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Numerical Integration Over A Linear Convex Polyhedron Using An All Hexahedral Discretisation And Gauss Legendre Formulas

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ABSTRACT

Numerical integration is an important ingradient within many techniques of applied mathematics,engineering and scinietific applications, this is due to the need for accurate and efficient integration schemes over complex integration domains and the arbitrary functions as their corresponding integrands. In this paper,we propose a method to discretise the physical domain in the shape of a linear polyhedron into an assemblage of all hexahedral finite elements. The idea is to generate a coarse mesh of all tetrahedrons for the given domain,Then divide each of these tetrahedron further into a refined mesh of all tetrahedrons, if necessary. Then finally, we divide each of these tetrahedron into four hexahedra.We have further demonstrated that each of these hexahedra can be divided into 2^3 and $(2^3)^2$ hexahedra. This generates an all hexahedral finite element mesh which can be used for various applications In order to achieve this we first establish a relation between the arbitrary linear tetrahedron and the standard tetrahedron.We then decompose the standard tetrahedron into four hexahedra. We transform each of these hexahedra into a 2-cube and discover an interesting fact that the Jacobian of these transformations is same and the transformations are also the same but in different order for all the four hexahedra.This fact can be used with great advantage to generate the numerical integration scheme for the standard tetrahedron and hence for the arbitrary linear tetrahedron. We have proposed three numerical schemes which decompose a arbitrary linear tetrahedron into 4, $4(2^3)$ *and* $4((2^3)^2)$ hexahedra.These numerical schemes are applied to solve typical integrals over a unit cube and irregular heptahedron using Gauss Legendre Quadrature Rules. Matlab codes are developed and appended to this paper.

Key Words: Numerical Integration,Finite Elements,Tetrhedron,Hexahedron,Polyhedron**,** Irregular Heptahedron ,Gauss Legendre Quadrature Rules,

1. INTRODUCTION

Finite element method is a general purpose method for numerical analysis in engineering applications and scientific investigations. It is an important component of computer-aided design and manufacture, and extensively applied in

the fields of hydraulics, hydrology, water resources, geotechnical engineering, solid/fluid mechanics, computer graphics, mechanics, and architecture [1], etc.

The finite element method (FEM) has become a central tool in computer graphics, and it is widely used for physically based animation of deformations, fracture, fluids, smoke, or other affects. Most methods discretize the computational domain by tetrahedral or hexahedral elements and linear or trilinear interpolants, respectively.

Finite element methods also play an important role in the present practice of numerically solving partial differential equations (PDEs).They inherit the use of piecewise polynomial spaces for the representation of the solution, the PDE coefficients and source functions. Another feature rooted in the finite element is the suitability for use with general and irregular grids, allowing the effective treatment of complex geometries, with a natural realization of local grid refinement and use of higher order polynomial interpolations.

In several earlier works[1-12], numerical integration rules for tetrahedron are already established. In a studies[13] , composites integration with all tetrahedron decomposition is also proposed. In a recent study[16] numerical integration over a standard tetrahedron is computed by decomposing it into four hexahedrons and applied to some typical integrals which were considered earlieris also presented. They have not shown the application of the method to compute integrals over a linear polyhedron. Finite element studies also require the use of hexahedron elements to represent the appropriate behaviour of physical quantities[14-15].

In this paper,we propose a method to discretise the physical domain in the shape of a linear polyhedron into an assemblage of all hexahedral finite elements. The idea is to generate a coarse mesh of all tetrahedrons for the given domain,Then divide each of these tetrahedron further into a refined mesh of all tetrahedrons, if necessary. Then finally, we divide each of these tetrahedron into four hexahedra.We have further demonstrated that each of these hexahedra can be divided into 2^3 and $(2^3)^2$ hexahedra. This generates an all hexahedral finite element mesh which can be used for various applications In order to achieve this we first establish a relation between the arbitrary linear tetrahedron and the standard tetrahedron.We then decompose the standard tetrahedron into four hexahedra. We transform each of these hexahedra into a 2-cube and discover an interesting fact that the Jacobian of these transformations is same and the transformations are also the same but in different order for all the four hexahedra.This fact can be used with great advantage to generate the numerical integration scheme for the standard tetrahedron and hence for the arbitrary linear tetrahedron. We have proposed three numerical schemes which decompose a arbitrary linear tetrahedron into 4, $4(2^3)$ and $4((2^3)^2$) hexahedra. This is proposed in sections 2-4 of this paper.In section 5,numerical integration scheme for a linear polyhedron which is partitioned into tetrahedra,pyramids and tetrahdra obtained by triangulating the surface of polyhedron is shown in figures. In section 6,the above numerical schemes are applied to solve typical integrals over a unit cube and irregular heptahedron using Gauss Legendre Quadrature Rules. Matlab codes are developed and appended to this paper.

2. VOLUME INTEGRATION OVER AN ARBITRARY LINEAR TETRAHEDRON

Let us consider the volume integral over an arbitrary linear tetrahedron T_{1234} as

$$III_{T_{1234}}(f) = \iiint_{T_{1234}^{\sim}} f(X,Y,Z)\ dXdYdZ \qquad (1)$$

Where,

T_{1234} is an arbitrary linear tetrahedron in Cartesian space with vertices ((X_i , Y_i , Z_i) , i=1,2,3.4).

We can transform the arbitrary linear tetrahedron into an orthogonal tetrahedron(standard tetrahedron) $T_{1234}^{\sim}$ by using the following affine coordinate transformation as shown in Fig.1

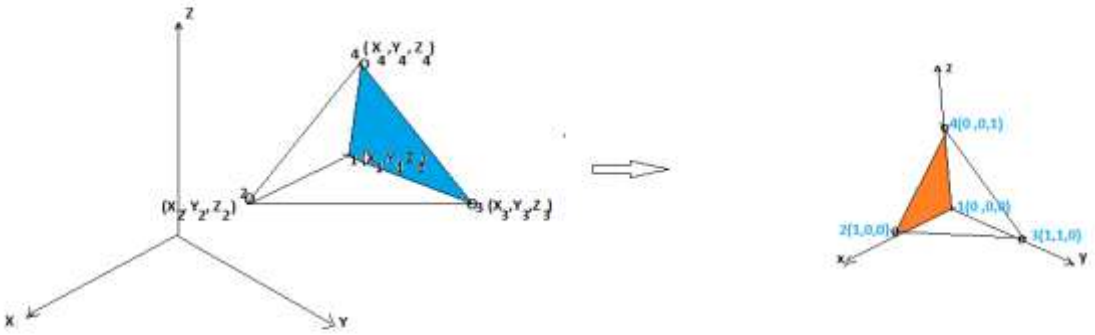

Fig.1 Three dimensional mapping of an arbitrary linear tetrahedron in XYZ-space into a unit orthogonal tetrahedron in xyz-space

and the transformation is

$$X=X_1+(X_2-X_1)x+(X_3-X_1)y+(X_4-X_1)z\,,$$

$$Y=Y_1+(Y_2-Y_1)x+(Y_3-Y_1)y+(Y_4-Y_1)z\,,$$

$$Z=Z_1+(Z_2-Z_1)\,x+(Z_2-Z_1)y+(Z_4-Z_1)z\,, \qquad (2)$$

We now evaluate the integral

$$\iiint_{T_{1234}} f(X,Y,Z)dXdYdZ = |\det J| \iiint_{T_{1234}^{\sim}} f(X(x,y,z),Y(x,y,z),Z(x,y,z))dxdydz \qquad (3)$$

Where,

$$\det J = \begin{vmatrix} X_{21} & X_{31} & X_{41} \\ Y_{21} & Y_{31} & Y_{41} \\ Z_{21} & Z_{31} & Z_{41} \end{vmatrix},$$

$$X_{pq}=X_p-X_q\,, Y_{pq}=Y_p-Y_q\,, Z_{pq}=Z_p-Z_q\,, \qquad (4)$$

$$p=2,3,4\,; q=1$$

In the above derivations, we have established a relation between an arbitrary linear tetrahedron and the unit orthogonal linear tetrahedron. In the next section we consider the composite integration over the unit orthogonal linear tetrahedron by decomposing it into four hexahedrons.

3.COMPOSITE INTEGRATION OVER A UNIT ORTHOGONAL LINEAR TETRAHEDRON

This paper proposes the composite integration by dividing the arbitrary linear tetrahedron into an all hexahedron finite element mesh.We first divide the arbitrary linear tetrahedron into four unique hexadrons,then we refine this division into a mesh of 32 hexahedrons and finally into 256 hexahdrons. These three proposals are connected and the give higher accuracies by using mathematical expressions of same order but higher rational constants. We

now obtain the necessary coordinate transformations and their Jacobians which will transform the integration over arbitrary linear hexahedrons to the integrals over a 2-cube.

3.1 Division of a tetrahedron into four hexahedrons

We divide tetrahedron into four hexahedron as shown in the following figure, Fig.1.This is done first by joining the centroid of the tetrahedron to the centroids of four triangular surfaces which form the tetrahedron. Then we locate te centroids of the four triangular surfaces which are further joined to the mid points of the respective triangular edges of the triangular surfaces. This creates four hexahedrons Ω_i $(i = 1,2,3,4)$ in the standard linear tetrahedron , Thus,we can write from eqn(3) for the triple integral over the arbitrary linear tetrahedron as

$$\iiint_{T_{1234}} f(X,Y,Z)dXdYdZ = |\det J| \iiint_{T_{1234}^{\sim}} f(X(x,y,z),Y(x,y,z),Z(x,y,z))dxdydz$$

$$=|detJ| \sum_{i=1}^{4} \iiint_{\Omega_i} f(X(x,y,z),Y(x,y,z),Z(x,y,z))dxdydz \quad \text{.....(5)}$$

Where, as discussed above both (X,Y,Z) and (x,y,z) are different Cartesian spaces and T_{1234} , $T_{1234}^{\sim}$ are the arbitrary linear tetrahedron and the standard orthogonal linear tetrahedron respectively.

As mentioned above Ω_i $(i = 1,2,3,4)$ are the hexahedrons created inside the standard linear orthogonal tetrahedron $T_{1234}^{\sim}$.

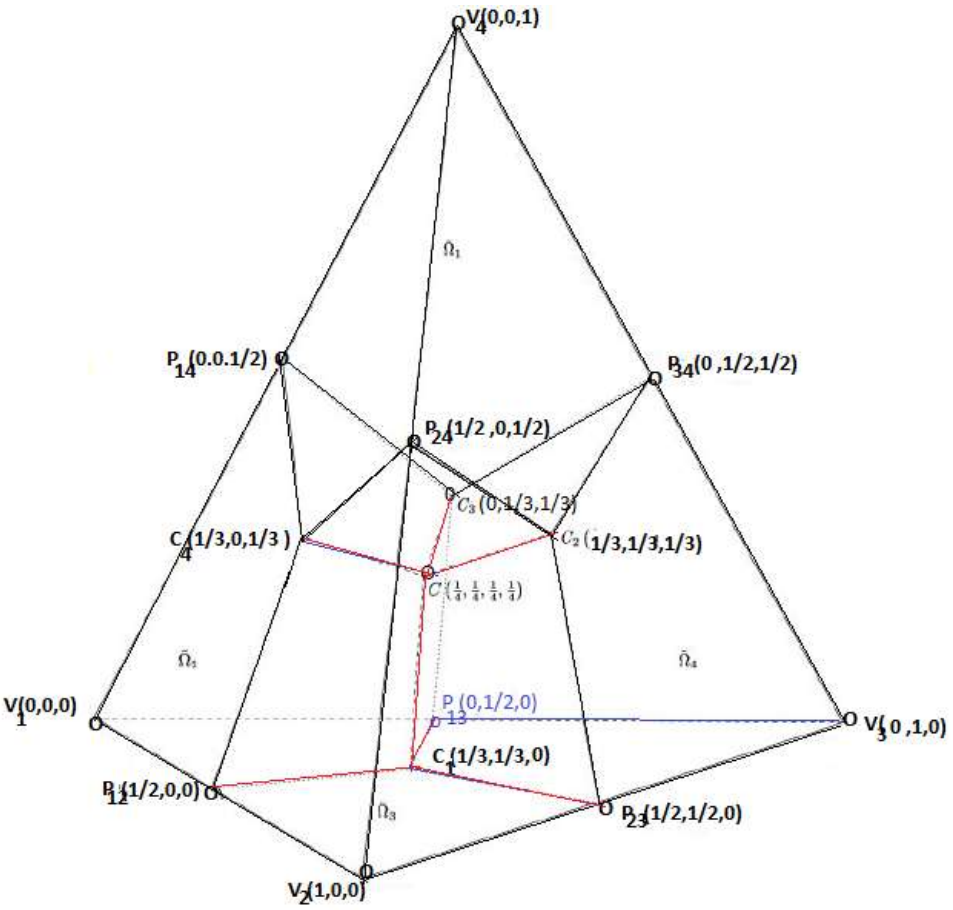

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Fig.2 DIVISION OF A STANDARD TETRAHEDRON INTO FOUR HEXAHEDRONS

In Fig.2 above,
Ω_1 : Hexahedron spanned by$\{C_1, P_{13}, V_1, P_{12}, C, C_3, P_{14}, C_4\}$
Ω_2 : : Hexahedron spanned by$\{C_1, P_{12}, V_2, P_{23}, C, C_4, P_{24}, C_2\}$
Ω_3 : : Hexahedron spanned by$\{C_1, P_{23}, V_3, P_{13}, C, C_2, P_{34}, C_3\}$
Ω_4 :Hexahedron spanned by$\{C, C_4, P_{24}, C_2, C_3, P_{34}, V_4, P_{14}\}$,

Where ,
C_i (i=1,2,3,4) are centroids of triangular faces spanned by vertices $\{V_1, V_2, V_3\}$,
$\{V_2, V_3, V_4\}$, $\{V_1, V_3, V_4\}$, and $\{V_1, V_2, V_4\}$ respectively
P_{ij} ,{(ij)= (12),(23),(13),(14},(24),(34)} are the midpoints of edges joining vertices V_i V_j and
C is the centroid of the standard tetrahedron
The nodal coordinates for the above four hexahedrons are mentioned below
Over the hexahedron Ω_1:
x_1 =0, x_2 =1/2, x_3= 1/3, $x_4 =$ 0, $x_5 =$ 0, $x_6 =$ 1/3, $x_7 =$ 1/4, $x_8 =$ 0
$y_1 =$ 0, $y_2 =$ 0, $y_3 =$ 1/3, $y_4 =$ 1/2, $y_5 =$ 0, $y_6 =$ 0, $y_7 =$ 1/4, $y_8 =$ 1/3
z_1 =0, z_2 = 0, z_3 = 0, z_4 = 0, z_5 =1/2, z_6 = 1/3, z_7 = 1/4, z_8 = 1/3 .....(6)

Over the hexahedron Ω_2:

x_1 = 1, $x_2 =$ 1/2, x_3= 1/3, $x_4 =$ 1/2, , $x_5 =$ 1/2, $x_6 =$ 1/3, $x_7 =$ 1/4, $x_8 =$ 1/3
$y_1 =$ 0, $y_2 =$ 1/2, $y_3 =$ 1/3, $y_4 =$ 0, $y_5 =$ 0, $y_6 =$ 1/3, $y_7 =$ 1/4, $y_8 =$ 0
z_1 = 0, z_2 = 0, z_3 = 0, z_4 = 0, z_5 =1/2, z_6 = 1/3, , z_7 = 1/4, , z_8 = 1/3 .....(7)

Over the hexahedron Ω_3:

$x_1 =$ 0, $x_2 =$ 0, x_3= 1/3, $x_4 =$ 1/2, $x_5 =$ 0, $x_6 =$ 0, $x_7 =$ 1/4, , $x_8 =$ 1/3
$y_1 =$ 1, $y_2 =$ 1/2, $y_3 =$ 1/3, $y_4 =$ 1/2, $y_5 =$ 1/2, $y_6 =$ 1/3, , $y_7 =$ 1/4, , $y_8 =$ 1/3
z_1 = 0, z_2 = 0, , z_3 = 0, z_4 = 0, z_5 =1/2, , z_6 = 1/3, , z_7 = 1/4, z_8 = 1/3 .....(8)

Over the hexahedron Ω_4:

$x_1 =$ 0, , $x_2 =$ 0, x_3= 1/3, $x_4 =$ 1/2, $x_5 =$ 0, $x_6 =$ 0, $x_7 =$ 1/4, $x_8 =$ 1/3
$y_1 =$ 0, $y_2 =$ 1/2, $y_3 =$ 1/3, $y_4 =$ 0, $y_5 =$ 0, $y_6 =$ 1/3, $y_7 =$ 1/4, $y_8 =$ 0
z_1 = 1, z_2 = 1/2, z_3 = 1/3, z_4 = 1/2, z_5 = 1/2, z_6 = 1/3, z_7 = 1/4, z_8 = 1/3 .....(9)

We can transform each of these hexahedrons in physical space (x,y ,z) into a standard 2-cube in a parametric space (r,s,t) by using the following coordinate transformations:
$x=\sum_{h=1}^{8} N_h(r,s,t)\, x_h$; $y=\sum_{h=1}^{8} N_h(r,s,t)\, y_h$; $z=\sum_{h=1}^{8} N_h(r,s,t)\, z_h$ .....(10)
Where, N_h(r,s,t) are the nodal shape functions for a the standard 2-cube ,$-1 \le r,s,t \le 1$ in the parametric space $(r,s,t)$and (x_h, y_h, z_h) are the nodal coordinates of the hexahedron in the
Cartesian space (x,y,z) and the shape functions N_h(r,s,t) are:
N_1(r,s,t)=$\frac{1}{8}(1-r)(1-s)(1-t)$,
N_2(r,s,t)=$\frac{1}{8}(1+r)(1-s)(1-t)$,
N_3(r,s,t)=$\frac{1}{8}(1+r)(1+s)(1-t)$,
N_4(r,s,t)=$\frac{1}{8}(1+r)(1-s)(1-t)$,
N_5(r,s,t)=$\frac{1}{8}(1-r)(1-s)(1+t)$,
N_6(r,s,t)=$\frac{1}{8}(1+r)(1-s)(1+t)$,
N_7(r,s,t)=$\frac{1}{8}(1+r)(1+s)(1+t)$,

N_8(r,s,t)=$\frac{1}{8}(1+r)(1-s)(1+t)$. .....(10)

The integrals over the hexahedrons Ω_i (i=1,2,3,4) in Cartesian space can be now expressed from the above transformations of eqn(9) as : $\iiint_{\Omega_i} f(X(x,y,z),Y(x,y,z),Z(x,y,z))dxdydz$

$=\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f(X(x^i,y^i,z^i),Y(x^i,y^i,z^i),Z(x^i,y^i,z^i))\frac{\partial(x^i,y^i,z^i)}{\partial(r,s,t)}\ drdsdt$ .....(11)

Hence from eqns(5) and (11),we obtain

$\iiint_{T_{1234}} f(X,Y,Z)dXdYdZ = |\det J| \iiint_{T_{1234}^{\sim}} f(X(x,y,z),Y(x,y,z),Z(x,y,z))dxdydz$

$=|detJ|\sum_{i=1}^{4}\iiint_{\Omega_i} f(X(x,y,z),Y(x,y,z),Z(x,y,z))dxdydz$

$=|detJ|\sum_{i=1}^{4}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f(X(x^i,y^i,z^i),Y(x^i,y^i,z^i),Z(x^i,y^i,z^i))\frac{\partial(x^i,y^i,z^i)}{\partial(r,s,t)}\ drdsdt$ .....(12)

In eqn(11) above x^i=x^i(r,s,t), y^i=y^i(r,s,t), z^i=z^i(r,s,t) are the coordinate transformations the hexahedrons Ω_i and they can be computed using eqns(5-10).Then we compute the Jacobian which can be obtained by computing the following equation

$$\frac{\partial(x^i,y^i,z^i)}{\partial(r,s,t)}=\begin{pmatrix}\frac{\partial x^i}{\partial r} & \frac{\partial x^i}{\partial s} & \frac{\partial x^i}{\partial t}\\ \frac{\partial y^i}{\partial r} & \frac{\partial y^i}{\partial s} & \frac{\partial y^i}{\partial t}\\ \frac{\partial z^i}{\partial r} & \frac{\partial z^i}{\partial s} & \frac{\partial z^i}{\partial t}\end{pmatrix}=J^i(r,s,t)=J^i \text{ (say)} \quad \text{.....(13)}$$

Using eqn(2),the the coordinate trans formations over a hexahedron Ω_i can be now rewritten as

$X(x^i,y^i,z^i) = X_1 w^i + X_2x^i + X_3y^i + X_4z^i$

$Y(x^i,y^i,z^i) = Y_1 w^i + Y_2x^i + Y_3y^i + Y_4z^i$

$Z(x^i,y^i,z^i) = Z_1 w^i + Z_2x^i + Z_3y^i + Z_4z^i$

$w^i = 1 - x^i - y^i - z^i$ .....(14)

Table-1

The computed expressions of x^i , y^i, z^i, w^i , J^i over a hexahedron are Ω_i,($i = 1,2.3.4$)

i	Ω_i	x^i	y^i	z^i	w^i	J^i
1	Ω_1	α	β	γ	δ	ϑ
2	Ω_2	δ	α	γ	β	ϑ
3	Ω_3	β	δ	γ	α	ϑ
4	Ω_4	β	α	δ	γ	ϑ

In the above table α , β , γ, δ and ϑ are trivariate polynomials in the variables r,s,t and they are computed by using MATLAB program which uses symbolic maths.

ϑ =(r^2*s*t)/55296 - (5*r^2*s)/55296 - (5*r^2*t)/55296 + (13*r^2)/55296 + (r*s^2*t)/55296 - (5*r*s^2)/55296 + (r*s*t^2)/55296 - (5*r*s*t)/27648 + (49*r*s)/55296 - (5*r*t^2)/55296 + (49*r*t)/55296 - (5*r)/2304 - (5*s^2*t)/55296 + (13*s^2)/55296 - (5*s*t^2)/55296 + (49*s*t)/55296 - (5*s)/2304 + (13*t^2)/55296 - (5*t)/2304 + 275/55296 .....(16a)

α =(17*r)/96 - s/32 - t/32 - (r*s)/32 - (r*t)/32 + (s*t)/96 + (r*s*t)/96 + 17/96 .....(16b)

β =(17*s)/96 - r/32 - t/32 - (r*s)/32 + (r*t)/96 - (s*t)/32 + (r*s*t)/96 + 17/96 .....(16c)

γ =(17*t)/96 - s/32 - r/32 + (r*s)/96 - (r*t)/32 - (s*t)/32 + (r*s*t)/96 + 17/96 .....(16d)

δ =(5*r*s)/96 -(11*s)/96 -(11*t)/96 -(11*r)/96 + (5*r*t)/96 +(5*s*t)/96 -(r*s*t)/32 + 15/32.....(16e)

We may now mention that the integration over an arbitrary linear tetrahedron can be now computed as a sum of four integrals over the 2-cube: $-1 \leq r, s, t \leq 1$ which is shown in Fig.3.These integrals can be computed numerically by applying Gaussian rules and it will be explained later

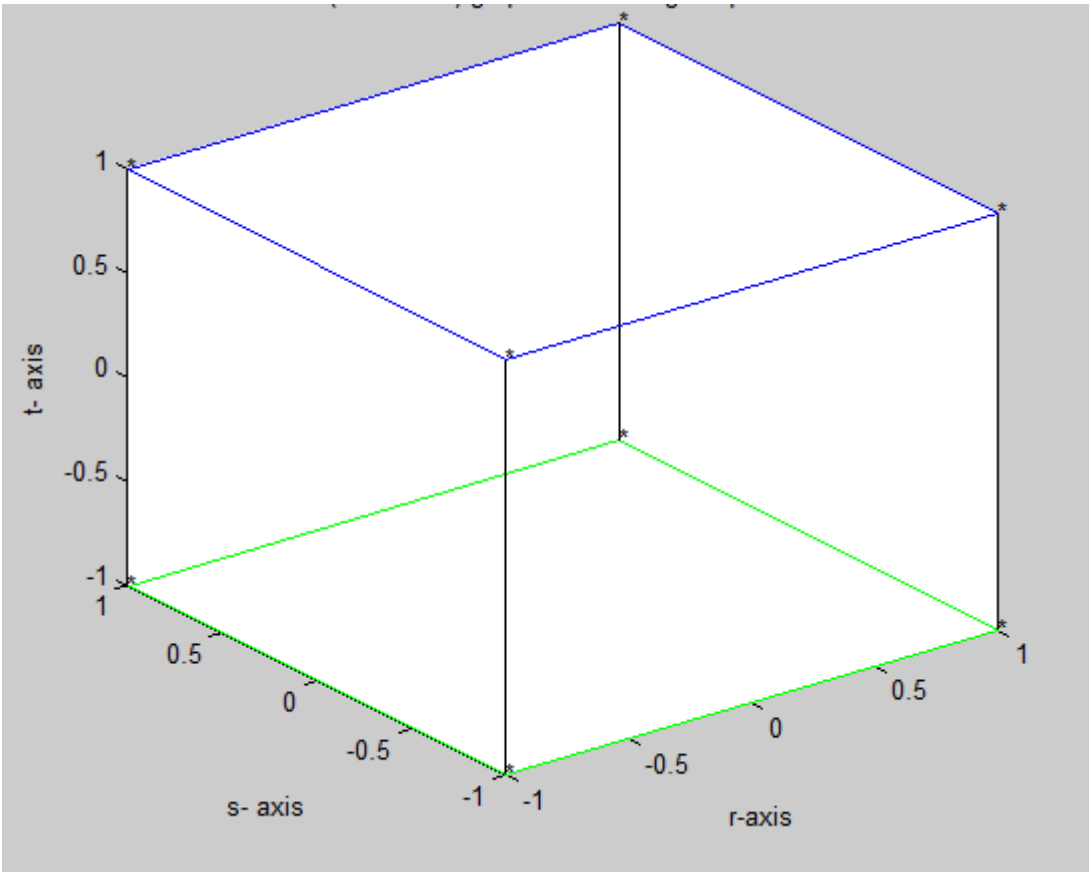

Fig.3 A 2-cube *over the domain* $-1 \leq r, s, t \leq 1$ in (r,s,t) space

3.2 Division of a tetrahedron into thirty two hexahedrons

We divide each hexahedron into eight hexahedrons .This can be done first by locating the centroids of the six faces of the hexahedrons and then joining the centroids to the midpoints of the respective edges We also locate the centroid of the hexahedron and join this to the centroids of the six faces .This process can be repeated for the remaining three hexahedrons as well.This divides the tetrahedron into thirty two hexahedrons.We can then integrate over all the hexahedrons.This straight forward process is very tedious,because one has to obtain the coordinates of transformations and the respective jacobian to apply numerical integration over the 2-cube.Instead of this we follow a more efficient method of finding the coordinates of transformations and their respective jacobians.

Using the transformations of eqn(14) and the expressions of x^i , y^i, z^i, w^i , J^i

shown in Table-1 ,we can map the hexahedrons Ω_i,($i = 1,2.3.4$), into a 2-cube in (r,s,t) space. It is well known that there is a one to one correspondence between 2- cube in (r,s,t) space and the hexahedron Ω_i, $i = 1,2.3.4$). Hence a division of 2-cube correspondences to a unique division of the hexahedra.Thus the division of hexahedra can be achieved by dividing the the 2-cube. The division of a 2-cube into eight cubes of unit dimension is displayed in Fig.4

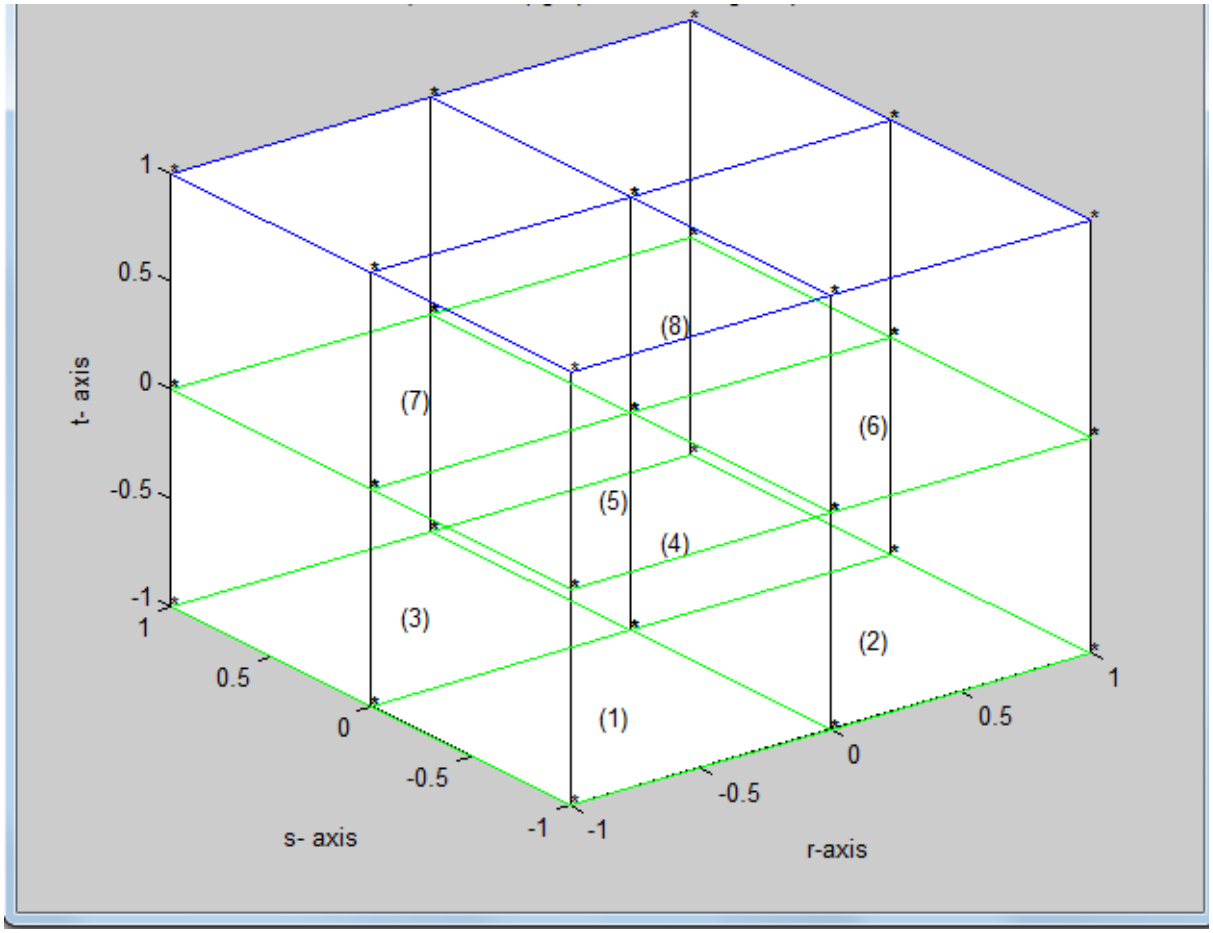

Fig.4 Division of a 2-cube into eight cubes of unit dimension

We now consider the integration over a 2-cube by dividing the2-cube into eight unit cubes.

Let F(r,s,t)= $f(X(x^i,y^i,z^i),Y(x^i,y^i,z^i),Z(x^i,y^i,z^i))\frac{\partial(x^i,y^i,z^i)}{\partial(r,s,t)}$

.....(17)

Clearly , *it is already assumed that* x^i=x^i(r,s,t), y^i=y^i(r,s,t), z^i=z^i(r,s,t) ,
so that we can write from eq(11):

$\iiint_{\Omega_i} f(X(x,y,z),Y(x,y,z),Z(x,y,z))dxdydz$

=$\iiint_{\Omega_i} f(X(x^i,y^i,z^i),Y(x^i,y^i,z^i),Z(x^i,y^i,z^i))\frac{\partial(x^i,y^i,z^i)}{\partial(r,s,t)}$ drdsdt .....(18a)

=$\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}$ F(r, s, t) $drdsdt$

=$\sum_{j=1}^{8}\iiint_{\Omega_{i,j}} f(X(x^{i,j},y^{i,j},z^{i,j}),Y(x^{i,j},y^{i,j},z^{i,j}),Z(x^{i,j},y^{i,j},z^{i,j}))J^{i,j}(r,s,t)$ drdsdt ..... (18b)

=$\int_{-1}^{0}\int_{-1}^{0}\int_{-1}^{0}$ F(r, s, t) $drdsdt$+$\int_{-1}^{0}\int_{-1}^{0}\int_{0}^{1}$ F(r, s, t) $drdsdt$

$$+\int_{-1}^{0}\int_{0}^{1}\int_{-1}^{0}\mathrm{F(r,s,t)}\,drdsdt+\int_{-1}^{0}\int_{0}^{1}\int_{0}^{1}\mathrm{F(r,s,t)}\,drdsdt$$
$$+\int_{0}^{1}\int_{-1}^{0}\int_{-1}^{0}\mathrm{F(r,s,t)}\,drdsdt+\int_{-1}^{0}\int_{-1}^{0}\int_{0}^{1}\mathrm{F(r,s,t)}\,drdsdt$$
$$+\int_{0}^{1}\int_{0}^{1}\int_{-1}^{0}\mathrm{F(r,s,t)}\,drdsdt+\int_{0}^{1}\int_{0}^{1}\int_{0}^{1}\mathrm{F(r,s,t)}\,drdsdt \qquad ....(18c)$$

$$=\frac{1}{8}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}F\left(\frac{-1}{2}+\frac{1}{2}r.\frac{-1}{2}+\frac{1}{2}s.\frac{-1}{2}+\frac{1}{2}t\right)drdsdt+\frac{1}{8}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}F\left(\frac{1}{2}+\frac{1}{2}r.\frac{-1}{2}+\frac{1}{2}s.\frac{-1}{2}+\frac{1}{2}t\right)drdsdt$$
$$+\frac{1}{8}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}F\left(\frac{-1}{2}+\frac{1}{2}r.\frac{1}{-2}+\frac{1}{2}s.\frac{-1}{2}+\frac{1}{2}t.\right)drdsdt+\frac{1}{8}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}F\left(\frac{1}{2}+\frac{1}{2}r.\frac{1}{2}+\frac{1}{2}s.\frac{-1}{2}+\frac{1}{2}t\right)drdsdt$$
$$+\frac{1}{8}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}F\left(\frac{-1}{2}+\frac{1}{2}r.\frac{-1}{2}+\frac{1}{2}s.\frac{1}{2}+\frac{1}{2}t\right)drdsdt+\frac{1}{8}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}F\left(\frac{1}{2}+\frac{1}{2}r.\frac{-1}{2}+\frac{1}{2}s.\frac{-1}{2}+\frac{1}{2}t\right)drdsdt$$
$$+\frac{1}{8}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}F\left(\frac{-1}{2}+\frac{1}{2}r.\frac{1}{2}+\frac{1}{2}s.\frac{1}{2}+\frac{1}{2}t\right)drdsdt+\frac{1}{8}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}F\left(\frac{1}{2}+\frac{1}{2}r.\frac{1}{2}+\frac{1}{2}s.\frac{1}{2}+\frac{1}{2}t\right)drdsdt$$
$$..........(18d)$$

Now referring to eqn(18),we can compute the new coordinate transformations ($x^{i,j}$, $y^{i,j}$, $z^{i,j}$),and the corresponding jacobians $J^{i,j}$ over the hexahedron $\Omega_{i,j}$ (i=1,2,3,4;j=1,2,3,4,5,6,7,8),which referes to jth division of the ith hexahedron Ω_i

Table-2

The computed expressions of $x^{i,j}$, $y^{i,j}$, $z^{i,j}$, $w^{i,j}$, $J^{i,j}$ over the 32 hexahedrons $\Omega_{i,j}$, ($i = 1,2.3.4$; $j = 1,2,3,4,5,6,7,8$) ,Note that $\Omega_{i,j}$ referes to the jth division of ith hexahedron Ω_i.

(i , j)	$\Omega_{i,j}$	$x^{i,j}$	$y^{i,j}$	$z^{i,j}$	$w^{i,j}$	$J^{i,j}$
(1,j)	$\Omega_{1,j}$	α^j	β^j	γ^j	$\delta^{,j}$	$\vartheta^{,j}$
(2,j)	$\Omega_{2,j}$	δ^j	α^j	γ^j	β^j	$\vartheta^{,j}$
(3,j)	$\Omega_{3,j}$	β^j	$\delta^{,j}$	γ^j	α^j	ϑ^j
(4 ,j)	$\Omega_{4,j}$	β^j	α^j	δ^j	γ^j	ϑ^j

..........(19)

From Table-2,it is clear that the computed values of ($\alpha^{,j}$, β^j , γ^j , δ^j, $\vartheta^{,j}$) , ($j = 1,2,3,4,5,6,7,8$)will be necessary.

