

Purelet approach and ICA Based Poisson Noise Reduction in MRI Data Set

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Abstract— In this paper we proposed a hybrid method for Poisson noise amputation in MRI (Magnetic Resonance Imaging) datasets. The awareness should be paid while enhancing the Poisson noise, because it is vastly signal dependent. The Independent Component Analysis and Purelet (Poisson Unbiased Risk Estimation) performances are assessed directly. In purelet we are using Linear Expansion of thresholding. We approximate the mean square error to bring a dutiful transform domain thresholding. The hybrid method is in cooperation with ICA and Purelet Method. Foremost the ICA bring into play intended for dimensionality reduction for Multivariate Data, Subsequently Wavelet threshold Method exploit for Denoising. Performance Comparison is exploited in requisite of PSNR (Peak Signal to Noise Ratio) and speed of Denoising.

Key words— Poisson Noise, Image Denoising, Purelet, Thresholding, Dimensionality, Reduction, Independent Component Analysis, Unbiased Estimation.

I.INTRODUCTION

The noise is occurred in images during the acquisition process, since the intrinsic and thermal fluctuations of acquisition devices. The other reason is only low count photon unruffled by the sensors while comparing others, the signal dependent noise is imperative. It should be uneasy. Image processing takes part in medical field of the essence. During the disease diagnosis, the consequences of many types of equipment in the medical field are in digital format. In the vein of X-Ray based scan reports, MRI (Magnetic resonance Imaging), CT (Computed tomography), PET (Positron Emission Tomography) furnish the digital form reports. There are many prehistoric methods are used for denoising which have its own annoyances. In this paper we introduce the constructive hybrid method for effectual separation of noise from data and eliminate it. There are multiscale methods and also non-multiscale methods are on hand for Poisson noise. At this juncture we are using the transform domain thresholding strategies which is a multiscale technique. The independent component analysis is employed for representing the multivariate data and to diminish the curse of dimensionality.

II. RELATED WORK

The general solution of Poisson noise is “Gaussianizing” this is carried out by non linear data that is applied to the raw data. It has been theorized by anscombe and applied first by donoho [1]. In Median filtering, transform domain methods such as Fourier and discrete Fourier transform will also be employed for denoising the medical images. But they introduce blur in the images. This will damage the

texture in the images. To encounter this problem the total variational method is utilized for medical image denoising. It preserves the edges in the medical images during denoising process. The process involved here is to reduce the total variation of noisy image and get a closer match to the original image. Split bregman method is from optimization theory is adapted to find the solution for non-linear convex optimization problem. The denoising strength is highly depending on regularization parameter. If this parameter increase denoising strength also increased. the TV method has the disadvantage that it will introduce the artifacts as a result of denoising process 1. We should compromise with the quality of the image 2.it will damage small scale structures with high curvature edges [2]. Wavelet based methods are now widely used in medical image denoising and disease diagnosis. Many of these algorithms are related to shrinkage/thresholding to wavelet image coefficients. The difficult task in this is to select appropriate threshold selection [3]. The unnormalised Haar transform is apposite for Poisson noise in the view of the fact that it is self-reproducing across scales. By capturing this advantage [4] consequently the Bayesian intensity estimate for multiscale multiplicative innovation model is applied. Multiscale analysis is a powerful tool in denoising procedures [4]. Non local Means Algorithm which is introduced by Buades [5]. It is a non local averaging technique which can be operated on all pixels in the image with the same characteristics. Regrettably this method is very slow. The variance stabilization transform which is an extension of anscombe transform [6] [7] combine with filter banks of wavelet, ridgelets and curvelet leading to multiscale VSTs. For different morphologies we cannot use this technique. We should introduce different multiscale transform.

For the different morphologies [8], the non local PCA with patch based algorithm is applied to eliminate the Poisson noise. It also reduces .the non local PCA with patch based algorithm is applied to eliminate the Poisson noise. It also reduces the artifacts. It has the disadvantage that the domain shape dependencies. To avoid this, ICA [9] (Independent Component Analysis) in which the reconstructed image has the clarity compare with other preprocessing filters.

