the solution of a special case of seventh order boundary value problems by using reproducing kernel Hilbert space method and reproducing kernel method respectively. So far, seventh order boundary value problems have not been solved by using collocation method with sextic B-splines as basis functions.

In this paper, we try to present a simple finite element method which involves collocation approach with sextic B-splines as basis functions to solve the seventh order boundary value problem of the type (1)-(2). This paper is organized as follows. In section 2 of this paper, the justification for using the collocation method has been mentioned. In section 3, the definition of sextic B-splines has been described. In section 4, description of the collocation method with sextic B-splines as basis functions has been presented and in section 5, solution procedure to find the nodal parameters is presented. In section 6, numerical examples of both linear and non-linear boundary value problems are presented. The solution to a nonlinear problem has been obtained as the limit of a sequence of solution of linear problems generated by the quasilinearization technique [12]. Finally, the last section is dealt with conclusions of the paper.

2. Justification for using Collocation method

In finite element method (FEM) the approximate solution can be written as a linear combination of basis functions which constitute a basis for the approximation space under consideration. FEM involves variational methods like Ritzs approach, Galerkins approach, least squares method and collocation method etc. The collocation method seeks an approximate solution by requiring the residual of the di_erential equation to be identically zero at N selected points in the given space variable domain where N is the number of basis functions in the basis [13]. That means, to get an accurate solution by the collocation method one needs a set of basis functions which in number match with the number of collocation points selected in the given space variable domain. Further, the collocation method is the easiest to implement among the variational methods of FEM. When a differential equation is approximated by m[th] order B-splines, it yields (m + 1)[th] order accurate results [14]. Hence this motivated us to solve a seventhorder boundary value problem of type (1)-(2) by collocation method with sextic B-splines as basis functions.

3. Definition of sextic B-spline

The sextic B-splines are defined in [14]-[16]. The existence of sextic spline interpolate s(x) to a function in a closed interval [c, d] for spaced knots (need not be evenly spaced) of a partition $c = x_0 < x_1 <\ldots< x_{n-1} < x_n = d$ is established by constructing it. The construction of $s(x)$ is done with the help of the sextic B-splines. Introduce twelve additional knots x_{-6}, x_{-5}, x_{-4}, x_{-3}, x_{-2}, x_{-1}, x_{n+1}, x_{n+2}, x_{n+3}, x_{n+4}, x_{n+5} and x_{n+6} in such a way that

$x_{-6}<x_{-5}<x_{-4}<x_{-3}<x_{-2}<x_{-1}<x_0$ and $x_n<x_{n+1}<x_{n+2}<x_{n+3}<x_{n+4}<x_{n+5}< x_{n+6}$.

Now the sextic B-splines $B_i(x)'s$ are defined by

$$B_i(x)=\begin{cases}\sum_{r=i-3}^{i+4}\dfrac{(x_r-x)_+^6}{\pi'(x_r)}, & x\in[x_{i-3},\ x_{i+4}]\\ 0, & otherwise\end{cases}$$

where

$$(x_r-x)_+^6=\begin{cases}(x_r-x)^6, & if\ x_r\ge x\\ 0, & if\ x_r\le x\end{cases}$$

and $$\pi(x)=\prod_{r=i-3}^{i+4}(x-x_r)$$

where $\{B_{-3}(x), B_{-2}(x), B_{-1}(x), B_0(x), B_1(x),\ldots,B_n(x), B_{n+1}(x), B_{n+2}(x)\}$ forms a basis for the space $s_6(\pi)$ of sextic polynomial splines. Schoenberg [16] has proved that sextic B-splines are the unique nonzero splines of smallest compact support with the knots at

$x_{-6}<x_{-5}<x_{-4}<x_{-3}<x_{-2}<x_{-1}<x_0<x_1<\ldots<x_{n-1}<x_n<x_{n+1}<x_{n+2}<x_{n+3}<x_{n+4}< x_{n+5}< x_{n+6}$.