They are listed below:

ϑ^1 =(r^2*s*t)/7077888 - (11*r^2*s)/7077888 - (11*r^2*t)/7077888 + (73*r^2)/7077888 + (r*s^2*t)/7077888 - (11*r*s^2)/7077888 + (r*s*t^2)/7077888 - (13*r*s*t)/3538944 + (29*r*s)/786432 - (11*r*t^2)/7077888 + (29*r*t)/786432 - (385*r)/1769472 - (11*s^2*t)/7077888 + (73*s^2)/7077888 - (11*s*t^2)/7077888 + (29*s*t)/786432 - (385*s)/1769472 + (73*t^2)/7077888 - (385*t)/1769472 + 8107/7077888;
..............(20a)

α^1 =(27*r)/256 - (7*s)/768 - (7*t)/768 - (7*r*s)/768 - (7*r*t)/768 + (s*t)/768 + (r*s*t)/768 + 27/256; ....(20b)

β^1=(27*s)/256 - (7*r)/768 - (7*t)/768 - (7*r*s)/768 + (r*t)/768 - (7*s*t)/768 + (r*s*t)/768 + 27/256; ....(20c)

$\gamma^{,1}$=(27*t)/256 - (7*s)/768 - (7*r)/768 + (r*s)/768 - (7*r*t)/768 - (7*s*t)/768 + (r*s*t)/768 + 27/256; ....(20d)

δ^1=(13*r*s)/768 - (67*s)/768 - (67*t)/768 - (67*r)/768 + (13*r*t)/768 + (13*s*t)/768 - (r*s*t)/256 + 175/256; ....(20e)

ϑ^2 =(r^2*s*t)/7077888 - (11*r^2*s)/7077888 - (11*r^2*t)/7077888 + (73*r^2)/7077888 + (r*s^2*t)/7077888 - (11*r*s^2)/7077888 + (r*s*t^2)/7077888 - (11*r*s*t)/3538944 + (217*r*s)/7077888 - (11*r*t^2)/7077888 + (217*r*t)/7077888 - (13*r)/73728 - (s^2*t)/786432 + (17*s^2)/2359296 - (s*t^2)/786432 + (71*s*t)/2359296 - (59*s)/393216 + (17*t^2)/2359296 - (59*t)/393216 + 197/262144; .....(21a)

α^2 =(27*r)/256 - (7*s)/256 - (7*t)/256 - (7*r*s)/768 - (7*r*t)/768 + (s*t)/256 + (r*s*t)/768 + 81/256; .....(21b)

β^2=(67*s)/768 - (7*r)/768 - (5*t)/768 - (7*r*s)/768 + (r*t)/768 - (5*s*t)/768 + (r*s*t)/768 + 67/768; .....(21c)

γ'^2 =(67*t)/768 - (5*s)/768 - (7*r)/768 + (r*s)/768 - (7*r*t)/768 - (5*s*t)/768 + (r*s*t)/768 + 67/768; .....(21d)

δ^2=(13*r*s)/768 - (41*s)/768 - (41*t)/768 - (67*r)/768 + (13*r*t)/768 + (7*s*t)/768 - (r*s*t)/256 + 391/768; .....(21e)

ϑ'^3 =(r^2*s*t)/7077888 - (11*r^2*s)/7077888 - (r^2*t)/786432 + (17*r^2)/2359296 + (r*s^2*t)/7077888 - (11*r*s^2)/7077888 + (r*s*t^2)/7077888 - (11*r*s*t)/3538944 + (217*r*s)/7077888 - (r*t^2)/786432 + (71*r*t)/2359296 - (59*r)/393216 - (11*s^2*t)/7077888 + (73*s^2)/7077888 - (11*s*t^2)/7077888 + (217*s*t)/7077888 - (13*s)/73728 + (17*t^2)/2359296 - (59*t)/393216 + 197/262144;
.....(22a)

α^3 =(67*r)/768 - (7*s)/768 - (5*t)/768 - (7*r*s)/768 - (5*r*t)/768 + (s*t)/768 + (r*s*t)/768 + 67/768; .....(22b)

β^3=(27*s)/256 - (7*r)/256 - (7*t)/256 - (7*r*s)/768 + (r*t)/256 - (7*s*t)/768 + (r*s*t)/768 + 81/256; .....(22c)

γ'^3=(67*t)/768 - (7*s)/768 - (5*r)/768 + (r*s)/768 - (5*r*t)/768 - (7*s*t)/768 + (r*s*t)/768 + 67/768; .....(22d)

δ^3 =(13*r*s)/768 - (67*s)/768 - (41*t)/768 - (41*r)/768 + (7*r*t)/768 + (13*s*t)/768 - (r*s*t)/256 + 391/768; .....(22e)

ϑ'^4=(r^2*s*t)/7077888 - (11*r^2*s)/7077888 - (r^2*t)/786432 + (17*r^2)/2359296 + (r*s^2*t)/7077888 - (11*r*s^2)/7077888 + (r*s*t^2)/7077888 - (r*s*t)/393216 + (173*r*s)/7077888 - (r*t^2)/786432 + (59*r*t)/2359296 - (143*r)/1179648 - (s^2*t)/786432 + (17*s^2)/2359296 - (s*t^2)/786432 + (59*s*t)/2359296 - (143*s)/1179648 + (11*t^2)/2359296 - (7*t)/73728 + 1133/2359296;
.....(23a)

α^4=(67*r)/768 - (7*s)/256 - (5*t)/256 - (7*r*s)/768 - (5*r*t)/768 + (s*t)/256 + (r*s*t)/768 + 67/256; .....(23b)

β^4=(=(67*s)/768 - (7*r)/256 - (5*t)/256 - (7*r*s)/768 + (r*t)/256 - (5*s*t)/768 + (r*s*t)/768 + 67/256; .....(23c)

γ'^4=(19*t)/256 - (5*s)/768 - (5*r)/768 + (r*s)/768 - (5*r*t)/768 - (5*s*t)/768 + (r*s*t)/768 + 19/256; .....(23d)

δ^4 =(13*r*s)/768 - (41*s)/768 - (9*t)/256 - (41*r)/768 + (7*r*t)/768 + (7*s*t)/768 - (r*s*t)/256 + 103/256; .....(23e)

ϑ'^5=(r^2*s*t)/7077888 - (r^2*s)/786432 - (11*r^2*t)/7077888 + (17*r^2)/2359296 + (r*s^2*t)/7077888 - (r*s^2)/786432 + (r*s*t^2)/7077888 - (11*r*s*t)/3538944 + (71*r*s)/2359296 - (11*r*t^2)/7077888 + (217*r*t)/7077888 - (59*r)/393216 - (11*s^2*t)/7077888 + (17*s^2)/2359296 - (11*s*t^2)/7077888 + (217*s*t)/7077888 - (59*s)/393216 + (73*t^2)/7077888 - (13*t)/73728 + 197/262144;
.....(24a)

α^5=(67*r)/768 - (5*s)/768 - (7*t)/768 - (5*r*s)/768 - (7*r*t)/768 + (s*t)/768 + (r*s*t)/768 + 67/768; .....(24b)

β^5=(67*s)/768 - (5*r)/768 - (7*t)/768 - (5*r*s)/768 + (r*t)/768 - (7*s*t)/768 + (r*s*t)/768 + 67/768; .....(24c)

γ'^5=(27*t)/256 - (7*s)/256 - (7*r)/256 + (r*s)/256 - (7*r*t)/768 - (7*s*t)/768 + (r*s*t)/768 + 81/256; .....(24d)

δ^5 =(7*r*s)/768 - (41*s)/768 - (67*t)/768 - (41*r)/768 + (13*r*t)/768 + (13*s*t)/768 - (r*s*t)/256 + 391/768; .....(24e)

ϑ'^6=(r^2*s*t)/7077888 - (r^2*s)/786432 - (11*r^2*t)/7077888 + (17*r^2)/2359296 + (r*s^2*t)/7077888 - (r*s^2)/786432 + (r*s*t^2)/7077888 - (r*s*t)/393216 + (59*r*s)/2359296 - (11*r*t^2)/7077888 + (173*r*t)/7077888 - (143*r)/1179648 -

```
(s^2*t)/786432 + (11*s^2)/2359296 - (s*t^2)/786432 + (59*s*t)/2359296 - (7*s)/73728 +
(17*t^2)/2359296 - (143*t)/1179648 + 1133/2359296;
.............(25a)
```

α^6=(67*r)/768 - (5*s)/256 - (7*t)/256 - (5*r*s)/768 - (7*r*t)/768 + (s*t)/256 + (r*s*t)/768 + 67/256; .....(25b)

β^6=(19*s)/256 - (5*r)/768 - (5*t)/768 - (5*r*s)/768 + (r*t)/768 - (5*s*t)/768 + (r*s*t)/768 + 19/256; .....(25c)

γ'^6=(67*t)/768 - (5*s)/256 - (7*r)/256 + (r*s)/256 - (7*r*t)/768 - (5*s*t)/768 + (r*s*t)/768 + 67/256; .....(25d)

δ^6=(7*r*s)/768 - (9*s)/256 - (41*t)/768 - (41*r)/768 + (13*r*t)/768 + (7*s*t)/768 - (r*s*t)/256 + 103/256; .....(25e)

ϑ'^7=(r^2*s*t)/7077888 - (r^2*s)/786432 - (r^2*t)/786432 + (11*r^2)/2359296 + (r*s^2*t)/7077888 - (r*s^2)/786432 + (r*s*t^2)/7077888 - (r*s*t)/393216 + (59*r*s)/2359296 - (r*t^2)/786432 + (59*r*t)/2359296 - (7*r)/73728 - (11*s^2*t)/7077888 + (17*s^2)/2359296 - (11*s*t^2)/7077888 + (173*s*t)/7077888 - (143*s)/1179648 + (17*t^2)/2359296 - (143*t)/1179648 + 1133/2359296;
.............(26a)

α^7 =(19*r)/256 - (5*s)/768 - (5*t)/768 - (5*r*s)/768 - (5*r*t)/768 + (s*t)/768 + (r*s*t)/768 + 19/256; .....(26b)

β^7=(67*s)/768 - (5*r)/256 - (7*t)/256 - (5*r*s)/768 + (r*t)/256 - (7*s*t)/768 + (r*s*t)/768 + 67/256; .....(26c)

γ'^7=(67*t)/768 - (7*s)/256 - (5*r)/256 + (r*s)/256 - (5*r*t)/768 - (7*s*t)/768 + (r*s*t)/768 + 67/256; .....(26d)

δ^7 =(7*r*s)/768 - (41*s)/768 - (41*t)/768 - (9*r)/256 + (7*r*t)/768 + (13*s*t)/768 - (r*s*t)/256 + 103/256; .....(26e)

ϑ'^8=(r^2*s*t)/7077888 - (r^2*s)/786432 - (r^2*t)/786432 + (11*r^2)/2359296 + (r*s^2*t)/7077888 - (r*s^2)/786432 + (r*s*t^2)/7077888 - (7*r*s*t)/3538944 + (47*r*s)/2359296 - (r*t^2)/786432 + (47*r*t)/2359296 - (5*r)/65536 - (s^2*t)/786432 + (11*s^2)/2359296 - (s*t^2)/786432 + (47*s*t)/2359296 - (5*s)/65536 + (11*t^2)/2359296 - (5*t)/65536 + 81/262144; .....(27a)

α^8=(19*r)/256 - (5*s)/256 - (5*t)/256 - (5*r*s)/768 - (5*r*t)/768 + (s*t)/256 + (r*s*t)/768 + 57/256; .....(27b)

β^8=(19*s)/256 - (5*r)/256 - (5*t)/256 - (5*r*s)/768 + (r*t)/256 - (5*s*t)/768 + (r*s*t)/768 + 57/256; .....(27c)

γ'^8=(19*t)/256 - (5*s)/256 - (5*r)/256 + (r*s)/256 - (5*r*t)/768 - (5*s*t)/768 + (r*s*t)/768 + 57/256; .....(27d)

δ^8=(7*r*s)/768 - (9*s)/256 - (9*t)/256 - (9*r)/256 + (7*r*t)/768 + (7*s*t)/768 - (r*s*t)/256 + 85/256; .....(27e)

We may now mention that the integration over an arbitrary linear tetrahedron can be now computed as a sum of thirty two integrals over the 2-cube: $-1 \leq r, s, t \leq 1$ which is shown in Fig.3.These integrals can be computed numerically by applying Gaussian rules and it will be explained later

3.3 Division of a tetrahedron into 256 hexahedrons

In the previous section, we have explained the division of each hexahedron Ω_i into eight hexahedrons $\Omega_{i,j}$ (j=1,2,3,4,5,6,7,8). The tetrahedron consists of four hexahedrons Ω_i (i=1,2,3,4) and so this procedure divides the tetrahedron into 32 hexahedrons .In this section ,each hexahedron $\Omega_{i,j}$ is further divided into eight hexahedrons $\Omega_{i,j,k}$ (k=1,2,3,4,5,6,7,8). This process divides the tetrahedron into 256 hexahedrons

Using the transformations of eqn(14) and the expressions of x^i , y^i, z^i, w^i , J^i shown in Table-1 ,we can map the hexahedrons Ω_i,($i = 1,2.3.4$), into a 2-cube in (r,s,t) space. It is well known that there is a one to one correspondence between 2- cube in (r,s,t) space and the hexahedron Ω_i, $i = 1,2.3.4$). Hence a division of 2-cube correspondences to a unique division of the hexahedra.Thus the division of hexahedra

can be achieved by dividing the the 2-cube. The division of a 2-cube into cubes of unit dimension is displayed in Fig.5

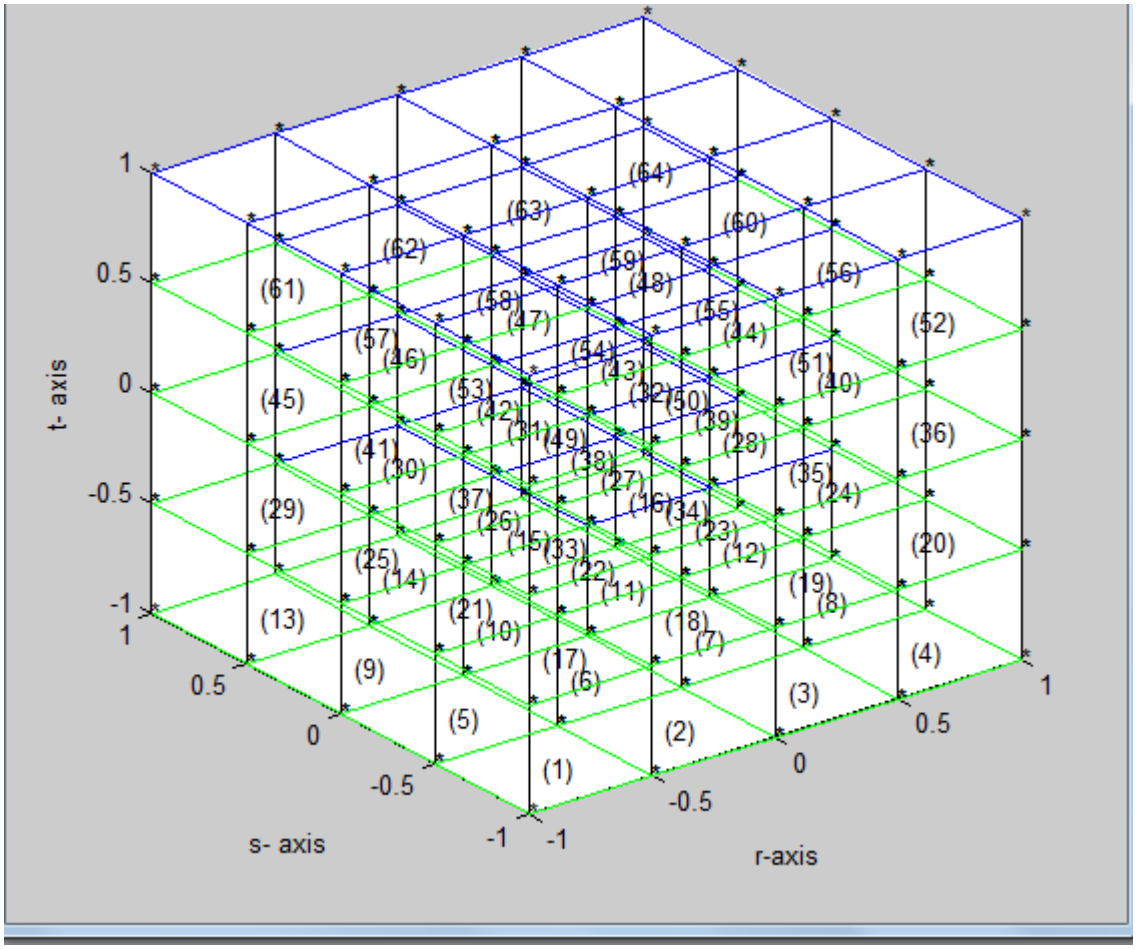

Fig.5 Division of a 2-cube into sixty four cubes of dimension 1/8 unit

We now consider the integration over a 2-cube by dividing the2-cube into sixty four cubes each of dimension 1/8 units

Using eqns(17-18), and modifying we have

Let F(r,s,t)= $f(X(x^i,y^i,z^i),Y(x^i,y^i,z^i),Z(x^i,y^i,z^i))\frac{\partial(x^i,y^i,z^i)}{\partial(r,s,t)}$

Clearly , *it is already assumed that* x^i=x^i(r,s,t), y^i=y^i(r,s,t), z^i=z^i(r,s,t) ,
so that we can write from eq(11):

$\iiint_{\Omega_i} f(X(x,y,z),Y(x,y,z),Z(x,y,z))dxdydz$

$=\iiint_{\Omega_i} f(X(x^i,y^i,z^i),Y(x^i,y^i,z^i),Z(x^i,y^i,z^i))\mathrm{d}x^i\mathrm{d}y^i\mathrm{d}z^i$

$=\sum_{j=1}^{8}\iiint_{\Omega_{i,j}} f(X(x^{i,j},y^{i,j},z^{i,j}),Y(x^{i,j},y^{i,j},z^{i,j}),Z(x^{i,j},y^{i,j},z^{i,j}))\,\mathrm{d}x^{i,j}\mathrm{d}y^{i,j}\mathrm{d}z^{i,j}$

$$=\sum_{k=1}^{8}\sum_{j=1}^{8}\iiint_{\Omega_{i,j,k}} f\left(X\left(x^{i,j,k},y^{i,j,k},z^{i,j,k}\right),Y\left(x^{i,j,k},y^{i,j,k},z^{i,j,k}\right),Z\left(x^{i,j,k},y^{i,j,k},z^{i,j,k}\right)\right)\,dx^{i,j,k}\,\mathrm{d}y^{i,j,k}\,\mathrm{d}z^{i,j,k}$$

$=\sum_{k=1}^{8}\sum_{j=1}^{8}\iiint_{[-1,1]^3} f\left(X\left(x^{i,j,k},y^{i,j,k},z^{i,j,k}\right),Y\left(x^{i,j,k},y^{i,j,k},z^{i,j,k}\right),Z\left(x^{i,j,k},y^{i,j,k},z^{i,j,k}\right)\right)J^{i,j,k}(r,s,t)\,\mathrm{drdsdt}$

$=\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\mathrm{F(r,s,t)}\;\;drdsdt$ .....(28)

$$=\int_{-1}^{0}\int_{-1}^{0}\int_{-1}^{0} \mathrm{F(r,s,t)}\, drdsdt+\int_{-1}^{0}\int_{-1}^{0}\int_{0}^{1} \mathrm{F(r,s,t)}\, drdsdt$$
$$+\int_{-1}^{0}\int_{0}^{1}\int_{-1}^{0} \mathrm{F(r,s,t)}\, drdsdt+\int_{-1}^{0}\int_{0}^{1}\int_{0}^{1} \mathrm{F(r,s,t)}\, drdsdt$$
$$+\int_{0}^{1}\int_{-1}^{0}\int_{-1}^{0} \mathrm{F(r,s,t)}\, drdsdt+\int_{-1}^{0}\int_{-1}^{0}\int_{0}^{1} \mathrm{F(r,s,t)}\, drdsdt$$
$$+\int_{0}^{1}\int_{0}^{1}\int_{-1}^{0} \mathrm{F(r,s,t)}\, drdsdt+\int_{0}^{1}\int_{0}^{1}\int_{0}^{1} \mathrm{F(r,s,t)}\, drdsdt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(29)$$

We note that each of the above integrals is defined over a cube of unit dimension. We can divide each of these cubes into eight cubes of dimension 1/8 unit.,this gives eight integrals defined over cubes of dimension 1/8 unit. We further transform these integral into eight integrals over the 2-cube domain..This is explained below.

For example, from eqn(29) we can write

$$\int_{-1}^{0}\int_{-1}^{0}\int_{-1}^{0} \mathrm{F(r,s,t)}\, drdsdt$$
$$=\int_{-1}^{-1/2}\int_{-1}^{-1/2}\int_{-1}^{-1/2} \mathrm{F(r,s,t)} drdsdt+\int_{-1}^{-1/2}\int_{-1}^{-1/2}\int_{-1/2}^{0} \mathrm{F(r,s,t)} drdsdt$$
$$+\int_{-1}^{-1/2}\int_{-1/2}^{0}\int_{-1}^{-1/2} \mathrm{F(r,s,t)} drdsdt+\int_{-1}^{-1/2}\int_{-1/2}^{0}\int_{-1/2}^{0} \mathrm{F(r,s,t)} drdsdt$$
$$+\int_{-1/2}^{0}\int_{-1}^{-1/2}\int_{-1}^{-1/2} \mathrm{F(r,s,t)} drdsdt+\int_{-1/2}^{0}\int_{-1}^{-1/2}\int_{-1/2}^{0} \mathrm{F(r,s,t)} drdsdt$$
$$+\int_{-1/2}^{0}\int_{-1/2}^{0}\int_{-1}^{-1/2} \mathrm{F(r,s,t)} drdsdt+\int_{-1/2}^{0}\int_{-1/2}^{0}\int_{-1/2}^{0} \mathrm{F(r,s,t)d} rdsdt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(30)$$
$$=\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} F(\tfrac{-3}{4}+\tfrac{1}{4}r,\tfrac{-3}{4}+\tfrac{1}{4}s,\tfrac{-3}{4}+\tfrac{1}{4}t)\tfrac{drdsdt}{64}+\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} F(\tfrac{-1}{4}+\tfrac{1}{4}r,\tfrac{-3}{4}+\tfrac{1}{4}s,\tfrac{-3}{4}+\tfrac{1}{4}t)\,\tfrac{drdsdt}{64}$$
$$+\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} F(\tfrac{-3}{4}+\tfrac{1}{4}r,\tfrac{-1}{4}+\tfrac{1}{4}s,\tfrac{-3}{4}+\tfrac{1}{4}t)\,\tfrac{drdsdt}{64}+\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} F(\tfrac{-1}{4}+\tfrac{1}{4}r,\tfrac{-1}{4}+\tfrac{1}{4}s,\tfrac{-3}{4}+\tfrac{1}{4}t)\,\tfrac{drdsdt}{64}$$
$$+\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} F(\tfrac{-3}{4}+\tfrac{1}{4}r,\tfrac{-3}{4}+\tfrac{1}{4}s,\tfrac{-1}{4}+\tfrac{1}{4}t)\,\tfrac{drdsdt}{64}+\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} F(\tfrac{-1}{4}+\tfrac{1}{4}r,\tfrac{-3}{4}+\tfrac{1}{4}s,\tfrac{-1}{4}+\tfrac{1}{4}t)\,\tfrac{drdsdt}{64}$$
$$+\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} F(\tfrac{-3}{4}+\tfrac{1}{4}r,\tfrac{-1}{4}+\tfrac{1}{4}s,\tfrac{-1}{4}+\tfrac{1}{4}t)\,\tfrac{drdsdt}{64}+\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} F(\tfrac{-1}{4}+\tfrac{1}{4}r,\tfrac{-1}{4}+\tfrac{1}{4}s,\tfrac{-1}{4}+\tfrac{1}{4}t)\,\tfrac{drdsdt}{64}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(31)$$

Similar expansions can be written for the other integrals of eqn{29)..This gives 64 integrals over the 2- cube.

The coordinate transfomations $x^{i,j,k}\,(\mathrm{r,s,t}), y^{\,i,j,k}\,(\mathrm{r,s,t}), z^{i,j,k}(\mathrm{r,s,t})$ and the corresponding Jacobians $J^{i,j,k}(r,s,t)$ over the 256 cubes $[-1,1]^3$, i=1,2,3,4 ; (j, k=1,2,3,4,5,6,7,8). are displayed in Table-3.

Table-3

The computed expressions of $x^{i,j,k}$, $y^{i,j}, k$, $z^{i,j,k}, w^{i,j,k}$, $J^{i,j,k}$ over the 256 hexahedrons $\Omega_{i,j,k}(\, i = 1,2.3.4\, ; j,k = 1,2,3,4,5,6,7,8)$,Note that $\Omega_{i,j,k}$ referes to the kth division of hexahedron $\Omega_{i,j}$.

(i , j,k)	$\Omega_{\mathrm{i,j,k}}$	$x^{i,j,k}$	$y^{i,j,k}$	$z^{i,j,k}$	$w^{i,j,k}$	$J^{i,j,k}$
(1,,j,k)	$\Omega_{1,\mathrm{j,k}}$	${\alpha_k}^j$	${\beta_k}^j$	${\gamma_k}^j$	${\delta_k}^j$	${\vartheta_k}^j$
(2,,j,k)	$\Omega_{2,\mathrm{j,k}}$	${\delta_k}^j$	${\alpha_k}^j$	${\gamma_k}^j$	${\beta_k}^j$	${\vartheta_k}^j$

(3,,j,k) $\Omega_{3,j,k}$ ${\beta_k}^j$ ${\delta_k}^j$ ${\gamma_k}^j$ ${\alpha_k}^j$ ${\vartheta_k}^j$

(4,,j,k) $\Omega_{4,j,k}$ ${\beta_k}^j$ ${\alpha_k}^j$ ${\delta_k}^j$ ${\gamma_k}^j$ ${\vartheta_k}^j$

From Table-3, it is clear that the computed values of $({\alpha_k}^j,\ {\beta_k}^j,\ {\gamma_k}^j,\ {\delta_k}^j, {\vartheta_k}^j)$, $(j,k = 1,2,3,4,5,6,7,8)$ will be necessary.

They are listed below

.For convenience,let $((a_m, b_m, c_m, d_m, w_m)$,m=1(1)64)=$({\alpha_k}^j,\ {\beta_k}^j,\ {\gamma_k}^j,\ {\delta_k}^j, {\vartheta_k}^j)$, j=1(1)8),k=1(1)8)

DIVISION 1

a_1=(353*r)/6144 - (5*s)/2048 - (5*t)/2048 - (5*r*s)/2048 - (5*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 + 353/6144;

b_1=(353*s)/6144 - (5*r)/2048 - (5*t)/2048 - (5*r*s)/2048 + (r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 353/6144;

c_1=(353*t)/6144 - (5*s)/2048 - (5*r)/2048 + (r*s)/6144 - (5*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 + 353/6144;

d_1=(29*r*s)/6144 - (323*s)/6144 - (323*t)/6144 - (323*r)/6144 + (29*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 1695/2048;

w_1=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (23*r^2*t)/905969664 + (337*r^2)/905969664 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (29*r*s*t)/452984832 + (1189*r*s)/905969664 - (23*r*t^2)/905969664 + (1189*r*t)/905969664 - (1265*r)/75497472 - (23*s^2*t)/905969664 + (337*s^2)/905969664 - (23*s*t^2)/905969664 + (1189*s*t)/905969664 - (1265*s)/75497472 + (337*t^2)/905969664 - (1265*t)/75497472 + 170867/905969664; .....(32a-e)

DIVISION 2

a_2=(353*r)/6144 - (15*s)/2048 - (15*t)/2048 - (5*r*s)/2048 - (5*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 + 353/2048;

b_2=(323*s)/6144 - (5*r)/2048 - (13*t)/6144 - (5*r*s)/2048 + (r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 323/6144;

c_2=(323*t)/6144 - (13*s)/6144 - (5*r)/2048 + (r*s)/6144 - (5*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 + 323/6144;

d_2=(29*r*s)/6144 - (265*s)/6144 - (265*t)/6144 - (323*r)/6144 + (29*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 4439/6144;

w_2=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (23*r^2*t)/905969664 + (337*r^2)/905969664 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (r*s*t)/16777216 + (1097*r*s)/905969664 - (23*r*t^2)/905969664 + (1097*r*t)/905969664 - (1729*r)/113246208 - (7*s^2*t)/301989888 + (97*s^2)/301989888 - (7*s*t^2)/301989888 + (359*s*t)/301989888 - (2149*s)/150994944 + (97*t^2)/301989888 - (2149*t)/150994944 + 47285/301989888; .....(33a-e)

DIVISION 3

a_3=(323*r)/6144 - (5*s)/2048 - (13*t)/6144 - (5*r*s)/2048 - (13*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 + 323/6144;

b_3=(353*s)/6144 - (15*r)/2048 - (15*t)/2048 - (5*r*s)/2048 + (r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 + 353/2048;

c_3=(323*t)/6144 - (5*s)/2048 - (13*r)/6144 + (r*s)/6144 - (13*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 323/6144;

d_3=(29*r*s)/6144 - (323*s)/6144 - (265*t)/6144 - (265*r)/6144 + (23*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 4439/6144;

w_3=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (7*r^2*t)/301989888 + (97*r^2)/301989888 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (r*s*t)/16777216 + (1097*r*s)/905969664 - (7*r*t^2)/301989888 + (359*r*t)/301989888 - (2149*r)/150994944 - (23*s^2*t)/905969664 + (337*s^2)/905969664 - (23*s*t^2)/905969664 + (1097*s*t)/905969664 - (1729*s)/113246208 + (97*t^2)/301989888 - (2149*t)/150994944 + 47285/301989888; .....(34a-e)

DIVISION 4

a_4=(323*r)/6144 - (15*s)/2048 - (13*t)/2048 - (5*r*s)/2048 - (13*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 + 323/2048;

b_4=(323*s)/6144 - (15*r)/2048 - (13*t)/2048 - (5*r*s)/2048 + (r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 + 323/2048;

c_4=(99*t)/2048 - (13*s)/6144 - (13*r)/6144 + (r*s)/6144 - (13*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 99/2048;

d_4=(29*r*s)/6144 - (265*s)/6144 - (73*t)/2048 - (265*r)/6144 + (23*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 1303/2048;

w_4=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (7*r^2*t)/301989888 + (97*r^2)/301989888 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (335*r*s)/301989888 - (7*r*t^2)/301989888 + (331*r*t)/301989888 - (1955*r)/150994944 - (7*s^2*t)/301989888 + (97*s^2)/301989888 - (7*s*t^2)/301989888 + (331*s*t)/301989888 - (1955*s)/150994944 + (83*t^2)/301989888 - (451*t)/37748736 + 39077/301989888; .....(35a-e)

DIVISION 5

a_5=(323*r)/6144 - (13*s)/6144 - (5*t)/2048 - (13*r*s)/6144 - (5*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 + 323/6144;

b_5=(323*s)/6144 - (13*r)/6144 - (5*t)/2048 - (13*r*s)/6144 + (r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 323/6144;

c_5=(353*t)/6144 - (15*s)/2048 - (15*r)/2048 + (r*s)/2048 - (5*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 + 353/2048;

d_5=(23*r*s)/6144 - (265*s)/6144 - (323*t)/6144 - (265*r)/6144 + (29*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 4439/6144;

w_5=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (23*r^2*t)/905969664 + (97*r^2)/301989888 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (r*s*t)/16777216 + (359*r*s)/301989888 - (23*r*t^2)/905969664 + (1097*r*t)/905969664 - (2149*r)/150994944 - (23*s^2*t)/905969664 + (97*s^2)/301989888 - (23*s*t^2)/905969664 + (1097*s*t)/905969664 - (2149*s)/150994944 + (337*t^2)/905969664 - (1729*t)/113246208 + 47285/301989888; .....(36a-e)

DIVISION 6

a_6=(323*r)/6144 - (13*s)/2048 - (15*t)/2048 - (13*r*s)/6144 - (5*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 + 323/2048;

b_6=(99*s)/2048 - (13*r)/6144 - (13*t)/6144 - (13*r*s)/6144 + (r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 99/2048;

c_6=(323*t)/6144 - (13*s)/2048 - (15*r)/2048 + (r*s)/2048 - (5*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 + 323/2048;

d_6=(23*r*s)/6144 - (73*s)/2048 - (265*t)/6144 - (265*r)/6144 + (29*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 1303/2048;

w_6=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (23*r^2*t)/905969664 + (97*r^2)/301989888 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (331*r*s)/301989888 - (23*r*t^2)/905969664 + (335*r*t)/301989888 - (1955*r)/150994944 - (7*s^2*t)/301989888 + (83*s^2)/301989888 - (7*s*t^2)/301989888 + (331*s*t)/301989888 - (451*s)/37748736 + (97*t^2)/301989888 - (1955*t)/150994944 + 39077/301989888; .....(37a-e)

DIVISION 7

a_7=(99*r)/2048 - (13*s)/6144 - (13*t)/6144 - (13*r*s)/6144 - (13*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 + 99/2048;

b_7=(323*s)/6144 - (13*r)/2048 - (15*t)/2048 - (13*r*s)/6144 + (r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 + 323/2048;

c_7=(323*t)/6144 - (15*s)/2048 - (13*r)/2048 + (r*s)/2048 - (13*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 323/2048;

d_7=(23*r*s)/6144 - (265*s)/6144 - (265*t)/6144 - (73*r)/2048 + (23*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 1303/2048;

w_7=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (7*r^2*t)/301989888 + (83*r^2)/301989888 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (331*r*s)/301989888 - (7*r*t^2)/301989888 + (331*r*t)/301989888 - (451*r)/37748736 - (23*s^2*t)/905969664 + (97*s^2)/301989888 - (23*s*t^2)/905969664 +

(335*s*t)/301989888 - (1955*s)/150994944 + (97*t^2)/301989888 - (1955*t)/150994944 + 39077/301989888; .....(38a-e)

DIVISION 8

a_8=(99*r)/2048 - (13*s)/2048 - (13*t)/2048 - (13*r*s)/6144 - (13*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 + 297/2048;

b_8=(99*s)/2048 - (13*r)/2048 - (13*t)/2048 - (13*r*s)/6144 + (r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 + 297/2048;

c_8=(99*t)/2048 - (13*s)/2048 - (13*r)/2048 + (r*s)/2048 - (13*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 297/2048;

d_8=(23*r*s)/6144 - (73*s)/2048 - (73*t)/2048 - (73*r)/2048 + (23*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 1157/2048;

w_8=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (7*r^2*t)/301989888 + (83*r^2)/301989888 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (101*r*s)/100663296 - (7*r*t^2)/301989888 + (101*r*t)/100663296 - (91*r)/8388608 - (7*s^2*t)/301989888 + (83*s^2)/301989888 - (7*s*t^2)/301989888 + (101*s*t)/100663296 - (91*s)/8388608 + (83*t^2)/301989888 - (91*t)/8388608 + 3577/33554432; .....(39a-e)

DIVISION 9

a_9=(353*r)/6144 - (25*s)/2048 - (25*t)/2048 - (5*r*s)/2048 - (5*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144 + 1765/6144;

b_9=(293*s)/6144 - (5*r)/2048 - (11*t)/6144 - (5*r*s)/2048 + (r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 293/6144;

c_9=(293*t)/6144 - (11*s)/6144 - (5*r)/2048 + (r*s)/6144 - (5*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 + 293/6144;

d_9=(29*r*s)/6144 - (69*s)/2048 - (69*t)/2048 - (323*r)/6144 + (29*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 3793/6144;

w_9=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (23*r^2*t)/905969664 + (337*r^2)/905969664 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (335*r*s)/301989888 - (23*r*t^2)/905969664 + (335*r*t)/301989888 - (3121*r)/226492416 - (19*s^2*t)/905969664 + (245*s^2)/905969664 - (19*s*t^2)/905969664 + (973*s*t)/905969664 - (1349*s)/113246208 + (245*t^2)/905969664 - (1349*t)/113246208 + 38513/301989888; .....(40a-e)

DIVISION 10

a_{10}=(353*r)/6144 - (35*s)/2048 - (35*t)/2048 - (5*r*s)/2048 - (5*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144 + 2471/6144;

b_{10}=(263*s)/6144 - (5*r)/2048 - (3*t)/2048 - (5*r*s)/2048 + (r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 263/6144;

c_{10}=(263*t)/6144 - (3*s)/2048 - (5*r)/2048 + (r*s)/6144 - (5*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 + 263/6144;

d_{10}=(29*r*s)/6144 - (149*s)/6144 - (149*t)/6144 - (323*r)/6144 + (29*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 1049/2048;

w_{10}=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (23*r^2*t)/905969664 + (337*r^2)/905969664 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (913*r*s)/905969664 - (23*r*t^2)/905969664 + (913*r*t)/905969664 - (29*r)/2359296 - (17*s^2*t)/905969664 + (199*s^2)/905969664 - (17*s*t^2)/905969664 + (877*s*t)/905969664 - (493*s)/50331648 + (199*t^2)/905969664 - (493*t)/50331648 + 91919/905969664; .....(41a-e)

DIVISION 11

a_{11}=(323*r)/6144 - (25*s)/2048 - (65*t)/6144 - (5*r*s)/2048 - (13*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144 + 1615/6144;

b_{11}=(293*s)/6144 - (15*r)/2048 - (11*t)/2048 - (5*r*s)/2048 + (r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 + 293/2048;

c_{11}=(271*t)/6144 - (11*s)/6144 - (13*r)/6144 + (r*s)/6144 - (13*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 271/6144;

d_{11}=(29*r*s)/6144 - (69*s)/2048 - (173*t)/6144 - (265*r)/6144 + (23*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 3379/6144;

w_{11}=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (7*r^2*t)/301989888 + (97*r^2)/301989888 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 -

(23*r*s*t)/452984832 + (913*r*s)/905969664 - (7*r*t^2)/301989888 + (101*r*t)/100663296 - (587*r)/50331648 - (19*s^2*t)/905969664 + (245*s^2)/905969664 - (19*s*t^2)/905969664 + (299*s*t)/301989888 - (2453*s)/226492416 + (23*t^2)/100663296 - (1487*t)/150994944 + 31645/301989888; .....(42a-e)

