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MR Image Reconstruction with L^1norm Minimization and Total Variation Denoising

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Abstract:***This paper is a meek venture to introduce the compressed sensing magnetic resonance image reconstruction. I have devised this algorithm by minimizing total variation (TV) and L1 norm regularization with total variation denoising. MR image reconstruction has gained a tremendous far-reaching impact in the present technologically advanced world. The original problem will be bifurcated into two entities; L^1norm and Total Variation [1,2]. Eventually it can be effectively solved. It helps the expedite reconstruction of the MR image through an iterative framework. An additional application of a denoising technique to this method was found to be very efficient and reliable. A comparative view of the computational complexity and reconstruction accuracy of the present method with the earlier approaches will open our eyes to the effectiveness of it based on the numerical results.***

Keywords: Compressed sensing, L^1norm, Total variation**.**

1. Introduction

MRI stimulates a signal from the object using magnetic fields and radio-frequency pulses. MRI reads data using magnetic gradients and places it into k-space which is the frequency domain. K-space is a space where MRI data is stored. Fourier transformation of k-space translates it into spatial domain. The problem that we come across usually in MRI reconstruction is our inference of the homogeneity of the static magnetic field and unidirectional nature of applied field gradients with constant magnitude [3]. Various efforts have been made to solve the inhomogeneity of the magnetic field. One of the remedies is to contrive a remote homogeneous field [4]. In the present article there is an attempt to tackle the issue of image reconstruction in high-field MRI, incorporating the inhomogeneity of static magnetic fields. The clinical applications usually require the direct reconstruction for the rapid imaging by completely evading the compensating techniques.

Many researchers are in an expedition to assuage the quantity of acquired data without contaminating the image quality. The undersampling of the k-space to an extend result in the infringement of Nyquist criterion, and also the reconstruction with fourier methods will cause artifacts [5]. In recent years some proposals are arise to eliminate the undersampling artifacts based on sparse models. MR images are not sparse in pixel representation. Medical images like angiograms are sparse in their pixel representation. Some of the medical images are sparse in their transform domain or there exists a transform Sparsity [6]. Certain MR images are sparse in the wavelet domain, for an instance the brain image we intend to recover occurs to have a sparse representation in the wavelet domain [5]. Sparsity in an image domain means there are hardly significant pixels with non-zero values. The constraints of Sparsity are not specified as non-zero coefficients do not possess a bunched form. The transform Sparsity is too have this quality as it needs only to be clear in some transform domain. Sparsity constraints can enable sparser undersampling of k-space under expedient situations [6,7]. The maximum exploitation of transform Sparsity is a great boon to the successful compression of data that encodes a few significant coefficients and stores them for the reconstruction process.

MR images are compressible in nature. So it is unnecessary to acquire all the transform coefficients rather than simply measuring the compressed information directly from lesser number of measurements and by using the theory of compressed sensing [6,7] we are able to reconstruct the same image from the fully sampled set. Compressed sensing was first introduced in the literature of Information Theory and Approximation Theory.

One does not measure the nominally defining signal samples, instead they are taking a linear combination of signal values that are much smaller. The signal reconstruction is carried out by certain nonlinear methods which exhibit good accuracy and precision. In MRI we glance over the method of compressed sensing, where the sampled linear combinations are the k-space samples or the individual fourier coefficients. By employing the principles of compressed sensing it is possible to reconstruct the entire image from a tiny subset of k-space.

2. Method

The non-uniformly sampled data is acquired and used for the reconstruction process. The reconstructed image will be corrupted by non-coherent artifacts. A sparse denoising technique can eliminate the noise contents. The noise is present in all the pixels. It is possible to remove the noise by using Split Bregman Total Variation method. Fourier transform of the denoised image is performed to obtain the k-space. The k-space data contains new information and merged with the measured data. Thus the interference level gets reduced and the reconstruction process continued until we get the image.

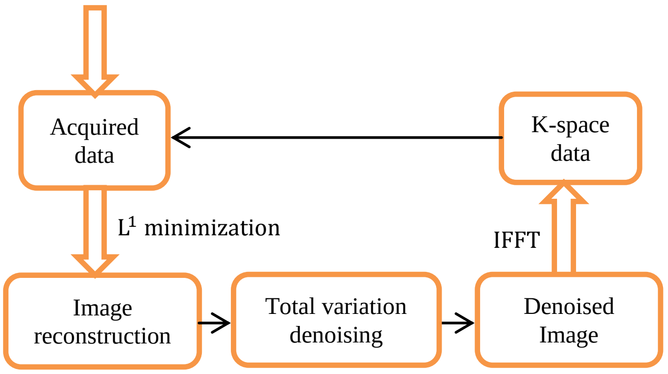


Fig. 2.1 Block diagram of MR Image reconstruction process with total variation denoising

3. L^1norm Minimization

To reduce the scan time we can resort to the principles of compressed sensing which makes use of the randomly undersampled k space. The missing data can be redeemed with sparsity promoting regularization. The image reconstruction problem or the constrained optimization problem can be defined as

$$argmin_v J(v) \text{subject to } Mv = c \qquad (1)$$

Where J represents a form of L_1 regularization [8]. M is the fourier matrix, c represents compressed sensing data and v stands for the unknown image that we intend to reconstruct.