4. Description of the method

To solve the boundary value problem (1) subject to boundary conditions (2) by the collocation method with sextic B-splines as basis functions, we define the approximation for $y(x)$ as

$$y(x)=\sum_{j=-3}^{n+2}\alpha_j B_j(x) \tag{3}$$

Where $\alpha_j's$ are the nodal parameters to be determined and $B_j(x)'s$ are sextic B-spline basis functions. In the present method, the mesh points $x_1, x_2, \ldots, x_{n-1}, x_{n-1}$ are selected as the collocation points. In collocation method, the number of basis functions in the approximation should match with the number of collocation points [13]. Here the number of basis functions in the approximation (3) is n+6, where as the number of selected collocation points is n-1. So, there is a need to redefine the basis functions into a new set of basis functions which in number match with the number of selected collocation points. The procedure for redefining the basis functions is as follows:

Using the definition of sextic B-splines, the Dirichlet, Neumann, second order derivatives boundary conditions and third order derivative boundary condition of (2), we get the approximate solution at the boundary points as

$$y(c)=y(x_0)=\sum_{j=-3}^{2}\alpha_j B_j(x_0)=A_0 \tag{4}$$

$$y(d)=y(x_n)=\sum_{j=n-3}^{n+2}\alpha_j B_j(x_n)=C_0 \tag{5}$$

$$y'(c)=y'(x_0)=\sum_{j=-3}^{2}\alpha_j B'_j(x_0)=A_1 \tag{6}$$

$$y'(d)=y'(x_n)=\sum_{j=n-3}^{n+2}\alpha_j B'_j(x_n)=C_1 \tag{7}$$

$$y''(c)=y''(x_0)=\sum_{j=-3}^{2}\alpha_j B''_j(x_0)=A_2 \tag{8}$$

$$y''(d)=y''(x_n)=\sum_{j=n-3}^{n+2}\alpha_j B_j''(x_n)=C_2$$
(9)

$$y'''(c)=y'''(x_0)=\sum_{j=-3}^{2}\alpha_j B_j'''(x_0)=A_3$$
(10)

Eliminating $\alpha_{-3},\alpha_{-2},\alpha_{-1},\alpha_0,\alpha_n,\alpha_{n+1}$ and α_{n+2} from the equations (3) to (10), we get the approximation for y(*x*) as

$$y(x)=w(x)+\sum_{j=1}^{n-1}\alpha_j S_j(x)$$
(11)

where

$$w(x)=w_3(x)+\frac{A_3-w_3'''(x_0)}{R_0'''(x_0)}R_0(x)$$

$$w_3(x)=w_2(x)+\frac{A_2-w_2''(x_0)}{Q_{-1}'(x_0)}Q_{-1}(x)+\frac{C_2-w_2''(x_n)}{Q_n''(x_n)}Q_n(x)$$

$$w_2(x)=w_1(x)+\frac{A_1-w_1'(x_0)}{P_{-2}'(x_0)}P_{-2}(x)+\frac{C_1-w_1'(x_n)}{P_{n+1}(x_n)}P_{n+1}(x)$$

$$w_1(x)=\frac{A_0}{B_{-3}(x_0)}B_{-3}(x)+\frac{C_0}{B_{n+2}(x_n)}B_{n+2}(x)$$

$$S_j(x)=\begin{cases}R_j(x)-\dfrac{R_j'''(x_0)}{R_0'''(x_0)}R_0(x), & j=1,2\\ R_j(x), & j=3,4,\ldots,n\end{cases} \qquad (12)$$

$$R_j(x)=\begin{cases}Q_j(x)-\dfrac{Q_j''(x_0)}{Q_{-1}''(x_0)}Q_{-1}(x), & j=0,1,2\\ Q_j(x), & j=3,4,\ldots,n-4\\ Q_j(x)-\dfrac{Q_j''(x_n)}{Q_n''(x_n)}Q_n(x), & j=n-3,n-2,n-1\end{cases}$$

$$Q_j(x)=\begin{cases}P_j(x)-\dfrac{P_j'(x_0)}{P_{-2}'(x_0)}P_{-2}(x), & j=-1,0,1,2\\ P_j(x), & j=3,4,\ldots,n-4\\ P_j(x)-\dfrac{P_j'(x_n)}{P_{n+1}'(x_n)}P_{n+1}(x), & j=n-3,n-2,n-1,n\end{cases}$$

$$P_j(x)=\begin{cases}B_j(x)-\dfrac{B_j(x_0)}{B_{-3}(x_0)}B_{-3}(x), & j=-2,-1,0,1,2\\ B_j(x), & j=3,4,\ldots,n-4\\ B_j(x)-\dfrac{B_j(x_n)}{B_{n+2}(x_n)}B_{n+2}(x), & j=n-3,n-2,n-1,n,n+1\end{cases}$$