DIVISION 12

a_{12}=(323*r)/6144 - (35*s)/2048 - (91*t)/6144 - (5*r*s)/2048 - (13*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144 + 2261/6144;

b_{12}=(263*s)/6144 - (15*r)/2048 - (9*t)/2048 - (5*r*s)/2048 + (r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 + 263/2048;

c_{12}=(245*t)/6144 - (3*s)/2048 - (13*r)/6144 + (r*s)/6144 - (13*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 245/6144;

d_{12}=(29*r*s)/6144 - (149*s)/6144 - (127*t)/6144 - (265*r)/6144 + (23*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 2849/6144;

w_{12}=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (7*r^2*t)/301989888 + (97*r^2)/301989888 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (821*r*s)/905969664 - (7*r*t^2)/301989888 + (275*r*t)/301989888 - (1567*r)/150994944 - (17*s^2*t)/905969664 + (199*s^2)/905969664 - (17*s*t^2)/905969664 + (809*s*t)/905969664 - (4039*s)/452984832 + (55*t^2)/301989888 - (599*t)/75497472 + 24989/301989888; .....(43a-e)

DIVISION 13

a_{13}=(323*r)/6144 - (65*s)/6144 - (25*t)/2048 - (13*r*s)/6144 - (5*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144 + 1615/6144;

b_{13}=(271*s)/6144 - (13*r)/6144 - (11*t)/6144 - (13*r*s)/6144 + (r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 271/6144;

c=(293*t)/6144 - (11*s)/2048 - (15*r)/2048 + (r*s)/2048 - (5*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 + 293/2048;

d_{13}=(23*r*s)/6144 - (173*s)/6144 - (69*t)/2048 - (265*r)/6144 + (29*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 3379/6144;

w_{13}=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (23*r^2*t)/905969664 + (97*r^2)/301989888 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (101*r*s)/100663296 - (23*r*t^2)/905969664 + (913*r*t)/905969664 - (587*r)/50331648 - (19*s^2*t)/905969664 + (23*s^2)/100663296 - (19*s*t^2)/905969664 + (299*s*t)/301989888 - (1487*s)/150994944 + (245*t^2)/905969664 - (2453*t)/226492416 + 31645/301989888; .....(44a-e)

DIVISION 14

a_{14}=(323*r)/6144 - (91*s)/6144 - (35*t)/2048 - (13*r*s)/6144 - (5*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144 + 2261/6144;

b_{14}=(245*s)/6144 - (13*r)/6144 - (3*t)/2048 - (13*r*s)/6144 + (r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 245/6144;

c_{14}=(263*t)/6144 - (9*s)/2048 - (15*r)/2048 + (r*s)/2048 - (5*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 + 263/2048;

d_{14}=(23*r*s)/6144 - (127*s)/6144 - (149*t)/6144 - (265*r)/6144 + (29*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 2849/6144;

w_{14}=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (23*r^2*t)/905969664 + (97*r^2)/301989888 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (275*r*s)/301989888 - (23*r*t^2)/905969664 + (821*r*t)/905969664 - (1567*r)/150994944 - (17*s^2*t)/905969664 + (55*s^2)/301989888 - (17*s*t^2)/905969664 + (809*s*t)/905969664 - (599*s)/75497472 + (199*t^2)/905969664 - (4039*t)/452984832 + 24989/301989888; .....(45a-e)

DIVISION 15

a_{15}=(99*r)/2048 - (65*s)/6144 - (65*t)/6144 - (13*r*s)/6144 - (13*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144 + 495/2048;

b_{15}=(271*s)/6144 - (13*r)/2048 - (11*t)/2048 - (13*r*s)/6144 + (r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 + 271/2048;

c_{15}=(271*t)/6144 - (11*s)/2048 - (13*r)/2048 + (r*s)/2048 - (13*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 271/2048;

d_{15}=(23*r*s)/6144 - (173*s)/6144 - (173*t)/6144 - (73*r)/2048 + (23*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 1011/2048;

w_{15}=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (7*r^2*t)/301989888 + (83*r^2)/301989888 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (275*r*s)/301989888 - (7*r*t^2)/301989888 + (275*r*t)/301989888 - (23*r)/2359296 - (19*s^2*t)/905969664 + (23*s^2)/100663296 - (19*s*t^2)/905969664 + (821*s*t)/905969664 - (1349*s)/150994944 + (23*t^2)/100663296 - (1349*t)/150994944 + 25973/301989888; ......(46a-e)

DIVISION 16

a_{16}=(99*r)/2048 - (91*s)/6144 - (91*t)/6144 - (13*r*s)/6144 - (13*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144 + 693/2048;

b_{16}=(245*s)/6144 - (13*r)/2048 - (9*t)/2048 - (13*r*s)/6144 + (r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 + 245/2048;

c_{16}=(245*t)/6144 - (9*s)/2048 - (13*r)/2048 + (r*s)/2048 - (13*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 245/2048;

d_{16}=(23*r*s)/6144 - (127*s)/6144 - (127*t)/6144 - (73*r)/2048 + (23*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 865/2048;

w_{16}=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (7*r^2*t)/301989888 + (83*r^2)/301989888 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (247*r*s)/301989888 - (7*r*t^2)/301989888 + (247*r*t)/301989888 - (653*r)/75497472 - (17*s^2*t)/905969664 + (55*s^2)/301989888 - (17*s*t^2)/905969664 + (247*s*t)/301989888 - (17*s)/2359296 + (55*t^2)/301989888 - (17*t)/2359296 + 20417/301989888; ......(47a-e)

DIVISION 17

a_{17}=(293*r)/6144 - (5*s)/2048 - (11*t)/6144 - (5*r*s)/2048 - (11*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 + 293/6144;

b_{17}=(353*s)/6144 - (25*r)/2048 - (25*t)/2048 - (5*r*s)/2048 + (5*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 1765/6144;

c_{17}=(293*t)/6144 - (5*s)/2048 - (11*r)/6144 + (r*s)/6144 - (11*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 293/6144;

d_{17}=(29*r*s)/6144 - (323*s)/6144 - (69*t)/2048 - (69*r)/2048 + (17*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 3793/6144;

w_{17}=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (19*r^2*t)/905969664 + (245*r^2)/905969664 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (335*r*s)/301989888 - (19*r*t^2)/905969664 + (973*r*t)/905969664 - (1349*r)/113246208 - (23*s^2*t)/905969664 + (337*s^2)/905969664 - (23*s*t^2)/905969664 + (335*s*t)/301989888 - (3121*s)/226492416 + (245*t^2)/905969664 - (1349*t)/113246208 + 38513/301989888; ......(48a-e)

DIVISION 18

a_{18}=(293*r)/6144 - (15*s)/2048 - (11*t)/2048 - (5*r*s)/2048 - (11*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 + 293/2048;

b_{18}=(323*s)/6144 - (25*r)/2048 - (65*t)/6144 - (5*r*s)/2048 + (5*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 1615/6144;

c_{18}=(271*t)/6144 - (13*s)/6144 - (11*r)/6144 + (r*s)/6144 - (11*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 271/6144;

d_{18}= (29*r*s)/6144 - (265*s)/6144 - (173*t)/6144 - (69*r)/2048 + (17*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 3379/6144;

w_{18}= (r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (19*r^2*t)/905969664 + (245*r^2)/905969664 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (913*r*s)/905969664 - (19*r*t^2)/905969664 + (299*r*t)/301989888 - (2453*r)/226492416 - (7*s^2*t)/301989888 + (97*s^2)/301989888 - (7*s*t^2)/301989888 + (101*s*t)/100663296 - (587*s)/50331648 + (23*t^2)/100663296 - (1487*t)/150994944 + 31645/301989888; ......(49a-e)

DIVISION 19

a_{19}=(263*r)/6144 - (5*s)/2048 - (3*t)/2048 - (5*r*s)/2048 - (3*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 + 263/6144;

b_{19}=(353*s)/6144 - (35*r)/2048 - (35*t)/2048 - (5*r*s)/2048 + (7*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 2471/6144;

c_{19}=(263*t)/6144 - (5*s)/2048 - (3*r)/2048 + (r*s)/6144 - (3*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 + 263/6144;

d_{19}=(29*r*s)/6144 - (323*s)/6144 - (149*t)/6144 - (149*r)/6144 + (11*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 1049/2048;

w_{19}=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (17*r^2*t)/905969664 + (199*r^2)/905969664 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (913*r*s)/905969664 - (17*r*t^2)/905969664 + (877*r*t)/905969664 - (493*r)/50331648 - (23*s^2*t)/905969664 + (337*s^2)/905969664 - (23*s*t^2)/905969664 + (913*s*t)/905969664 - (29*s)/2359296 + (199*t^2)/905969664 - (493*t)/50331648 + 91919/905969664; .....(50a-e)

DIVISION 20

a_{20}=(263*r)/6144 - (15*s)/2048 - (9*t)/2048 - (5*r*s)/2048 - (3*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 + 263/2048;

b_{20}=(323*s)/6144 - (35*r)/2048 - (91*t)/6144 - (5*r*s)/2048 + (7*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 2261/6144;

c_{20}=(245*t)/6144 - (13*s)/6144 - (3*r)/2048 + (r*s)/6144 - (3*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 + 245/6144;

d_{20}=(29*r*s)/6144 - (265*s)/6144 - (127*t)/6144 - (149*r)/6144 + (11*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 2849/6144;

w_{20}=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (17*r^2*t)/905969664 + (199*r^2)/905969664 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (821*r*s)/905969664 - (17*r*t^2)/905969664 + (809*r*t)/905969664 - (4039*r)/452984832 - (7*s^2*t)/301989888 + (97*s^2)/301989888 - (7*s*t^2)/301989888 + (275*s*t)/301989888 - (1567*s)/150994944 + (55*t^2)/301989888 - (599*t)/75497472 + 24989/301989888; .....(51a-e)

DIVISION 21

a_{21}=(271*r)/6144 - (13*s)/6144 - (11*t)/6144 - (13*r*s)/6144 - (11*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 + 271/6144;

b_{21}=(323*s)/6144 - (65*r)/6144 - (25*t)/2048 - (13*r*s)/6144 + (5*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 1615/6144;

c_{21}=(293*t)/6144 - (15*s)/2048 - (11*r)/2048 + (r*s)/2048 - (11*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 293/2048;

d_{21}=(23*r*s)/6144 - (265*s)/6144 - (69*t)/2048 - (173*r)/6144 + (17*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 3379/6144;

w_{21}=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (19*r^2*t)/905969664 + (23*r^2)/100663296 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (101*r*s)/100663296 - (19*r*t^2)/905969664 + (299*r*t)/301989888 - (1487*r)/150994944 - (23*s^2*t)/905969664 + (97*s^2)/301989888 - (23*s*t^2)/905969664 + (913*s*t)/905969664 - (587*s)/50331648 + (245*t^2)/905969664 - (2453*t)/226492416 + 31645/301989888; .....(52a-e)

DIVISION 22

a_{22}=(271*r)/6144 - (13*s)/2048 - (11*t)/2048 - (13*r*s)/6144 - (11*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 + 271/2048;

b_{22}=(99*s)/2048 - (65*r)/6144 - (65*t)/6144 - (13*r*s)/6144 + (5*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 495/2048;

c_{22}=(271*t)/6144 - (13*s)/2048 - (11*r)/2048 + (r*s)/2048 - (11*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 271/2048;

d_{22}=(23*r*s)/6144 - (73*s)/2048 - (173*t)/6144 - (173*r)/6144 + (17*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 1011/2048;

w_{22}=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (19*r^2*t)/905969664 + (23*r^2)/100663296 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (275*r*s)/301989888 - (19*r*t^2)/905969664 + (821*r*t)/905969664 - (1349*r)/150994944 - (7*s^2*t)/301989888 + (83*s^2)/301989888 - (7*s*t^2)/301989888 + (275*s*t)/301989888 - (23*s)/2359296 + (23*t^2)/100663296 - (1349*t)/150994944 + 25973/301989888; .....(53a-e)

DIVISION 23

a_{23}=(245*r)/6144 - (13*s)/6144 - (3*t)/2048 - (13*r*s)/6144 - (3*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 + 245/6144;

b_{23}=(323*s)/6144 - (91*r)/6144 - (35*t)/2048 - (13*r*s)/6144 + (7*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 2261/6144;

c_{23}=(263*t)/6144 - (15*s)/2048 - (9*r)/2048 + (r*s)/2048 - (3*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 + 263/2048;

d_{23}=(23*r*s)/6144 - (265*s)/6144 - (149*t)/6144 - (127*r)/6144 + (11*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 2849/6144;

w_{23}=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (17*r^2*t)/905969664 + (55*r^2)/301989888 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (275*r*s)/301989888 - (17*r*t^2)/905969664 + (809*r*t)/905969664 - (599*r)/75497472 - (23*s^2*t)/905969664 + (97*s^2)/301989888 - (23*s*t^2)/905969664 + (821*s*t)/905969664 - (1567*s)/150994944 + (199*t^2)/905969664 - (4039*t)/452984832 + 24989/301989888; .....(54a-e)

DIVISION 24

a_{24}=(245*r)/6144 - (13*s)/2048 - (9*t)/2048 - (13*r*s)/6144 - (3*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 + 245/2048;

b_{24}=(99*s)/2048 - (91*r)/6144 - (91*t)/6144 - (13*r*s)/6144 + (7*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 693/2048;

c_{24}=(245*t)/6144 - (13*s)/2048 - (9*r)/2048 + (r*s)/2048 - (3*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 + 245/2048;

d_{24}=(23*r*s)/6144 - (73*s)/2048 - (127*t)/6144 - (127*r)/6144 + (11*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 865/2048;

w_{24}=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (17*r^2*t)/905969664 + (55*r^2)/301989888 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (247*r*s)/301989888 - (17*r*t^2)/905969664 + (247*r*t)/301989888 - (17*r)/2359296 - (7*s^2*t)/301989888 + (83*s^2)/301989888 - (7*s*t^2)/301989888 + (247*s*t)/301989888 - (653*s)/75497472 + (55*t^2)/301989888 - (17*t)/2359296 + 20417/301989888; .....(55a-e)

DIVISION 25

a_{25}=(293*r)/6144 - (25*s)/2048 - (55*t)/6144 - (5*r*s)/2048 - (11*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144 + 1465/6144;

b_{25}=(293*s)/6144 - (25*r)/2048 - (55*t)/6144 - (5*r*s)/2048 + (5*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 1465/6144;

c_{25}=(83*t)/2048 - (11*s)/6144 - (11*r)/6144 + (r*s)/6144 - (11*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 83/2048;

d_{25}=(29*r*s)/6144 - (69*s)/2048 - (139*t)/6144 - (69*r)/2048 + (17*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 2965/6144;

w_{25}=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (19*r^2*t)/905969664 + (245*r^2)/905969664 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (821*r*s)/905969664 - (19*r*t^2)/905969664 + (821*r*t)/905969664 - (23*r)/2359296 - (19*s^2*t)/905969664 + (245*s^2)/905969664 - (19*s*t^2)/905969664 + (821*s*t)/905969664 - (23*s)/2359296 + (169*t^2)/905969664 - (1801*t)/226492416 + 76291/905969664; .....(56a-e)

DIVISION 26

a_{26}=(293*r)/6144 - (35*s)/2048 - (77*t)/6144 - (5*r*s)/2048 - (11*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144 + 2051/6144;

b_{26}=(263*s)/6144 - (25*r)/2048 - (15*t)/2048 - (5*r*s)/2048 + (5*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 1315/6144;

c_{26}=(227*t)/6144 - (3*s)/2048 - (11*r)/6144 + (r*s)/6144 - (11*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 227/6144;

d_{26}=(29*r*s)/6144 - (149*s)/6144 - (35*t)/2048 - (69*r)/2048 + (17*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 2551/6144;

w_{26}=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (19*r^2*t)/905969664 + (245*r^2)/905969664 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (27*r*s)/33554432 - (19*r*t^2)/905969664 + (745*r*t)/905969664 - (1963*r)/226492416 - (17*s^2*t)/905969664 + (199*s^2)/905969664 - (17*s*t^2)/905969664 + (247*s*t)/301989888 - (3641*s)/452984832 + (131*t^2)/905969664 - (2819*t)/452984832 + 6623/100663296; .....(57a-e)

DIVISION 27

a_{27}=(263*r)/6144 - (25*s)/2048 - (15*t)/2048 - (5*r*s)/2048 - (3*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144 + 1315/6144;

b_{27}=(293*s)/6144 - (35*r)/2048 - (77*t)/6144 - (5*r*s)/2048 + (7*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 2051/6144;

c_{27}=(227*t)/6144 - (11*s)/6144 - (3*r)/2048 + (r*s)/6144 - (3*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 + 227/6144;

d_{27}=(29*r*s)/6144 - (69*s)/2048 - (35*t)/2048 - (149*r)/6144 + (11*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 2551/6144;

w_{27}=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (17*r^2*t)/905969664 + (199*r^2)/905969664 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (27*r*s)/33554432 - (17*r*t^2)/905969664 + (247*r*t)/301989888 - (3641*r)/452984832 - (19*s^2*t)/905969664 + (245*s^2)/905969664 - (19*s*t^2)/905969664 + (745*s*t)/905969664 - (1963*s)/226492416 + (131*t^2)/905969664 - (2819*t)/452984832 + 6623/100663296; .....(58a-e)

DIVISION 28

a_{28}=(263*r)/6144 - (35*s)/2048 - (21*t)/2048 - (5*r*s)/2048 - (3*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144 + 1841/6144;

b_{28}=(263*s)/6144 - (35*r)/2048 - (21*t)/2048 - (5*r*s)/2048 + (7*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 1841/6144;

c_{28}=(209*t)/6144 - (3*s)/2048 - (3*r)/2048 + (r*s)/6144 - (3*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 + 209/6144;

d_{28}=(29*r*s)/6144 - (149*s)/6144 - (83*t)/6144 - (149*r)/6144 + (11*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 751/2048;

w_{28}=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (17*r^2*t)/905969664 + (199*r^2)/905969664 + (r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (637*r*s)/905969664 - (17*r*t^2)/905969664 + (673*r*t)/905969664 - (1081*r)/150994944 - (17*s^2*t)/905969664 + (199*s^2)/905969664 - (17*s*t^2)/905969664 + (673*s*t)/905969664 - (1081*s)/150994944 + (97*t^2)/905969664 - (11*t)/2359296 + 45839/905969664; .....(59a-e)

DIVISION 29

a_{29}=(271*r)/6144 - (65*s)/6144 - (55*t)/6144 - (13*r*s)/6144 - (11*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144 + 1355/6144;

b_{29}=(271*s)/6144 - (65*r)/6144 - (55*t)/6144 - (13*r*s)/6144 + (5*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 1355/6144;

c_{29}=(83*t)/2048 - (11*s)/2048 - (11*r)/2048 + (r*s)/2048 - (11*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 249/2048;

d_{29}=(23*r*s)/6144 - (173*s)/6144 - (139*t)/6144 - (173*r)/6144 + (17*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 2687/6144;

w_{29}=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (19*r^2*t)/905969664 + (23*r^2)/100663296 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (247*r*s)/301989888 - (19*r*t^2)/905969664 + (745*r*t)/905969664 - (1211*r)/150994944 - (19*s^2*t)/905969664 + (23*s^2)/100663296 - (19*s*t^2)/905969664 + (745*s*t)/905969664 - (1211*s)/150994944 + (169*t^2)/905969664 - (17*t)/2359296 + 2317/33554432; .....(60a-e)

DIVISION 30

a_{30}=(271*r)/6144 - (91*s)/6144 - (77*t)/6144 - (13*r*s)/6144 - (11*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144 + 1897/6144;

b_{30}=(245*s)/6144 - (65*r)/6144 - (15*t)/2048 - (13*r*s)/6144 + (5*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 1225/6144;

c_{30}=(227*t)/6144 - (9*s)/2048 - (11*r)/2048 + (r*s)/2048 - (11*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 227/2048;

d_{30}=(23*r*s)/6144 - (127*s)/6144 - (35*t)/2048 - (173*r)/6144 + (17*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 2341/6144;

w_{30}=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (19*r^2*t)/905969664 + (23*r^2)/100663296 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (73*r*s)/100663296 - (19*r*t^2)/905969664 + (223*r*t)/301989888 - (1073*r)/150994944 - (17*s^2*t)/905969664 + (55*s^2)/301989888 - (17*s*t^2)/905969664 + (673*s*t)/905969664 - (163*s)/25165824 + (131*t^2)/905969664 - (2557*t)/452984832 + 16285/301989888; .....(61a-e)

DIVISION 31

a_{31}=(245*r)/6144 - (65*s)/6144 - (15*t)/2048 - (13*r*s)/6144 - (3*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144 + 1225/6144;

b_{31}=(271*s)/6144 - (91*r)/6144 - (77*t)/6144 - (13*r*s)/6144 + (7*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 1897/6144;

c_{31}=(227*t)/6144 - (11*s)/2048 - (9*r)/2048 + (r*s)/2048 - (3*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 + 227/2048;

d_{31}=(23*r*s)/6144 - (173*s)/6144 - (35*t)/2048 - (127*r)/6144 + (11*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 2341/6144;

w_{31}=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (17*r^2*t)/905969664 + (55*r^2)/301989888 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (73*r*s)/100663296 - (17*r*t^2)/905969664 + (673*r*t)/905969664 - (163*r)/25165824 - (19*s^2*t)/905969664 + (23*s^2)/100663296 - (19*s*t^2)/905969664 + (223*s*t)/301989888 - (1073*s)/150994944 + (131*t^2)/905969664 - (2557*t)/452984832 + 16285/301989888; .....(62a-e)

DIVISION 32

a_{32}=(245*r)/6144 - (91*s)/6144 - (21*t)/2048 - (13*r*s)/6144 - (3*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144 + 1715/6144;

b_{32}=(245*s)/6144 - (91*r)/6144 - (21*t)/2048 - (13*r*s)/6144 + (7*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 1715/6144;

c_{32}=(209*t)/6144 - (9*s)/2048 - (9*r)/2048 + (r*s)/2048 - (3*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 + 209/2048;

d_{32}=(23*r*s)/6144 - (127*s)/6144 - (83*t)/6144 - (127*r)/6144 + (11*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 2087/6144;

w_{32}=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (17*r^2*t)/905969664 + (55*r^2)/301989888 + (r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (5*r*s*t)/150994944 + (191*r*s)/301989888 - (17*r*t^2)/905969664 + (605*r*t)/905969664 - (217*r)/37748736 - (17*s^2*t)/905969664 + (55*s^2)/301989888 - (17*s*t^2)/905969664 + (605*s*t)/905969664 - (217*s)/37748736 + (97*t^2)/905969664 - (959*t)/226492416 + 12593/301989888; .....(63a-e)

DIVISION 33

a_{33}=(293*r)/6144 - (11*s)/6144 - (5*t)/2048 - (11*r*s)/6144 - (5*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 + 293/6144;

b_{33}=(293*s)/6144 - (11*r)/6144 - (5*t)/2048 - (11*r*s)/6144 + (r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 293/6144;

c_{33}=(353*t)/6144 - (25*s)/2048 - (25*r)/2048 + (5*r*s)/6144 - (5*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 + 1765/6144;

d_{33}=(17*r*s)/6144 - (69*s)/2048 - (323*t)/6144 - (69*r)/2048 + (29*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 3793/6144;

w_{33}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (23*r^2*t)/905969664 + (245*r^2)/905969664 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (973*r*s)/905969664 - (23*r*t^2)/905969664 + (335*r*t)/301989888 - (1349*r)/113246208 - (23*s^2*t)/905969664 + (245*s^2)/905969664 - (23*s*t^2)/905969664 + (335*s*t)/301989888 - (1349*s)/113246208 + (337*t^2)/905969664 - (3121*t)/226492416 + 38513/301989888; .....(64a-e)

DIVISION 34

a_{34}=(293*r)/6144 - (11*s)/2048 - (15*t)/2048 - (11*r*s)/6144 - (5*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 + 293/2048;

b_{34}=(271*s)/6144 - (11*r)/6144 - (13*t)/6144 - (11*r*s)/6144 + (r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 271/6144;

c_{34}=(323*t)/6144 - (65*s)/6144 - (25*r)/2048 + (5*r*s)/6144 - (5*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 + 1615/6144;

d_{34}=(17*r*s)/6144 - (173*s)/6144 - (265*t)/6144 - (69*r)/2048 + (29*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 3379/6144;

w_{34}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (23*r^2*t)/905969664 + (245*r^2)/905969664 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (299*r*s)/301989888 - (23*r*t^2)/905969664 + (913*r*t)/905969664 - (2453*r)/226492416 - (7*s^2*t)/301989888 + (23*s^2)/100663296 - (7*s*t^2)/301989888 + (101*s*t)/100663296 - (1487*s)/150994944 + (97*t^2)/301989888 - (587*t)/50331648 + 31645/301989888; .....(65a-e)

DIVISION 35

a_{35}=(271*r)/6144 - (11*s)/6144 - (13*t)/6144 - (11*r*s)/6144 - (13*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 + 271/6144;

b_{35}=(293*s)/6144 - (11*r)/2048 - (15*t)/2048 - (11*r*s)/6144 + (r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 + 293/2048;

c_{35}=(323*t)/6144 - (25*s)/2048 - (65*r)/6144 + (5*r*s)/6144 - (13*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 1615/6144;

d_{35}=(17*r*s)/6144 - (69*s)/2048 - (265*t)/6144 - (173*r)/6144 + (23*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 3379/6144;

w_{35}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (7*r^2*t)/301989888 + (23*r^2)/100663296 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (299*r*s)/301989888 - (7*r*t^2)/301989888 + (101*r*t)/100663296 - (1487*r)/150994944 - (23*s^2*t)/905969664 + (245*s^2)/905969664 - (23*s*t^2)/905969664 + (913*s*t)/905969664 - (2453*s)/226492416 + (97*t^2)/301989888 - (587*t)/50331648 + 31645/301989888; .....(66a-e)

DIVISION 36

a_{36}=(271*r)/6144 - (11*s)/2048 - (13*t)/2048 - (11*r*s)/6144 - (13*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 + 271/2048;

b_{36}=(271*s)/6144 - (11*r)/2048 - (13*t)/2048 - (11*r*s)/6144 + (r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 + 271/2048;

c_{36}=(99*t)/2048 - (65*s)/6144 - (65*r)/6144 + (5*r*s)/6144 - (13*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 495/2048;

d_{36}=(17*r*s)/6144 - (173*s)/6144 - (73*t)/2048 - (173*r)/6144 + (23*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 1011/2048;

w_{36}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (7*r^2*t)/301989888 + (23*r^2)/100663296 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (821*r*s)/905969664 - (7*r*t^2)/301989888 + (275*r*t)/301989888 - (1349*r)/150994944 - (7*s^2*t)/301989888 + (23*s^2)/100663296 - (7*s*t^2)/301989888 + (275*s*t)/301989888 - (1349*s)/150994944 + (83*t^2)/301989888 - (23*t)/2359296 + 25973/301989888; .....(67a-e)

DIVISION 37

a_{37}=(263*r)/6144 - (3*s)/2048 - (5*t)/2048 - (3*r*s)/2048 - (5*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 + 263/6144;

b_{37}=(263*s)/6144 - (3*r)/2048 - (5*t)/2048 - (3*r*s)/2048 + (r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 263/6144;

c_{37}=(353*t)/6144 - (35*s)/2048 - (35*r)/2048 + (7*r*s)/6144 - (5*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 + 2471/6144;

d_{37}=(11*r*s)/6144 - (149*s)/6144 - (323*t)/6144 - (149*r)/6144 + (29*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 1049/2048;

w_{37}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (23*r^2*t)/905969664 + (199*r^2)/905969664 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (877*r*s)/905969664 - (23*r*t^2)/905969664 + (913*r*t)/905969664 - (493*r)/50331648 - (23*s^2*t)/905969664 + (199*s^2)/905969664 - (23*s*t^2)/905969664 + (913*s*t)/905969664 - (493*s)/50331648 + (337*t^2)/905969664 - (29*t)/2359296 + 91919/905969664; .....(68a-e)

DIVISION 38

a_{38}=(263*r)/6144 - (9*s)/2048 - (15*t)/2048 - (3*r*s)/2048 - (5*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 + 263/2048;

b_{38}=(245*s)/6144 - (3*r)/2048 - (13*t)/6144 - (3*r*s)/2048 + (r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 245/6144;

c_{38}=(323*t)/6144 - (91*s)/6144 - (35*r)/2048 + (7*r*s)/6144 - (5*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 + 2261/6144;

d_{38}=(11*r*s)/6144 - (127*s)/6144 - (265*t)/6144 - (149*r)/6144 + (29*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 2849/6144;

w_{38}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (23*r^2*t)/905969664 + (199*r^2)/905969664 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (809*r*s)/905969664 - (23*r*t^2)/905969664 + (821*r*t)/905969664 - (4039*r)/452984832 - (7*s^2*t)/301989888 + (55*s^2)/301989888 - (7*s*t^2)/301989888 + (275*s*t)/301989888 - (599*s)/75497472 + (97*t^2)/301989888 - (1567*t)/150994944 + 24989/301989888; .....(69a-e)

DIVISION 39

a_{39}=(245*r)/6144 - (3*s)/2048 - (13*t)/6144 - (3*r*s)/2048 - (13*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 + 245/6144;

b_{39}=(263*s)/6144 - (9*r)/2048 - (15*t)/2048 - (3*r*s)/2048 + (r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 + 263/2048;

c_{39}=(323*t)/6144 - (35*s)/2048 - (91*r)/6144 + (7*r*s)/6144 - (13*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 2261/6144;

d_{39}=(11*r*s)/6144 - (149*s)/6144 - (265*t)/6144 - (127*r)/6144 + (23*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 2849/6144;

w_{39}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (7*r^2*t)/301989888 + (55*r^2)/301989888 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (809*r*s)/905969664 - (7*r*t^2)/301989888 + (275*r*t)/301989888 - (599*r)/75497472 - (23*s^2*t)/905969664 + (199*s^2)/905969664 - (23*s*t^2)/905969664 + (821*s*t)/905969664 - (4039*s)/452984832 + (97*t^2)/301989888 - (1567*t)/150994944 + 24989/301989888; .....(70a-e)

DIVISION 40

a_{40}=(245*r)/6144 - (9*s)/2048 - (13*t)/2048 - (3*r*s)/2048 - (13*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 + 245/2048;

b_{40}=(245*s)/6144 - (9*r)/2048 - (13*t)/2048 - (3*r*s)/2048 + (r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 + 245/2048;

c_{40}=(99*t)/2048 - (91*s)/6144 - (91*r)/6144 + (7*r*s)/6144 - (13*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 693/2048;

d_{40}=(11*r*s)/6144 - (127*s)/6144 - (73*t)/2048 - (127*r)/6144 + (23*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 865/2048;

w_{40}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (7*r^2*t)/301989888 + (55*r^2)/301989888 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (247*r*s)/301989888 - (7*r*t^2)/301989888 + (247*r*t)/301989888 - (17*r)/2359296 - (7*s^2*t)/301989888 + (55*s^2)/301989888 - (7*s*t^2)/301989888 + (247*s*t)/301989888 - (17*s)/2359296 + (83*t^2)/301989888 - (653*t)/75497472 + 20417/301989888; .....(71a-e)

DIVISION 41

a_{41}=(293*r)/6144 - (55*s)/6144 - (25*t)/2048 - (11*r*s)/6144 - (5*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144 + 1465/6144;

b_{41}=(83*s)/2048 - (11*r)/6144 - (11*t)/6144 - (11*r*s)/6144 + (r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 83/2048;

c_{41}=(293*t)/6144 - (55*s)/6144 - (25*r)/2048 + (5*r*s)/6144 - (5*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 + 1465/6144;

d_{41}=(17*r*s)/6144 - (139*s)/6144 - (69*t)/2048 - (69*r)/2048 + (29*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 2965/6144;

w_{41}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (23*r^2*t)/905969664 + (245*r^2)/905969664 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (821*r*s)/905969664 - (23*r*t^2)/905969664 + (821*r*t)/905969664 - (23*r)/2359296 - (19*s^2*t)/905969664 + (169*s^2)/905969664 - (19*s*t^2)/905969664 + (821*s*t)/905969664 - (1801*s)/226492416 + (245*t^2)/905969664 - (23*t)/2359296 + 76291/905969664; .....(72a-e)

DIVISION 42

a_{42}=(293*r)/6144 - (77*s)/6144 - (35*t)/2048 - (11*r*s)/6144 - (5*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144 + 2051/6144;

b_{42}=(227*s)/6144 - (11*r)/6144 - (3*t)/2048 - (11*r*s)/6144 + (r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 227/6144;

c_{42}=(263*t)/6144 - (15*s)/2048 - (25*r)/2048 + (5*r*s)/6144 - (5*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 + 1315/6144;

d_{42}=(17*r*s)/6144 - (35*s)/2048 - (149*t)/6144 - (69*r)/2048 + (29*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 2551/6144;

w_{42}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (23*r^2*t)/905969664 + (245*r^2)/905969664 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (745*r*s)/905969664 - (23*r*t^2)/905969664 + (27*r*t)/33554432 - (1963*r)/226492416 - (17*s^2*t)/905969664 + (131*s^2)/905969664 - (17*s*t^2)/905969664 + (247*s*t)/301989888 - (2819*s)/452984832 + (199*t^2)/905969664 - (3641*t)/452984832 + 6623/100663296; .....(73a-e)

DIVISION 43

a_{43}=(271*r)/6144 - (55*s)/6144 - (65*t)/6144 - (11*r*s)/6144 - (13*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144 + 1355/6144;

b_{43}=(83*s)/2048 - (11*r)/2048 - (11*t)/2048 - (11*r*s)/6144 + (r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 + 249/2048;

c_{43}=(271*t)/6144 - (55*s)/6144 - (65*r)/6144 + (5*r*s)/6144 - (13*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 1355/6144;

d_{43}=(17*r*s)/6144 - (139*s)/6144 - (173*t)/6144 - (173*r)/6144 + (23*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 2687/6144;

w_{43}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (7*r^2*t)/301989888 + (23*r^2)/100663296 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (745*r*s)/905969664 - (7*r*t^2)/301989888 + (247*r*t)/301989888 - (1211*r)/150994944 - (19*s^2*t)/905969664 + (169*s^2)/905969664 - (19*s*t^2)/905969664 + (745*s*t)/905969664 - (17*s)/2359296 + (23*t^2)/100663296 - (1211*t)/150994944 + 2317/33554432; .....(74a-e)

DIVISION 44

a_{44}=(271*r)/6144 - (77*s)/6144 - (91*t)/6144 - (11*r*s)/6144 - (13*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144 + 1897/6144;

b_{44}=(227*s)/6144 - (11*r)/2048 - (9*t)/2048 - (11*r*s)/6144 + (r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 + 227/2048;

c_{44}=(245*t)/6144 - (15*s)/2048 - (65*r)/6144 + (5*r*s)/6144 - (13*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 1225/6144;

d_{44}=(17*r*s)/6144 - (35*s)/2048 - (127*t)/6144 - (173*r)/6144 + (23*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 2341/6144;

w_{44}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (7*r^2*t)/301989888 + (23*r^2)/100663296 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (223*r*s)/301989888 - (7*r*t^2)/301989888 + (73*r*t)/100663296 - (1073*r)/150994944 - (17*s^2*t)/905969664 + (131*s^2)/905969664 - (17*s*t^2)/905969664 + (673*s*t)/905969664 - (2557*s)/452984832 + (55*t^2)/301989888 - (163*t)/25165824 + 16285/301989888; . .....(75a-e)

DIVISION 45

a_{45}=(263*r)/6144 - (15*s)/2048 - (25*t)/2048 - (3*r*s)/2048 - (5*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144 + 1315/6144;

b_{45}=(227*s)/6144 - (3*r)/2048 - (11*t)/6144 - (3*r*s)/2048 + (r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 227/6144;

c_{45}=(293*t)/6144 - (77*s)/6144 - (35*r)/2048 + (7*r*s)/6144 - (5*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 + 2051/6144;

d_{45}=(11*r*s)/6144 - (35*s)/2048 - (69*t)/2048 - (149*r)/6144 + (29*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 2551/6144;

w_{45}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (23*r^2*t)/905969664 + (199*r^2)/905969664 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (247*r*s)/301989888 - (23*r*t^2)/905969664 + (27*r*t)/33554432 - (3641*r)/452984832 - (19*s^2*t)/905969664 + (131*s^2)/905969664 - (19*s*t^2)/905969664 + (745*s*t)/905969664 - (2819*s)/452984832 + (245*t^2)/905969664 - (1963*t)/226492416 + 6623/100663296; .....(76a-e)

DIVISION 46

a_{46}=(263*r)/6144 - (21*s)/2048 - (35*t)/2048 - (3*r*s)/2048 - (5*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144 + 1841/6144;

b_{46}=(209*s)/6144 - (3*r)/2048 - (3*t)/2048 - (3*r*s)/2048 + (r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 209/6144;

c_{46}=(263*t)/6144 - (21*s)/2048 - (35*r)/2048 + (7*r*s)/6144 - (5*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 + 1841/6144;

d_{46}=(11*r*s)/6144 - (83*s)/6144 - (149*t)/6144 - (149*r)/6144 + (29*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 751/2048;

w_{46}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (23*r^2*t)/905969664 + (199*r^2)/905969664 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (673*r*s)/905969664 - (23*r*t^2)/905969664 + (637*r*t)/905969664 - (1081*r)/150994944 - (17*s^2*t)/905969664 + (97*s^2)/905969664 - (17*s*t^2)/905969664 + (673*s*t)/905969664 - (11*s)/2359296 + (199*t^2)/905969664 - (1081*t)/150994944 + 45839/905969664; .....(77a-e)