3.1 Split Bregman Iteration

Split Bregman algorithm [9] has a good convergence rate. It converts the constrained optimization [10] problem into an unconstrained problem.

$$\begin{cases} v^{k+1} = \min_v |\nabla v| + \frac{\alpha}{2}\|Mv - c^k\|^2 \\ \quad c^{k+1} = c^k + c - Mv^{k+1} \end{cases} \qquad (2)$$

α is the convergence parameter. v^k and c^k are updated in an alternate way. It produces a solution $v^k \rightarrow v$ to the constrained optimization problem. The update of c^k is in a direct manner. The minimization of unconstrained problem can be done by introducing new variables $b_x = \nabla_x v$, $b_y = \nabla_y v$, and $r = Mv$ and update equation can be rewrite as

$$v^{k+1} = min_v |\nabla_b| + |r| + \frac{\alpha}{2}\|Mv - c^k\|^2 \qquad (3)$$

The constrained optimization problem can be converted to unconstrained problem by using Bregman algorithm.

Algorithm: Split Bregman Iteration

Initialize: $v^0 = M^t c$, and
$g_x^0 = g_y^0 = z^0 = a_x^0 = a_y^0 = a_w^0 = 0$
while $\|Mv^k - c\|_2^2 >$ threshold **do**
for i = 1 to N_{inn} **do**

$$v^{k+1} = min_v \frac{\alpha}{2}\|Mv - c^k\|^2 + \frac{\tau}{2}\|g_x^k - \nabla_x v - a_x^k\|^2 + \frac{\tau}{2}\|g_y^k - \nabla_y v - a_y^k\|^2 + \frac{\mu}{2}\|w^k - Wv - g_w^k\|^2$$

$$g_x^{k+1} = shrink(\nabla_x v^{k+1} + g_x^k, {}^1/_\tau)$$

$$g_y^{k+1} = shrink(\nabla_y v^{k+1} + g_y^k, {}^1/_\tau)$$

$$w_x^{k+1} = shrink(Wv^{k+1} + g_x^k, {}^1/_\mu)$$

$$a_x^{k+1} = a_x^k + (\nabla_x v^{k+1} - g_x^{k+1})$$

$$a_y^{k+1} = a_y^k + (\nabla_y v^{k+1} - g_y^{k+1})$$

$$a_w^{k+1} = a_w^k + (Wv^{k+1} - w^{k+1})$$

end

$$c^{k+1} = c^k + c - Mv^{k+1}$$

end

the function *shrink* is defined as

$$shrink(u,v) = \frac{u}{|u|}\max(|u| - v, 0) \qquad (4)$$

The parameters τ and μ are affecting the convergence rate. The tempo of the image reconstruction problem is entirely depends on the speed of optimization or minimization problem. Most of the computational bottlenecks are occurring in the update of v^{k+1}. Conjugate gradient method is used for the updating v^{k+1}.

$$(\alpha M^t M + \tau \nabla_x^t \nabla_x + \tau + \mu W^t W)v^{k+1} = rh^k \qquad (5)$$

Where

$$rh^k = \alpha M^t c^k + \tau \nabla_x^t (g_x^k - a_x) + \tau \nabla_y^t + \mu W^t (w - a_w) \qquad (6)$$

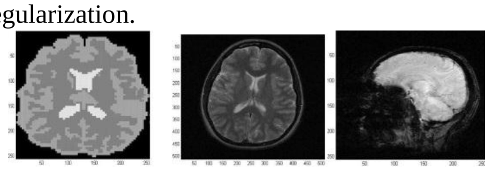
we can write the above equation to the form

$$(\alpha M^t M - \tau\Delta + \mu I)v^{k+1} = rh^k \qquad (7)$$

Equation is solved by using conjugate gradient method.

4. Experimental Result

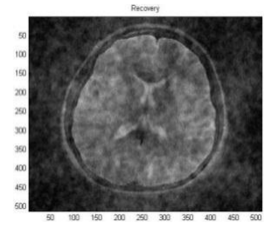
The image reconstruction method is compared with the Non Uniform Fast Fourier Transform and Total Variation regularization.



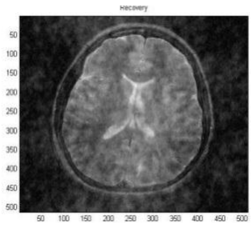
(a) Phantom image (b) Brain image (c) Exact brain image

Figure. 4.1 Images used for simulation

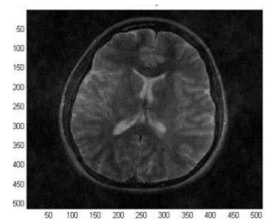
Figure. 4.1 shows the images which are used for the simulation purpose. The phantom image and the brain image had a pixel size of 256×256 and 512×512 respectively.