Now the new basis functions for the approximation y(*x*) are {S*j*(*x*), *j*=1, 2,…, n-1} and they are in number match with the number of selected collocated points. Since the approximation for y(*x*) in (11) is a sextic approximation, let us approximate $y^{(6)}$ and $y^{(7)}$ at the selected collocation points with central differences as

$$y_i^{(6)}=\frac{y_{i+1}^{(5)}-y_{i-1}^{(5)}}{2h}\quad \text{for } i=1,2,\ldots,n-1$$
(13)

$$y_i^{(7)}=\frac{y_{i+1}^{(5)}-2y_i^{(5)}+y_{i-1}^{(5)}}{h^2}\quad \text{for } i=1,2,\ldots.,n-1$$
(14)

where

$$y_i=y(x_i)=w(x_i)+\sum_{j=1}^{n-1}\alpha_j S_j(x_i)$$
(15)

Now applying the collocation method to (1), we get

$$a_0(x_i)y_i^{(7)}+a_1(x_i)y_i^{(6)}+a_2(x_i)y_i^{(5)}+a_3(x_i)y_i^{(4)}$$
$$+a_4(x_i)y_i'''+a_5(x_i)y_i''+a_6(x_i)y_i'$$
$$+a_7(x_i)y_i=b(x_i)\quad \text{for } i=1,2,\cdots,n-1.$$
(16)

Substituting (13), (14) and (15) in (16), we get

$$\frac{a_0(x_i)}{h^2}\left[\begin{array}{l} w^{(5)}(x_{i+1})+\sum_{j=1}^{n-1}\alpha_j S_j^{(5)}(x_{i+1})-2w^{(5)}(x_i)\\ -2\sum_{j=1}^{n-1}\alpha_j S_j^{(5)}(x_i)+w^{(5)}(x_{i-1})+\sum_{j=1}^{n-1}\alpha_j S_j^{(5)}(x_{i-1})\end{array}\right]$$
$$\frac{a_1(x_i)}{2h}\left[w^{(5)}(x_{i+1})+\sum_{j=1}^{n-1}\alpha_j S_j^{(5)}(x_{i+1})-w^{(5)}(x_{i-1})-\sum_{j=1}^{n-1}\alpha_j S_j^{(5)}(x_{i-1})\right]$$
$$+a_2(x_i)\left[w^{(5)}(x_i)+\sum_{j=1}^{n-1}\alpha_j S_j^{(5)}(x_i)\right]+a_3(x_i)\left[w^{(4)}(x_i)+\sum_{j=1}^{n-1}\alpha_j S_j^{(4)}(x_i)\right]$$
$$+a_4(x_i)\left[w'''(x_i)+\sum_{j=1}^{n-1}\alpha_j S_j'''(x_i)\right]+a_5(x_i)\left[w''(x_i)+\sum_{j=1}^{n-1}\alpha_j S_j''(x_i)\right]$$
$$+a_6(x_i)\left[w'(x_i)+\sum_{j=1}^{n-1}\alpha_j S_j'(x_i)\right]+a_7(x_i)\left[w(x_i)+\sum_{j=1}^{n-1}\alpha_j S_j(x_i)\right]$$

for i=1, 2,…, n-1.

(17)

Rearranging the terms and writing the system of equations (17) in matrix form, we get

$$\mathbf{A}\alpha=\mathbf{B}$$
(18)

where $\mathbf{A}=[a_{ij}]$;

$$a_{ij}=\frac{a_0(x_i)}{h^2}\left[S_j^{(5)}(x_{i+1})-2S_j^{(5)}(x_i)+S_j^{(5)}(x_{i-1})\right]$$
$$+\frac{a_1(x_i)}{2h}\left[S_j^{(5)}(x_{i+1})-S_j^{(5)}(x_{i-1})\right]+a_2(x_i)S_j^{(5)}(x_i)$$
$$+a_3(x_i)S_j^{(4)}(x_i)+a_4(x_i)S_j'''(x_i)+a_5(x_i)S_j''(x_i)$$
$$+a_6(x_i)S_j'(x_i)+a_7(x_i)S_j(x_i)$$

for i= 1, 2, …., n-1; j=1, 2,…, n-1.