DIVISION 47

a_{47}=(245*r)/6144 - (15*s)/2048 - (65*t)/6144 - (3*r*s)/2048 - (13*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144 + 1225/6144;

b_{47}=(227*s)/6144 - (9*r)/2048 - (11*t)/2048 - (3*r*s)/2048 + (r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 + 227/2048;

c_{47}=(271*t)/6144 - (77*s)/6144 - (91*r)/6144 + (7*r*s)/6144 - (13*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 1897/6144;

d_{47}=(11*r*s)/6144 - (35*s)/2048 - (173*t)/6144 - (127*r)/6144 + (23*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 2341/6144;

w_{47}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (7*r^2*t)/301989888 + (55*r^2)/301989888 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (673*r*s)/905969664 - (7*r*t^2)/301989888 + (73*r*t)/100663296 - (163*r)/25165824 - (19*s^2*t)/905969664 + (131*s^2)/905969664 - (19*s*t^2)/905969664 + (223*s*t)/301989888 - (2557*s)/452984832 + (23*t^2)/100663296 - (1073*t)/150994944 + 16285/301989888; .....(78a-e)

DIVISION 48

a_{48}=(245*r)/6144 - (21*s)/2048 - (91*t)/6144 - (3*r*s)/2048 - (13*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144 + 1715/6144;

b_{48}=(209*s)/6144 - (9*r)/2048 - (9*t)/2048 - (3*r*s)/2048 + (r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 + 209/2048;

c_{48}=(245*t)/6144 - (21*s)/2048 - (91*r)/6144 + (7*r*s)/6144 - (13*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 1715/6144;

d_{48}=(11*r*s)/6144 - (83*s)/6144 - (127*t)/6144 - (127*r)/6144 + (23*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 2087/6144;

w_{48}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (7*r^2*t)/301989888 + (55*r^2)/301989888 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (5*r*s*t)/150994944 + (605*r*s)/905969664 - (7*r*t^2)/301989888 + (191*r*t)/301989888 - (217*r)/37748736 - (17*s^2*t)/905969664 + (97*s^2)/905969664 - (17*s*t^2)/905969664 + (605*s*t)/905969664 - (959*s)/226492416 + (55*t^2)/301989888 - (217*t)/37748736 + 12593/301989888; .....(79a-e)

DIVISION 49

a_{49}=(83*r)/2048 - (11*s)/6144 - (11*t)/6144 - (11*r*s)/6144 - (11*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 + 83/2048;

b_{49}=(293*s)/6144 - (55*r)/6144 - (25*t)/2048 - (11*r*s)/6144 + (5*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 1465/6144;

c_{49}=(293*t)/6144 - (25*s)/2048 - (55*r)/6144 + (5*r*s)/6144 - (11*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 1465/6144;

d_{49})=(17*r*s)/6144 - (69*s)/2048 - (69*t)/2048 - (139*r)/6144 + (17*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 2965/6144;

w_{49}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (19*r^2*t)/905969664 + (169*r^2)/905969664 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (821*r*s)/905969664 - (19*r*t^2)/905969664 + (821*r*t)/905969664 - (1801*r)/226492416 - (23*s^2*t)/905969664 + (245*s^2)/905969664 - (23*s*t^2)/905969664 + (821*s*t)/905969664 - (23*s)/2359296 + (245*t^2)/905969664 - (23*t)/2359296 + 76291/905969664; .....(80a-e)

DIVISION 50

a_{50}=(83*r)/2048 - (11*s)/2048 - (11*t)/2048 - (11*r*s)/6144 - (11*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 + 249/2048;

b_{50}=(271*s)/6144 - (55*r)/6144 - (65*t)/6144 - (11*r*s)/6144 + (5*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 1355/6144;

c_{50}=(271*t)/6144 - (65*s)/6144 - (55*r)/6144 + (5*r*s)/6144 - (11*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 1355/6144;

d_{50}=(17*r*s)/6144 - (173*s)/6144 - (173*t)/6144 - (139*r)/6144 + (17*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 2687/6144;

w_{50}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (19*r^2*t)/905969664 + (169*r^2)/905969664 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (745*r*s)/905969664 - (19*r*t^2)/905969664 + (745*r*t)/905969664 - (17*r)/2359296 - (7*s^2*t)/301989888 + (23*s^2)/100663296 - (7*s*t^2)/301989888 + (247*s*t)/301989888 - (1211*s)/150994944 + (23*t^2)/100663296 - (1211*t)/150994944 + 2317/33554432; .....(81a-e)

DIVISION 51

a_{51}=(227*r)/6144 - (11*s)/6144 - (3*t)/2048 - (11*r*s)/6144 - (3*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 + 227/6144;

b_{51}=(293*s)/6144 - (77*r)/6144 - (35*t)/2048 - (11*r*s)/6144 + (7*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 2051/6144;

c_{51}=(263*t)/6144 - (25*s)/2048 - (15*r)/2048 + (5*r*s)/6144 - (3*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 + 1315/6144;

d_{51}=(17*r*s)/6144 - (69*s)/2048 - (149*t)/6144 - (35*r)/2048 + (11*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 2551/6144;

w_{51}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (17*r^2*t)/905969664 + (131*r^2)/905969664 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (745*r*s)/905969664 - (17*r*t^2)/905969664 + (247*r*t)/301989888 - (2819*r)/452984832 - (23*s^2*t)/905969664 + (245*s^2)/905969664 - (23*s*t^2)/905969664 + (27*s*t)/33554432 - (1963*s)/226492416 + (199*t^2)/905969664 - (3641*t)/452984832 + 6623/100663296; .....(82a-e)

DIVISION 52

a_{52}=(227*r)/6144 - (11*s)/2048 - (9*t)/2048 - (11*r*s)/6144 - (3*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 + 227/2048;

b_{52}=(271*s)/6144 - (77*r)/6144 - (91*t)/6144 - (11*r*s)/6144 + (7*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 1897/6144;

c_{52}=(245*t)/6144 - (65*s)/6144 - (15*r)/2048 + (5*r*s)/6144 - (3*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 + 1225/6144;

d_{52}=(17*r*s)/6144 - (173*s)/6144 - (127*t)/6144 - (35*r)/2048 + (11*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 2341/6144;

w_{52}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (17*r^2*t)/905969664 + (131*r^2)/905969664 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (223*r*s)/301989888 - (17*r*t^2)/905969664 + (673*r*t)/905969664 - (2557*r)/452984832 - (7*s^2*t)/301989888 + (23*s^2)/100663296 - (7*s*t^2)/301989888 + (73*s*t)/100663296 - (1073*s)/150994944 + (55*t^2)/301989888 - (163*t)/25165824 + 16285/301989888; .....(83a-e)

DIVISION 53

a_{53}=(227*r)/6144 - (3*s)/2048 - (11*t)/6144 - (3*r*s)/2048 - (11*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 + 227/6144;

b_{53}=(263*s)/6144 - (15*r)/2048 - (25*t)/2048 - (3*r*s)/2048 + (5*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 1315/6144;

c_{53}=(293*t)/6144 - (35*s)/2048 - (77*r)/6144 + (7*r*s)/6144 - (11*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 2051/6144;

d_{53}=(11*r*s)/6144 - (149*s)/6144 - (69*t)/2048 - (35*r)/2048 + (17*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 2551/6144;

w_{53}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (19*r^2*t)/905969664 + (131*r^2)/905969664 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (247*r*s)/301989888 - (19*r*t^2)/905969664 + (745*r*t)/905969664 - (2819*r)/452984832 - (23*s^2*t)/905969664 + (199*s^2)/905969664 - (23*s*t^2)/905969664 + (27*s*t)/33554432 - (3641*s)/452984832 + (245*t^2)/905969664 - (1963*t)/226492416 + 6623/100663296; .....(84a-e)

DIVISION 54

a_{54}=(227*r)/6144 - (9*s)/2048 - (11*t)/2048 - (3*r*s)/2048 - (11*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 + 227/2048;

b_{54}=(245*s)/6144 - (15*r)/2048 - (65*t)/6144 - (3*r*s)/2048 + (5*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 1225/6144;

c_{54}=(271*t)/6144 - (91*s)/6144 - (77*r)/6144 + (7*r*s)/6144 - (11*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 1897/6144;

d_{54}=(11*r*s)/6144 - (127*s)/6144 - (173*t)/6144 - (35*r)/2048 + (17*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 2341/6144;

w_{54}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (19*r^2*t)/905969664 + (131*r^2)/905969664 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (673*r*s)/905969664 - (19*r*t^2)/905969664 + (223*r*t)/301989888 - (2557*r)/452984832 - (7*s^2*t)/301989888 + (55*s^2)/301989888 - (7*s*t^2)/301989888 + (73*s*t)/100663296 - (163*s)/25165824 + (23*t^2)/100663296 - (1073*t)/150994944 + 16285/301989888; .....(85a-e)

DIVISION 55

a_{55}=(209*r)/6144 - (3*s)/2048 - (3*t)/2048 - (3*r*s)/2048 - (3*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 + 209/6144;

b_{55}=(263*s)/6144 - (21*r)/2048 - (35*t)/2048 - (3*r*s)/2048 + (7*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 + 1841/6144;

c_{55}=(263*t)/6144 - (35*s)/2048 - (21*r)/2048 + (7*r*s)/6144 - (3*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 + 1841/6144;

d_{55}=(11*r*s)/6144 - (149*s)/6144 - (149*t)/6144 - (83*r)/6144 + (11*r*t)/6144 + (29*s*t)/6144 - (r*s*t)/2048 + 751/2048;

w_{55}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (17*r^2*t)/905969664 + (97*r^2)/905969664 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (673*r*s)/905969664 - (17*r*t^2)/905969664 + (673*r*t)/905969664 - (11*r)/2359296 - (23*s^2*t)/905969664 + (199*s^2)/905969664 - (23*s*t^2)/905969664 + (637*s*t)/905969664 - (1081*s)/150994944 + (199*t^2)/905969664 - (1081*t)/150994944 + 45839/905969664; .....(86a-e)

DIVISION 56

a_{56}=(209*r)/6144 - (9*s)/2048 - (9*t)/2048 - (3*r*s)/2048 - (3*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 + 209/2048;

b_{56}=(245*s)/6144 - (21*r)/2048 - (91*t)/6144 - (3*r*s)/2048 + (7*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 + 1715/6144;

c_{56}=(245*t)/6144 - (91*s)/6144 - (21*r)/2048 + (7*r*s)/6144 - (3*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 + 1715/6144;

d_{56}=(11*r*s)/6144 - (127*s)/6144 - (127*t)/6144 - (83*r)/6144 + (11*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048 + 2087/6144;

w_{56}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (17*r^2*t)/905969664 + (97*r^2)/905969664 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (5*r*s*t)/150994944 + (605*r*s)/905969664 - (17*r*t^2)/905969664 + (605*r*t)/905969664 - (959*r)/226492416 - (7*s^2*t)/301989888 + (55*s^2)/301989888 - (7*s*t^2)/301989888 + (191*s*t)/301989888 - (217*s)/37748736 + (55*t^2)/301989888 - (217*t)/37748736 + 12593/301989888; .....(87a-e)

DIVISION 57

a_{57}=(83*r)/2048 - (55*s)/6144 - (55*t)/6144 - (11*r*s)/6144 - (11*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144 + 415/2048;

b_{57}=(83*s)/2048 - (55*r)/6144 - (55*t)/6144 - (11*r*s)/6144 + (5*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 415/2048;

c_{57}=(83*t)/2048 - (55*s)/6144 - (55*r)/6144 + (5*r*s)/6144 - (11*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 415/2048;

d_{57}=(17*r*s)/6144 - (139*s)/6144 - (139*t)/6144 - (139*r)/6144 + (17*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 803/2048;

w_{57}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (19*r^2*t)/905969664 + (169*r^2)/905969664 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (223*r*s)/301989888 - (19*r*t^2)/905969664 + (223*r*t)/301989888 - (1463*r)/226492416 - (19*s^2*t)/905969664 + (169*s^2)/905969664 - (19*s*t^2)/905969664 + (223*s*t)/301989888 - (1463*s)/226492416 + (169*t^2)/905969664 - (1463*t)/226492416 + 50179/905969664; .....(88a-e)

DIVISION 58

a_{58}=(83*r)/2048 - (77*s)/6144 - (77*t)/6144 - (11*r*s)/6144 - (11*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144 + 581/2048;

b_{58}=(227*s)/6144 - (55*r)/6144 - (15*t)/2048 - (11*r*s)/6144 + (5*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 1135/6144;

c_{58}=(227*t)/6144 - (15*s)/2048 - (55*r)/6144 + (5*r*s)/6144 - (11*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 1135/6144;

d_{58}=(17*r*s)/6144 - (35*s)/2048 - (35*t)/2048 - (139*r)/6144 + (17*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 2131/6144;

w_{58}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (19*r^2*t)/905969664 + (169*r^2)/905969664 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (5*r*s*t)/150994944 + (593*r*s)/905969664 - (19*r*t^2)/905969664 + (593*r*t)/905969664 - (647*r)/113246208 - (17*s^2*t)/905969664 + (131*s^2)/905969664 - (17*s*t^2)/905969664 + (605*s*t)/905969664 - (85*s)/16777216 + (131*t^2)/905969664 - (85*t)/16777216 + 39151/905969664; .....(89a-e)

DIVISION 59

a_{59}=(227*r)/6144 - (55*s)/6144 - (15*t)/2048 - (11*r*s)/6144 - (3*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144 + 1135/6144;

b_{59}=(83*s)/2048 - (77*r)/6144 - (77*t)/6144 - (11*r*s)/6144 + (7*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 581/2048;

c_{59}=(227*t)/6144 - (55*s)/6144 - (15*r)/2048 + (5*r*s)/6144 - (3*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 + 1135/6144;

d_{59}=(17*r*s)/6144 - (139*s)/6144 - (35*t)/2048 - (35*r)/2048 + (11*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 2131/6144;

w_{59}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (17*r^2*t)/905969664 + (131*r^2)/905969664 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (5*r*s*t)/150994944 + (593*r*s)/905969664 - (17*r*t^2)/905969664 + (605*r*t)/905969664 - (85*r)/16777216 - (19*s^2*t)/905969664 + (169*s^2)/905969664 - (19*s*t^2)/905969664 + (593*s*t)/905969664 - (647*s)/113246208 + (131*t^2)/905969664 - (85*t)/16777216 + 39151/905969664; .....(90a-e)

DIVISION 60

a_{60}=(227*r)/6144 - (77*s)/6144 - (21*t)/2048 - (11*r*s)/6144 - (3*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144 + 1589/6144;

b_{60}=(227*s)/6144 - (77*r)/6144 - (21*t)/2048 - (11*r*s)/6144 + (7*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 1589/6144;

c_{60}=(209*t)/6144 - (15*s)/2048 - (15*r)/2048 + (5*r*s)/6144 - (3*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 + 1045/6144;

d_{60}=(17*r*s)/6144 - (35*s)/2048 - (83*t)/6144 - (35*r)/2048 + (11*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 1921/6144;

w_{60}=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (17*r^2*t)/905969664 + (131*r^2)/905969664 + (r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (13*r*s*t)/452984832 + (517*r*s)/905969664 - (17*r*t^2)/905969664 + (179*r*t)/301989888 - (2033*r)/452984832 - (17*s^2*t)/905969664 + (131*s^2)/905969664 - (17*s*t^2)/905969664 + (179*s*t)/301989888 - (2033*s)/452984832 + (97*t^2)/905969664 - (431*t)/113246208 + 10165/301989888; .....(91a-e)

DIVISION 61

a_{61}=(227*r)/6144 - (15*s)/2048 - (55*t)/6144 - (3*r*s)/2048 - (11*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144 + 1135/6144;

b_{61}=(227*s)/6144 - (15*r)/2048 - (55*t)/6144 - (3*r*s)/2048 + (5*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 1135/6144;

c_{61}=(83*t)/2048 - (77*s)/6144 - (77*r)/6144 + (7*r*s)/6144 - (11*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 581/2048;

d_{61}=(11*r*s)/6144 - (35*s)/2048 - (139*t)/6144 - (35*r)/2048 + (17*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 2131/6144;

w_{61}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (19*r^2*t)/905969664 + (131*r^2)/905969664 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (5*r*s*t)/150994944 + (605*r*s)/905969664 - (19*r*t^2)/905969664 + (593*r*t)/905969664 - (85*r)/16777216 - (19*s^2*t)/905969664 + (131*s^2)/905969664 - (19*s*t^2)/905969664 + (593*s*t)/905969664 - (85*s)/16777216 + (169*t^2)/905969664 - (647*t)/113246208 + 39151/905969664; .....(92a-e)

DIVISION 62

a_{62}=(227*r)/6144 - (21*s)/2048 - (77*t)/6144 - (3*r*s)/2048 - (11*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144 + 1589/6144;

b_{62}=(209*s)/6144 - (15*r)/2048 - (15*t)/2048 - (3*r*s)/2048 + (5*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 1045/6144;

c_{62}=(227*t)/6144 - (21*s)/2048 - (77*r)/6144 + (7*r*s)/6144 - (11*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 1589/6144;

d_{62}=(11*r*s)/6144 - (83*s)/6144 - (35*t)/2048 - (35*r)/2048 + (17*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 1921/6144;

w_{62}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (19*r^2*t)/905969664 + (131*r^2)/905969664 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (13*r*s*t)/452984832 + (179*r*s)/301989888 - (19*r*t^2)/905969664 + (517*r*t)/905969664 - (2033*r)/452984832 - (17*s^2*t)/905969664 + (97*s^2)/905969664 - (17*s*t^2)/905969664 + (179*s*t)/301989888 - (431*s)/113246208 + (131*t^2)/905969664 - (2033*t)/452984832 + 10165/301989888; .....(93a-e)

DIVISION 63

a_{63}=(209*r)/6144 - (15*s)/2048 - (15*t)/2048 - (3*r*s)/2048 - (3*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144 + 1045/6144;

b_{63}=(227*s)/6144 - (21*r)/2048 - (77*t)/6144 - (3*r*s)/2048 + (7*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 + 1589/6144;

c_{63}=(227*t)/6144 - (77*s)/6144 - (21*r)/2048 + (7*r*s)/6144 - (3*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 + 1589/6144;
d_{63}=(11*r*s)/6144 - (35*s)/2048 - (35*t)/2048 - (83*r)/6144 + (11*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048 + 1921/6144;
w_{63}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (17*r^2*t)/905969664 + (97*r^2)/905969664 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (13*r*s*t)/452984832 + (179*r*s)/301989888 - (17*r*t^2)/905969664 + (179*r*t)/301989888 - (431*r)/113246208 - (19*s^2*t)/905969664 + (131*s^2)/905969664 - (19*s*t^2)/905969664 + (517*s*t)/905969664 - (2033*s)/452984832 + (131*t^2)/905969664 - (2033*t)/452984832 + 10165/301989888; .....(94a-e)

DIVISION 64

a_{64}=(209*r)/6144 - (21*s)/2048 - (21*t)/2048 - (3*r*s)/2048 - (3*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144 + 1463/6144;
b_{64}=(209*s)/6144 - (21*r)/2048 - (21*t)/2048 - (3*r*s)/2048 + (7*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 + 1463/6144;
c_{64}=(209*t)/6144 - (21*s)/2048 - (21*r)/2048 + (7*r*s)/6144 - (3*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 + 1463/6144;
d_{64}=(11*r*s)/6144 - (83*s)/6144 - (83*t)/6144 - (83*r)/6144 + (11*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048 + 585/2048;
w_{64}=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (17*r^2*t)/905969664 + (97*r^2)/905969664 + (r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (11*r*s*t)/452984832 + (469*r*s)/905969664 - (17*r*t^2)/905969664 + (469*r*t)/905969664 - (85*r)/25165824 - (17*s^2*t)/905969664 + (97*s^2)/905969664 - (17*s*t^2)/905969664 + (469*s*t)/905969664 - (85*s)/25165824 + (97*t^2)/905969664 - (85*t)/25165824 + 23987/905969664; .....(95a-e)

4 Numerical Integration Schemes For Linear Tetrahedra

4.1 Decomposition into four hexahedrons

We now refer to the derivation of section 3.1.

Letting

$$\boldsymbol{T^i}(\boldsymbol{r},\boldsymbol{s},\boldsymbol{t}) = \boldsymbol{T}\left(x^i,y^i,z^i,w^i\right) = \left(X\left(x^i,y^i,z^i\right), Y\left(x^i,y^i,z^i\right), Z\left(x^i,y^i,z^i\right)\right) \quad \text{.....(96a)}$$

$$f\left(X\left(x^i,y^i,z^i\right),Y\left(x^i,y^i,z^i\right),Z\left(x^i,y^i,z^i\right)\right) = f\left(\boldsymbol{T^i}(\boldsymbol{r},\boldsymbol{s},\boldsymbol{t})\right) = f\left(\boldsymbol{T}\left(x^i,y^i,z^i,w^i\right)\right) \quad \text{.....(96b)}$$

where it is already mentioned above that

$$x^i=x^i(r,s,t),\ y^i=y^i(r,s,t),\ z^i=z^i(r,s,t) \quad \text{.....(96c)}$$

and

$$\frac{\partial(x^i,y^i,z^i)}{\partial(r,s,t)} = = J^i(r,s,t) = J^i \text{ (say)} \quad \text{..... (96d)}$$

We have also shown in eqns(11-14) and Table-1 that

$$\iiint_{T_{1234}} f(X,Y,Z)dXdYdZ = |\det J| \iiint_{T_{1234}^{\sim}} f(X(x,y,z),Y(x,y,z),Z(x,y,z))dxdydz$$

$$= |detJ| \sum_{i=1}^{4} \iiint_{\Omega_i} f(X(x,y,z),Y(x,y,z),Z(x,y,z))dxdydz$$

$$= |detJ| \sum_{i=1}^{4} \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f\left(X\left(x^i,y^i,z^i\right),Y\left(x^i,y^i,z^i\right),Z\left(x^i,y^i,z^i\right)\right) \frac{\partial(x^i,y^i,z^i)}{\partial(r,s,t)}\ drdsdt$$

$$= |detJ| \sum_{i=1}^{4} \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f\left(\boldsymbol{T^i}(\boldsymbol{r},\boldsymbol{s},\boldsymbol{t})\right) J^i(r,s,t)\ drdsdt$$

$$= |detJ| \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \{\sum_{i=1}^{4} f\left(\boldsymbol{T^i}(\boldsymbol{r},\boldsymbol{s},\boldsymbol{t})\right) J^i(r,s,t)\}\ drdsdt$$

$$= |detJ| \sum_{p=1}^{p=N}\sum_{n=1}^{n=N}\sum_{m=1}^{m=N} \left[\{\sum_{i=1}^{i=4} f\left(\boldsymbol{T^i}(\boldsymbol{r_m}^{(N)},\boldsymbol{s_n}^{(N)},\boldsymbol{t_p}^{(N)})\right)\} \vartheta(\boldsymbol{r_m}^{(N)},\boldsymbol{s_n}^{(N)},\boldsymbol{t_p}^{(N)}) W_m^{(N)} W_n^{(N)} W_p^{(N)}\right]$$

.....(97)

Where, $((\boldsymbol{r_m}^{(N)},\ W_m^{(N)}), (\boldsymbol{s_n}^{(N)}, W_n^{(N)}), (\boldsymbol{t_p}^{(N)}, W_p^{(N)})), m = 1(1)\text{N}, n = 1(1)N, p = 1(1)N$ are the pairs of sampling points and weights of the N th order Gauss Legendre Quadrature rules

4.2 Decomposition into thirty two hexahedrons

We now refer to the derivation of section 3.2.

Letting

$$\boldsymbol{T^{i,j}(r,s,t)} = \boldsymbol{T}\left(x^{i,j},y^{i,j},z^{i,j},w^{i,j}\right) = \left(X\left(x^{i,j},y^{i,j},z^{i,j}\right), Y\left(x^{i,j},y^{i,j},z^{i,j}\right), Z\left(x^{i,j},y^{i,j},z^{i,j}\right)\right) \quad \text{.....(98a)}$$

$$f\left(X\left(x^{i,j},y^{i,j},z^{i,j}\right),Y\left(x^{i,j},y^{i,j},z^{i,j}\right),Z\left(x^{i,j},y^{i,j},z^{i,j}\right)\right) = f\left(\boldsymbol{T^{i,j}(r,s,t)}\right) = f\left(\boldsymbol{T}\left(x^{i,j},y^{i,j},z^{i,j},w^{i,j}\right)\right) \quad \text{.....(98b)}$$

where it is already mentioned above that

$x^{i,j}=x^{i,j}(r,s,t)$, $y^{i,j}=y^{i,j}(r,s,t)$, $z^{i,j}=z^{i,j}(r,s,t)$ .....(98c)

and

$$\frac{\partial(x^{i,j},y^{i,j},z^{i,j})}{\partial(r,s,t)} = J^{i,j}(r,s,t) \quad \text{..... (98d)}$$

Let F(r,s,t)= $f\left(X\left(x^i,y^i,z^i\right),Y\left(x^i,y^i,z^i\right),Z\left(x^i,y^i,z^i\right)\right)\frac{\partial(x^i,y^i,z^i)}{\partial(r,s,t)}$ .....(17)

Clearly , *it is already assumed that* $x^i=x^i(r,s,t)$, $y^i=y^i(r,s,t)$, $z^i=z^i(r,s,t)$,
so that we can write from eq(11):

$$\iiint_{T_{1234}} f(X,Y,Z)dXdYdZ = |\det J| \iiint_{T_{1234}^{\sim}} f(X(x,y,z),Y(x,y,z),Z(x,y,z))dxdydz$$

$$=|detJ| \sum_{i=1}^{4} \iiint_{\Omega_i} f(X(x,y,z),Y(x,y,z),Z(x,y,z))dxdydz$$

$$=|detJ| \sum_{i=1}^{i=4} \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f\left(X\left(x^i,y^i,z^i\right),Y\left(x^i,y^i,z^i\right),Z\left(x^i,y^i,,z^i\right)\right)\frac{\partial(x^i,y^i,z^i)}{\partial(r,s,t)}\ drdsdt$$

$$=|detJ| \sum_{j=1}^{j=8}\sum_{i=1}^{i=4} \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f\left(X\left(x^{i,j},y^{i,j},z^{i,j}\right),Y\left(x^{i,j},y^{i,j},z^{i,j}\right),Z\left(x^{i,j},y^{i,j},z^{i,j}\right)\right)\frac{\partial(x^{i,j},y^{i,j},z^{i,j})}{\partial(r,s,t)}\ drdsdt$$

$$=|detJ| \sum_{j=1}^{j=8}\sum_{i=1}^{i=4} \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f\left(X\left(x^{i,j},y^{i,j},z^{i,j}\right),Y\left(x^{i,j},y^{i,j},z^{i,j}\right),Z\left(x^{i,j},y^{i,j},z^{i,j}\right)\right) J^{i,j}(r,s,t)drdsdt$$

$$=|detJ| \sum_{j=1}^{j=8}\sum_{i=1}^{i=4} \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f\left(\boldsymbol{T^{i,j}(r,s,t)}\right)\vartheta^j(r,s,t)drdsdt$$

$$=|detJ|. \sum_{p=1}^{p=N}\sum_{n=1}^{n=N}\sum_{m=1}^{m=N}\sum_{j=1}^{j=8}\left[\{\sum_{i=1}^{i=4} f\left(\boldsymbol{T^{i,j}}(\boldsymbol{r_m}^{(N)},\boldsymbol{s_n}^{(N)},\boldsymbol{t_p}^{(N)})\right)\}\vartheta^j(\boldsymbol{r_m}^{(N)},\boldsymbol{s_n}^{(N)},\boldsymbol{t_p}^{(N)})W_m^{(N)}W_n^{(N)}W_p^{(N)}\right] \quad \text{.....(99)}$$

Where, $((\boldsymbol{r_m}^{(N)},\ W_m^{(N)}),(\boldsymbol{s_n}^{(N)},W_n^{(N)}),(\boldsymbol{t_p}^{(N)},W_p^{(N)})), m=1(1)\mathrm{N}, n=1(1)N, p=1(1)N$
are the pairs of sampling points and weights of the N th order Gauss Legendre Quadrature rules

4.3 **Decomposition into thirty two hexahedrons**

In a similar,we refer derivation section 3.3 and write

$$\iiint_{T_{1234}} f(X,Y,Z)dXdYdZ = |\det J| \iiint_{T_{1234}^{\sim}} f(X(x,y,z),Y(x,y,z),Z(x,y,z))dxdydz$$

$$=|detJ| \sum_{p=1}^{p=N}\sum_{n=1}^{n=N}\sum_{m=1}^{m=N}\sum_{k=1}^{k=8}\cdot \sum_{j=1}^{j=8}\left[\{\sum_{i=1}^{i=4} f\left(\boldsymbol{T^{i,j,k}}(\boldsymbol{r_m}^{(N)},\boldsymbol{s_n}^{(N)},\boldsymbol{t_p}^{(N)})\right)\}\vartheta^{j,k}(\boldsymbol{r_m}^{(N)},\boldsymbol{s_n}^{(N)},\boldsymbol{t_p}^{(N)})W_m^{(N)}W_n^{(N)}W_p^{(N)}\right] \quad \text{.....(100)}$$

Where, $((\boldsymbol{r_m}^{(N)},\ W_m^{(N)}),(\boldsymbol{s_n}^{(N)},W_n^{(N)}),(\boldsymbol{t_p}^{(N)},W_p^{(N)})), m=1(1)\mathrm{N}, n=1(1)N, p=1(1)N$
are the pairs of sampling points and weights of the N th order Gauss Legendre Quadrature rules

5 NUMERICAL INTEGRATION OVER A LINEAR POLYHEDRON

Numerical integration over a n arbitrary linear hexahedron will be very tedious and complicated if trilinear transformations are directly used. However,the alternative is to divide the hexahedron into an assemblage of tetrahedrons and then sum the contributions to get the desired accuracy.This procedure can also be applied to the numerical integratiom over a linear polyhedron.

Let P denote the linear polyhedron.We can write $\mathrm{P}=\cup_{i=1}^{M}\boldsymbol{T}^{\boldsymbol{e}}_{\boldsymbol{a^e,b^e,c^e,d^e}}$, where $\boldsymbol{T}^{\boldsymbol{e}}_{\boldsymbol{a^e,b^e,c^e,d^e}}$ is a linear tetrahedron element 'e' with nodal addresses a^e,b^e,c^e,d^e *and M is the total number of tetrahedral elements made in P*.

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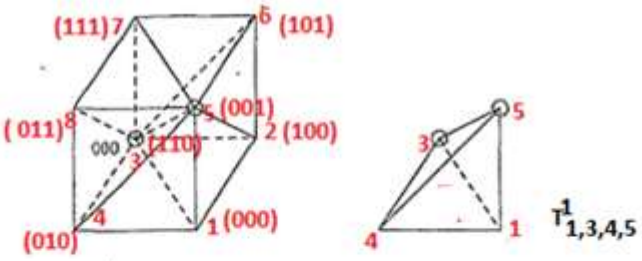
Let us consider the volume integral over an arbitrary linear polyhedron $P=\cup_{i=1}^{M} T^{e}_{a^e,b^e,c^e,d^e}$, defined as

$$III_P(f) = \iiint_{P=\cup_{i=1}^{M} T^{e}_{a^e,b^e,c^e,d^e}} f(X,Y,Z)\ dXdYdZ = \sum_{e=1}^{M} \iiint_{T^{e}_{a^e,b^e,c^e,d^e}} f(X,Y,Z)\ dXdYdZ \quad \ldots\ldots\ldots\ldots\ldots(101)$$

We illustrate this procedure to integrate over a unit cube and an irregular heptahedron. We first consider a unie cube.We can divide a unit cube either into five tetrahedra or more. We first choose to divide the unit cube into six linear tetrahedra which is shown in Fig.6. We have next shown the division of a unit cube into 24 tetrahedra which can be applied to integrate a linear convex polyhedron. This is done by partitioning the unit cube first into six pyramids and then divide each of these pyramids further into four unique tetrahedra. Base of the pyramid is a square which is divide into four isoscles right triangles.Then join the corner nodal points of these triangles to the centroid of the unit cube.This division is shown in Fig.7.We may note that the procedure applied is equivalent to first triangulating the six faces of unit cube,We then select an interior point in the unit cube.By joining the three corner nodes of a triangle to this interior point creats a tetrahedron.We repeat this process for all the triangles of triangulated faces of the unit cube We next consider the integration region which is a unit cube minus a tetrahedron constructed over one of the vertices of the cube. The region can be defined using 10 vertices and 7 faces with their coordinates and connectivity shown as in Fig.8.



$T^1_{1,3,4,5}$ FIRST TETRAHEDRON

$T^2_{1,2,3,5}$ SECOND TETRAHEDRON

$T^3_{3,2,6,5}$ THIRD TETRAHEDRON

$T^4_{3,6,7,5}$ FOURTH TETRAHEDRON

$T^5_{3,7,8,5}$ FIFTH TETRAHEDRON

$T^6_{3,8,4,5}$ SIXTH TETRAHEDRON

Fig.6 DIVISION OF A CUBE INTO SIX TETRAHEDRA

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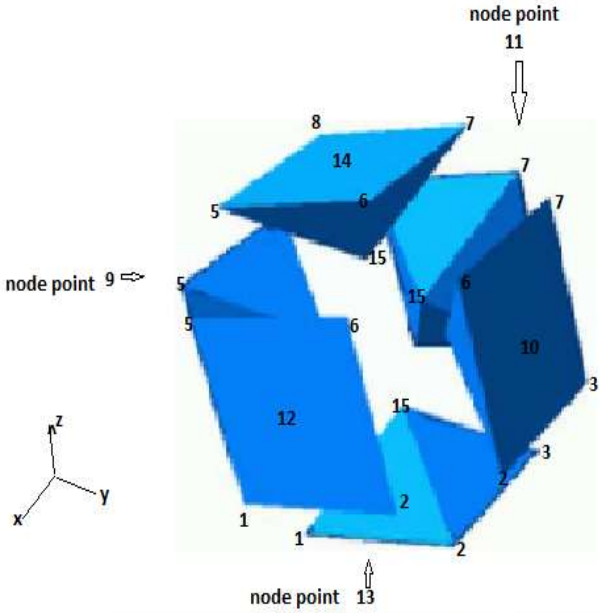
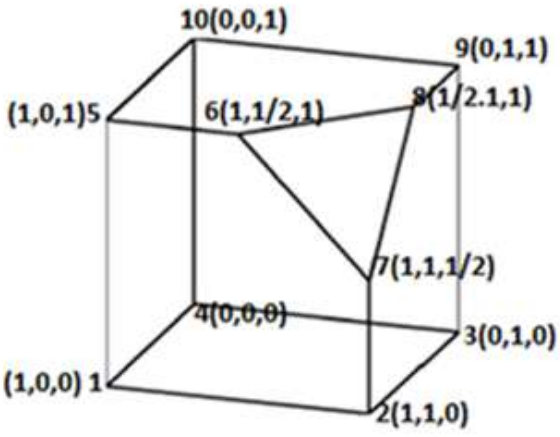


Fig.7-DIVISION OF A CUBE INTO SIX PYRAMIDS OF SAME SIZE MEETING AT A CENTRE POINT



FACES	CONNECTIVITY
1	1,2,7,6,5
2	2,3,9,8,7
3	3,4,10,9
4	5,10,4,1
5	1,4 3,2
6	5,6,8,9,10
7	6,7,8

Fig.8 Irregular heptahedron:unit cube minus a tetrahedron

6.NUMERICAL EXAMPLES

We now consider some typical integrals on the regions described above in Figs.6–8

The integral are:

$$III_j^i = \int\int\int_{V_i} f_j(x,y,z)dxdydz \; .i = 0,1; j = 1,2,3,4$$

Where V_0 *is a unit cube and* V_1 *is a irregular heptahedron*;and the integrands $f_j(x,y,z), j = 1,2,3,4$ are defined as

$$f_1(x,y,z) = x^3\sin(\pi y)\sin(\pi z)$$

$f_2\ (x,y,z) = \sin(\pi x)\sin(\pi y)\sin(\pi z)$

$f_3\ (x,y,z) = e^{(-((x-0{,}5)^2+(y-0.5)^2+(z-0.5)^2))}$

$f_4\ (x,y,z) = (\frac{27}{8})\sqrt{(1-|2x-1|)}\sqrt{(1-|2y-1|)}\sqrt{(1-|2z-1|)}$

EXACT VALUES OF INTEGRALS

$III_1^0 = \frac{1}{\pi^2}$=0.10132118364233778397171412865964

$III_2^0 = \frac{8}{\pi^3}$ =0.25801227546559596132817993937 3

III_3^0 = 0.78521159617436901020962024602291

III_j^0 =1

We may note that the exact values of integrals (III_j^1 ,j=1,2,3,4) are not available,because the domain is an irregular heptahedron which is complicated. In this paper,we have computed these integrals by two method (1)domain is taken as a unit cube minus a tetrahedron and (2) the domain is obtained by first triangulating the faces of irregular heptahedron and then joining the corner nodes of the triangles to an interior point of this domain which divides the irregular heptahedron into a domain of all tetrahedra.We have then compared the computed values of the integrals (III_j^1 ,j=1,2,3,4) by both the methods which are in agreement.