III. WAVELET BASED DENOISING

The wavelet based denoising model minimizes the unbiased risk estimation for Poisson noise. There are mainly three steps are involved in the procedure. The first step is the transform method which includes discrete wavelet transform and the second step is thresholding.

A. Discrete wavelet transform

There are variety of wavelets are available for transform technique. They are haar, Daubeschies, Coiflits, Symlets, Morelet, Maxican Hat and biorthogonal wavelet. We are taking discrete wavelet transform of the application divide the image into four subbands. The LH, HL, and HH represent detailed features of image. The LL subband represents approximation of the image. The LL subband can be further decomposed. Based on the application we can restrict the level of decomposition.

B. Thresholding

The selection of appropriate thresholding technique is the major problem in the case of wavelet transform. This will remove the noise by shrinking coefficients. The efficiency of the wavelet transform mainly based on the threshold selection. There are two types of thresholding .hard and soft thresholding.

$$T_{hard}[I, \lambda] = \begin{cases} I & \text{for all } |I| > \lambda \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$T_{soft}[I, \lambda] = \begin{cases} \text{sign}(I)\max(0, |I| - \lambda) & \text{for all } |I| > \lambda \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

C. Selection of threshold

There are two types of thresholding types. Universal thresholding and adaptive thresholding the universal thresholding was proposed by Dohono and Johnstone in 1995.the threshold λ is calculated by using the following equation.

$$\lambda = \sigma\sqrt{2 \log(M)} \quad (3)$$

σ is the local noise variance of each subband of the poisson image

The noise variance can be estimated by,

$$\sigma = \sum_{i=0}^{N-1} X_j^2 \quad (4)$$

IV. ICA DENOISING MODEL

ICA denoising model is introducing by Hyvarinen 1999.for data sets which are contaminated by Gaussian noise can perform well by applying soft thresholding. For Poisson noise corrupted data sets we have to develop new filtering to adapt to the property of noise that is signal dependent.

For the n dimensional vector

$$X=S+V \quad (5)$$

Where S is the orthogonal signal, V is the noise. We have to find the V' such that $V=V'$

Step1: Evaluate the orthogonal ICA algorithm. 'W' is evaluated by using set of noise free data Y.

Step2: $S_i=w_i^T Z$ for $i=1, \dots, n$ by using the weight vector the non linear shrinkage is calculated.

Step3: for each X, ICA performs

$$Y=WX \quad (6)$$

The inverse transform

$$S = W^T S' \quad (7)$$

It is the unsupervised learning so it is need additional data to train the data.

V. HYBRID METHOD

Hybrid method uses the Independent Component Analysis for noise separation from the data. The wavelet transform is used to remove the noise.

A. Independent component analysis

The ICA method is used to find the components that are both statistical independent and non Gaussian. It is one type of non supervised learning. The m-dimensional space is reduced to an n dimensional space. So that the hidden information from the large data sets are obtained.

The every component y_i is given by

$$y_i = \sum_j w_{ij} x_{ij}(t), \quad (8)$$

for $i=1, \dots, n, j=1, \dots, m$

The problem is there is to determine the coefficient w_{ij} .the coefficient can be represented as matrix.

$$\begin{pmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_n(t) \end{pmatrix} = w \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix} \quad (9)$$

The original signal should find from the mixers $x_1(t), x_2(t), x_3(t)$ this is the blind sourceseparation problem.merely we know the petite amount of information

regarding the original signal, the 'w' matrix can be calculated from the inverse of the matrix that capable of mixing coefficient a_{ij}

B. Forbidding of Gaussian variables

The restraint in ICA method is that the independent components ought to non Gaussian. If Gaussian variable present in our data, formulate that ICA is impossible. Their joint density function is given by

$$p(x_1, x_2) = \frac{1}{2\pi} e^{- (x_1^2 + x_2^2)/2} \quad (10)$$

From the equation we can see that the density is absolutely symmetric. So it does not have any information on the directions of the columns of the mixing matrix W. For that reason W cannot be approximated.