(19)

$$\mathbf{B}=[b_i];$$

$$b_i = \frac{a_0(x_i)}{h^2}\left[w^{(5)}(x_{i+1}) - 2w^{(5)}(x_i) + w^{(5)}(x_{i-1})\right]$$
$$+\frac{a_1(x_i)}{2h}\left[w^{(5)}(x_{i+1}) - w^{(5)}(x_{i-1})\right] + a_2(x_i)w^{(5)}(x_i)$$
$$+a_3(x_i)w^{(4)}(x_i) + a_4(x_i)w'''(x_i) + a_5(x_i)w''(x_i)$$
$$+a_6(x_i)w'(x_i) + a_7(x_i)w(x_i)$$

for i= 1, 2, …, n-1.

(20)

and $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_{n-1}]^T$.

5. Solution procedure to find the nodal parameters

The basis function $S_i(x)$ is defined only in the interval $[x_{i-3}, x_{i+4}]$ and outside of this interval it is zero. Also at the end points of the interval $[x_{i-3}, x_{i+4}]$ the basis function $S_i(x)$ vanishes. Therefore, $S_i(x)$ is having non-vanishing values at the mesh points $x_{i-2}, x_{i-1}, x_i, x_{i+1}, x_{i+2}, x_{i+3}$ and zero at the other mesh points. The first five derivatives of $S_i(x)$ also have the same nature at the mesh points as in the case of $S_i(x)$. Using these facts, we can say that the Thus the stiff matrix $\mathbf{A}$ is a eight diagonal band matrix. Therefore, the system of equations (18) is a eight band system in α_i's. The nodal parameters α_i's can be obtained by using band matrix solution package. We have used the FORTRAN-90 programming to solve the boundary value problem (1)-(2) by the proposed method

6. **Numerical results**

To demonstrate the applicability of the proposed method for solving the seventh order boundary value problems of the type (1) and (2), we considered two linear and two nonlinear boundary value problems. The obtained numerical results for each problem are presented in tabular forms and compared with the exact solutions available in the literature.

Example 1: Consider the linear boundary value problem

$$y^{(7)} + y = -(35 + 12x + 12x^2)e^x, \quad 0 < x < 1$$

(21)

subject to

$$y(0) = 0,\ y(1) = 0,\ y'(0) = 1,\ y'(1) = -e,$$
$$y''(0) = 0,\ y''(1) = -4e,\ y'''(0) = -3.$$

The exact solution for the above problem is $y = x(1-x)e^x$.

The proposed method is tested on this problem where the domain [0, 1] is divided into 10 equal subintervals. The obtained numerical results for this problem are given in Table 1. The maximum absolute error obtained by the proposed method is 8.821487×10^{-6}.

Table 1: Numerical results for Example 1

x	Absolute error by the proposed method
0.1	3.054738E-07
0.2	5.811453E-07
0.3	2.592802E-06
0.4	8.940697E-07
0.5	3.218651E-06
0.6	1.877546E-06
0.7	2.950430E-06
0.8	5.155802E-06
0.9	8.821487E-06

Example 2: Consider the linear boundary value problem

$$y^{(7)} - xy = (x^2 - 2x - 6)e^x, \quad 0 \le x \le 1$$

(22)

subject to

$$y(0) = 1,\ y(1) = 0,\ y'(0) = 0,\ y'(1) = -e,$$
$$y''(0) = -1,\ y''(1) = -2e,\ y'''(0) = -2.$$

The exact solution for the above problem is $y = e^x(x-1)$.

The proposed method is tested on this problem where the domain [0, 1] is divided into 10 equal subintervals. The obtained numerical results for this problem are given in Table 2. The maximum absolute error obtained by the proposed method is 7.927418×10^{-6}.

Table 2: Numerical results for Example 2

x	Absolute error by the proposed method
0.1	8.940697E-07
0.2	2.145767E-06
0.3	1.847744E-06
0.4	2.622604E-06
0.5	7.927418E-06
0.6	1.788139E-07
0.7	2.622604E-06
0.8	4.947186E-06
0.9	7.659197E-06

Example 3: Consider the linear boundary value problem

$$y^{(7)} + \cos x\, y''' + (1-x)y = (2 + \cos x - x)e^x, \quad 0 \le x \le 1 \qquad (23)$$

subject to

$$y(0) = 1, y(1) = e, y'(0) = 1, y'(1) = e, y''(0) = 1, y'''(0) = e.$$

The exact solution for the above problem is $y = e^x$.