The computed numerical values of the above integrals are presented in TABLES 1-12

TABLE 1
DOMAIN IS A UNIT CUBE DISCRITISED BY SIX TETRA HEDRONS
(EACH TETRAHEDRON IS DECOMPOSED INTO F0UR HEXAHEDRA)
OGLR=ORDER OF GAUSS LEGENDRE RULE

OGLR	III_1^0	III_2^0	III_3^0	III_4^0
5.	1.013211806444221e-001	2.580122959339014e-001	7.852115963588021e-001	1.001478584900792e+000
10	1.013211836423378e-001	2.580122754655960e-001	7.852115961743690e-001	1.000227646249210e+000
15	1.013211836423378e-001	2.580122754655954e-001	7.852115961743692e-001	1.000061678153538e+000
20	1.013211836423378e-001	2.580122754655963e-001	7.852115961743685e-001	1.000025804554168e+000
25	1.013211836423379e-001	2.580122754655957e-001	7.852115961743715e-001	1.000016478982331e+000
30	1.013211836423380e-001	2.580122754655955e-001	7.852115961743695e-001	1.000007373245699e+000
35	1.013211836423377e-001	2.580122754655954e-001	7.852115961743712e-001	1.000006273893386e+000
40	1.013211836423377e-001	2.580122754655946e-001	7.852115961743718e-001	1.000004744754559e+000

TABLE 2
DOMAIN IS A UNIT CUBE DISCRITISED BY SIX PYRAMIDS(=24 TETRAHEDRA)
(EACH TETRAHEDRON IS DECOMPOSED INTO F0UR HEXAHEDRA)
OGLR=ORDER OF GAUSS LEGENDRE RULE

OGLR	III_1^0	III_2^0	III_3^0	III_4^0
5	1.013211836443502e-001	2.580122754044406e-001	7.852115961737480e-001	1.002158759290186e+000
10	1.013211836423378e-001	2.580122754655962e-001	7.852115961743691e-001	1.000509125714392e+000
15	1.013211836423376e-001	2.580122754655944e-001	7.852115961743688e-001	1.000220551380930e+000
20	1.013211836423378e-001	2.580122754655957e-001	7.852115961743686e-001	1.000122286256523e+000
25	1.013211836423376e-001	2.580122754655948e-001	7.852115961743690e-001	1.000077545110675e+000
30	1.013211836423374e-001	2.580122754655982e-001	7.852115961743688e-001	1.000053507040659e+000
35	1.013211836423377e-001	2.580122754655942e-001	7.852115961743681e-001	1.000039126709146e+000
40	1.013211836423389e-001	2.580122754655933e-001	7.852115961743797e-001	1.000029848591206e+000

TABLE 3
DOMAIN IS AN IRREGULAR HEPTAHEDRON DISCRETISED AS A UNIT CUBE (6-PYRAMIDS) MINUS A TETRAHEDRON AND EACH PYRAMID IS DISCRITISED BY FOUR TETRAHEDRA
(EACH TETRAHEDRON IS DECOMPOSED INTO F0UR HEXAHEDRA)
OGLR=ORDER OF GAUSS LEGENDRE RULE

OGLR	III_1^1	III_2^1	III_3^1	III_4^1
5	9.954581672978760e-002	2.574236108595807e-001	7.718974082731483e-001	9.965258219759986e-001
10	9.954581672778172e-002	2.574236109206273e-001	7.718974082737680e-001	9.948959571274587e-001

15	9.954581672778155e-002	2.574236109206255e-001	7.718974082737676e-001	9.946095823321248e-001
20	9.954581672778166e-002	2.574236109206268e-001	7.718974082737675e-001	9.945118759015000e-001
25	9.954581672778147e-002	2.574236109206259e-001	7.718974082737677e-001	9.944673384663093e-001
30	9.954581672778129e-002	2.574236109206293e-001	7.718974082737676e-001	9.944433918097072e-001
35	9.954581672778159e-002	2.574236109206252e-001	7.718974082737671e-001	9.944290584626422e-001
40	9.954581672778282e-002	2.574236109206244e-001	7.718974082737785e-001	9.944198069266682e-001

TABLE 4

DOMAIN IS AN IRREGULAR HEPTAHEDRON DISCRETISED AS A DOMAIN (6-PYRAMIDS) PLUS A TETRAHEDRON WHERE EACH OF THE 3 PYRAMIDS(SQUARE BASE) ARE DISCRITISED BY FOUR TETRAHEDRA AND EACH OF THE OTHER 3 PYRAMIDS(PENTAGON BASE) ARE DISCRITISED BY FIVE TETRAHEDRA (EACH TETRAHEDRON IS DECOMPOSED INTO F0UR HEXAHEDRA)
OGLR=ORDER OF GAUSS LEGENDRE RULE

OGLR	III_1^1	III_2^1	III_3^1	III_4^1
5	9.954581673034073e-002	2.574236108720406e-001	7.718974082732784e-001	9.961442283927878e-001
10	9.954581672778178e-002	2.574236109206273e-001	7.718974082737680e-001	9.947907683783336e-001
15	9.954581672778159e-002	2.574236109206260e-001	7.718974082737677e-001	9.945612745394171e-001
20	9.954581672778172e-002	2.574236109206268e-001	7.718974082737675e-001	9.944842507710737e-001
25	9.954581672778151e-002	2.574236109206263e-001	7.718974082737672e-001	9.944494816323988e-001
30	9.954581672778128e-002	2.574236109206290e-001	7.718974082737674e-001	9.944309084228089e-001
35	9.954581672778176e-002	2.574236109206255e-001	7.718974082737669e-001	9.944198431719347e-001
40	9.954581672778221e-002	2.574236109206248e-001	7.718974082737751e-001	9.944127261477582e-001

TABLE 5
DOMAIN IS A UNIT CUBE DISCRITISED BY SIX TETRA HEDRA
(EACH TETRAHEDRON IS DECOMPOSED INTO THIRTY TWO HEXAHEDRA)
OGLR=ORDER OF GAUSS LEGENDRE RULE

OGLR	$\boldsymbol{III}_1^0$	III_2^0	III_3^0	III_4^0
5	1.013211836405128e-001	2.580122754750177e-001	7.852115961744891e-001	1.000541558072360e+000
10	1.013211836423382e-001	2.580122754655960e-001	7.852115961743676e-001	1.000080161885487e+000
15	1.013211836423376e-001	2.580122754655955e-001	7.852115961743696e-001	1.000026674687432e+000
20	1.013211836423380e-001	2.580122754655961e-001	7.852115961743671e-001	1.000010625685798e+000
25	1.013211836423373e-001	2.580122754655977e-001	7.852115961743734e-001	1.000006073926073e+000
30	1.013211836423391e-001	2.580122754655951e-001	7.852115961743696e-001	1.000003166084095e+000
35	1.013211836423367e-001	2.580122754655963e-001	7.852115961743693e-001	1.000002209277418e+000
40	1.013211836423374e-001	2.580122754655967e-001	7.852115961743480e-001	1.000001470999350e+000

TABLE 6
DOMAIN IS A UNIT CUBE DISCRITISED BY SIX PYRAMIDS(=24 TETRAHEDRA)
(EACH TETRAHEDRON IS DECOMPOSED INTO THIRTY TWO HEXAHEDRA)
OGLR=ORDER OF GAUSS LEGENDRE RULE

OGLR	$\boldsymbol{III}_1^0$	III_2^0	III_3^0	III_4^0
5	1.013211836423394e-001	2.580122754655437e-001	7.852115961743670e-001	1.000628783127993e+000
10	1.013211836423379e-001	2.580122754655969e-001	7.852115961743691e-001	1.000143497901664e+000
15	1.013211836423384e-001	2.580122754655957e-001	7.852115961743734e-001	1.000061249380577e+000
20	1.013211836423362e-001	2.580122754655962e-001	7.852115961743638e-001	1.000033676653095e+000
25	1.013211836423390e-001	2.580122754656025e-001	7.852115961743521e-001	1.000021240415194e+000
30	1.013211836423383e-001	2.580122754655928e-001	7.852115961743789e-001	1.000014601102844e+000
35	1.013211836423372e-001	2.580122754655936e-001	7.852115961743749e-001	1.000010647400004e+000
40	1.013211836423384e-001	2.580122754656105e-001	7.852115961743768e-001	1.000008105305021e+000

TABLE 7
DOMAIN IS AN IRREGULAR HEPTAHEDRON DISCRETISED AS A UNIT CUBE (6-PYRAMIDS) MINUS A TETRAHEDRON

AND EACH PYRAMID IS DISCRITISED BY FOUR TETRAHEDRA
(EACH TETRAHEDRON IS DECOMPOSED INTO THIRTY TWO HEXAHEDRA)
OGLR=ORDER OF GAUSS LEGENDRE RULE

OGLR	III_1^1	III_2^1	III_3^1	III_4^1
5	9.954581672778327e-002	2.574236109205748e-001	7.718974082737659e-001	9.950108079772150e-001
10	9.954581672778182e-002	2.574236109206280e-001	7.718974082737680e-001	9.945323906004613e-001
15	9.954581672778233e-002	2.574236109206268e-001	7.718974082737725e-001	9.944509158750174e-001
20	9.954581672778010e-002	2.574236109206273e-001	7.718974082737629e-001	9.944235402039482e-001

TABLE 8
DOMAIN IS AN IRREGULAR HEPTAHEDRON DISCRETISED AS A DOMAIN (6-PYRAMIDS) PLUS A TETRAHEDRON WHERE EACH OF THE 3 PYRAMID S(SQUARE BASE) ARE DISCRITISED BY FOUR TETRAHEDRA AND EACH OF THE OTHER 3 PYRAMIDS(PENTAGON BASE) ARE DISCRITISED BY FIVE TETRAHEDRA
(EACH TETRAHEDRON IS DECOMPOSED INTO THIRTY TWO HEXAHEDRA)
OGLR=ORDER OF GAUSS LEGENDRE RULE

OGLR	III_1^1	III_2^1	III_3^1	III_4^1
5	9.954581672771182e-002	2.574236109202270e-001	7.718974082737642e-001	9.949173386872332e-001
10	9.954581672778166e-002	2.574236109206279e-001	7.718974082737675e-001	9.945054967379960e-001
15	9.954581672778215e-002	2.574236109206268e-001	7.718974082737715e-001	9.944383845310796e-001
20	9.954581672778097e-002	2.574236109206279e-001	7.718974082737632e-001	9.944163214583467e-001

TABLE 9
DOMAIN IS A UNIT CUBE DISCRITISED BY SIX TETRA HEDRONS
(EACH TETRAHEDRON IS DECOMPOSED INTO TWO HUNDRED FIFTY SIX HEXAHEDRA)
OGLR=ORDER OF GAUSS LEGENDRE RULE

OGLR	$\boldsymbol{III}_1^0$	III_2^0	III_3^0	III_4^0
2	1.013203674348838e-001	2.580043877734050e-001	7.852106348288163e-001	1.002371551801550e+000
3	1.013211828522880e-001	2.580122858947788e-001	7.852115967789256e-001	1.000807760221705e+000
4	1.013211836442704e-001	2.580122754551041e-001	7.852115961740829e-001	1.000375868033130e+000
5	1.013211836423364e-001	2.580122754656039e-001	7.852115961743680e-001	1.000205374689130e+000
10	1.013211836423384e-001	2.580122754655955e-001	7.852115961743679e-001	1.000029631458212e+000

TABLE 10
DOMAIN IS A UNIT CUBE DISCRITISED BY SIX PYRAMIDS(=24 TETRAHEDRA)
(EACH TETRAHEDRON IS DECOMPOSED INTO TWO HUNDRED FIFTY SIX HEXAHEDRA)
OGLR=ORDER OF GAUSS LEGENDRE RULE

OGLR	$\boldsymbol{III}_1^0$	III_2^0	III_3^0	III_4^0
2	1.013208334385053e-001	2.580140862513543e-001	7.852119020811539e-001	1.001418294412512e+000
3	1.013211837125683e-001	2.580122744592550e-001	7.852115961249017e-001	1.000566005640103e+000
4	1.013211836423264e-001	2.580122754658746e-001	7.852115961743749e-001	1.000297938031433e+000
5	1.013211836423378e-001	2.580122754655975e-001	7.852115961743655e-001	1.000181368230410e+000
10	1.013211836423384e-001	2.580122754655989e-001	7.852115961743788e-001	1.000039539963383e+000

TABLE 11
DOMAIN IS AN IRREGULAR HEPTAHEDRON DISCRETISED AS A UNIT CUBE (6-PYRAMIDS) MINUS A TETRAHEDRON AND EACH PYRAMID IS DISCRITISED BY FOUR TETRAHEDRA
(EACH TETRAHEDRON IS DECOMPOSED INTO TWO HUNDRED FIFTY SIX HEXAHEDRA)
OGLR=ORDER OF GAUSS LEGENDRE RULE

OGLR	III_1^1	III_2^1	III_3^1	III_4^1
2	9.954547917158518e-002	2.574254145988052e-001	7.718977109978557e-001	9.957744565404281e-001
3	9.954581679577371e-002	2.574236099185355e-001	7.718974082245863e-001	9.949446912791101e-001
4	9.954581672777049e-002	2.574236109209048e-001	7.718974082737736e-001	9.946827660534204e-001

5	9.954581672778173e-002	2.574236109206285e-001	7.718974082737644e-001	9.945685785969760e-001
10	9.954581672778227e-002	2.574236109206299e-001	7.718974082737774e-001	9.944291577364538e-001

TABLE 12

DOMAIN IS AN IRREGULAR HEPTAHEDRON DISCRETISED AS A DOMAIN (6-PYRAMIDS) PLUS A TETRAHEDRON WHERE EACH OF THE 3 PYRAMIDS(SQUARE BASE) ARE DISCRITISED BY FOUR TETRAHEDRA AND EACH OF THE OTHER 3 PYRAMIDS(PENTAGON BASE) ARE DISCRITISED BY FIVE TETRAHEDRA (EACH TETRAHEDRON IS DECOMPOSED INTO TWO HUNDRED FIFTY SIX HEXAHEDRA)
OGLR=ORDER OF GAUSS LEGENDRE RULE

OGLR	III_1^1	III_2^1	III_3^1	III_4^1
2	9.954558848258124e-002	2.574251230689288e-001	7.718976913002109e-001	9.956517643192921e-001
3	9.954581679990247e-002	2.574236101022193e-001	7.718974082318901e-001	9.948815276829097e-001
4	9.954581672776572e-002	2.574236109208508e-001	7.718974082737725e-001	9.946442438102825e-001
5	9.954581672778187e-002	2.574236109206282e-001	7.718974082737651e-001	9.945427017244719e-001
10	9.954581672778218e-002	2.574236109206295e-001	7.718974082737756e-001	9.944220235521168e-001

We have coded the numerical schemes in MATLAB and some of these are listed below:

(1)**hexahedrasampleptsweights**

(2)**hexahedrasampleptsweights4eightdivisions**

(3)**hexahedrasampleptsweights4sixtyfourdivisions1X1**

(4)**integration_tetrahedron**

(5)**integration_tetrahedron32hexahedron**

(6)**integration_tetrahedron256hexahedron**

(7)**integration_tetrahedron_hexahedron**

(8)**integration_6pyramids_hexahedron**

(9)**integration_irregularheptahedron_hexahedron**

The above codes are appended to this paper

7. CONCLUSIONS

In this paper,we propose a method to discretise the physical domain in the shape of a linear polyhedron into an assemblage of all hexahedral finite elements. The idea is to generate a coarse mesh of all tetrahedrons for the given domain,Then divide each of these tetrahedron further into a refined mesh of all tetrahedrons, if necessary. Then finally, we divide each of these tetrahedron into four hexahedra.We have further demonstrated that each of these hexahedra can be divided into 2^3 and $(2^3)^2$ hexahedra. This generates an all hexahedral finite element mesh which can be used for various applications In order to achieve this we first establish a relation between the arbitrary linear tetrahedron and the standard tetrahedron.We then decompose the standard tetrahedron into four hexahedra. We transform each of these hexahedra into a 2-cube and discover an interesting fact that the Jacobian of these transformations is same and the transformations are also the same but in different order for all the four hexahedra.This fact can be used with great advantage to generate the numerical integration scheme for the standard tetrahedron and hence for the arbitrary linear tetrahedron. We have proposed three numerical schemes which decompose a arbitrary linear tetrahedron into 4, $4(2^3)$ and $4((2^3)^2$) hexahedra.These numerical schemes are applied to solve typical integrals over a unit cube and irregular heptahedron using Gauss Legendre Quadrature Rules. Matlab codes are developed and appended to this paper.We hope that this paper has ample ideas for applications to real life problems where numerical integration is of a prime importance

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APPENDIX

(1)PROGRAM-1

```
function[A,B,C,D,W]=hexahedrasampleptsweights(N)
format long e
[sn,cn]=glsampleptsweights(N);
%disp([sn cn])
M=N*N*N;
A(1:M,1)=zeros(M,1);B(1:M,1)=zeros(M,1);C(1:M,1)=zeros(M,1);D(1:M,1)=zeros(M,1);W(1:M,1)=zeros(M,1);
m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
 W(m,1)=a*b*c*abs(275/55296-(5/2304)*(r+s+t)-(5/27648)*r*s*t+49/55296*(r*s+s*t+r*t)-
(5/55296)*(r^2*s+r*s^2+r^2*t+r*t^2+s^2*t+s*t^2)+(13/55296)*(r^2+s^2+t^2)+1/55296*(r^2*s*t+r*s^2*t+r*s*t^2));
         A(m,1)=17/96-1/32*t-1/32*s+1/96*s*t+17/96*r-1/32*r*t-1/32*r*s+1/96*r*s*t;
         B(m,1)=17/96-1/32*r+17/96*s-1/32*t-1/32*r*s-1/32*s*t+1/96*r*t+1/96*r*s*t;
         C(m,1)=17/96+17/96*t-1/32*s-1/32*s*t-1/32*r-1/32*r*t+1/96*r*s+1/96*r*s*t;
         D(m,1)=1-(A(m,1)+B(m,1)+C(m,1));
         P(m,1)=m;
 end
end
end
%disp([P A B C D W])
table=[A B C D W];
```

(2)PROGRAM-2

```
function[A,B,C,D,W]=hexahedrasampleptsweights4eightdivisions(N)
format long e
[sn,cn]=glsampleptsweights(N);
%disp([sn cn])
M=N*N*N;
A=zeros(M,8);B=zeros(M,8);C=zeros(M,8);D=zeros(M,8);W=zeros(M,8);
m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;

%JACOBIAN  FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 1
WW =(r^2*s*t)/7077888 - (11*r^2*s)/7077888 - (11*r^2*t)/7077888 + (73*r^2)/7077888 + (r*s^2*t)/7077888 -
(11*r*s^2)/7077888 + (r*s*t^2)/7077888 - (13*r*s*t)/3538944 + (29*r*s)/786432 - (11*r*t^2)/7077888 +
(29*r*t)/786432 - (385*r)/1769472 - (11*s^2*t)/7077888 + (73*s^2)/7077888 - (11*s*t^2)/7077888 +
(29*s*t)/786432 - (385*s)/1769472 + (73*t^2)/7077888 - (385*t)/1769472 + 8107/7077888;%wght
```

```
W(m,1)=WW*a*b*c;
%COORDINATE  TRANSFORMATIONS    FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 1
A(m,1) =(27*r)/256 - (7*s)/768 - (7*t)/768 - (7*r*s)/768 - (7*r*t)/768 + (s*t)/768 + (r*s*t)/768 +
27/256;%phai
B(m,1) =(27*s)/256 - (7*r)/768 - (7*t)/768 - (7*r*s)/768 + (r*t)/768 - (7*s*t)/768 + (r*s*t)/768 +
27/256;%xhai
C(m,1) =(27*t)/256 - (7*s)/768 - (7*r)/768 + (r*s)/768 - (7*r*t)/768 - (7*s*t)/768 + (r*s*t)/768 +
27/256;%chai
D(m,1) =(13*r*s)/768 - (67*s)/768 - (67*t)/768 - (67*r)/768 + (13*r*t)/768 + (13*s*t)/768 - (r*s*t)/256 +
175/256;%dlta

%JACOBIAN  FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 2
WW =(r^2*s*t)/7077888 - (11*r^2*s)/7077888 - (11*r^2*t)/7077888 + (73*r^2)/7077888 + (r*s^2*t)/7077888 -
(11*r*s^2)/7077888 + (r*s*t^2)/7077888 - (11*r*s*t)/3538944 + (217*r*s)/7077888 - (11*r*t^2)/7077888 +
(217*r*t)/7077888 - (13*r)/73728 - (s^2*t)/786432 + (17*s^2)/2359296 - (s*t^2)/786432 + (71*s*t)/2359296 -
(59*s)/393216 + (17*t^2)/2359296 - (59*t)/393216 + 197/262144;%wght
W(m,2)=WW*a*b*c;
%COORDINATE  TRANSFORMATIONS    FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 2
A(m,2) =(27*r)/256 - (7*s)/256 - (7*t)/256 - (7*r*s)/768 - (7*r*t)/768 + (s*t)/256 + (r*s*t)/768 +
81/256;%phai
B(m,2) =(67*s)/768 - (7*r)/768 - (5*t)/768 - (7*r*s)/768 + (r*t)/768 - (5*s*t)/768 + (r*s*t)/768 +
67/768;%xhai
C(m,2) =(67*t)/768 - (5*s)/768 - (7*r)/768 + (r*s)/768 - (7*r*t)/768 - (5*s*t)/768 + (r*s*t)/768 +
67/768;%chai
D(m,2)=(13*r*s)/768 - (41*s)/768 - (41*t)/768 - (67*r)/768 + (13*r*t)/768 + (7*s*t)/768 - (r*s*t)/256 +
391/768;%dlta
%x=phi,y=xai,z=chai
%JACOBIAN  FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 3
WW =(r^2*s*t)/7077888 - (11*r^2*s)/7077888 - (r^2*t)/786432 + (17*r^2)/2359296 + (r*s^2*t)/7077888 -
(11*r*s^2)/7077888 + (r*s*t^2)/7077888 - (11*r*s*t)/3538944 + (217*r*s)/7077888 - (r*t^2)/786432 +
(71*r*t)/2359296 - (59*r)/393216 - (11*s^2*t)/7077888 + (73*s^2)/7077888 - (11*s*t^2)/7077888 +
(217*s*t)/7077888 - (13*s)/73728 + (17*t^2)/2359296 - (59*t)/393216 + 197/262144;
W(m,3)=WW*a*b*c;
%COORDINATE  TRANSFORMATIONS    FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 3
A(m,3) =(67*r)/768 - (7*s)/768 - (5*t)/768 - (7*r*s)/768 - (5*r*t)/768 + (s*t)/768 + (r*s*t)/768 + 67/768;
B(m,3) =(27*s)/256 - (7*r)/256 - (7*t)/256 - (7*r*s)/768 + (r*t)/256 - (7*s*t)/768 + (r*s*t)/768 + 81/256;
C(m,3) =(67*t)/768 - (7*s)/768 - (5*r)/768 + (r*s)/768 - (5*r*t)/768 - (7*s*t)/768 + (r*s*t)/768 + 67/768;
D(m,3) =(13*r*s)/768 - (67*s)/768 - (41*t)/768 - (41*r)/768 + (7*r*t)/768 + (13*s*t)/768 - (r*s*t)/256 +
391/768;
%x=phi,y=xai,z=chai
%JACOBIAN  FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 4
WW=(r^2*s*t)/7077888 - (11*r^2*s)/7077888 - (r^2*t)/786432 + (17*r^2)/2359296 + (r*s^2*t)/7077888 -
(11*r*s^2)/7077888 + (r*s*t^2)/7077888 - (r*s*t)/393216 + (173*r*s)/7077888 - (r*t^2)/786432 +
(59*r*t)/2359296 - (143*r)/1179648 - (s^2*t)/786432 + (17*s^2)/2359296 - (s*t^2)/786432 + (59*s*t)/2359296 -
(143*s)/1179648 + (11*t^2)/2359296 - (7*t)/73728 + 1133/2359296;
W(m,4)=WW*a*b*c;
%COORDINATE  TRANSFORMATIONS    FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 4
A(m,4) =(67*r)/768 - (7*s)/256 - (5*t)/256 - (7*r*s)/768 - (5*r*t)/768 + (s*t)/256 + (r*s*t)/768 + 67/256;
B(m,4)=(67*s)/768 - (7*r)/256 - (5*t)/256 - (7*r*s)/768 + (r*t)/256 - (5*s*t)/768 + (r*s*t)/768 + 67/256;
C(m,4) =(19*t)/256 - (5*s)/768 - (5*r)/768 + (r*s)/768 - (5*r*t)/768 - (5*s*t)/768 + (r*s*t)/768 + 19/256;
D(m,4) =(13*r*s)/768 - (41*s)/768 - (9*t)/256 - (41*r)/768 + (7*r*t)/768 + (7*s*t)/768 - (r*s*t)/256 +
103/256;
%x=phi,y=xai,z=chai
% ON HEXAHEDRA H1    X=X1+(X2-X1)*phi+(X3-X1)*xai+(X4-X1)*chai
%.....................................................................................................
.....................................
%JACOBIAN  FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 5
WW=(r^2*s*t)/7077888 - (r^2*s)/786432 - (11*r^2*t)/7077888 + (17*r^2)/2359296 + (r*s^2*t)/7077888 -
(r*s^2)/786432 + (r*s*t^2)/7077888 - (11*r*s*t)/3538944 + (71*r*s)/2359296 - (11*r*t^2)/7077888 +
(217*r*t)/7077888 - (59*r)/393216 - (11*s^2*t)/7077888 + (17*s^2)/2359296 - (11*s*t^2)/7077888 +
(217*s*t)/7077888 - (59*s)/393216 + (73*t^2)/7077888 - (13*t)/73728 + 197/262144;
W(m,5)=WW*a*b*c;
%COORDINATE  TRANSFORMATIONS    FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 5
A(m,5)=(67*r)/768 - (5*s)/768 - (7*t)/768 - (5*r*s)/768 - (7*r*t)/768 + (s*t)/768 + (r*s*t)/768 + 67/768;
B(m,5)=(67*s)/768 - (5*r)/768 - (7*t)/768 - (5*r*s)/768 + (r*t)/768 - (7*s*t)/768 + (r*s*t)/768 + 67/768;
C(m,5)=(27*t)/256 - (7*s)/256 - (7*r)/256 + (r*s)/256 - (7*r*t)/768 - (7*s*t)/768 + (r*s*t)/768 + 81/256;
D(m,5) =(7*r*s)/768 - (41*s)/768 - (67*t)/768 - (41*r)/768 + (13*r*t)/768 + (13*s*t)/768 - (r*s*t)/256 +
391/768;

%JACOBIAN  FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 6
WW=(r^2*s*t)/7077888 - (r^2*s)/786432 - (11*r^2*t)/7077888 + (17*r^2)/2359296 +
(r*s^2*t)/7077888 - (r*s^2)/786432 + (r*s*t^2)/7077888 - (r*s*t)/393216 + (59*r*s)/2359296 -
(11*r*t^2)/7077888 + (173*r*t)/7077888 - (143*r)/1179648 - (s^2*t)/786432 + (11*s^2)/2359296 - (s*t^2)/786432
+ (59*s*t)/2359296 - (7*s)/73728 + (17*t^2)/2359296 - (143*t)/1179648 + 1133/2359296;
W(m,6)=WW*a*b*c;
%COORDINATE  TRANSFORMATIONS    FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 6
A(m,6)=(67*r)/768 - (5*s)/256 - (7*t)/256 - (5*r*s)/768 - (7*r*t)/768 + (s*t)/256 + (r*s*t)/768 + 67/256;
B(m,6)=(19*s)/256 - (5*r)/768 - (5*t)/768 - (5*r*s)/768 + (r*t)/768 - (5*s*t)/768 + (r*s*t)/768 + 19/256;
```

```
C(m,6)=(67*t)/768 - (5*s)/256 - (7*r)/256 + (r*s)/256 - (7*r*t)/768 - (5*s*t)/768 + (r*s*t)/768 + 67/256;
D(m,6) =(7*r*s)/768 - (9*s)/256 - (41*t)/768 - (41*r)/768 + (13*r*t)/768 + (7*s*t)/768 - (r*s*t)/256 +
103/256;

%JACOBIAN  FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION  7
WW=(r^2*s*t)/7077888 - (r^2*s)/786432 - (r^2*t)/786432 + (11*r^2)/2359296 + (r*s^2*t)/7077888 -
(r*s^2)/786432 + (r*s*t^2)/7077888 - (r*s*t)/393216 + (59*r*s)/2359296 - (r*t^2)/786432 + (59*r*t)/2359296 -
(7*r)/73728 - (11*s^2*t)/7077888 + (17*s^2)/2359296 - (11*s*t^2)/7077888 + (173*s*t)/7077888 -
(143*s)/1179648 + (17*t^2)/2359296 - (143*t)/1179648 + 1133/2359296;
W(m,7)=WW*a*b*c;
%COORDINATE  TRANSFORMATIONS    FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 7
A(m,7) =(19*r)/256 - (5*s)/768 - (5*t)/768 - (5*r*s)/768 - (5*r*t)/768 + (s*t)/768 + (r*s*t)/768 + 19/256;
B(m,7)=(67*s)/768 - (5*r)/256 - (7*t)/256 - (5*r*s)/768 + (r*t)/256 - (7*s*t)/768 + (r*s*t)/768 + 67/256;
C(m,7)=(67*t)/768 - (7*s)/256 - (5*r)/256 + (r*s)/256 - (5*r*t)/768 - (7*s*t)/768 + (r*s*t)/768 + 67/256;
D(m,7) =(7*r*s)/768 - (41*s)/768 - (41*t)/768 - (9*r)/256 + (7*r*t)/768 + (13*s*t)/768 - (r*s*t)/256 +
103/256;

%JACOBIAN  FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION  8
WW=(r^2*s*t)/7077888 - (r^2*s)/786432 - (r^2*t)/786432 + (11*r^2)/2359296 + (r*s^2*t)/7077888 -
(r*s^2)/786432 + (r*s*t^2)/7077888 - (7*r*s*t)/3538944 + (47*r*s)/2359296 - (r*t^2)/786432 + (47*r*t)/2359296
- (5*r)/65536 - (s^2*t)/786432 + (11*s^2)/2359296 - (s*t^2)/786432 + (47*s*t)/2359296 - (5*s)/65536 +
(11*t^2)/2359296 - (5*t)/65536 + 81/262144;
W(m,8)=WW*a*b*c;
%COORDINATE  TRANSFORMATIONS    FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 8
A(m,8)=(19*r)/256 - (5*s)/256 - (5*t)/256 - (5*r*s)/768 - (5*r*t)/768 + (s*t)/256 + (r*s*t)/768 + 57/256;
B(m,8)=(19*s)/256 - (5*r)/256 - (5*t)/256 - (5*r*s)/768 + (r*t)/256 - (5*s*t)/768 + (r*s*t)/768 + 57/256;
C(m,8)=(19*t)/256 - (5*s)/256 - (5*r)/256 + (r*s)/256 - (5*r*t)/768 - (5*s*t)/768 + (r*s*t)/768 + 57/256;
D(m,8) =(7*r*s)/768 - (9*s)/256 - (9*t)/256 - (9*r)/256 + (7*r*t)/768 + (7*s*t)/768 - (r*s*t)/256 + 85/256;
end
end
end
```