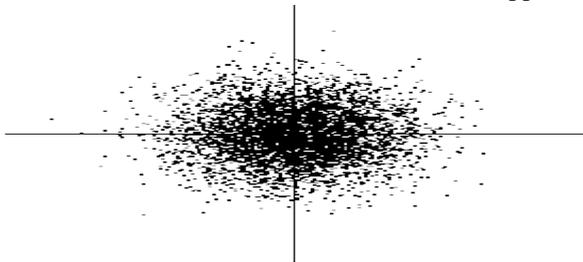


Fig 1. The multivariate distribution of two independent Gaussian variables.

More austere, we can prove that the distribution of any orthogonal transformation of the Gaussian (x_1, x_2) has extremely the same distribution as (x_1, x_2) . For the Gaussian variable we can just estimate the ICA model up to an orthogonal transformation. The matrix W is not exclusive for Gaussian independent component.

C. Source separation based on independent property

The estimation of coefficients w_{ij} is difficult, for the reason that the estimation method works in many different state of affairs. The solution is good representation of multivariate data. If we consider the statistical independence of the data the problem is easily solved. When comparing uncorrelatedness, the independence is the stronger property. Uncorrelation between the components does not show that the components are independent. The failure of the PCA and factor analysis cannot separate the components in actual fact, because it follows the uncorrelatedness property.

D. Measures of non gaussianity

The measure of nongaussianity is given by negentropy. It is based on the information theoretic quantity of (differential) entropy. It is the basic concept of information theory. The random variable entropy can be construed as the degree of information that the observation of the variable give unpredictable and unconstructed the variable is the larger its entropy. Entropy is intimately related to the coding length of the random variable.

Entropy H is defined for a discrete random variable Y as

$$H(Y) = - \sum_i p(y = a_i) \log p(y = a_i) \quad (11)$$

Where p, a_i are the possible values of Y. This can be also generalized for continuous valued random variable and vectors.

The differential entropy H of a random vector Y with density $f(y)$ is given by

$$H(y) = - \int f(y) \log f(y) dy \quad (12)$$

The result of the information theory is that the largest entropy among all will be for the Gaussian variables of equal variance. So nongaussianity measured by entropy. The measure of non gaussianity is always nonnegative. A slightly modified version of the definition of differential entropy called negentropy J is defined as follows

$$J(y) = h(y_{gauss}) - H(y) \quad (13)$$

y_{gauss} is the Gaussian random variable of the same covariance matrix as y. By means of above mentioned Properties of negentropy is always non negative and it is zero only if y has a Gaussian distribution. It has the additional property is that it is invariant for invertible linear transformations. The computations are complex.

E. Preprocessing of ICA

Before applying practical algorithm we should do preprocessing.

1) Centering:

The basic and necessary of preprocessing is to center X. That is subtract its mean vector $m = E(x)$, so as to make X a zero mean variable. This implies that is zero mean as well as can be seen by taking expectations on both sides on the following equation.

$$S = XW \quad (14)$$

After estimating the mixing matrix A with centered data we can complete the estimation by adding the mean vector of S back to the centered estimates of S. The mean vector of S is given by

$$A^{-1}m \quad (15)$$

Where, 'm' is the mean that was subtracted in the preprocessing steps.

2) Whitening:

Another useful preprocessing strategy ICA is to first whiten the observed variables. Before the algorithm transfer the observed vector linearly. The new vector \bar{X} which is white is obtained. It has the uncorrelated component whose variances are unity.

$$E[\bar{x}\bar{x}^T] = I \quad (16)$$

Covariance matrix is equal to the identity matrix. One of the popular methods of whitening is to use Eigen value decomposition .EVD of the covariance matrix

$$E[XX^T] = EDE^T \quad (17)$$

E is the orthogonal matrix of Eigen vector of $E[XX^T]$, d is the diagonal matrix. Whitening reduces the number of parameters to be estimated.