The proposed method is tested on this problem where the domain [0, 1] is divided into 10 equal subintervals. The obtained numerical results for this problem are given in Table 3. The maximum absolute error obtained by the proposed method is 3.743172×10^{-5}.

Table 3: Numerical results for Example 3

x	Absolute error by the proposed method
0.1	1.549721E-06
0.2	8.344650E-07
0.3	1.204014E-05
0.4	2.264977E-06
0.5	2.717972E-05
0.6	1.335144E-05
0.7	3.361702E-05
0.8	2.336502E-05
0.9	3.743172E-05

Example 4: Consider the nonlinear boundary value problem

$$y^{(7)} - yy' = e^{-2x}(2+e^{x}(x-8)-3x+x^{2}),\quad 0\le x\le 1 \tag{24}$$

subject to

$$y(0)=1,\, y(1)=0,\, y'(0)=-2,\, y'(1)=-e^{-1},$$

$$y''(0)=3,\, y''(1)=2e^{-1},\, y'''(0)=-4.$$

The exact solution for the above problem is $y=(1-x)e^{-x}$.

The nonlinear boundary value problem (24) is converted into a sequence of linear boundary value problems generated by quasilinearization technique [12] as

$$\begin{aligned} &y^{(7)}_{(n+1)} - y_{(n)}y'_{(n+1)} - y'_{(n)}y_{(n+1)} \\ &= e^{-2x}(2+e^{x}(x-8)-3x+x^{2}) - y_{(n)}y'_{(n)},\quad n=0,1,2,\ldots \end{aligned} \tag{25}$$

Subject to

$$y_{(n+1)}(0)=1,\, y_{(n+1)}(1)=0,\, y_{(n+1)'}(0)=-2,\, y_{(n+1)'}(1)=-e^{-1},$$

$$y''_{(n+1)}(0)=3,\, y''(1)_{(n+1)}=2e^{-1},\, y'''_{(n+1)}(0)=-4.$$

Here $y_{(n+1)}$ is the $(n+1)^{th}$ approximation for $y(x)$. The domain [0, 1] is divided into 10 equal subintervals and the proposed method is applied to the sequence of linear problems (25). The obtained numerical results for this problem are presented in Table 4. The maximum absolute error obtained by the proposed method is $1.108646\text{x}10^{-5}$.

Table 4: Numerical results for Example 4

x	Absolute error by the proposed method
0.1	5.960464E-08
0.2	3.039837E-06
0.3	4.708767E-06
0.4	8.851290E-06
0.5	1.108646E-05
0.6	8.225441E-06
0.7	4.798174E-06
0.8	1.586974E-06
0.9	3.725290E-07

Example 5: Consider the nonlinear boundary value problem

$$y^{(7)} + e^{-x}y^{2} = e^{-3x} - e^{-x},\quad 0<x<1$$

(26)

subject to

$$y(0)=1,\, y(1)=\frac{1}{e},\, y'(0)=-1,\, y'(1)=-\frac{1}{e},$$

$$y''(0)=1,\, y''(1)=\frac{1}{e},\, y'''(1)=-1.$$

The exact solution for the above problem is $y=e^{-x}$.

The nonlinear boundary value problem (29) is converted into a sequence of linear boundary value problems generated by quasilinearization technique [12] as

$$y^{(7)}_{(n+1)} + 2e^{-x}y_{(n)}y_{(n+1)} = e^{-x}y^{2}_{(n)}$$

$$-e^{-x}+e^{-3x},\quad n=0,1,2,\ldots$$

(27)

subject to

$$y_{(n+1)}(0)=1,\, y_{(n+1)}(1)=\frac{1}{e},\, y_{(n+1)'}(0)=-1,\, y_{(n+1)'}(1)=-\frac{1}{e},$$

$$y''_{(n+1)}(0)=1,\, y''_{(n+1)}(1)=\frac{1}{e},\, y'''_{(n+1)}(0)=-1.$$

Here $y_{(n+1)}$ is the $(n+1)^{th}$ approximation for $y(x)$. The domain [0, 1] is divided into 10 equal subintervals and the proposed method is applied to the sequence of linear problems (27). The obtained numerical results for this problem are presented in Table 5. The maximum absolute error obtained by the proposed method is $1.144409\text{x}10^{-5}$.