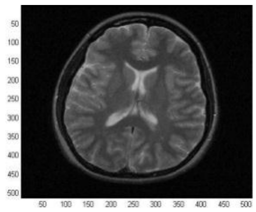
(a) Reconstruction from 4.25x undersampled k-space



(b) Reconstruction from 3.75x undersampled k-space



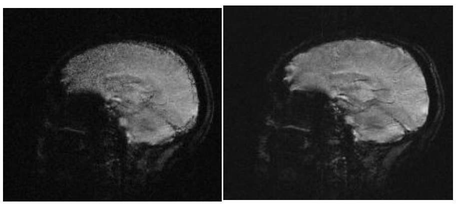
(c) Reconstruction from 3.25x undersampled k-space



(d) Reconstruction from 2.25x undersampled k-space

Figure.4.2 MR Image reconstruction from differently undersampled k-space data.

Figure 4.2 shows the reconstructed MR brain images with different undersampled k-space values. (a) shows the reconstruction with 23.5% k-space data. (b) using 27% of samples. In (b) the edges are more cleared compared to (a). If we are using more k-space samples like (c), the SNR and quality of image increases.(d) shows the reconstructed image which uses almost 45% of the k-space values. The reconstruction error for the (d) is very less and thereconstructed image is almost an exact replica of the brain image.



(a)reconstructed image (b)TV denoised image

Fig.4.3 Reconstructed and total variation denoised image

Figure 4.3 represent the importance of total variation denoising in MRI reconstruction. The reconstruction is done with 30% of k-space samples. Fine structers are not visible in the presence of artifacts. Total variation denoising helps to overcome the difficulties in the presence of artifacts, arised due to the field inhomogenity and off-resonanc effects. The fine structures and the edge portions are more visible in denoised image.

Min max interpolation method is used to find the Non Uniform Fast Fourier Transform(NUFFT) with 6 level interpolation. MRI is often acquired on non-uniform grids in k-space, the Non Uniform Fast Fourier Transform can be used for the reconstruction purpose. In this reconstruction the magnetic inhomogenity effects are not considered. Field correction is applied for the total variation regularization. The NUFFT and Total variation methods are compared to the proposed method with different k-space undersampled values. It is found that L^1 norm minimization with Total variation denoising shows the lowest reconstruction error and highest SNR.

Table I. Brain image reconstruction error $\|\boldsymbol{v}_{exact} - \boldsymbol{v}\|_2$

Reduction factor	NUFFT	TV	L^1norm with TV denoising
1.25x	1.4954e+04	.5147	.4953
2.25x	1.6214e+04	36.0482	14.31
3.25x	1.665e+04	39.8761	19.47
4.25x	1.683e+04	49.7351	29.83

The table I gives a comparison of the L^1norm with TV denoising method with total variation and NUFFT. The computational complexity of the implemented method is some what higher than the other two methods. But the SNR is so high and able to produce the reconstructed image with less number of input samples.

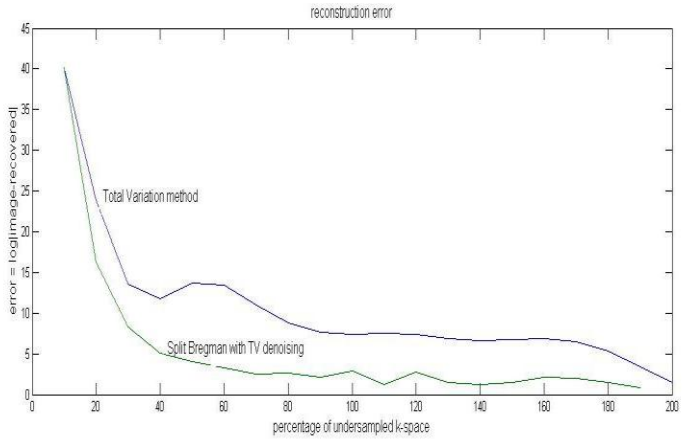


Figure.4.3 Brain image reconstruction error after 200 iterationsat 2.25x undersampling. The error, $\|\boldsymbol{v}_{exact} - \boldsymbol{v}\|_2$.

Figure. 4.3 represents the reconstruction errors per iterations. From the graph it can be shown that the Split Bregman with TV denoising is more efficient than the Total Variation method. Also the method shows good convergence rate.

5. Conclusion

We had discussed here the principle of compressed sensing along with its application for the fast MR Imaging. We showed how to reduce the scanning time by the maximum exploitation of the sparsity. This method is so fast that it subjects to expedite convergence in the presence of magnetic inhomogeneous fields. It necessities the knowledge of k-space trajectory and magnetic field inhomogeneity profile. The application of this method can be successfully carried out to certain types of issues where the signal is a linear function of the spin density.

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