(3)PROGRAM-3

```
function[A,B,C,D,W]=hexahedrasampleptsweights4sixtyfourdivisions1X1(N,L)
%global A B C D W
format long e
%skip=1%donot display the sample points and weights  when skip=0
%any nonzero value of skip will display the sample points and weights
[sn,cn]=glsampleptsweights(N);
%disp([sn cn])
M=N*N*N;

switch L
%1=============================================================================================================
==========================================
    case 1
  A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;

%COORDINATE  TRANSFORMATIONS AND JACOBIANS FOR  HEXAHEDRA  H1 H2 H3 H4  AT  DIVISION 1
A(m,1)=(353*r)/6144 - (5*s)/2048 - (5*t)/2048 - (5*r*s)/2048 - (5*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 +
353/6144;
B(m,1)=(353*s)/6144 - (5*r)/2048 - (5*t)/2048 - (5*r*s)/2048 + (r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 +
353/6144;
C(m,1)=(353*t)/6144 - (5*s)/2048 - (5*r)/2048 + (r*s)/6144 - (5*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 +
353/6144;
D(m,1)=(29*r*s)/6144 - (323*s)/6144 - (323*t)/6144 - (323*r)/6144 + (29*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 1695/2048;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (23*r^2*t)/905969664 + (337*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (29*r*s*t)/452984832 +
(1189*r*s)/905969664 - (23*r*t^2)/905969664 + (1189*r*t)/905969664 - (1265*r)/75497472 - (23*s^2*t)/905969664
+ (337*s^2)/905969664 - (23*s*t^2)/905969664 + (1189*s*t)/905969664 - (1265*s)/75497472 + (337*t^2)/905969664
- (1265*t)/75497472 + 170867/905969664;
   %(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (23*r^2*t)/905969664 + (337*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (29*r*s*t)/452984832 +
(1189*r*s)/905969664 - (23*r*t^2)/905969664 + (1189*r*t)/905969664 - (1265*r)/75497472 - (23*s^2*t)/905969664
+ (337*s^2)/905969664 - (23*s*t^2)/905969664 + (1189*s*t)/905969664 - (1265*s)/75497472 + (337*t^2)/905969664
- (1265*t)/75497472 + 170867/905969664
 W(m,1)=WT*a*b*c;
end
end
end
 case 2
```

```
 A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;

%2=============================================================================================================
=========================================
%COORDINATE  TRANSFORMATIONS AND JACOBIANS FOR  HEXAHEDRA  H1 H2 H3 H4  AT DIVISION 2
A(m,1)=(353*r)/6144 - (15*s)/2048 - (15*t)/2048 - (5*r*s)/2048 - (5*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 +
353/2048;
B(m,1)=(323*s)/6144 - (5*r)/2048 - (13*t)/6144 - (5*r*s)/2048 + (r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 +
323/6144;
C(m,1)=(323*t)/6144 - (13*s)/6144 - (5*r)/2048 + (r*s)/6144 - (5*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 +
323/6144;
D(m,1)=(29*r*s)/6144 - (265*s)/6144 - (265*t)/6144 - (323*r)/6144 + (29*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 4439/6144;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (23*r^2*t)/905969664 + (337*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (r*s*t)/16777216 + (1097*r*s)/905969664 -
(23*r*t^2)/905969664 + (1097*r*t)/905969664 - (1729*r)/113246208 - (7*s^2*t)/301989888 + (97*s^2)/301989888 -
(7*s*t^2)/301989888 + (359*s*t)/301989888 - (2149*s)/150994944 + (97*t^2)/301989888 - (2149*t)/150994944 +
47285/301989888;
 %  (r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (23*r^2*t)/905969664 + (337*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (r*s*t)/16777216 + (1097*r*s)/905969664 -
(23*r*t^2)/905969664 + (1097*r*t)/905969664 - (1729*r)/113246208 - (7*s^2*t)/301989888 + (97*s^2)/301989888 -
(7*s*t^2)/301989888 + (359*s*t)/301989888 - (2149*s)/150994944 + (97*t^2)/301989888 - (2149*t)/150994944 +
47285/301989888
W(m,1)=WT*a*b*c;
end
end
end
 case 3
     A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%3=============================================================================================================
=========================================
%COORDINATE  TRANSFORMATIONS AND JACOBIANS FOR  HEXAHEDRA  H1 H2 H3 H4  AT DIVISION 3
A(m,1)=(323*r)/6144 - (5*s)/2048 - (13*t)/6144 - (5*r*s)/2048 - (13*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 +
323/6144;
B(m,1)=(353*s)/6144 - (15*r)/2048 - (15*t)/2048 - (5*r*s)/2048 + (r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 +
353/2048;
C(m,1)=(323*t)/6144 - (5*s)/2048 - (13*r)/6144 + (r*s)/6144 - (13*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 +
323/6144;
D(m,1)=(29*r*s)/6144 - (323*s)/6144 - (265*t)/6144 - (265*r)/6144 + (23*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 4439/6144;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (7*r^2*t)/301989888 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (r*s*t)/16777216 + (1097*r*s)/905969664 -
(7*r*t^2)/301989888 + (359*r*t)/301989888 - (2149*r)/150994944 - (23*s^2*t)/905969664 + (337*s^2)/905969664 -
(23*s*t^2)/905969664 + (1097*s*t)/905969664 - (1729*s)/113246208 + (97*t^2)/301989888 - (2149*t)/150994944 +
47285/301989888;
   %(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (7*r^2*t)/301989888 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (r*s*t)/16777216 + (1097*r*s)/905969664 -
(7*r*t^2)/301989888 + (359*r*t)/301989888 - (2149*r)/150994944 - (23*s^2*t)/905969664 + (337*s^2)/905969664 -
(23*s*t^2)/905969664 + (1097*s*t)/905969664 - (1729*s)/113246208 + (97*t^2)/301989888 - (2149*t)/150994944 +
47285/301989888
W(m,1)=WT*a*b*c;
end
end
end
 case 4
    A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%4=============================================================================================================
=========================================
%COORDINATE  TRANSFORMATIONS AND JACOBIANS FOR  HEXAHEDRA  H1 H2 H3 H4  AT DIVISION 4
```

```
A(m,1)=(323*r)/6144 - (15*s)/2048 - (13*t)/2048 - (5*r*s)/2048 - (13*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 +
323/2048;
B(m,1)=(323*s)/6144 - (15*r)/2048 - (13*t)/2048 - (5*r*s)/2048 + (r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 +
323/2048;
C(m,1)=(99*t)/2048 - (13*s)/6144 - (13*r)/6144 + (r*s)/6144 - (13*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 +
99/2048;
D(m,1)=(29*r*s)/6144 - (265*s)/6144 - (73*t)/2048 - (265*r)/6144 + (23*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 1303/2048;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (7*r^2*t)/301989888 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (335*r*s)/301989888
- (7*r*t^2)/301989888 + (331*r*t)/301989888 - (1955*r)/150994944 - (7*s^2*t)/301989888 + (97*s^2)/301989888 -
(7*s*t^2)/301989888 + (331*s*t)/301989888 - (1955*s)/150994944 + (83*t^2)/301989888 - (451*t)/37748736 +
39077/301989888;
  % (r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (7*r^2*t)/301989888 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (335*r*s)/301989888
- (7*r*t^2)/301989888 + (331*r*t)/301989888 - (1955*r)/150994944 - (7*s^2*t)/301989888 + (97*s^2)/301989888 -
(7*s*t^2)/301989888 + (331*s*t)/301989888 - (1955*s)/150994944 + (83*t^2)/301989888 - (451*t)/37748736 +
39077/301989888
W(m,1)=WT*a*b*c;
end
end
end
%end%if skip==0
%if skip==1
 case 5
        A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%5=======================================================================================================
=========================================
%COORDINATE  TRANSFORMATIONS AND JACOBIANS FOR  HEXAHEDRA  H1 H2 H3 H4  AT DIVISION 5
A(m,1)=(323*r)/6144 - (13*s)/6144 - (5*t)/2048 - (13*r*s)/6144 - (5*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 +
323/6144;
B(m,1)=(323*s)/6144 - (13*r)/6144 - (5*t)/2048 - (13*r*s)/6144 + (r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 +
323/6144;
C(m,1)=(353*t)/6144 - (15*s)/2048 - (15*r)/2048 + (r*s)/2048 - (5*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 +
353/2048;
D(m,1)=(23*r*s)/6144 - (265*s)/6144 - (323*t)/6144 - (265*r)/6144 + (29*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 4439/6144;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (23*r^2*t)/905969664 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (r*s*t)/16777216 + (359*r*s)/301989888 -
(23*r*t^2)/905969664 + (1097*r*t)/905969664 - (2149*r)/150994944 - (23*s^2*t)/905969664 + (97*s^2)/301989888
- (23*s*t^2)/905969664 + (1097*s*t)/905969664 - (2149*s)/150994944 + (337*t^2)/905969664 - (1729*t)/113246208
+ 47285/301989888;
  % (r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (23*r^2*t)/905969664 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (r*s*t)/16777216 + (359*r*s)/301989888 -
(23*r*t^2)/905969664 + (1097*r*t)/905969664 - (2149*r)/150994944 - (23*s^2*t)/905969664 + (97*s^2)/301989888
- (23*s*t^2)/905969664 + (1097*s*t)/905969664 - (2149*s)/150994944 + (337*t^2)/905969664 - (1729*t)/113246208
+ 47285/301989888
W(m,1)=WT*a*b*c;
end
end
end
 case 6
        A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%6=======================================================================================================
=========================================
A(m,1)=(323*r)/6144 - (13*s)/2048 - (15*t)/2048 - (13*r*s)/6144 - (5*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 +
323/2048;
B(m,1)=(99*s)/2048 - (13*r)/6144 - (13*t)/6144 - (13*r*s)/6144 + (r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 +
99/2048;
C(m,1)=(323*t)/6144 - (13*s)/2048 - (15*r)/2048 + (r*s)/2048 - (5*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 +
323/2048;
D(m,1)=(23*r*s)/6144 - (73*s)/2048 - (265*t)/6144 - (265*r)/6144 + (29*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 1303/2048;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (23*r^2*t)/905969664 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (331*r*s)/301989888
- (23*r*t^2)/905969664 + (335*r*t)/301989888 - (1955*r)/150994944 - (7*s^2*t)/301989888 + (83*s^2)/301989888
```

```
- (7*s*t^2)/301989888 + (331*s*t)/301989888 - (451*s)/37748736 + (97*t^2)/301989888 - (1955*t)/150994944 +
39077/301989888;
  % (r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (23*r^2*t)/905969664 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (331*r*s)/301989888
- (23*r*t^2)/905969664 + (335*r*t)/301989888 - (1955*r)/150994944 - (7*s^2*t)/301989888 + (83*s^2)/301989888
- (7*s*t^2)/301989888 + (331*s*t)/301989888 - (451*s)/37748736 + (97*t^2)/301989888 - (1955*t)/150994944 +
39077/301989888
W(m,1)=WT*a*b*c;
end
end
end
 case 7
        A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%7=======================================================================================================
=======================================
A(m,1)=(99*r)/2048 - (13*s)/6144 - (13*t)/6144 - (13*r*s)/6144 - (13*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 +
99/2048;
B(m,1)=(323*s)/6144 - (13*r)/2048 - (15*t)/2048 - (13*r*s)/6144 + (r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 +
323/2048;
C(m,1)=(323*t)/6144 - (15*s)/2048 - (13*r)/2048 + (r*s)/2048 - (13*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 +
323/2048;
D(m,1)=(23*r*s)/6144 - (265*s)/6144 - (265*t)/6144 - (73*r)/2048 + (23*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 1303/2048;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (7*r^2*t)/301989888 + (83*r^2)/301989888 + (r*s^2*t)/905969664
- (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (331*r*s)/301989888 -
(7*r*t^2)/301989888 + (331*r*t)/301989888 - (451*r)/37748736 - (23*s^2*t)/905969664 + (97*s^2)/301989888 -
(23*s*t^2)/905969664 + (335*s*t)/301989888 - (1955*s)/150994944 + (97*t^2)/301989888 - (1955*t)/150994944 +
39077/301989888;
 %  (r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (7*r^2*t)/301989888 + (83*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (331*r*s)/301989888
- (7*r*t^2)/301989888 + (331*r*t)/301989888 - (451*r)/37748736 - (23*s^2*t)/905969664 + (97*s^2)/301989888 -
(23*s*t^2)/905969664 + (335*s*t)/301989888 - (1955*s)/150994944 + (97*t^2)/301989888 - (1955*t)/150994944 +
39077/301989888
W(m,1)=WT*a*b*c;
end
end
end
 case 8
        A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%8=======================================================================================================
=======================================
A(m,1)=(99*r)/2048 - (13*s)/2048 - (13*t)/2048 - (13*r*s)/6144 - (13*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 +
297/2048;
B(m,1)=(99*s)/2048 - (13*r)/2048 - (13*t)/2048 - (13*r*s)/6144 + (r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 +
297/2048;
C(m,1)=(99*t)/2048 - (13*s)/2048 - (13*r)/2048 + (r*s)/2048 - (13*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 +
297/2048;
D(m,1)=(23*r*s)/6144 - (73*s)/2048 - (73*t)/2048 - (73*r)/2048 + (23*r*t)/6144 + (23*s*t)/6144 - (r*s*t)/2048
+ 1157/2048;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (7*r^2*t)/301989888 + (83*r^2)/301989888 + (r*s^2*t)/905969664
- (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (101*r*s)/100663296 -
(7*r*t^2)/301989888 + (101*r*t)/100663296 - (91*r)/8388608 - (7*s^2*t)/301989888 + (83*s^2)/301989888 -
(7*s*t^2)/301989888 + (101*s*t)/100663296 - (91*s)/8388608 + (83*t^2)/301989888 - (91*t)/8388608 +
3577/33554432;
%   (r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (7*r^2*t)/301989888 + (83*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (101*r*s)/100663296
- (7*r*t^2)/301989888 + (101*r*t)/100663296 - (91*r)/8388608 - (7*s^2*t)/301989888 + (83*s^2)/301989888 -
(7*s*t^2)/301989888 + (101*s*t)/100663296 - (91*s)/8388608 + (83*t^2)/301989888 - (91*t)/8388608 +
3577/33554432
W(m,1)=WT*a*b*c;
end
end
end
%end%if skip==1
%if skip==0
 case 9
```

```
        A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%9=======================================================================================================================
=========================================
A(m,1)=(353*r)/6144 - (25*s)/2048 - (25*t)/2048 - (5*r*s)/2048 - (5*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144 +
1765/6144;
B(m,1)=(293*s)/6144 - (5*r)/2048 - (11*t)/6144 - (5*r*s)/2048 + (r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 +
293/6144;
C(m,1)=(293*t)/6144 - (11*s)/6144 - (5*r)/2048 + (r*s)/6144 - (5*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 +
293/6144;
D(m,1)=(29*r*s)/6144 - (69*s)/2048 - (69*t)/2048 - (323*r)/6144 + (29*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 3793/6144;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (23*r^2*t)/905969664 + (337*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (335*r*s)/301989888
- (23*r*t^2)/905969664 + (335*r*t)/301989888 - (3121*r)/226492416 - (19*s^2*t)/905969664 +
(245*s^2)/905969664 - (19*s*t^2)/905969664 + (973*s*t)/905969664 - (1349*s)/113246208 + (245*t^2)/905969664 -
(1349*t)/113246208 + 38513/301989888;
 %  (r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (23*r^2*t)/905969664 + (337*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (335*r*s)/301989888
- (23*r*t^2)/905969664 + (335*r*t)/301989888 - (3121*r)/226492416 - (19*s^2*t)/905969664 +
(245*s^2)/905969664 - (19*s*t^2)/905969664 + (973*s*t)/905969664 - (1349*s)/113246208 + (245*t^2)/905969664 -
(1349*t)/113246208 + 38513/301989888
W(m,1)=WT*a*b*c;
end
end
end
 case 10
        A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%10======================================================================================================================
=========================================
A(m,1)=(353*r)/6144 - (35*s)/2048 - (35*t)/2048 - (5*r*s)/2048 - (5*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144 +
2471/6144;
B(m,1)=(263*s)/6144 - (5*r)/2048 - (3*t)/2048 - (5*r*s)/2048 + (r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 +
263/6144;
C(m,1)=(263*t)/6144 - (3*s)/2048 - (5*r)/2048 + (r*s)/6144 - (5*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 +
263/6144;
D(m,1)=(29*r*s)/6144 - (149*s)/6144 - (149*t)/6144 - (323*r)/6144 + (29*r*t)/6144 + (11*s*t)/6144 -
(r*s*t)/2048 + 1049/2048;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (23*r^2*t)/905969664 + (337*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (913*r*s)/905969664
- (23*r*t^2)/905969664 + (913*r*t)/905969664 - (29*r)/2359296 - (17*s^2*t)/905969664 + (199*s^2)/905969664 -
(17*s*t^2)/905969664 + (877*s*t)/905969664 - (493*s)/50331648 + (199*t^2)/905969664 - (493*t)/50331648 +
91919/905969664;
 %  (r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (23*r^2*t)/905969664 + (337*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (913*r*s)/905969664
- (23*r*t^2)/905969664 + (913*r*t)/905969664 - (29*r)/2359296 - (17*s^2*t)/905969664 + (199*s^2)/905969664 -
(17*s*t^2)/905969664 + (877*s*t)/905969664 - (493*s)/50331648 + (199*t^2)/905969664 - (493*t)/50331648 +
91919/905969664
W(m,1)=WT*a*b*c;
end
end
end
 case 11
        A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%11======================================================================================================================
=========================================
A(m,1)=(323*r)/6144 - (25*s)/2048 - (65*t)/6144 - (5*r*s)/2048 - (13*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144
+ 1615/6144;
B(m,1)=(293*s)/6144 - (15*r)/2048 - (11*t)/2048 - (5*r*s)/2048 + (r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 +
293/2048;
```

```
C(m,1)=(271*t)/6144 - (11*s)/6144 - (13*r)/6144 + (r*s)/6144 - (13*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 +
271/6144;
D(m,1)=(29*r*s)/6144 - (69*s)/2048 - (173*t)/6144 - (265*r)/6144 + (23*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 3379/6144;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (7*r^2*t)/301989888 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (913*r*s)/905969664
- (7*r*t^2)/301989888 + (101*r*t)/100663296 - (587*r)/50331648 - (19*s^2*t)/905969664 + (245*s^2)/905969664 -
(19*s*t^2)/905969664 + (299*s*t)/301989888 - (2453*s)/226492416 + (23*t^2)/100663296 - (1487*t)/150994944 +
31645/301989888;
 %  (r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (7*r^2*t)/301989888 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (913*r*s)/905969664
- (7*r*t^2)/301989888 + (101*r*t)/100663296 - (587*r)/50331648 - (19*s^2*t)/905969664 + (245*s^2)/905969664 -
(19*s*t^2)/905969664 + (299*s*t)/301989888 - (2453*s)/226492416 + (23*t^2)/100663296 - (1487*t)/150994944 +
31645/301989888
W(m,1)=WT*a*b*c;
end
end
end
 case 12
        A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%12=============================================================================================================
=========================================
A(m,1)=(323*r)/6144 - (35*s)/2048 - (91*t)/6144 - (5*r*s)/2048 - (13*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144
+ 2261/6144;
B(m,1)=(263*s)/6144 - (15*r)/2048 - (9*t)/2048 - (5*r*s)/2048 + (r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 +
263/2048;
C(m,1)=(245*t)/6144 - (3*s)/2048 - (13*r)/6144 + (r*s)/6144 - (13*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 +
245/6144;
D(m,1)=(29*r*s)/6144 - (149*s)/6144 - (127*t)/6144 - (265*r)/6144 + (23*r*t)/6144 + (11*s*t)/6144 -
(r*s*t)/2048 + 2849/6144;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (7*r^2*t)/301989888 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (821*r*s)/905969664
- (7*r*t^2)/301989888 + (275*r*t)/301989888 - (1567*r)/150994944 - (17*s^2*t)/905969664 + (199*s^2)/905969664
- (17*s*t^2)/905969664 + (809*s*t)/905969664 - (4039*s)/452984832 + (55*t^2)/301989888 - (599*t)/75497472 +
24989/301989888;
%   (r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (7*r^2*t)/301989888 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (821*r*s)/905969664
- (7*r*t^2)/301989888 + (275*r*t)/301989888 - (1567*r)/150994944 - (17*s^2*t)/905969664 + (199*s^2)/905969664
- (17*s*t^2)/905969664 + (809*s*t)/905969664 - (4039*s)/452984832 + (55*t^2)/301989888 - (599*t)/75497472 +
24989/301989888
W(m,1)=WT*a*b*c;
end
end
end
 case 13
        A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%13=============================================================================================================
=========================================
A(m,1)=(323*r)/6144 - (65*s)/6144 - (25*t)/2048 - (13*r*s)/6144 - (5*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144
+ 1615/6144;
B(m,1)=(271*s)/6144 - (13*r)/6144 - (11*t)/6144 - (13*r*s)/6144 + (r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 +
271/6144;
C(m,1)=(293*t)/6144 - (11*s)/2048 - (15*r)/2048 + (r*s)/2048 - (5*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 +
293/2048;
D(m,1)=(23*r*s)/6144 - (173*s)/6144 - (69*t)/2048 - (265*r)/6144 + (29*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 3379/6144;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (23*r^2*t)/905969664 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (101*r*s)/100663296
- (23*r*t^2)/905969664 + (913*r*t)/905969664 - (587*r)/50331648 - (19*s^2*t)/905969664 + (23*s^2)/100663296 -
(19*s*t^2)/905969664 + (299*s*t)/301989888 - (1487*s)/150994944 + (245*t^2)/905969664 - (2453*t)/226492416 +
31645/301989888;
%   (r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (23*r^2*t)/905969664 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (101*r*s)/100663296
- (23*r*t^2)/905969664 + (913*r*t)/905969664 - (587*r)/50331648 - (19*s^2*t)/905969664 + (23*s^2)/100663296 -
(19*s*t^2)/905969664 + (299*s*t)/301989888 - (1487*s)/150994944 + (245*t^2)/905969664 - (2453*t)/226492416 +
31645/301989888
```

```
W(m,1)=WT*a*b*c;
end
end
end
 case 14
        A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%14=============================================================================================================
=========================================
A(m,1)=(323*r)/6144 - (91*s)/6144 - (35*t)/2048 - (13*r*s)/6144 - (5*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144
+ 2261/6144;
B(m,1)=(245*s)/6144 - (13*r)/6144 - (3*t)/2048 - (13*r*s)/6144 + (r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 +
245/6144;
C(m,1)=(263*t)/6144 - (9*s)/2048 - (15*r)/2048 + (r*s)/2048 - (5*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 +
263/2048;
D(m,1)=(23*r*s)/6144 - (127*s)/6144 - (149*t)/6144 - (265*r)/6144 + (29*r*t)/6144 + (11*s*t)/6144 -
(r*s*t)/2048 + 2849/6144;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (23*r^2*t)/905969664 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (275*r*s)/301989888 -
(23*r*t^2)/905969664 + (821*r*t)/905969664 - (1567*r)/150994944 - (17*s^2*t)/905969664 + (55*s^2)/301989888 -
(17*s*t^2)/905969664 + (809*s*t)/905969664 - (599*s)/75497472 + (199*t^2)/905969664 - (4039*t)/452984832 +
24989/301989888;
 %  (r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (23*r^2*t)/905969664 + (97*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (275*r*s)/301989888 -
(23*r*t^2)/905969664 + (821*r*t)/905969664 - (1567*r)/150994944 - (17*s^2*t)/905969664 + (55*s^2)/301989888 -
(17*s*t^2)/905969664 + (809*s*t)/905969664 - (599*s)/75497472 + (199*t^2)/905969664 - (4039*t)/452984832 +
24989/301989888
W(m,1)=WT*a*b*c;
end
end
end
 case 15
        A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%15=============================================================================================================
=========================================
A(m,1)=(99*r)/2048 - (65*s)/6144 - (65*t)/6144 - (13*r*s)/6144 - (13*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144
+ 495/2048;
B(m,1)=(271*s)/6144 - (13*r)/2048 - (11*t)/2048 - (13*r*s)/6144 + (r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 +
271/2048;
C(m,1)=(271*t)/6144 - (11*s)/2048 - (13*r)/2048 + (r*s)/2048 - (13*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 +
271/2048;
D(m,1)=(23*r*s)/6144 - (173*s)/6144 - (173*t)/6144 - (73*r)/2048 + (23*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 1011/2048;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (7*r^2*t)/301989888 + (83*r^2)/301989888 + (r*s^2*t)/905969664
- (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (275*r*s)/301989888 - (7*r*t^2)/301989888
+ (275*r*t)/301989888 - (23*r)/2359296 - (19*s^2*t)/905969664 + (23*s^2)/100663296 - (19*s*t^2)/905969664 +
(821*s*t)/905969664 - (1349*s)/150994944 + (23*t^2)/100663296 - (1349*t)/150994944 + 25973/301989888;
 %  (r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (7*r^2*t)/301989888 + (83*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (275*r*s)/301989888 -
(7*r*t^2)/301989888 + (275*r*t)/301989888 - (23*r)/2359296 - (19*s^2*t)/905969664 + (23*s^2)/100663296 -
(19*s*t^2)/905969664 + (821*s*t)/905969664 - (1349*s)/150994944 + (23*t^2)/100663296 - (1349*t)/150994944 +
25973/301989888
W(m,1)=WT*a*b*c;
end
end
end
 case 16
        A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
```

```
%16=============================================================================================A(m,1)=(99
*r)/2048 - (91*s)/6144 - (91*t)/6144 - (13*r*s)/6144 - (13*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144 +
693/2048;
B(m,1)=(245*s)/6144 - (13*r)/2048 - (9*t)/2048 - (13*r*s)/6144 + (r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 +
245/2048;
C(m,1)=(245*t)/6144 - (9*s)/2048 - (13*r)/2048 + (r*s)/2048 - (13*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 +
245/2048;
D(m,1)=(23*r*s)/6144 - (127*s)/6144 - (127*t)/6144 - (73*r)/2048 + (23*r*t)/6144 + (11*s*t)/6144 -
(r*s*t)/2048 + 865/2048;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (7*r^2*t)/301989888 + (83*r^2)/301989888 + (r*s^2*t)/905969664
- (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (247*r*s)/301989888 -
(7*r*t^2)/301989888 + (247*r*t)/301989888 - (653*r)/75497472 - (17*s^2*t)/905969664 + (55*s^2)/301989888 -
(17*s*t^2)/905969664 + (247*s*t)/301989888 - (17*s)/2359296 + (55*t^2)/301989888 - (17*t)/2359296 +
20417/301989888;
 %  (r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (7*r^2*t)/301989888 + (83*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (247*r*s)/301989888
- (7*r*t^2)/301989888 + (247*r*t)/301989888 - (653*r)/75497472 - (17*s^2*t)/905969664 + (55*s^2)/301989888 -
(17*s*t^2)/905969664 + (247*s*t)/301989888 - (17*s)/2359296 + (55*t^2)/301989888 - (17*t)/2359296 +
20417/301989888
W(m,1)=WT*a*b*c;
end
end
end
 case 17
        A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%17=============================================================================================================
=========================================
A(m,1)=(293*r)/6144 - (5*s)/2048 - (11*t)/6144 - (5*r*s)/2048 - (11*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 +
293/6144;
B(m,1)=(353*s)/6144 - (25*r)/2048 - (25*t)/2048 - (5*r*s)/2048 + (5*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 +
1765/6144;
C(m,1)=(293*t)/6144 - (5*s)/2048 - (11*r)/6144 + (r*s)/6144 - (11*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 +
293/6144;
D(m,1)=(29*r*s)/6144 - (323*s)/6144 - (69*t)/2048 - (69*r)/2048 + (17*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 3793/6144;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (19*r^2*t)/905969664 + (245*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (335*r*s)/301989888
- (19*r*t^2)/905969664 + (973*r*t)/905969664 - (1349*r)/113246208 - (23*s^2*t)/905969664 +
(337*s^2)/905969664 - (23*s*t^2)/905969664 + (335*s*t)/301989888 - (3121*s)/226492416 + (245*t^2)/905969664 -
(1349*t)/113246208 + 38513/301989888;
W(m,1)=WT*a*b*c;
end
end
end
 case 18
        A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%18=============================================================================================================
=========================================
A(m,1)=(293*r)/6144 - (15*s)/2048 - (11*t)/2048 - (5*r*s)/2048 - (11*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 +
293/2048;
B(m,1)=(323*s)/6144 - (25*r)/2048 - (65*t)/6144 - (5*r*s)/2048 + (5*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144
+ 1615/6144;
C(m,1)=(271*t)/6144 - (13*s)/6144 - (11*r)/6144 + (r*s)/6144 - (11*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 +
271/6144;
D(m,1)=(29*r*s)/6144 - (265*s)/6144 - (173*t)/6144 - (69*r)/2048 + (17*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 3379/6144;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (19*r^2*t)/905969664 + (245*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (913*r*s)/905969664
- (19*r*t^2)/905969664 + (299*r*t)/301989888 - (2453*r)/226492416 - (7*s^2*t)/301989888 + (97*s^2)/301989888
- (7*s*t^2)/301989888 + (101*s*t)/100663296 - (587*s)/50331648 + (23*t^2)/100663296 - (1487*t)/150994944 +
31645/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 19
```

```
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%19=============================================================================================================
=========================================
A(m,1)=(263*r)/6144 - (5*s)/2048 - (3*t)/2048 - (5*r*s)/2048 - (3*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 +
263/6144;
B(m,1)=(353*s)/6144 - (35*r)/2048 - (35*t)/2048 - (5*r*s)/2048 + (7*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 +
2471/6144;
C(m,1)=(263*t)/6144 - (5*s)/2048 - (3*r)/2048 + (r*s)/6144 - (3*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 +
263/6144;
D(m,1)=(29*r*s)/6144 - (323*s)/6144 - (149*t)/6144 - (149*r)/6144 + (11*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 1049/2048;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (17*r^2*t)/905969664 + (199*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (913*r*s)/905969664
- (17*r*t^2)/905969664 + (877*r*t)/905969664 - (493*r)/50331648 - (23*s^2*t)/905969664 + (337*s^2)/905969664
- (23*s*t^2)/905969664 + (913*s*t)/905969664 - (29*s)/2359296 + (199*t^2)/905969664 - (493*t)/50331648 +
91919/905969664;
W(m,1)=WT*a*b*c;
end
end
end
case 20
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%=============================================================================================================
=========================================
A(m,1)=(263*r)/6144 - (15*s)/2048 - (9*t)/2048 - (5*r*s)/2048 - (3*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 +
263/2048;
B(m,1)=(323*s)/6144 - (35*r)/2048 - (91*t)/6144 - (5*r*s)/2048 + (7*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144
+ 2261/6144;
C(m,1)=(245*t)/6144 - (13*s)/6144 - (3*r)/2048 + (r*s)/6144 - (3*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 +
245/6144;
D(m,1)=(29*r*s)/6144 - (265*s)/6144 - (127*t)/6144 - (149*r)/6144 + (11*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 2849/6144;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (17*r^2*t)/905969664 + (199*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (821*r*s)/905969664
- (17*r*t^2)/905969664 + (809*r*t)/905969664 - (4039*r)/452984832 - (7*s^2*t)/301989888 + (97*s^2)/301989888
- (7*s*t^2)/301989888 + (275*s*t)/301989888 - (1567*s)/150994944 + (55*t^2)/301989888 - (599*t)/75497472 +
24989/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 21
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%21=============================================================================================================
=========================================
A(m,1)=(271*r)/6144 - (13*s)/6144 - (11*t)/6144 - (13*r*s)/6144 - (11*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 +
271/6144;
B(m,1)=(323*s)/6144 - (65*r)/6144 - (25*t)/2048 - (13*r*s)/6144 + (5*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144
+ 1615/6144;
C(m,1)=(293*t)/6144 - (15*s)/2048 - (11*r)/2048 + (r*s)/2048 - (11*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 +
293/2048;
D(m,1)=(23*r*s)/6144 - (265*s)/6144 - (69*t)/2048 - (173*r)/6144 + (17*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 3379/6144;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (19*r^2*t)/905969664 + (23*r^2)/100663296 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (101*r*s)/100663296
- (19*r*t^2)/905969664 + (299*r*t)/301989888 - (1487*r)/150994944 - (23*s^2*t)/905969664 + (97*s^2)/301989888
- (23*s*t^2)/905969664 + (913*s*t)/905969664 - (587*s)/50331648 + (245*t^2)/905969664 - (2453*t)/226492416 +
31645/301989888;
W(m,1)=WT*a*b*c;
```

```
end
end
end
case 22
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%22=========================================================================================================
=========================================
A(m,1)=(271*r)/6144 - (13*s)/2048 - (11*t)/2048 - (13*r*s)/6144 - (11*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 +
271/2048;
B(m,1)=(99*s)/2048 - (65*r)/6144 - (65*t)/6144 - (13*r*s)/6144 + (5*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144
+ 495/2048;
C(m,1)=(271*t)/6144 - (13*s)/2048 - (11*r)/2048 + (r*s)/2048 - (11*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 +
271/2048;
D(m,1)=(23*r*s)/6144 - (73*s)/2048 - (173*t)/6144 - (173*r)/6144 + (17*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 1011/2048;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (19*r^2*t)/905969664 + (23*r^2)/100663296 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (275*r*s)/301989888 -
(19*r*t^2)/905969664 + (821*r*t)/905969664 - (1349*r)/150994944 - (7*s^2*t)/301989888 + (83*s^2)/301989888 -
(7*s*t^2)/301989888 + (275*s*t)/301989888 - (23*s)/2359296 + (23*t^2)/100663296 - (1349*t)/150994944 +
25973/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 23
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%=========================================================================================================
=========================================
A(m,1)=(245*r)/6144 - (13*s)/6144 - (3*t)/2048 - (13*r*s)/6144 - (3*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 +
245/6144;
B(m,1)=(323*s)/6144 - (91*r)/6144 - (35*t)/2048 - (13*r*s)/6144 + (7*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144
+ 2261/6144;
C(m,1)=(263*t)/6144 - (15*s)/2048 - (9*r)/2048 + (r*s)/2048 - (3*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 +
263/2048;
D(m,1)=(23*r*s)/6144 - (265*s)/6144 - (149*t)/6144 - (127*r)/6144 + (11*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 2849/6144;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (17*r^2*t)/905969664 + (55*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (275*r*s)/301989888 -
(17*r*t^2)/905969664 + (809*r*t)/905969664 - (599*r)/75497472 - (23*s^2*t)/905969664 + (97*s^2)/301989888 -
(23*s*t^2)/905969664 + (821*s*t)/905969664 - (1567*s)/150994944 + (199*t^2)/905969664 - (4039*t)/452984832 +
24989/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 24
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%24=========================================================================================================
=========================================
A(m,1)=(245*r)/6144 - (13*s)/2048 - (9*t)/2048 - (13*r*s)/6144 - (3*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 +
245/2048;
B(m,1)=(99*s)/2048 - (91*r)/6144 - (91*t)/6144 - (13*r*s)/6144 + (7*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144
+ 693/2048;
C(m,1)=(245*t)/6144 - (13*s)/2048 - (9*r)/2048 + (r*s)/2048 - (3*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 +
245/2048;
D(m,1)=(23*r*s)/6144 - (73*s)/2048 - (127*t)/6144 - (127*r)/6144 + (11*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 865/2048;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (17*r^2*t)/905969664 + (55*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (247*r*s)/301989888
```

```
- (17*r*t^2)/905969664 + (247*r*t)/301989888 - (17*r)/2359296 - (7*s^2*t)/301989888 + (83*s^2)/301989888 -
(7*s*t^2)/301989888 + (247*s*t)/301989888 - (653*s)/75497472 + (55*t^2)/301989888 - (17*t)/2359296 +
20417/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 25
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%25=============================================================================================================
=========================================
A(m,1)=(293*r)/6144 - (25*s)/2048 - (55*t)/6144 - (5*r*s)/2048 - (11*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144
+ 1465/6144;
B(m,1)=(293*s)/6144 - (25*r)/2048 - (55*t)/6144 - (5*r*s)/2048 + (5*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144
+ 1465/6144;
C(m,1)=(83*t)/2048 - (11*s)/6144 - (11*r)/6144 + (r*s)/6144 - (11*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 +
83/2048;
D(m,1)=(29*r*s)/6144 - (69*s)/2048 - (139*t)/6144 - (69*r)/2048 + (17*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 2965/6144;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (19*r^2*t)/905969664 + (245*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (821*r*s)/905969664
- (19*r*t^2)/905969664 + (821*r*t)/905969664 - (23*r)/2359296 - (19*s^2*t)/905969664 + (245*s^2)/905969664 -
(19*s*t^2)/905969664 + (821*s*t)/905969664 - (23*s)/2359296 + (169*t^2)/905969664 - (1801*t)/226492416 +
76291/905969664;
W(m,1)=WT*a*b*c;
end
end
end
case 26
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%26=============================================================================================================
=========================================
A(m,1)=(293*r)/6144 - (35*s)/2048 - (77*t)/6144 - (5*r*s)/2048 - (11*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144
+ 2051/6144;
B(m,1)=(263*s)/6144 - (25*r)/2048 - (15*t)/2048 - (5*r*s)/2048 + (5*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 +
1315/6144;
C(m,1)=(227*t)/6144 - (3*s)/2048 - (11*r)/6144 + (r*s)/6144 - (11*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 +
227/6144;
D(m,1)=(29*r*s)/6144 - (149*s)/6144 - (35*t)/2048 - (69*r)/2048 + (17*r*t)/6144 + (11*s*t)/6144 -
(r*s*t)/2048 + 2551/6144;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (19*r^2*t)/905969664 + (245*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (27*r*s)/33554432 -
(19*r*t^2)/905969664 + (745*r*t)/905969664 - (1963*r)/226492416 - (17*s^2*t)/905969664 + (199*s^2)/905969664
- (17*s*t^2)/905969664 + (247*s*t)/301989888 - (3641*s)/452984832 + (131*t^2)/905969664 - (2819*t)/452984832
+ 6623/100663296;
W(m,1)=WT*a*b*c;
end
end
end
case 27
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%=============================================================================================================
=========================================
A(m,1)=(263*r)/6144 - (25*s)/2048 - (15*t)/2048 - (5*r*s)/2048 - (3*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144 +
1315/6144;
B(m,1)=(293*s)/6144 - (35*r)/2048 - (77*t)/6144 - (5*r*s)/2048 + (7*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144
+ 2051/6144;
C(m,1)=(227*t)/6144 - (11*s)/6144 - (3*r)/2048 + (r*s)/6144 - (3*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 +
227/6144;
```

```
D(m,1)=(29*r*s)/6144 - (69*s)/2048 - (35*t)/2048 - (149*r)/6144 + (11*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 2551/6144;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (17*r^2*t)/905969664 + (199*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (27*r*s)/33554432 -
(17*r*t^2)/905969664 + (247*r*t)/301989888 - (3641*r)/452984832 - (19*s^2*t)/905969664 + (245*s^2)/905969664
- (19*s*t^2)/905969664 + (745*s*t)/905969664 - (1963*s)/226492416 + (131*t^2)/905969664 - (2819*t)/452984832
+ 6623/100663296;
W(m,1)=WT*a*b*c;
end
end
end
case 28
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%28=============================================================================================================
=========================================
A(m,1)=(263*r)/6144 - (35*s)/2048 - (21*t)/2048 - (5*r*s)/2048 - (3*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144 +
1841/6144;
B(m,1)=(263*s)/6144 - (35*r)/2048 - (21*t)/2048 - (5*r*s)/2048 + (7*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 +
1841/6144;
C(m,1)=(209*t)/6144 - (3*s)/2048 - (3*r)/2048 + (r*s)/6144 - (3*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 +
209/6144;
D(m,1)=(29*r*s)/6144 - (149*s)/6144 - (83*t)/6144 - (149*r)/6144 + (11*r*t)/6144 + (11*s*t)/6144 -
(r*s*t)/2048 + 751/2048;
WT=(r^2*s*t)/905969664 - (23*r^2*s)/905969664 - (17*r^2*t)/905969664 + (199*r^2)/905969664 +
(r*s^2*t)/905969664 - (23*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (637*r*s)/905969664
- (17*r*t^2)/905969664 + (673*r*t)/905969664 - (1081*r)/150994944 - (17*s^2*t)/905969664 +
(199*s^2)/905969664 - (17*s*t^2)/905969664 + (673*s*t)/905969664 - (1081*s)/150994944 + (97*t^2)/905969664 -
(11*t)/2359296 + 45839/905969664;
W(m,1)=WT*a*b*c;
end
end
end
case 29
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%29=============================================================================================================
=========================================
A(m,1)=(271*r)/6144 - (65*s)/6144 - (55*t)/6144 - (13*r*s)/6144 - (11*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144
+ 1355/6144;
B(m,1)=(271*s)/6144 - (65*r)/6144 - (55*t)/6144 - (13*r*s)/6144 + (5*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144
+ 1355/6144;
C(m,1)=(83*t)/2048 - (11*s)/2048 - (11*r)/2048 + (r*s)/2048 - (11*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 +
249/2048;
D(m,1)=(23*r*s)/6144 - (173*s)/6144 - (139*t)/6144 - (173*r)/6144 + (17*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 2687/6144;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (19*r^2*t)/905969664 + (23*r^2)/100663296 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (247*r*s)/301989888
- (19*r*t^2)/905969664 + (745*r*t)/905969664 - (1211*r)/150994944 - (19*s^2*t)/905969664 + (23*s^2)/100663296
- (19*s*t^2)/905969664 + (745*s*t)/905969664 - (1211*s)/150994944 + (169*t^2)/905969664 - (17*t)/2359296 +
2317/33554432;
W(m,1)=WT*a*b*c;
end
end
end
case 30
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%30=============================================================================================================
=========================================
A(m,1)=(271*r)/6144 - (91*s)/6144 - (77*t)/6144 - (13*r*s)/6144 - (11*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144
+ 1897/6144;
```

```
B(m,1)=(245*s)/6144 - (65*r)/6144 - (15*t)/2048 - (13*r*s)/6144 + (5*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144
+ 1225/6144;
C(m,1)=(227*t)/6144 - (9*s)/2048 - (11*r)/2048 + (r*s)/2048 - (11*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 +
227/2048;
D(m,1)=(23*r*s)/6144 - (127*s)/6144 - (35*t)/2048 - (173*r)/6144 + (17*r*t)/6144 + (11*s*t)/6144 -
(r*s*t)/2048 + 2341/6144;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (19*r^2*t)/905969664 + (23*r^2)/100663296 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (73*r*s)/100663296 -
(19*r*t^2)/905969664 + (223*r*t)/301989888 - (1073*r)/150994944 - (17*s^2*t)/905969664 + (55*s^2)/301989888 -
(17*s*t^2)/905969664 + (673*s*t)/905969664 - (163*s)/25165824 + (131*t^2)/905969664 - (2557*t)/452984832 +
16285/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 31
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%31=========================================================================================================
=========================================
A(m,1)=(245*r)/6144 - (65*s)/6144 - (15*t)/2048 - (13*r*s)/6144 - (3*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144
+ 1225/6144;
B(m,1)=(271*s)/6144 - (91*r)/6144 - (77*t)/6144 - (13*r*s)/6144 + (7*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144
+ 1897/6144;
C(m,1)=(227*t)/6144 - (11*s)/2048 - (9*r)/2048 + (r*s)/2048 - (3*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 +
227/2048;
D(m,1)=(23*r*s)/6144 - (173*s)/6144 - (35*t)/2048 - (127*r)/6144 + (11*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 2341/6144;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (17*r^2*t)/905969664 + (55*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (73*r*s)/100663296 -
(17*r*t^2)/905969664 + (673*r*t)/905969664 - (163*r)/25165824 - (19*s^2*t)/905969664 + (23*s^2)/100663296 -
(19*s*t^2)/905969664 + (223*s*t)/301989888 - (1073*s)/150994944 + (131*t^2)/905969664 - (2557*t)/452984832 +
16285/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 32
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%32=========================================================================================================
=========================================
A(m,1)=(245*r)/6144 - (91*s)/6144 - (21*t)/2048 - (13*r*s)/6144 - (3*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144
+ 1715/6144;
B(m,1)=(245*s)/6144 - (91*r)/6144 - (21*t)/2048 - (13*r*s)/6144 + (7*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144
+ 1715/6144;
C(m,1)=(209*t)/6144 - (9*s)/2048 - (9*r)/2048 + (r*s)/2048 - (3*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 +
209/2048;
D(m,1)=(23*r*s)/6144 - (127*s)/6144 - (83*t)/6144 - (127*r)/6144 + (11*r*t)/6144 + (11*s*t)/6144 -
(r*s*t)/2048 + 2087/6144;
WT=(r^2*s*t)/905969664 - (7*r^2*s)/301989888 - (17*r^2*t)/905969664 + (55*r^2)/301989888 +
(r*s^2*t)/905969664 - (7*r*s^2)/301989888 + (r*s*t^2)/905969664 - (5*r*s*t)/150994944 + (191*r*s)/301989888 -
(17*r*t^2)/905969664 + (605*r*t)/905969664 - (217*r)/37748736 - (17*s^2*t)/905969664 + (55*s^2)/301989888 -
(17*s*t^2)/905969664 + (605*s*t)/905969664 - (217*s)/37748736 + (97*t^2)/905969664 - (959*t)/226492416 +
12593/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 33
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
```

```
%33=============================================================================================================
=========================================
A(m,1)=(293*r)/6144 - (11*s)/6144 - (5*t)/2048 - (11*r*s)/6144 - (5*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 +
293/6144;
B(m,1)=(293*s)/6144 - (11*r)/6144 - (5*t)/2048 - (11*r*s)/6144 + (r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 +
293/6144;
C(m,1)=(353*t)/6144 - (25*s)/2048 - (25*r)/2048 + (5*r*s)/6144 - (5*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 +
1765/6144;
D(m,1)=(17*r*s)/6144 - (69*s)/2048 - (323*t)/6144 - (69*r)/2048 + (29*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 3793/6144;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (23*r^2*t)/905969664 + (245*r^2)/905969664 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (25*r*s*t)/452984832 + (973*r*s)/905969664
- (23*r*t^2)/905969664 + (335*r*t)/301989888 - (1349*r)/113246208 - (23*s^2*t)/905969664 +
(245*s^2)/905969664 - (23*s*t^2)/905969664 + (335*s*t)/301989888 - (1349*s)/113246208 + (337*t^2)/905969664 -
(3121*t)/226492416 + 38513/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 34
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%34=============================================================================================================
=========================================
A(m,1)=(293*r)/6144 - (11*s)/2048 - (15*t)/2048 - (11*r*s)/6144 - (5*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 +
293/2048;
B(m,1)=(271*s)/6144 - (11*r)/6144 - (13*t)/6144 - (11*r*s)/6144 + (r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 +
271/6144;
C(m,1)=(323*t)/6144 - (65*s)/6144 - (25*r)/2048 + (5*r*s)/6144 - (5*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144
+ 1615/6144;
D(m,1)=(17*r*s)/6144 - (173*s)/6144 - (265*t)/6144 - (69*r)/2048 + (29*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 3379/6144;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (23*r^2*t)/905969664 + (245*r^2)/905969664 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (299*r*s)/301989888
- (23*r*t^2)/905969664 + (913*r*t)/905969664 - (2453*r)/226492416 - (7*s^2*t)/301989888 + (23*s^2)/100663296
- (7*s*t^2)/301989888 + (101*s*t)/100663296 - (1487*s)/150994944 + (97*t^2)/301989888 - (587*t)/50331648 +
31645/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 35
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%35=============================================================================================================
=========================================
A(m,1)=(271*r)/6144 - (11*s)/6144 - (13*t)/6144 - (11*r*s)/6144 - (13*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 +
271/6144;
B(m,1)=(293*s)/6144 - (11*r)/2048 - (15*t)/2048 - (11*r*s)/6144 + (r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 +
293/2048;
C(m,1)=(323*t)/6144 - (25*s)/2048 - (65*r)/6144 + (5*r*s)/6144 - (13*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144
+ 1615/6144;
D(m,1)=(17*r*s)/6144 - (69*s)/2048 - (265*t)/6144 - (173*r)/6144 + (23*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 3379/6144;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (7*r^2*t)/301989888 + (23*r^2)/100663296 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (299*r*s)/301989888
- (7*r*t^2)/301989888 + (101*r*t)/100663296 - (1487*r)/150994944 - (23*s^2*t)/905969664 + (245*s^2)/905969664
- (23*s*t^2)/905969664 + (913*s*t)/905969664 - (2453*s)/226492416 + (97*t^2)/301989888 - (587*t)/50331648 +
31645/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 36
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
```

```
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%36=============================================================================================A(m,1)=(27
1*r)/6144 - (11*s)/2048 - (13*t)/2048 - (11*r*s)/6144 - (13*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 + 271/2048;
B(m,1)=(271*s)/6144 - (11*r)/2048 - (13*t)/2048 - (11*r*s)/6144 + (r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 +
271/2048;
C(m,1)=(99*t)/2048 - (65*s)/6144 - (65*r)/6144 + (5*r*s)/6144 - (13*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144
+ 495/2048;
D(m,1)=(17*r*s)/6144 - (173*s)/6144 - (73*t)/2048 - (173*r)/6144 + (23*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 1011/2048;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (7*r^2*t)/301989888 + (23*r^2)/100663296 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (821*r*s)/905969664
- (7*r*t^2)/301989888 + (275*r*t)/301989888 - (1349*r)/150994944 - (7*s^2*t)/301989888 + (23*s^2)/100663296 -
(7*s*t^2)/301989888 + (275*s*t)/301989888 - (1349*s)/150994944 + (83*t^2)/301989888 - (23*t)/2359296 +
25973/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 37
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%37=============================================================================================================
=========================================
A(m,1)=(263*r)/6144 - (3*s)/2048 - (5*t)/2048 - (3*r*s)/2048 - (5*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 +
263/6144;
B(m,1)=(263*s)/6144 - (3*r)/2048 - (5*t)/2048 - (3*r*s)/2048 + (r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 +
263/6144;
C(m,1)=(353*t)/6144 - (35*s)/2048 - (35*r)/2048 + (7*r*s)/6144 - (5*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 +
2471/6144;
D(m,1)=(11*r*s)/6144 - (149*s)/6144 - (323*t)/6144 - (149*r)/6144 + (29*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 1049/2048;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (23*r^2*t)/905969664 + (199*r^2)/905969664 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (23*r*s*t)/452984832 + (877*r*s)/905969664
- (23*r*t^2)/905969664 + (913*r*t)/905969664 - (493*r)/50331648 - (23*s^2*t)/905969664 + (199*s^2)/905969664
- (23*s*t^2)/905969664 + (913*s*t)/905969664 - (493*s)/50331648 + (337*t^2)/905969664 - (29*t)/2359296 +
91919/905969664;
W(m,1)=WT*a*b*c;
end
end
end
case 38
    A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%38=============================================================================================================
=========================================
A(m,1)=(263*r)/6144 - (9*s)/2048 - (15*t)/2048 - (3*r*s)/2048 - (5*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 +
263/2048;
B(m,1)=(245*s)/6144 - (3*r)/2048 - (13*t)/6144 - (3*r*s)/2048 + (r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144 +
245/6144;
C(m,1)=(323*t)/6144 - (91*s)/6144 - (35*r)/2048 + (7*r*s)/6144 - (5*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144
+ 2261/6144;
D(m,1)=(11*r*s)/6144 - (127*s)/6144 - (265*t)/6144 - (149*r)/6144 + (29*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 2849/6144;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (23*r^2*t)/905969664 + (199*r^2)/905969664 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (809*r*s)/905969664
- (23*r*t^2)/905969664 + (821*r*t)/905969664 - (4039*r)/452984832 - (7*s^2*t)/301989888 + (55*s^2)/301989888
- (7*s*t^2)/301989888 + (275*s*t)/301989888 - (599*s)/75497472 + (97*t^2)/301989888 - (1567*t)/150994944 +
24989/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 39
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
```

```
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%39=============================================================================================================
=========================================
A(m,1)=(245*r)/6144 - (3*s)/2048 - (13*t)/6144 - (3*r*s)/2048 - (13*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 +
245/6144;
B(m,1)=(263*s)/6144 - (9*r)/2048 - (15*t)/2048 - (3*r*s)/2048 + (r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 +
263/2048;
C(m,1)=(323*t)/6144 - (35*s)/2048 - (91*r)/6144 + (7*r*s)/6144 - (13*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144
+ 2261/6144;
D(m,1)=(11*r*s)/6144 - (149*s)/6144 - (265*t)/6144 - (127*r)/6144 + (23*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 2849/6144;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (7*r^2*t)/301989888 + (55*r^2)/301989888 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (809*r*s)/905969664
- (7*r*t^2)/301989888 + (275*r*t)/301989888 - (599*r)/75497472 - (23*s^2*t)/905969664 + (199*s^2)/905969664 -
(23*s*t^2)/905969664 + (821*s*t)/905969664 - (4039*s)/452984832 + (97*t^2)/301989888 - (1567*t)/150994944 +
24989/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 40
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%40=============================================================================================================
=========================================
A(m,1)=(245*r)/6144 - (9*s)/2048 - (13*t)/2048 - (3*r*s)/2048 - (13*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 +
245/2048;
B(m,1)=(245*s)/6144 - (9*r)/2048 - (13*t)/2048 - (3*r*s)/2048 + (r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144 +
245/2048;
C(m,1)=(99*t)/2048 - (91*s)/6144 - (91*r)/6144 + (7*r*s)/6144 - (13*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144
+ 693/2048;
D(m,1)=(11*r*s)/6144 - (127*s)/6144 - (73*t)/2048 - (127*r)/6144 + (23*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 865/2048;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (7*r^2*t)/301989888 + (55*r^2)/301989888 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (247*r*s)/301989888
- (7*r*t^2)/301989888 + (247*r*t)/301989888 - (17*r)/2359296 - (7*s^2*t)/301989888 + (55*s^2)/301989888 -
(7*s*t^2)/301989888 + (247*s*t)/301989888 - (17*s)/2359296 + (83*t^2)/301989888 - (653*t)/75497472 +
20417/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 41
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%=============================================================================================================
=========================================
A(m,1)=(293*r)/6144 - (55*s)/6144 - (25*t)/2048 - (11*r*s)/6144 - (5*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144
+ 1465/6144;
B(m,1)=(83*s)/2048 - (11*r)/6144 - (11*t)/6144 - (11*r*s)/6144 + (r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 +
83/2048;
C(m,1)=(293*t)/6144 - (55*s)/6144 - (25*r)/2048 + (5*r*s)/6144 - (5*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144
+ 1465/6144;
D(m,1)=(17*r*s)/6144 - (139*s)/6144 - (69*t)/2048 - (69*r)/2048 + (29*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 2965/6144;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (23*r^2*t)/905969664 + (245*r^2)/905969664 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (821*r*s)/905969664
- (23*r*t^2)/905969664 + (821*r*t)/905969664 - (23*r)/2359296 - (19*s^2*t)/905969664 + (169*s^2)/905969664 -
(19*s*t^2)/905969664 + (821*s*t)/905969664 - (1801*s)/226492416 + (245*t^2)/905969664 - (23*t)/2359296 +
76291/905969664;
W(m,1)=WT*a*b*c;
end
end
```

```
end
case 42
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%42=============================================================================================
A(m,1)=(293*r)/6144 - (77*s)/6144 - (35*t)/2048 - (11*r*s)/6144 - (5*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144
+ 2051/6144;
B(m,1)=(227*s)/6144 - (11*r)/6144 - (3*t)/2048 - (11*r*s)/6144 + (r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 +
227/6144;
C(m,1)=(263*t)/6144 - (15*s)/2048 - (25*r)/2048 + (5*r*s)/6144 - (5*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 +
1315/6144;
D(m,1)=(17*r*s)/6144 - (35*s)/2048 - (149*t)/6144 - (69*r)/2048 + (29*r*t)/6144 + (11*s*t)/6144 -
(r*s*t)/2048 + 2551/6144;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (23*r^2*t)/905969664 + (245*r^2)/905969664 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (745*r*s)/905969664
- (23*r*t^2)/905969664 + (27*r*t)/33554432 - (1963*r)/226492416 - (17*s^2*t)/905969664 + (131*s^2)/905969664
- (17*s*t^2)/905969664 + (247*s*t)/301989888 - (2819*s)/452984832 + (199*t^2)/905969664 - (3641*t)/452984832
+ 6623/100663296;
W(m,1)=WT*a*b*c;
end
end
end
case 43
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%43=============================================================================================
=========================================
A(m,1)=(271*r)/6144 - (55*s)/6144 - (65*t)/6144 - (11*r*s)/6144 - (13*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144
+ 1355/6144;
B(m,1)=(83*s)/2048 - (11*r)/2048 - (11*t)/2048 - (11*r*s)/6144 + (r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 +
249/2048;
C(m,1)=(271*t)/6144 - (55*s)/6144 - (65*r)/6144 + (5*r*s)/6144 - (13*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144
+ 1355/6144;
D(m,1)=(17*r*s)/6144 - (139*s)/6144 - (173*t)/6144 - (173*r)/6144 + (23*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 2687/6144;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (7*r^2*t)/301989888 + (23*r^2)/100663296 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (745*r*s)/905969664
- (7*r*t^2)/301989888 + (247*r*t)/301989888 - (1211*r)/150994944 - (19*s^2*t)/905969664 + (169*s^2)/905969664
- (19*s*t^2)/905969664 + (745*s*t)/905969664 - (17*s)/2359296 + (23*t^2)/100663296 - (1211*t)/150994944 +
2317/33554432;
W(m,1)=WT*a*b*c;
end
end
end
case 44
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%44=============================================================================================
=========================================
A(m,1)=(271*r)/6144 - (77*s)/6144 - (91*t)/6144 - (11*r*s)/6144 - (13*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144
+ 1897/6144;
B(m,1)=(227*s)/6144 - (11*r)/2048 - (9*t)/2048 - (11*r*s)/6144 + (r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 +
227/2048;
C(m,1)=(245*t)/6144 - (15*s)/2048 - (65*r)/6144 + (5*r*s)/6144 - (13*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144
+ 1225/6144;
D(m,1)=(17*r*s)/6144 - (35*s)/2048 - (127*t)/6144 - (173*r)/6144 + (23*r*t)/6144 + (11*s*t)/6144 -
(r*s*t)/2048 + 2341/6144;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (7*r^2*t)/301989888 + (23*r^2)/100663296 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (223*r*s)/301989888
- (7*r*t^2)/301989888 + (73*r*t)/100663296 - (1073*r)/150994944 - (17*s^2*t)/905969664 + (131*s^2)/905969664
- (17*s*t^2)/905969664 + (673*s*t)/905969664 - (2557*s)/452984832 + (55*t^2)/301989888 - (163*t)/25165824 +
16285/301989888;
```

```
W(m,1)=WT*a*b*c;
end
end
end
case 45
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%45=============================================================================================================
=======================================
A(m,1)=(263*r)/6144 - (15*s)/2048 - (25*t)/2048 - (3*r*s)/2048 - (5*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144 +
1315/6144;
B(m,1)=(227*s)/6144 - (3*r)/2048 - (11*t)/6144 - (3*r*s)/2048 + (r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144 +
227/6144;
C(m,1)=(293*t)/6144 - (77*s)/6144 - (35*r)/2048 + (7*r*s)/6144 - (5*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144
+ 2051/6144;
D(m,1)=(11*r*s)/6144 - (35*s)/2048 - (69*t)/2048 - (149*r)/6144 + (29*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 2551/6144;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (23*r^2*t)/905969664 + (199*r^2)/905969664 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (247*r*s)/301989888
- (23*r*t^2)/905969664 + (27*r*t)/33554432 - (3641*r)/452984832 - (19*s^2*t)/905969664 + (131*s^2)/905969664
- (19*s*t^2)/905969664 + (745*s*t)/905969664 - (2819*s)/452984832 + (245*t^2)/905969664 - (1963*t)/226492416
+ 6623/100663296;
W(m,1)=WT*a*b*c;
end
end
end
case 46
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%46=============================================================================================================
=======================================
A(m,1)=(263*r)/6144 - (21*s)/2048 - (35*t)/2048 - (3*r*s)/2048 - (5*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144 +
1841/6144;
B(m,1)=(209*s)/6144 - (3*r)/2048 - (3*t)/2048 - (3*r*s)/2048 + (r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 +
209/6144;
C(m,1)=(263*t)/6144 - (21*s)/2048 - (35*r)/2048 + (7*r*s)/6144 - (5*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 +
1841/6144;
D(m,1)=(11*r*s)/6144 - (83*s)/6144 - (149*t)/6144 - (149*r)/6144 + (29*r*t)/6144 + (11*s*t)/6144 -
(r*s*t)/2048 + 751/2048;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (23*r^2*t)/905969664 + (199*r^2)/905969664 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (673*r*s)/905969664
- (23*r*t^2)/905969664 + (637*r*t)/905969664 - (1081*r)/150994944 - (17*s^2*t)/905969664 + (97*s^2)/905969664
- (17*s*t^2)/905969664 + (673*s*t)/905969664 - (11*s)/2359296 + (199*t^2)/905969664 - (1081*t)/150994944 +
45839/905969664;
W(m,1)=WT*a*b*c;
end
end
end
case 47
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%47=============================================================================================================
=======================================
A(m,1)=(245*r)/6144 - (15*s)/2048 - (65*t)/6144 - (3*r*s)/2048 - (13*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144
+ 1225/6144;
B(m,1)=(227*s)/6144 - (9*r)/2048 - (11*t)/2048 - (3*r*s)/2048 + (r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144 +
227/2048;
C(m,1)=(271*t)/6144 - (77*s)/6144 - (91*r)/6144 + (7*r*s)/6144 - (13*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144
+ 1897/6144;
D(m,1)=(11*r*s)/6144 - (35*s)/2048 - (173*t)/6144 - (127*r)/6144 + (23*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 2341/6144;
```

```
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (7*r^2*t)/301989888 + (55*r^2)/301989888 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (673*r*s)/905969664
- (7*r*t^2)/301989888 + (73*r*t)/100663296 - (163*r)/25165824 - (19*s^2*t)/905969664 + (131*s^2)/905969664 -
(19*s*t^2)/905969664 + (223*s*t)/301989888 - (2557*s)/452984832 + (23*t^2)/100663296 - (1073*t)/150994944 +
16285/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 48
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%48=============================================================================================
A(m,1)=(245*r)/6144 - (21*s)/2048 - (91*t)/6144 - (3*r*s)/2048 - (13*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144
+ 1715/6144;
B(m,1)=(209*s)/6144 - (9*r)/2048 - (9*t)/2048 - (3*r*s)/2048 + (r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 +
209/2048;
C(m,1)=(245*t)/6144 - (21*s)/2048 - (91*r)/6144 + (7*r*s)/6144 - (13*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144
+ 1715/6144;
D(m,1)=(11*r*s)/6144 - (83*s)/6144 - (127*t)/6144 - (127*r)/6144 + (23*r*t)/6144 + (11*s*t)/6144 -
(r*s*t)/2048 + 2087/6144;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (7*r^2*t)/301989888 + (55*r^2)/301989888 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (5*r*s*t)/150994944 + (605*r*s)/905969664
- (7*r*t^2)/301989888 + (191*r*t)/301989888 - (217*r)/37748736 - (17*s^2*t)/905969664 + (97*s^2)/905969664 -
(17*s*t^2)/905969664 + (605*s*t)/905969664 - (959*s)/226492416 + (55*t^2)/301989888 - (217*t)/37748736 +
12593/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 49
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%49=============================================================================================================
=========================================
A(m,1)=(83*r)/2048 - (11*s)/6144 - (11*t)/6144 - (11*r*s)/6144 - (11*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 +
83/2048;
B(m,1)=(293*s)/6144 - (55*r)/6144 - (25*t)/2048 - (11*r*s)/6144 + (5*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144
+ 1465/6144;
C(m,1)=(293*t)/6144 - (25*s)/2048 - (55*r)/6144 + (5*r*s)/6144 - (11*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144
+ 1465/6144;
D(m,1)=(17*r*s)/6144 - (69*s)/2048 - (69*t)/2048 - (139*r)/6144 + (17*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 2965/6144;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (19*r^2*t)/905969664 + (169*r^2)/905969664 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (7*r*s*t)/150994944 + (821*r*s)/905969664
- (19*r*t^2)/905969664 + (821*r*t)/905969664 - (1801*r)/226492416 - (23*s^2*t)/905969664 +
(245*s^2)/905969664 - (23*s*t^2)/905969664 + (821*s*t)/905969664 - (23*s)/2359296 + (245*t^2)/905969664 -
(23*t)/2359296 + 76291/905969664;
W(m,1)=WT*a*b*c;
end
end
end
case 50
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%50=============================================================================================A(m,1)=(83
*r)/2048 - (11*s)/2048 - (11*t)/2048 - (11*r*s)/6144 - (11*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 + 249/2048;
B(m,1)=(271*s)/6144 - (55*r)/6144 - (65*t)/6144 - (11*r*s)/6144 + (5*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144
+ 1355/6144;
C(m,1)=(271*t)/6144 - (65*s)/6144 - (55*r)/6144 + (5*r*s)/6144 - (11*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144
+ 1355/6144;
```

```
D(m,1)=(17*r*s)/6144 - (173*s)/6144 - (173*t)/6144 - (139*r)/6144 + (17*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 2687/6144;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (19*r^2*t)/905969664 + (169*r^2)/905969664 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (745*r*s)/905969664
- (19*r*t^2)/905969664 + (745*r*t)/905969664 - (17*r)/2359296 - (7*s^2*t)/301989888 + (23*s^2)/100663296 -
(7*s*t^2)/301989888 + (247*s*t)/301989888 - (1211*s)/150994944 + (23*t^2)/100663296 - (1211*t)/150994944 +
2317/33554432;
W(m,1)=WT*a*b*c;
end
end
end
case 51
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%51=============================================================================================================
=========================================
A(m,1)=(227*r)/6144 - (11*s)/6144 - (3*t)/2048 - (11*r*s)/6144 - (3*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 +
227/6144;
B(m,1)=(293*s)/6144 - (77*r)/6144 - (35*t)/2048 - (11*r*s)/6144 + (7*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144
+ 2051/6144;
C(m,1)=(263*t)/6144 - (25*s)/2048 - (15*r)/2048 + (5*r*s)/6144 - (3*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 +
1315/6144;
D(m,1)=(17*r*s)/6144 - (69*s)/2048 - (149*t)/6144 - (35*r)/2048 + (11*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 2551/6144;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (17*r^2*t)/905969664 + (131*r^2)/905969664 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (745*r*s)/905969664
- (17*r*t^2)/905969664 + (247*r*t)/301989888 - (2819*r)/452984832 - (23*s^2*t)/905969664 +
(245*s^2)/905969664 - (23*s*t^2)/905969664 + (27*s*t)/33554432 - (1963*s)/226492416 + (199*t^2)/905969664 -
(3641*t)/452984832 + 6623/100663296;
W(m,1)=WT*a*b*c;
end
end
end
case 52
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%52=============================================================================================================
=========================================
A(m,1)=(227*r)/6144 - (11*s)/2048 - (9*t)/2048 - (11*r*s)/6144 - (3*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 +
227/2048;
B(m,1)=(271*s)/6144 - (77*r)/6144 - (91*t)/6144 - (11*r*s)/6144 + (7*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144
+ 1897/6144;
C(m,1)=(245*t)/6144 - (65*s)/6144 - (15*r)/2048 + (5*r*s)/6144 - (3*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144
+ 1225/6144;
D(m,1)=(17*r*s)/6144 - (173*s)/6144 - (127*t)/6144 - (35*r)/2048 + (11*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 2341/6144;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (17*r^2*t)/905969664 + (131*r^2)/905969664 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (223*r*s)/301989888
- (17*r*t^2)/905969664 + (673*r*t)/905969664 - (2557*r)/452984832 - (7*s^2*t)/301989888 + (23*s^2)/100663296
- (7*s*t^2)/301989888 + (73*s*t)/100663296 - (1073*s)/150994944 + (55*t^2)/301989888 - (163*t)/25165824 +
16285/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 53
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%53=============================================================================================================
=========================================
A(m,1)=(227*r)/6144 - (3*s)/2048 - (11*t)/6144 - (3*r*s)/2048 - (11*r*t)/6144 + (s*t)/6144 + (r*s*t)/6144 +
227/6144;
```

```
B(m,1)=(263*s)/6144 - (15*r)/2048 - (25*t)/2048 - (3*r*s)/2048 + (5*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 +
1315/6144;
C(m,1)=(293*t)/6144 - (35*s)/2048 - (77*r)/6144 + (7*r*s)/6144 - (11*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144
+ 2051/6144;
D(m,1)=(11*r*s)/6144 - (149*s)/6144 - (69*t)/2048 - (35*r)/2048 + (17*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 2551/6144;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (19*r^2*t)/905969664 + (131*r^2)/905969664 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (19*r*s*t)/452984832 + (247*r*s)/301989888
- (19*r*t^2)/905969664 + (745*r*t)/905969664 - (2819*r)/452984832 - (23*s^2*t)/905969664 +
(199*s^2)/905969664 - (23*s*t^2)/905969664 + (27*s*t)/33554432 - (3641*s)/452984832 + (245*t^2)/905969664 -
(1963*t)/226492416 + 6623/100663296;
W(m,1)=WT*a*b*c;
end
end
end
case 54
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%54=============================================================================================================
=========================================
A(m,1)=(227*r)/6144 - (9*s)/2048 - (11*t)/2048 - (3*r*s)/2048 - (11*r*t)/6144 + (s*t)/2048 + (r*s*t)/6144 +
227/2048;
B(m,1)=(245*s)/6144 - (15*r)/2048 - (65*t)/6144 - (3*r*s)/2048 + (5*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144
+ 1225/6144;
C(m,1)=(271*t)/6144 - (91*s)/6144 - (77*r)/6144 + (7*r*s)/6144 - (11*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144
+ 1897/6144;
D(m,1)=(11*r*s)/6144 - (127*s)/6144 - (173*t)/6144 - (35*r)/2048 + (17*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 2341/6144;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (19*r^2*t)/905969664 + (131*r^2)/905969664 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (673*r*s)/905969664
- (19*r*t^2)/905969664 + (223*r*t)/301989888 - (2557*r)/452984832 - (7*s^2*t)/301989888 + (55*s^2)/301989888
- (7*s*t^2)/301989888 + (73*s*t)/100663296 - (163*s)/25165824 + (23*t^2)/100663296 - (1073*t)/150994944 +
16285/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 55
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%55=============================================================================================================
=========================================
A(m,1)=(209*r)/6144 - (3*s)/2048 - (3*t)/2048 - (3*r*s)/2048 - (3*r*t)/2048 + (s*t)/6144 + (r*s*t)/6144 +
209/6144;
B(m,1)=(263*s)/6144 - (21*r)/2048 - (35*t)/2048 - (3*r*s)/2048 + (7*r*t)/6144 - (5*s*t)/2048 + (r*s*t)/6144 +
1841/6144;
C(m,1)=(263*t)/6144 - (35*s)/2048 - (21*r)/2048 + (7*r*s)/6144 - (3*r*t)/2048 - (5*s*t)/2048 + (r*s*t)/6144 +
1841/6144;
D(m,1)=(11*r*s)/6144 - (149*s)/6144 - (149*t)/6144 - (83*r)/6144 + (11*r*t)/6144 + (29*s*t)/6144 -
(r*s*t)/2048 + 751/2048;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (17*r^2*t)/905969664 + (97*r^2)/905969664 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (673*r*s)/905969664
- (17*r*t^2)/905969664 + (673*r*t)/905969664 - (11*r)/2359296 - (23*s^2*t)/905969664 + (199*s^2)/905969664 -
(23*s*t^2)/905969664 + (637*s*t)/905969664 - (1081*s)/150994944 + (199*t^2)/905969664 - (1081*t)/150994944 +
45839/905969664;
W(m,1)=WT*a*b*c;
end
end
end
case 56
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
```

```
%56=============================================================================================================================
=========================================
A(m,1)=(209*r)/6144 - (9*s)/2048 - (9*t)/2048 - (3*r*s)/2048 - (3*r*t)/2048 + (s*t)/2048 + (r*s*t)/6144 +
209/2048;
B(m,1)=(245*s)/6144 - (21*r)/2048 - (91*t)/6144 - (3*r*s)/2048 + (7*r*t)/6144 - (13*s*t)/6144 + (r*s*t)/6144
+ 1715/6144;
C(m,1)=(245*t)/6144 - (91*s)/6144 - (21*r)/2048 + (7*r*s)/6144 - (3*r*t)/2048 - (13*s*t)/6144 + (r*s*t)/6144
+ 1715/6144;
D(m,1)=(11*r*s)/6144 - (127*s)/6144 - (127*t)/6144 - (83*r)/6144 + (11*r*t)/6144 + (23*s*t)/6144 -
(r*s*t)/2048 + 2087/6144;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (17*r^2*t)/905969664 + (97*r^2)/905969664 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (5*r*s*t)/150994944 + (605*r*s)/905969664
- (17*r*t^2)/905969664 + (605*r*t)/905969664 - (959*r)/226492416 - (7*s^2*t)/301989888 + (55*s^2)/301989888 -
(7*s*t^2)/301989888 + (191*s*t)/301989888 - (217*s)/37748736 + (55*t^2)/301989888 - (217*t)/37748736 +
12593/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 57
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%57=============================================================================================================================
=========================================
A(m,1)=(83*r)/2048 - (55*s)/6144 - (55*t)/6144 - (11*r*s)/6144 - (11*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144
+ 415/2048;
B(m,1)=(83*s)/2048 - (55*r)/6144 - (55*t)/6144 - (11*r*s)/6144 + (5*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144
+ 415/2048;
C(m,1)=(83*t)/2048 - (55*s)/6144 - (55*r)/6144 + (5*r*s)/6144 - (11*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144
+ 415/2048;
D(m,1)=(17*r*s)/6144 - (139*s)/6144 - (139*t)/6144 - (139*r)/6144 + (17*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 803/2048;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (19*r^2*t)/905969664 + (169*r^2)/905969664 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (17*r*s*t)/452984832 + (223*r*s)/301989888
- (19*r*t^2)/905969664 + (223*r*t)/301989888 - (1463*r)/226492416 - (19*s^2*t)/905969664 +
(169*s^2)/905969664 - (19*s*t^2)/905969664 + (223*s*t)/301989888 - (1463*s)/226492416 + (169*t^2)/905969664 -
(1463*t)/226492416 + 50179/905969664;
W(m,1)=WT*a*b*c;
end
end
end
case 58
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%58=============================================================================================================================
=========================================
A(m,1)=(83*r)/2048 - (77*s)/6144 - (77*t)/6144 - (11*r*s)/6144 - (11*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144
+ 581/2048;
B(m,1)=(227*s)/6144 - (55*r)/6144 - (15*t)/2048 - (11*r*s)/6144 + (5*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144
+ 1135/6144;
C(m,1)=(227*t)/6144 - (15*s)/2048 - (55*r)/6144 + (5*r*s)/6144 - (11*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144
+ 1135/6144;
D(m,1)=(17*r*s)/6144 - (35*s)/2048 - (35*t)/2048 - (139*r)/6144 + (17*r*t)/6144 + (11*s*t)/6144 -
(r*s*t)/2048 + 2131/6144;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (19*r^2*t)/905969664 + (169*r^2)/905969664 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (5*r*s*t)/150994944 + (593*r*s)/905969664
- (19*r*t^2)/905969664 + (593*r*t)/905969664 - (647*r)/113246208 - (17*s^2*t)/905969664 + (131*s^2)/905969664
- (17*s*t^2)/905969664 + (605*s*t)/905969664 - (85*s)/16777216 + (131*t^2)/905969664 - (85*t)/16777216 +
39151/905969664;
W(m,1)=WT*a*b*c;
end
end
end
case 59
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
```

```
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%59=============================================================================================================
=========================================
A(m,1)=(227*r)/6144 - (55*s)/6144 - (15*t)/2048 - (11*r*s)/6144 - (3*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144
+ 1135/6144;
B(m,1)=(83*s)/2048 - (77*r)/6144 - (77*t)/6144 - (11*r*s)/6144 + (7*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144
+ 581/2048;
C(m,1)=(227*t)/6144 - (55*s)/6144 - (15*r)/2048 + (5*r*s)/6144 - (3*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144
+ 1135/6144;
D(m,1)=(17*r*s)/6144 - (139*s)/6144 - (35*t)/2048 - (35*r)/2048 + (11*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 2131/6144;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (17*r^2*t)/905969664 + (131*r^2)/905969664 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (5*r*s*t)/150994944 + (593*r*s)/905969664
- (17*r*t^2)/905969664 + (605*r*t)/905969664 - (85*r)/16777216 - (19*s^2*t)/905969664 + (169*s^2)/905969664 -
(19*s*t^2)/905969664 + (593*s*t)/905969664 - (647*s)/113246208 + (131*t^2)/905969664 - (85*t)/16777216 +
39151/905969664;
W(m,1)=WT*a*b*c;
end
end
end
case 60
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%60=============================================================================================================
=========================================
A(m,1)=(227*r)/6144 - (77*s)/6144 - (21*t)/2048 - (11*r*s)/6144 - (3*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144
+ 1589/6144;
B(m,1)=(227*s)/6144 - (77*r)/6144 - (21*t)/2048 - (11*r*s)/6144 + (7*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144
+ 1589/6144;
C(m,1)=(209*t)/6144 - (15*s)/2048 - (15*r)/2048 + (5*r*s)/6144 - (3*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 +
1045/6144;
D(m,1)=(17*r*s)/6144 - (35*s)/2048 - (83*t)/6144 - (35*r)/2048 + (11*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048
+ 1921/6144;
WT=(r^2*s*t)/905969664 - (19*r^2*s)/905969664 - (17*r^2*t)/905969664 + (131*r^2)/905969664 +
(r*s^2*t)/905969664 - (19*r*s^2)/905969664 + (r*s*t^2)/905969664 - (13*r*s*t)/452984832 + (517*r*s)/905969664
- (17*r*t^2)/905969664 + (179*r*t)/301989888 - (2033*r)/452984832 - (17*s^2*t)/905969664 +
(131*s^2)/905969664 - (17*s*t^2)/905969664 + (179*s*t)/301989888 - (2033*s)/452984832 + (97*t^2)/905969664 -
(431*t)/113246208 + 10165/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 61
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%61=============================================================================================================
=========================================
A(m,1)=(227*r)/6144 - (15*s)/2048 - (55*t)/6144 - (3*r*s)/2048 - (11*r*t)/6144 + (5*s*t)/6144 + (r*s*t)/6144
+ 1135/6144;
B(m,1)=(227*s)/6144 - (15*r)/2048 - (55*t)/6144 - (3*r*s)/2048 + (5*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144
+ 1135/6144;
C(m,1)=(83*t)/2048 - (77*s)/6144 - (77*r)/6144 + (7*r*s)/6144 - (11*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144
+ 581/2048;
D(m,1)=(11*r*s)/6144 - (35*s)/2048 - (139*t)/6144 - (35*r)/2048 + (17*r*t)/6144 + (17*s*t)/6144 -
(r*s*t)/2048 + 2131/6144;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (19*r^2*t)/905969664 + (131*r^2)/905969664 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (5*r*s*t)/150994944 + (605*r*s)/905969664
- (19*r*t^2)/905969664 + (593*r*t)/905969664 - (85*r)/16777216 - (19*s^2*t)/905969664 + (131*s^2)/905969664 -
(19*s*t^2)/905969664 + (593*s*t)/905969664 - (85*s)/16777216 + (169*t^2)/905969664 - (647*t)/113246208 +
39151/905969664;
W(m,1)=WT*a*b*c;
end
end
end
case 62
```

```
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%62=============================================================================================================
=========================================
A(m,1)=(227*r)/6144 - (21*s)/2048 - (77*t)/6144 - (3*r*s)/2048 - (11*r*t)/6144 + (7*s*t)/6144 + (r*s*t)/6144
+ 1589/6144;
B(m,1)=(209*s)/6144 - (15*r)/2048 - (15*t)/2048 - (3*r*s)/2048 + (5*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 +
1045/6144;
C(m,1)=(227*t)/6144 - (21*s)/2048 - (77*r)/6144 + (7*r*s)/6144 - (11*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144
+ 1589/6144;
D(m,1)=(11*r*s)/6144 - (83*s)/6144 - (35*t)/2048 - (35*r)/2048 + (17*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048
+ 1921/6144;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (19*r^2*t)/905969664 + (131*r^2)/905969664 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (13*r*s*t)/452984832 + (179*r*s)/301989888
- (19*r*t^2)/905969664 + (517*r*t)/905969664 - (2033*r)/452984832 - (17*s^2*t)/905969664 + (97*s^2)/905969664
- (17*s*t^2)/905969664 + (179*s*t)/301989888 - (431*s)/113246208 + (131*t^2)/905969664 - (2033*t)/452984832 +
10165/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 63
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%63=============================================================================================================
=========================================
A(m,1)=(209*r)/6144 - (15*s)/2048 - (15*t)/2048 - (3*r*s)/2048 - (3*r*t)/2048 + (5*s*t)/6144 + (r*s*t)/6144 +
1045/6144;
B(m,1)=(227*s)/6144 - (21*r)/2048 - (77*t)/6144 - (3*r*s)/2048 + (7*r*t)/6144 - (11*s*t)/6144 + (r*s*t)/6144
+ 1589/6144;
C(m,1)=(227*t)/6144 - (77*s)/6144 - (21*r)/2048 + (7*r*s)/6144 - (3*r*t)/2048 - (11*s*t)/6144 + (r*s*t)/6144
+ 1589/6144;
D(m,1)=(11*r*s)/6144 - (35*s)/2048 - (35*t)/2048 - (83*r)/6144 + (11*r*t)/6144 + (17*s*t)/6144 - (r*s*t)/2048
+ 1921/6144;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (17*r^2*t)/905969664 + (97*r^2)/905969664 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (13*r*s*t)/452984832 + (179*r*s)/301989888
- (17*r*t^2)/905969664 + (179*r*t)/301989888 - (431*r)/113246208 - (19*s^2*t)/905969664 + (131*s^2)/905969664
- (19*s*t^2)/905969664 + (517*s*t)/905969664 - (2033*s)/452984832 + (131*t^2)/905969664 - (2033*t)/452984832
+ 10165/301989888;
W(m,1)=WT*a*b*c;
end
end
end
case 64
       A=zeros(M,1);B=zeros(M,1);C=zeros(M,1);D=zeros(M,1);W=zeros(M,1);
  m=0;
for I=1:N
for J=1:N
for K=1:N
    r=sn(I,1);s=sn(J,1);t=sn(K,1);
    a=cn(I,1);b=cn(J,1);c=cn(K,1);
    m=m+1;
%64=============================================================================================================
=========================================
A(m,1)=(209*r)/6144 - (21*s)/2048 - (21*t)/2048 - (3*r*s)/2048 - (3*r*t)/2048 + (7*s*t)/6144 + (r*s*t)/6144 +
1463/6144;
B(m,1)=(209*s)/6144 - (21*r)/2048 - (21*t)/2048 - (3*r*s)/2048 + (7*r*t)/6144 - (3*s*t)/2048 + (r*s*t)/6144 +
1463/6144;
C(m,1)=(209*t)/6144 - (21*s)/2048 - (21*r)/2048 + (7*r*s)/6144 - (3*r*t)/2048 - (3*s*t)/2048 + (r*s*t)/6144 +
1463/6144;
D(m,1)=(11*r*s)/6144 - (83*s)/6144 - (83*t)/6144 - (83*r)/6144 + (11*r*t)/6144 + (11*s*t)/6144 - (r*s*t)/2048
+ 585/2048;
WT=(r^2*s*t)/905969664 - (17*r^2*s)/905969664 - (17*r^2*t)/905969664 + (97*r^2)/905969664 +
(r*s^2*t)/905969664 - (17*r*s^2)/905969664 + (r*s*t^2)/905969664 - (11*r*s*t)/452984832 + (469*r*s)/905969664
- (17*r*t^2)/905969664 + (469*r*t)/905969664 - (85*r)/25165824 - (17*s^2*t)/905969664 + (97*s^2)/905969664 -
(17*s*t^2)/905969664 + (469*s*t)/905969664 - (85*s)/25165824 + (97*t^2)/905969664 - (85*t)/25165824 +
23987/905969664;
W(m,1)=WT*a*b*c;
```

```
end
end
end
%=============================================================================================================
==========================================
%
%end%if skip==0
end%switch L

%---------------------------
%%
%n=0;
%for L=17:20
 %   m=0;
%for I=1:N
%for J=1:N
%for K=1:N
%    m=m+1;n=n+1;
%disp('-----------------------------------------------------------------------------------------------------
-------------------------------------------------------')
%disp(['n=',num2str(n) ,'           ','A(', num2str(m),',', num2str(L), ')','             ','B(',
num2str(m),',', num2str(L), ')','                ','C(', num2str(m),',', num2str(L),')','
','D(', num2str(m),',', num2str(L), ')','                      ','W(', num2str(m),',', num2str(L), ')',])
%disp([A(m,L) B(m,L)  C(m,L) D(m,L) W(m,L)])
%end
%end
%end
%end
%disp('-----------------------------------------------------------------------------------------------------
-------------------------------------------------------')
%end%if skip==0
%return
%LL=L;
%[LL m]
%M=N*N*N;
%for L=17:20
%for K=1:M
%fprintf('%20.16e %20.16e %20.16e %20.16e %20.16e;...\n', A(K,L), B(K,L),C(K,L),D(K,L),W(K,L))
%end
%end
%[LL m n]
%[L M]
disp(['case ',num2str(L)])
disp('table=[ ')
for K=1:M
fprintf('%20.16e %20.16e %20.16e %20.16e %20.16e;...\n', A(K,1), B(K,1),C(K,1),D(K,1),W(K,1))
enddisp(']; ')
```