F. ICA algorithm

Here the ICA algorithm for one unit is given. The unit is referred as the single neuron having weight vector W. So as to the neuron is able to update the learning rule. The unit vector 'W' such that the projection $w^T x$ which maximizes nongaussianity. non gaussianity is measured by the approximations of negentropy $Jw^T x$.the variance of $w^T x$ must be unity. For the whitened data this is corresponding to restraining the norm of W to be unity. The fixed point iteration scheme is used to find a maximum of the nongaussianity of $w^T X$.it can be also derived as an approximate the newton iteration, denote by g. The derivation of the nonquadratic function G for example,

$$g_1(u) = \tanh(a, u) \quad (18)$$

$$g_2(u) = u \frac{\exp(-u^2)}{2} \quad (19)$$

Where $1 \leq a_1 \geq 2$ is constant $a_1 = 1$

Steps:

1. choose an initial weight vector W(any random)
2. let $W^+ = E[x_g(W^T X)] - E[g'(W^T X)]W$
3. let $W = W^+ / \|W^+\|$
4. If not converged go back 2

Convergence means that the old and new values of W point in the same direction. That is the dot product is equal to one. There is no necessity that the vector converges to a single point. Since W and W^+ define the same direction.

G. Purelet

Haar wavelet has the property which conserving Poisson statistics in its low pass channel. The second property of unnormalized haar transform applied to Poisson data. It is the statistical relation between a scaling coefficient (parent) and its child is very simple.

The efficiency of this approach came from the following two points.

1. The stein's unbiased estimate states that a prior free unbiased estimate of the predicted (MSE) that is flanked by the unknown original image and the denoised one. In the case of Poisson data, It is called PURE (Poisson unbiased risk estimate).
2. The linear parameterization in the denoising process carried in the course of a linear expansion of Threshold (LTE). The parameters from these expansions are the solution of linear equations when we minimize the subband dependent quadratic unbiased estimate of the MSE.

H. Properties of Poisson noise

Property 1

The sum of independent Poisson random variables is also a Poisson random variable. The intensity is equal to the sum of intensities

$$m_1 + m_2 \sim p(\mu_1 + \mu_2) \quad (20)$$

Property 2

If $m \sim p(\mu)$ and $\theta: R \rightarrow R$ is a real fuction such that $E\{\theta(m)\} < \infty$ then

$$E\{\mu\theta(m)\} = E\{m\theta(m-1)\} \quad (21)$$

This is the Poisson equivalent of stem's lemma for Gaussian statistics.

Property 3

Let $m \sim B(\ell, \eta)$ be the binomial random variable, where $\eta \in [0,1]$ represents the probability of success. If the number of trails $\ell \in N$ is random and follows a Poisson distribution with mean λ , then m is itself Poisson distributed with mean $\mu = \eta\lambda$

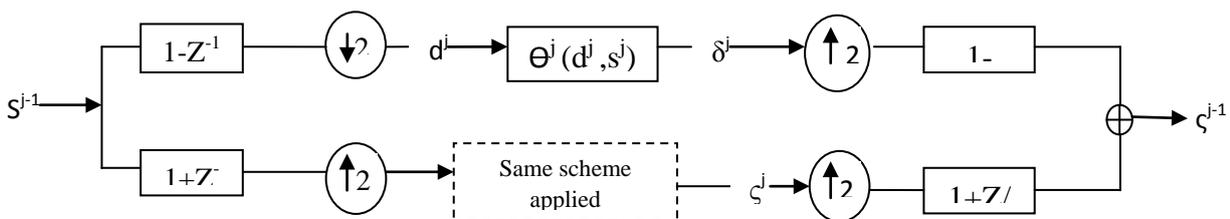


Fig 1.filter bank implementation of unnormalised discrete wavelet transform.the level of decomposition is defined by the subscript j=1....J.S is the noisy vector coefficients.Θ is the thresholding level. δ^j is the noise free wavelet estimated vector coefficients

I. The unnormalised haar discrete wavelet transform

The unnormalised haar discrete wavelet transform can be represented by a standard two channel filter bank. The low pass and high pass filter is represented by z transform

$$\begin{cases} H_a(z) = 1 + z^{-1} \\ G_a(z) = 1 - z^{-1} \end{cases} \quad (22)$$

The corresponding synthesis pair is

$$\begin{cases} H_s(Z) = 1/2Ha(Z^{-1}) \\ G_s(Z) = 1/2Ga(Z^{-1}) \end{cases} \quad (23)$$

J. Wavelet thresholding (pure)

The new estimator wavelet domain estimate which has the soft threshold with pure optimized threshold. The transposition of Donoho and Johnstone's Gaussian SURE shrink to Poisson noise removal are regarded as PURE shrink estimator. Here the threshold optimization is the minimization of PURE.