Table 5: Numerical results for Example 5

x	Absolute error by the proposed method
0.1	4.172325E-07
0.2	1.192093E-07
0.3	6.675720E-06
0.4	8.106232E-06
0.5	6.020069E-06
0.6	1.144409E-05
0.7	7.182360E-06
0.8	4.172325E-06
0.9	1.758337E-06

7. Conclusions

In this paper, we have developed a collocation method with sextic B-splines as basis functions to solve seventh order boundary value problems. Here we have taken mesh points $x_1, x_2, ..., x_{n-1}, x_{n-1}$ as the collocation points. The sextic B-spline basis set has been redefined into a new set of basis functions which in number match with the number of selected collocation points. The proposed method is applied to solve several number of linear and non-linear problems to test the efficiency of the method. The numerical results obtained by the proposed method are in good agreement with the exact solutions available in the literature. The objective of this paper is to present a simple method to solve a seventh order boundary value problem and its easiness for implementation

References

[1] Richards G and P.R.R.Sarma, ‘‘Reduced order models for induction motors with two rotor circuits’’, IEEE Transactions on Energy Conversion, X1, pp. 673-678, 1994.

[2] R.P. Agarwal, Boundary Value Problems for Higher Order Differential Equations, World Scientific, Singapore, 1986.

[3] Shahid S.Siddiqi, Ghazala Akram and Muzammal Iftikhar, ‘‘Solution of seventh order boundary value problem by Differential Transformation method’’, World Applied Sciences Journal XVI, pp.1521-1526, 2012.

[4] Shahid S.Siddiqi, Ghazala Akram and Muzammal Iftikhar, ‘‘Solution of seventh order boundary value problems by Variational iteration technique’’, Applied Mathematical Sciences, VI, pp. 4663-4672, 2012.

[5] Shahid S.Siddiqi and Muzammal Iftikhar, ‘‘Variational Iteration method for the solution of seventh-order boundary value problems using He’s polynomials’’, Journal of the Association of Arab Universities for Basic and Applied Sciences http://dx.doi.org/10.1016/j.jaubas.2014.03.001, 2014.

[6] Shahid S.Siddiqi and Muzammal Iftikhar, ‘‘Solution of seventh-order boundary value problems by the Adomain decomposition method’’,http://arxiv.org/abs/1301.3603V1 2013.

[7] Shahid S.Siddiqi and Muzammal Iftikhar, ‘‘Numerical solution of Higher order boundary value problems by Homotopy analysis method’’, Abstract and Applied Analysis, Article Id 427521, pp. 1-12, 2013.

[8] Shahid S.Siddiqi and Muzammal Iftikhar, ‘‘Solution of seventh order boundary value problems by Variation parameters method’’, Research Journal of Applied Sciences, Engineering and Technology, V, pp. 176-179, 2013.

[9] Shahid S.Siddiqi and Muzammal Iftikhar, ‘‘Variational iteration Homotopy perturbation method for the Solution of seventh-order boundary value problems’’, http://arxiv.org/abs/1310.2915V1, 2103.

[10] Mustafa Inc and Ali Akgul, ‘‘Numerical solution of seventh-order boundary value problems by a Novel method’’, Abstract and Applied Analysis, Article Id 745287, pp. 1-9, 2014.

[11] Ghazala Akram and Hamood Ur Rehman, ‘‘Numerical solution of seventh-order boundary value problems using the Reproducing kernel space’’, Research Journal of Applied Sciences, Engineering and Technology, VII , pp. 892-896, 2014.

[12] R.E.Bellman and R.E. Kalaba, Quasilinearzation and Nonlinear Boundary Value Problems, American Elsevier, New York, 1965.

[13] J.N.Reddy, An Introduction to thr Finite Element Method, 3rd Edition, Tata Mc-GrawHill Publishing Company Ltd., New Delhi, 2005

[14] P.M. Prenter, Splines and Variational Methods, John-Wiley and Sons, New York, 1989.

[15] Carl de-Boor, A Pratical Guide to Splines, Springer-Verlag, 1978.

[16] I.J. Schoenberg, ‘‘On Spline Functions’’, MRC Report 625, University of Wisconsin, 1966.

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