(4)program-4

```
function[ii]=integration_tetrahedron(N,FN,CC)
%COMPOSITE INTEGRATION ON AN ARBITRARY TETRAHEDRON
%USE OF AFFINE TRANSFORMATION TO MAP AN ARBITRARY TETRAHEDRON INTO A UNIT TETRAHEDRON
%A UNIT TETRAHEDRON IS DIVIDED INTO FOUR UNIQUE HEXAHEDRA
%N:ORDER OF QUADRATRE
%FN:N_TH FUNCTION NUMBER
% MATRIX OF COORDINATES FOR VERTICES AS COLUMN
%FOR EXAMPLE PROBLEM:CC=[5 5 0;10 10 0;10 5 0;8 7 8];
%X=CC(:,1);Y=CC(:,2);Z=CC(:,3);
%integration_tetrahedron(5,9,[1 0 0;0 1 0;0 0 1;0 0 0])
%integration_tetrahedron(5,1,[5 5 0;10 10 0;10 5 0;8 7 8])%HTR PAPER
%integration_tetrahedron(5,7,[1 1/2 1;1 1 1/2;1/2 1 1;1/2 1/2 1/2])%MINUS TETRAHEDRON
%global A B C D W
%global A B C D W
%syms x y z X Y Z J
%syms r s t  f1 f2 f3 f4
%syms ss ii
%(5,5,0),(10,10,0),(10,5,0),(8,7,8)
%X=[5;10;10;8];Y=[5;10;5;7];Z=[0;0;0;8];
X=CC(:,1);Y=CC(:,2);Z=CC(:,3);
%disp([X Y Z])
X1=X(1,1);X2=X(2,1);X3=X(3,1);X4=X(4,1);
Y1=Y(1,1);Y2=Y(2,1);Y3=Y(3,1);Y4=Y(4,1);
Z1=Z(1,1);Z2=Z(2,1);Z3=Z(3,1);Z4=Z(4,1);
J=det([(X4-X1),(X4-X2),(X4-X3);(Y4-Y1),(Y4-Y2),(Y4-Y3);(Z4-Z1),(Z4-Z2),(Z4-Z3)]);
%disp(J)
[A,B,C,D,W]=hexahedrasampleptsweights(N);
```

```
%x=X4+(X1-X4)*r+(X2-X4)*s+(X3-X4)*t;
%y=Y4+(Y1-Y4)*r+(Y2-Y4)*s+(Y3-Y4)*t;
%z=Z4+(Z1-Z4)*r+(Z2-Z4)*s+(Z3-Z4)*t;
%disp([x;y;z])
M=N*N*N;
J=abs(J);
ss=0;
for L=1:M
a=A(L,1);b=B(L,1);c=C(L,1);d=D(L,1);
 %xx1=subs(x,{r,s,t},{a,b,c});xx2=subs(x,{r,s,t},{d,a,c});xx3=subs(x,{r,s,t},{b,d,c});xx4=subs(x,{r,s,t},{b,a,d});
 %yy1=subs(y,{r,s,t},{a,b,c});yy2=subs(y,{r,s,t},{d,a,c});yy3=subs(y,{r,s,t},{b,d,c});yy4=subs(y,{r,s,t},{b,a,d});
 %zz1=subs(z,{r,s,t},{a,b,c});zz2=subs(z,{r,s,t},{d,a,c});zz3=subs(z,{r,s,t},{b,d,c});zz4=subs(z,{r,s,t},{b,a,d});
 rp=a;sp=b;tp=c;
xx1=X4+(X1-X4)*rp+(X2-X4)*sp+(X3-X4)*tp;
yy1=Y4+(Y1-Y4)*rp+(Y2-Y4)*sp+(Y3-Y4)*tp;
zz1=Z4+(Z1-Z4)*rp+(Z2-Z4)*sp+(Z3-Z4)*tp;
rp=d;sp=a;tp=c;
xx2=X4+(X1-X4)*rp+(X2-X4)*sp+(X3-X4)*tp;
yy2=Y4+(Y1-Y4)*rp+(Y2-Y4)*sp+(Y3-Y4)*tp;
zz2=Z4+(Z1-Z4)*rp+(Z2-Z4)*sp+(Z3-Z4)*tp;
rp=b;sp=d;tp=c;
xx3=X4+(X1-X4)*rp+(X2-X4)*sp+(X3-X4)*tp;
yy3=Y4+(Y1-Y4)*rp+(Y2-Y4)*sp+(Y3-Y4)*tp;
zz3=Z4+(Z1-Z4)*rp+(Z2-Z4)*sp+(Z3-Z4)*tp;
rp=b;sp=a;tp=d;
xx4=X4+(X1-X4)*rp+(X2-X4)*sp+(X3-X4)*tp;
yy4=Y4+(Y1-Y4)*rp+(Y2-Y4)*sp+(Y3-Y4)*tp;
zz4=Z4+(Z1-Z4)*rp+(Z2-Z4)*sp+(Z3-Z4)*tp;
 f1=(FNXYZ(FN,xx1,yy1,zz1));
 f2=(FNXYZ(FN,xx2,yy2,zz2));
 f3=(FNXYZ(FN,xx3,yy3,zz3));
 f4=(FNXYZ(FN,xx4,yy4,zz4));
   ss=ss+W(L,1)*(f1+f2+f3+f4);
end
ii=J*ss;
%disp(ii)
%disp(47165/3)%exact value for case 1
```