Compare to Gaussian noise the Poisson noise in this case is stationary and utterly described by its variance for Poisson data. The amount of noise is directly proportional to the intensity what we are going to estimate. The amount of shrinkage is measured by setting the threshold T as proportional to the square root of the scaling coefficient at the same location and scale. This is the standard deviation. It will bring the substance of wavelet coefficient. Every wavelet coefficient of the unnormalised Haar transform tag along a skellm distribution.

The pure shrink estimator is

$$\theta^{PUREshrink}(d, s; a) = \text{sign}(dn) \max(|dn|a\sqrt{|sn|}, 0) \quad (24)$$

K. Purelet

The PURE shrink produce which follows the SURELET strategy,

The wavelet estimator which is devised as a linear expansion of threshold

$$\theta^{LET}(d, s; a) = \sum_{k=1}^k a_k \theta_k(d, s) \quad (25)$$

θ_k is the generic estimators which should be chosen as arbitrarily.

L. Thresholding function

The linearly parameterized thresholding function with $k = 2$ then the n^{th} component is defined by

$$\theta_n^{LET}(d, s; [a1a2]^T) = a1d_n + a2 \left(1 - e^{-\left(\frac{d_n^2}{2T^2}\right)}\right) d_n \quad (26)$$

For denoting the compromising the parameter $a1, a2$ are to be defined.

VI. RESULTS

In this paper, we deals with two approaches. First we separate the Poisson noise and then denoised the image. In second method first we separate the Poisson noise from the image and then denoised the image and the performance analysis are measured in the mean of PSNR

The MSE value is calculated.

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \|I(i, j) - k(i, j)\|^2 \quad (27)$$

The PSNR value is calculated by

$$PSNR = 10 \log_{10} \frac{MAX_I^2}{MSE} = 20 \log_{10} \frac{MAX_I}{\sqrt{MSE}} \quad (28)$$

When compared to first method the second method provide the better results and is denoted by means of PSNR value and mean square value.

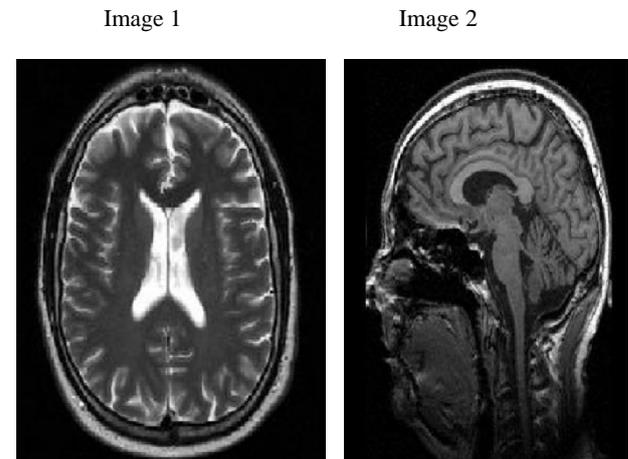


Fig. 3 Original Images

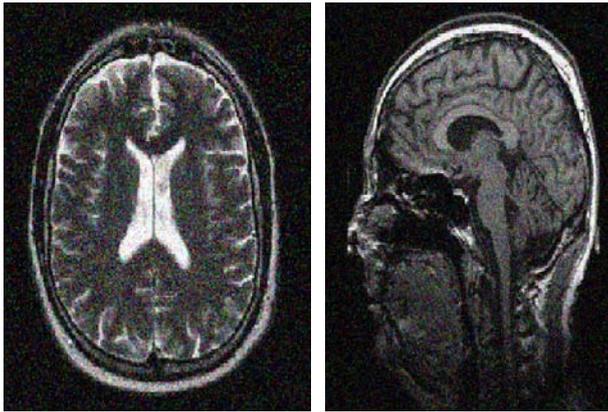


Fig. 4 Noisy images

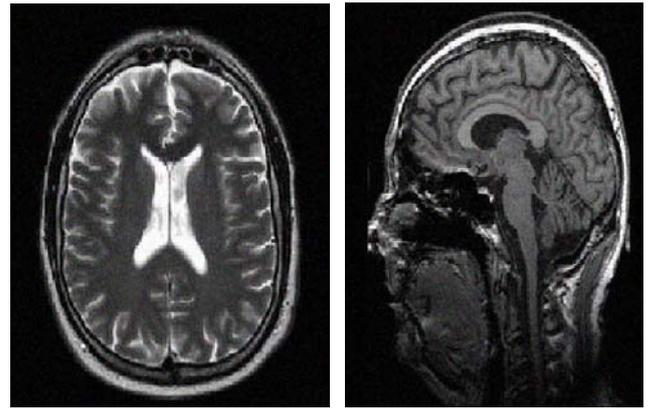


Fig. 6 Denoising and then noise separated output

TABLE I

	PSNR		MSE	
	Denoised and then noise separated image	Noise separated and then Denoised image	Denoised and then noise separated image	Noise separated and then Denoised image
Image 1	27.89	29.56	79.82	72.64
Image 2	31.04	33.47	82.31	78.37

TABLE 2 PERFORMANCE ANALYSES

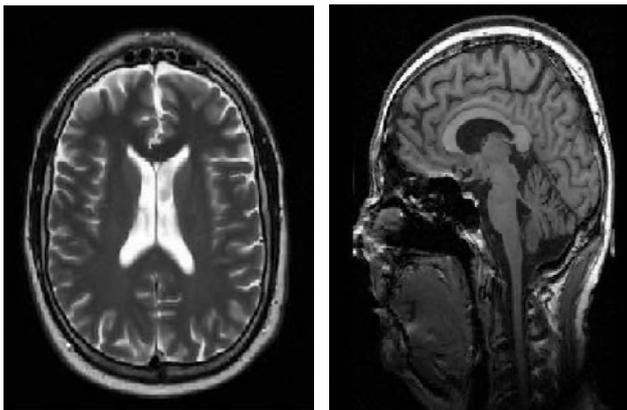
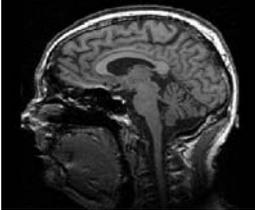
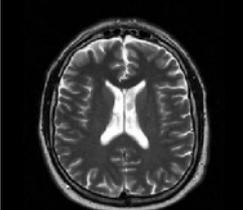
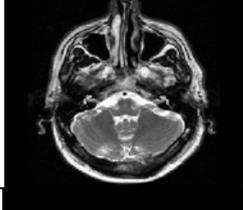
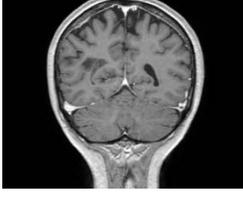
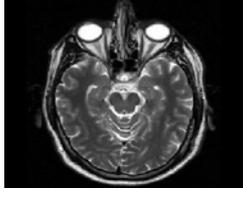


Fig. 5 Noise separation then denoised output

Input Images	Method	Performance analysis			MSE
		For $\sigma = 5$ PSNR	MSE	For $\sigma = 15$ PSNR	
	PURELET	34.6905	166.4567	25.5046	185.4756
	ICA with PURELET	38.2875	160.1754	29.7634	179.3465
	PURELET	34.2772	165.7640	25.44312	184.2534
	ICA with PURELET	38.8377	159.3764	28.0986	178.3645
	PURELET	34.3059	167.2704	25.3638	187.6323
	ICA with PURELET	39.3562	157.7463	28.0876	179.7321
	PURELET	33.1252	169.5242	24.7463	184.7635
	ICA with PURELET	40.6734	156.1241	27.7262	180.7326
	PURELET	34.3059	169.2704	25.4248	183.5552
	ICA with PURELET	39.5342	158.3645	26.6534	179.9635

VII. CONCLUSION

Thus from the above discussed two methods, the noise separation and then denoised method shows best results. It has the minimum distortion in terms of PSNR and MSE. For denoising the Poisson noise, Purelet approach is taken. In this method the selection of thresholding problem is rectified by means of linear expansion of thresholding.

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