(5)program-5

```
function[ii]=integration_tetrahedron32hexahedron(N,FN,CC)
%COMPOSITE INTEGRATION ON AN ARBITRARY TETRAHEDRON
%USE OF AFFINE TRANSFORMATION TO MAP AN ARBITRARY TETRAHEDRON INTO A UNIT TETRAHEDRON
%A UNIT TETRAHEDRON IS DIVIDED INTO FOUR UNIQUE HEXAHEDRA
%AND THEN EACH OF THESE HEXAHEDRA IS DIVIDED INTO EIGHT HEXAHEDRA
%N:ORDER OF QUADRATRE
%FN:N_TH FUNCTION NUMBER
% MATRIX OF COORDINATES FOR VERTICES AS COLUMN
%FOR EXAMPLE PROBLEM:CC=[5 5 0;10 10 0;10 5 0;8 7 8];
%X=CC(:,1);Y=CC(:,2);Z=CC(:,3);
%integration_tetrahedron(5,9,[1 0 0;0 1 0;0 0 1;0 0 0])
%global A B C D W
%syms x y z X Y Z J
%syms r s t  f1 f2 f3 f4
%syms ss ii
%(5,5,0),(10,10,0),(10,5,0),(8,7,8)
%X=[5;10;10;8];Y=[5;10;5;7];Z=[0;0;0;8];
X=CC(:,1);Y=CC(:,2);Z=CC(:,3);
%disp([X Y Z])
X1=X(1,1);X2=X(2,1);X3=X(3,1);X4=X(4,1);
Y1=Y(1,1);Y2=Y(2,1);Y3=Y(3,1);Y4=Y(4,1);
Z1=Z(1,1);Z2=Z(2,1);Z3=Z(3,1);Z4=Z(4,1);
J=det([(X4-X1),(X4-X2),(X4-X3);(Y4-Y1),(Y4-Y2),(Y4-Y3);(Z4-Z1),(Z4-Z2),(Z4-Z3)]);
%disp(J)
%[A,B,C,D,W]=hexahedrasampleptsweights(N);

[A,B,C,D,W]=hexahedrasampleptsweights4eightdivisions(N);
%x=X4+(X1-X4)*r+(X2-X4)*s+(X3-X4)*t;
%y=Y4+(Y1-Y4)*r+(Y2-Y4)*s+(Y3-Y4)*t;
%z=Z4+(Z1-Z4)*r+(Z2-Z4)*s+(Z3-Z4)*t;
%disp([x;y;z])
M=N*N*N;
J=abs(J);
ss=0;
for K=1:8
for L=1:M
a=A(L,K);b=B(L,K);c=C(L,K);d=D(L,K);
```

```
%xx1=subs(x,{r,s,t},{a,b,c});xx2=subs(x,{r,s,t},{d,a,c});xx3=subs(x,{r,s,t},{b,d,c});xx4=subs(x,{r,s,t},{b,a,
d});

%yy1=subs(y,{r,s,t},{a,b,c});yy2=subs(y,{r,s,t},{d,a,c});yy3=subs(y,{r,s,t},{b,d,c});yy4=subs(y,{r,s,t},{b,a,
d});

%zz1=subs(z,{r,s,t},{a,b,c});zz2=subs(z,{r,s,t},{d,a,c});zz3=subs(z,{r,s,t},{b,d,c});zz4=subs(z,{r,s,t},{b,a,
d});
 rp=a;sp=b;tp=c;
xx1=X4+(X1-X4)*rp+(X2-X4)*sp+(X3-X4)*tp;
yy1=Y4+(Y1-Y4)*rp+(Y2-Y4)*sp+(Y3-Y4)*tp;
zz1=Z4+(Z1-Z4)*rp+(Z2-Z4)*sp+(Z3-Z4)*tp;
rp=d;sp=a;tp=c;
xx2=X4+(X1-X4)*rp+(X2-X4)*sp+(X3-X4)*tp;
yy2=Y4+(Y1-Y4)*rp+(Y2-Y4)*sp+(Y3-Y4)*tp;
zz2=Z4+(Z1-Z4)*rp+(Z2-Z4)*sp+(Z3-Z4)*tp;
rp=b;sp=d;tp=c;
xx3=X4+(X1-X4)*rp+(X2-X4)*sp+(X3-X4)*tp;
yy3=Y4+(Y1-Y4)*rp+(Y2-Y4)*sp+(Y3-Y4)*tp;
zz3=Z4+(Z1-Z4)*rp+(Z2-Z4)*sp+(Z3-Z4)*tp;
rp=b;sp=a;tp=d;
xx4=X4+(X1-X4)*rp+(X2-X4)*sp+(X3-X4)*tp;
yy4=Y4+(Y1-Y4)*rp+(Y2-Y4)*sp+(Y3-Y4)*tp;
zz4=Z4+(Z1-Z4)*rp+(Z2-Z4)*sp+(Z3-Z4)*tp;
 f1=(FNXYZ(FN,xx1,yy1,zz1));
 f2=(FNXYZ(FN,xx2,yy2,zz2));
 f3=(FNXYZ(FN,xx3,yy3,zz3));
 f4=(FNXYZ(FN,xx4,yy4,zz4));
    ss=ss+W(L,K)*(f1+f2+f3+f4);
end
end
ii=J*ss;
%disp(ii)
%disp(47165/3)%exact value for case 1
```

(6)program-6

```
function[ii]=integration_tetrahedron256hexahedron(N,FN,CC)
%global A B C D W
%COMPOSITE INTEGRATION ON AN ARBITRARY TETRAHEDRON
%USE OF AFFINE TRANSFORMATION TO MAP AN ARBITRARY TETRAHEDRON INTO A UNIT TETRAHEDRON
%A UNIT TETRAHEDRON IS DIVIDED INTO FOUR UNIQUE HEXAHEDRA
%AND THEN EACH OF THESE HEXAHEDRA IS DIVIDED INTO EIGHT HEXAHEDRA
%N:ORDER OF QUADRATRE
%FN:N_TH FUNCTION NUMBER
% MATRIX OF COORDINATES FOR VERTICES AS COLUMN
%FOR EXAMPLE PROBLEM:CC=[5 5 0;10 10 0;10 5 0;8 7 8];
%X=CC(:,1);Y=CC(:,2);Z=CC(:,3);
%integration_tetrahedron(5,9,[1 0 0;0 1 0;0 0 1;0 0 0])
%global A B C D W
%syms x y z X Y Z J
%syms r s t  f1 f2 f3 f4
%syms ss ii
%(5,5,0),(10,10,0),(10,5,0),(8,7,8)
%X=[5;10;10;8];Y=[5;10;5;7];Z=[0;0;0;8];
format long e
X=CC(:,1);Y=CC(:,2);Z=CC(:,3);
%disp([X Y Z])
X1=X(1,1);X2=X(2,1);X3=X(3,1);X4=X(4,1);
Y1=Y(1,1);Y2=Y(2,1);Y3=Y(3,1);Y4=Y(4,1);
Z1=Z(1,1);Z2=Z(2,1);Z3=Z(3,1);Z4=Z(4,1);
J=det([(X4-X1),(X4-X2),(X4-X3);(Y4-Y1),(Y4-Y2),(Y4-Y3);(Z4-Z1),(Z4-Z2),(Z4-Z3)]);
%disp(J)
%[A,B,C,D,W]=hexahedrasampleptsweights(N);

%[A,B,C,D,W]=hexahedrasampleptsweights4eightdivisions(N);
%%[A,B,C,D,W]=hexahedrasampleptsweights4sixtyfourdivisions(N);
%for LL=1:64
 %   [AA,BB,CC,DD,WW]=hexahedrasampleptsweights4sixtyfourdivisions1X1(N,LL);
  %  A(:,LL)=AA(:,1);
   % B(:,LL)=BB(:,1);
   % C(:,LL)=CC(:,1);
   % D(:,LL)=DD(:,1);
   % W(:,LL)=WW(:,1);
%end
%[A,B,C,D,W]=hexahedraexplicitsampleptsweights4sixtyfourdivisions(N);
%x=X4+(X1-X4)*r+(X2-X4)*s+(X3-X4)*t;
%y=Y4+(Y1-Y4)*r+(Y2-Y4)*s+(Y3-Y4)*t;
%z=Z4+(Z1-Z4)*r+(Z2-Z4)*s+(Z3-Z4)*t;
%disp([x;y;z])
```

```
M=N*N*N;
J=abs(J);
ss=0;
for K=1:64
     %[AA,BB,CC,DD,WW]=hexahedrasampleptsweights4sixtyfourdivisions1X1(N,K);
    %A(:,K)=AA(:,1);
    %B(:,K)=BB(:,1);
    %C(:,K)=CC(:,1);
    %D(:,K)=DD(:,1);
    %W(:,K)=WW(:,1);
  %[A,B,C,D,W]=hexahedrasampleptsweights4sixtyfourdivisions1X1(N,K);
  switch N
   case 2
  [A,B,C,D,W]=hexahedrasampleptsweights4sixtyfourdivisions1X1explicit2(K);
  case 3
  [A,B,C,D,W]=hexahedrasampleptsweights4sixtyfourdivisions1X1explicit3(K);
  case 4
  [A,B,C,D,W]=hexahedrasampleptsweights4sixtyfourdivisions1X1explicit4(K);
  case 5
  [A,B,C,D,W]=hexahedrasampleptsweights4sixtyfourdivisions1X1explicit5(K);
  case 10
  [A,B,C,D,W]=hexahedrasampleptsweights4sixtyfourdivisions1X1explicit10(K);

  end%switch N
for L=1:M

%a=A(L,K);b=B(L,K);c=C(L,K);d=D(L,K);
a=A(L,1);b=B(L,1);c=C(L,1);d=D(L,1);

%xx1=subs(x,{r,s,t},{a,b,c});xx2=subs(x,{r,s,t},{d,a,c});xx3=subs(x,{r,s,t},{b,d,c});xx4=subs(x,{r,s,t},{b,a,
d});

%yy1=subs(y,{r,s,t},{a,b,c});yy2=subs(y,{r,s,t},{d,a,c});yy3=subs(y,{r,s,t},{b,d,c});yy4=subs(y,{r,s,t},{b,a,
d});

%zz1=subs(z,{r,s,t},{a,b,c});zz2=subs(z,{r,s,t},{d,a,c});zz3=subs(z,{r,s,t},{b,d,c});zz4=subs(z,{r,s,t},{b,a,
d});
 rp=a;sp=b;tp=c;
xx1=X4+(X1-X4)*rp+(X2-X4)*sp+(X3-X4)*tp;
yy1=Y4+(Y1-Y4)*rp+(Y2-Y4)*sp+(Y3-Y4)*tp;
zz1=Z4+(Z1-Z4)*rp+(Z2-Z4)*sp+(Z3-Z4)*tp;
rp=d;sp=a;tp=c;
xx2=X4+(X1-X4)*rp+(X2-X4)*sp+(X3-X4)*tp;
yy2=Y4+(Y1-Y4)*rp+(Y2-Y4)*sp+(Y3-Y4)*tp;
zz2=Z4+(Z1-Z4)*rp+(Z2-Z4)*sp+(Z3-Z4)*tp;
rp=b;sp=d;tp=c;
xx3=X4+(X1-X4)*rp+(X2-X4)*sp+(X3-X4)*tp;
yy3=Y4+(Y1-Y4)*rp+(Y2-Y4)*sp+(Y3-Y4)*tp;
zz3=Z4+(Z1-Z4)*rp+(Z2-Z4)*sp+(Z3-Z4)*tp;
rp=b;sp=a;tp=d;
xx4=X4+(X1-X4)*rp+(X2-X4)*sp+(X3-X4)*tp;
yy4=Y4+(Y1-Y4)*rp+(Y2-Y4)*sp+(Y3-Y4)*tp;
zz4=Z4+(Z1-Z4)*rp+(Z2-Z4)*sp+(Z3-Z4)*tp;
 f1=(FNXYZ(FN,xx1,yy1,zz1));
 f2=(FNXYZ(FN,xx2,yy2,zz2));
 f3=(FNXYZ(FN,xx3,yy3,zz3));
 f4=(FNXYZ(FN,xx4,yy4,zz4));
    %ss=ss+W(L,K)*(f1+f2+f3+f4);
    ss=ss+W(L,1)*(f1+f2+f3+f4);
end
end
ii=J*ss;
%disp(ii)
%disp(47165/3)%exact value for case 1
```

(7)program-7

```
function[]=integration_tetrahedron_hexahedron(N1,NC,N2,FN1,FN2)
%integration_tetrahedron_hexahedron(5,5,40,7,10)
%coordinates of the nodes on hexahedron
%global A B C D W
%[A,B,C,D,W]=hexahedrasampleptsweights(N);
%[A,B,C,D,W]=sampleptsweightshexatetrahedron(N);
Q=[1 0 0;1 1 0;0 1 0;0 0 0;...
  1 0 1;1 1 1;0 1 1;0 0 1];
[L M]=size(Q);
v=[1 2 4 6 1 6 4 5 5 6 4 8 ...%triangular prism-1
  2 3 4 7 2 4 6 7 4 7 8 6];%triangular prism-2
[p q]=size(v);
IFN=0;
```

```
for FN=FN1:FN2
   IFN=IFN+1;IN=0;
for N=N1:NC:N2
   IN=IN+1;
mm=0;
for pp=1:4:q
 CC=Q(v(pp:pp+3),:);
 [ii]=integration_tetrahedron(N,FN,CC);
 mm=mm+1;
 II(mm,IFN)=ii;
end
iii(IN,IFN)=0;
for nn=1:mm
   iii(IN,IFN)=iii(IN,IFN)+II(nn,IFN);
end
[N FN iii(IN,IFN)]
end%FOR N
%
end%FOR FN
disp('-------------------------------------------------------------------------------------------------------------------------------')
disp(['gauss legendre rule N=',' TRIPLE INTEGRALS:    iii1        iii2          iii3          iii4'])
[(N1:NC:N2)' iii]
```

(8)program-8

```
function[]=integration_6pyramids_hexahedron(N1,NC,N2,FN1,FN2)
%integration_6pyramids_hexahedron(5,40,7,10)
%integration_6pyramids_hexahedron(4,4,32,7,10)
IFN=0;
for FN=FN1:FN2%7:10FOR UNIT CUBE
    IFN=IFN+1;
switch FN
case 1 %TETRAHEDRA HTR PAPER
 disp('FF=x^2*y;')

case 2%UNIT ORTHOGONAL TETRAHEDRON
    %FF=x^2*y^2;
  disp('FF=(x+y+z)^(1/2);')
    %0.142857142857143
case 3%UNIT ORTHOGONAL TETRAHEDRON
   % FF=x^4*y^4;
 disp('FF=(x+y+z)^(-1/2);')
   %0.2
case 4%UNIT ORTHOGONAL TETRAHEDRON
   disp('FF=x^3*sin(pi*y)*sin(pi*z);')
   %0.0011802154211291
case 5%UNIT ORTHOGONAL TETRAHEDRON

 disp(' FF=sin(x+2*y+4*z);')
  %0.13190232689018
case 6%UNIT ORTHOGONAL TETRAHEDRON

disp('FF=1/((1+x+y+z)^4);')
%0.020833333333333
case 7%unit cube or %UNIT ORTHOGONAL TETRAHEDRON
 disp('FF=x^3*sin(pi*y)*sin(pi*z);% 1/(pi^2)= 1.013211836423378e-001')
case 8%unit cube or %UNIT ORTHOGONAL TETRAHEDRON
   disp(' FF=sin(pi*x)*sin(pi*y)*sin(pi*z);  %ii=8/pi^3 %ii =.258012275465595913475376421508 51 OR')
  case 9%unit cube or %UNIT ORTHOGONAL TETRAHEDRON
     disp('   FF=exp(-((x-.5)^2+(y-.5)^2+(z-.5)^2));%ii= 7.852115961743685e-001 for GL ORDER=20 OR')
 case 10%unit cube or %UNIT ORTHOGONAL TETRAHEDRON
disp('FF=(27/8)*sqrt(1-abs(2*x-1))*sqrt(1-abs(2*y-1))*sqrt(1-abs(2*z-1));')
    case 11%%TETRAHEDRA F.TONON PAPER
      disp('  FF=y^2+z^2;%43520.33257 ')
    case 12%%TETRAHEDRA F.TONON PAPER
       disp(' FF=x^2+z^2;')

     case 13%%TETRAHEDRA F.TONON PAPER
       disp(' FF=y^2+x^2;')

    case 14%%TETRAHEDRA F.TONON PAPER
    disp(' FF=y*z;')

 case 15%%TETRAHEDRA F.TONON PAPER
     disp('FF=x*z;')
  case 16%%TETRAHEDRA F.TONON PAPER
    disp('FF=x*y;')

end
```

```
%
Q=[1 0 0;1 1 0;0 1 0;0 0 0;...
   1 0 1;1 1 1;0 1 1;0 0 1;...
   0 1/2 1/2;1 1/2 1/2;1/2 0 1/2;1/2 1 1/2;...
   1/2 1/2 0;1/2 1/2 1;1/2 1/2 1/2];
[L M]=size(Q);
v=[1 2 13 15 2 3 13 15 4 1 13 15 3 4 13 15 ...%prmd-1
   5 6 14 15 6 7 14 15 7 8 14 15 8 5 14 15 ...%prmd-2
   2 3 12 15 3 7 12 15 7 6 12 15 6 2 12 15 ...%prmd-3
   1 4 11 15 4 8 11 15 8 5 11 15 5 1 11 15 ...%prmd-4
   4 3 9 15 3 7 9 15 7 8 9 15 8 4 9 15 ...    %prmd-5
   5 1 10 15 1 2 10 15 2 6 10 15 6 5 10 15];%prmd-6
[p q]=size(v);
MM1=Q(v(1:4),:);
MM2=Q(v(5:8),:);
MM3=Q(v(9:12),:);
MM4=Q(v(13:16),:);
MM5=Q(v(17:20),:);
MM6=Q(v(21:24),:);
MM7=Q(v(25:28),:);
MM8=Q(v(29:32),:);
MM9=Q(v(33:36),:);
MM10=Q(v(37:40),:);
MM11=Q(v(41:44),:);
MM12=Q(v(45:48),:);
MM13=Q(v(49:52),:);
MM14=Q(v(53:56),:);
MM15=Q(v(57:60),:);
MM16=Q(v(61:64),:);
MM17=Q(v(65:68),:);
MM18=Q(v(69:72),:);
MM19=Q(v(73:76),:);
MM20=Q(v(77:80),:);
MM21=Q(v(81:84),:);
MM22=Q(v(85:88),:);
MM23=Q(v(89:92),:);
MM24=Q(v(93:96),:);
IN=0;
for N=N1:NC:N2
IN=IN+1;
NN(IN,1)=N;
iii=0;
%PYRAMID-1
[ii1]=integration_tetrahedron(N,FN,MM1);II(1,1)=ii1;
[ii2]=integration_tetrahedron(N,FN,MM2);II(2,1)=ii2;
[ii3]=integration_tetrahedron(N,FN,MM3);II(3,1)=ii3;
[ii4]=integration_tetrahedron(N,FN,MM4);II(4,1)=ii4;
%PYRAMID-2;
[ii5]=integration_tetrahedron(N,FN,MM5);II(5,1)=ii5;
[ii6]=integration_tetrahedron(N,FN,MM6);II(6,1)=ii6;
[ii7]=integration_tetrahedron(N,FN,MM7);II(7,1)=ii7;
[ii8]=integration_tetrahedron(N,FN,MM8);II(8,1)=ii8;
%PYRAMID-3
[ii9]=integration_tetrahedron(N,FN,MM9);II(9,1)=ii9;
[ii10]=integration_tetrahedron(N,FN,MM10);II(10,1)=ii10;
[ii11]=integration_tetrahedron(N,FN,MM11);II(11,1)=ii11;
[ii12]=integration_tetrahedron(N,FN,MM12);II(12,1)=ii12;
%PYRAMID-4
[ii13]=integration_tetrahedron(N,FN,MM13);II(13,1)=ii13;
[ii14]=integration_tetrahedron(N,FN,MM14);II(14,1)=ii14;
[ii15]=integration_tetrahedron(N,FN,MM15);II(15,1)=ii15;
[ii16]=integration_tetrahedron(N,FN,MM16);II(16,1)=ii16;
%PYRAMID-5
[ii17]=integration_tetrahedron(N,FN,MM17);II(17,1)=ii17;
[ii18]=integration_tetrahedron(N,FN,MM18);II(18,1)=ii18;
[ii19]=integration_tetrahedron(N,FN,MM19);II(19,1)=ii19;
[ii20]=integration_tetrahedron(N,FN,MM20);II(20,1)=ii20;
%PYRAMID-6
[ii21]=integration_tetrahedron(N,FN,MM21);II(21,1)=ii21;
[ii22]=integration_tetrahedron(N,FN,MM22);II(22,1)=ii22;
[ii23]=integration_tetrahedron(N,FN,MM23);II(23,1)=ii23;
[ii24]=integration_tetrahedron(N,FN,MM24);II(24,1)=ii24;
for n=1:24
    iii=iii+II(n,1);
end
III(IN,IFN)=iii;
disp(iii)
end%for N
%
end%for FN
```

```
disp([NN III])

(9)program-9
function[]=integration_irregularheptahedron_hexahedron(N1,NC,N2,FN1,FN2)
%integration_6pyramids_hexahedron(5,40,7,10)
%integration_6pyramids_hexahedron(4,4,32,7,10)
%integration_6pyramidsminustetrahedron_hexahedron(4,4,32,7,10)
%integration_irregularheptahedron_hexahedron(4,4,32,7,10)
IFN=0;
for FN=FN1:FN2%7:10FOR UNIT CUBE
    IFN=IFN+1;
switch FN
case 1 %TETRAHEDRA HTR PAPER
 disp('FF=x^2*y;')

case 2%UNIT ORTHOGONAL TETRAHEDRON
    %FF=x^2*y^2;
  disp('FF=(x+y+z)^(1/2);')
    %0.142857142857143
case 3%UNIT ORTHOGONAL TETRAHEDRON
   % FF=x^4*y^4;
 disp('FF=(x+y+z)^(-1/2);')
   %0.2
case 4%UNIT ORTHOGONAL TETRAHEDRON
   disp('FF=x^3*sin(pi*y)*sin(pi*z);')
   %0.0011802154211291
case 5%UNIT ORTHOGONAL TETRAHEDRON

 disp(' FF=sin(x+2*y+4*z);')
  %0.13190232689018
case 6%UNIT ORTHOGONAL TETRAHEDRON

disp('FF=1/((1+x+y+z)^4);')
%0.020833333333333
case 7%unit cube or %UNIT ORTHOGONAL TETRAHEDRON
 disp('FF=x^3*sin(pi*y)*sin(pi*z);% 1/(pi^2)= 1.013211836423378e-001')
case 8%unit cube or %UNIT ORTHOGONAL TETRAHEDRON
   disp(' FF=sin(pi*x)*sin(pi*y)*sin(pi*z);  %ii=8/pi^3 %ii =.258012275465595913475376421508 51 OR')
  case 9%unit cube or %UNIT ORTHOGONAL TETRAHEDRON
     disp('   FF=exp(-((x-.5)^2+(y-.5)^2+(z-.5)^2));%ii= 7.852115961743685e-001 for GL ORDER=20 OR')
 case 10%unit cube or %UNIT ORTHOGONAL TETRAHEDRON
disp('FF=(27/8)*sqrt(1-abs(2*x-1))*sqrt(1-abs(2*y-1))*sqrt(1-abs(2*z-1));')
    case 11%%TETRAHEDRA F.TONON PAPER
      disp('  FF=y^2+z^2;%43520.33257 ')
    case 12%%TETRAHEDRA F.TONON PAPER
       disp(' FF=x^2+z^2;')

     case 13%%TETRAHEDRA F.TONON PAPER
       disp(' FF=y^2+x^2;')

    case 14%%TETRAHEDRA F.TONON PAPER
    disp(' FF=y*z;')

 case 15%%TETRAHEDRA F.TONON PAPER
     disp('FF=x*z;')
  case 16%%TETRAHEDRA F.TONON PAPER
    disp('FF=x*y;')

end
%
Q=[1 0 0;1 1 0;0 1 0;0 0 0;...%1-4
   1 0 1;1 1/2 1;1 1 1/2;1/2 1 1;0 1 1;0 0 1;...%5-10
   1 1/2 1/2;1/2 1 1/2;0 1/2 1/2;1/2 0 1/2;...%11-14
   1/2 1/2 0;1/2 1/2 1;1/2 1/2 1/2];%15-17
[L M]=size(Q);
v=[7 6 11 17 6 5 11 17 5 1 11 17 1 2 11 17 2 7 11 17 ...%face-1=20
   2 3 12 17 3 9 12 17 9 8 12 17 8 7 12 17 7 2 12 17 ...%face-2=20
   4 3 13 17 3 9 13 17 9 10 13 17 10 4 13 17 ...%face-3=16
   5 1 14 17 1 4 14 17 4 10 14 17 10 5 14 17 ...%face-4=16
   3 4 15 17 4 1 15 17 1  2 15 17  2 3 15 17 ...%face-5=16
   5 6 16 17 6 8 16 17 8  9 16 17  9 10 16 17 10 5 16 17 ...%face-6=20
   6 7 8 17];%face-7
[p q]=size(v);
MM1=Q(v(1:4),:);
MM2=Q(v(5:8),:);
MM3=Q(v(9:12),:);
MM4=Q(v(13:16),:);
MM5=Q(v(17:20),:);
MM6=Q(v(21:24),:);
MM7=Q(v(25:28),:);
```

```
MM8=Q(v(29:32),:);
MM9=Q(v(33:36),:);
MM10=Q(v(37:40),:);
MM11=Q(v(41:44),:);
MM12=Q(v(45:48),:);
MM13=Q(v(49:52),:);
MM14=Q(v(53:56),:);
MM15=Q(v(57:60),:);
MM16=Q(v(61:64),:);
MM17=Q(v(65:68),:);
MM18=Q(v(69:72),:);
MM19=Q(v(73:76),:);
MM20=Q(v(77:80),:);
MM21=Q(v(81:84),:);
MM22=Q(v(85:88),:);
MM23=Q(v(89:92),:);
MM24=Q(v(93:96),:);
MM25=Q(v(97:100),:);
MM26=Q(v(101:104),:);
MM27=Q(v(105:108),:);
MM28=Q(v(109:112),:);
IN=0;
for N=N1:NC:N2
IN=IN+1;
NN(IN,1)=N;
iii=0;
%
[ii1]=integration_tetrahedron(N,FN,MM1);II(1,1)=ii1;
[ii2]=integration_tetrahedron(N,FN,MM2);II(2,1)=ii2;
[ii3]=integration_tetrahedron(N,FN,MM3);II(3,1)=ii3;
[ii4]=integration_tetrahedron(N,FN,MM4);II(4,1)=ii4;
%
[ii5]=integration_tetrahedron(N,FN,MM5);II(5,1)=ii5;
[ii6]=integration_tetrahedron(N,FN,MM6);II(6,1)=ii6;
[ii7]=integration_tetrahedron(N,FN,MM7);II(7,1)=ii7;
[ii8]=integration_tetrahedron(N,FN,MM8);II(8,1)=ii8;
%
[ii9]=integration_tetrahedron(N,FN,MM9);II(9,1)=ii9;
[ii10]=integration_tetrahedron(N,FN,MM10);II(10,1)=ii10;
[ii11]=integration_tetrahedron(N,FN,MM11);II(11,1)=ii11;
[ii12]=integration_tetrahedron(N,FN,MM12);II(12,1)=ii12;
%
[ii13]=integration_tetrahedron(N,FN,MM13);II(13,1)=ii13;
[ii14]=integration_tetrahedron(N,FN,MM14);II(14,1)=ii14;
[ii15]=integration_tetrahedron(N,FN,MM15);II(15,1)=ii15;
[ii16]=integration_tetrahedron(N,FN,MM16);II(16,1)=ii16;
%
[ii17]=integration_tetrahedron(N,FN,MM17);II(17,1)=ii17;
[ii18]=integration_tetrahedron(N,FN,MM18);II(18,1)=ii18;
[ii19]=integration_tetrahedron(N,FN,MM19);II(19,1)=ii19;
[ii20]=integration_tetrahedron(N,FN,MM20);II(20,1)=ii20;
%
[ii21]=integration_tetrahedron(N,FN,MM21);II(21,1)=ii21;
[ii22]=integration_tetrahedron(N,FN,MM22);II(22,1)=ii22;
[ii23]=integration_tetrahedron(N,FN,MM23);II(23,1)=ii23;
[ii24]=integration_tetrahedron(N,FN,MM24);II(24,1)=ii24;
%
[ii25]=integration_tetrahedron(N,FN,MM25);II(25,1)=ii25;
[ii26]=integration_tetrahedron(N,FN,MM26);II(26,1)=ii26;
[ii27]=integration_tetrahedron(N,FN,MM27);II(27,1)=ii27;
[ii28]=integration_tetrahedron(N,FN,MM28);II(28,1)=ii28;
for n=1:28
    iii=iii+II(n,1);
end
III(IN,IFN)=iii;
%disp([iii,ii25,iiii])
end%for N
%
end%for FN
disp([NN III])
```