www.ijecs.in
International Journal Of Engineering And Computer Science ISSN:2319-7242
Volume 3 Issue 5 may, 2014 Page No. 6043-6047

Mtbf Analysis Of The Two-Unit Series System With Repair Facility

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Abstract:

This paper discusses mean time between failure and availability analysis in a two-unit series repairable system with three states under consideration. First two states are taken as up states and the third state is down state. A single repair facility is available which repairs the units on first-come-first-serve basis. Taking failure rates as identically distributed exponential random variable and repair rate as exponential, Mean time between failure and Steady State Availability are calculated. Denoting the failure times of the components as λ and repair time as μ expression for steady state availability and MTBF have been derived using semi-markov process and regenerative point technique. The difference equations are developed and solved using Laplace transform.

Keywords: Series System, Steady State Availability, MTBF, Semi-markov process, Regenerative point technique, Difference Equation, Laplace Transform.

Introduction:

Two-unit standby repairable system has been discussed by many authors under different assumptions in the past. To name a few authors such as Gray and Lewis (1967) , Butterworth and Nikolaison (1973), Lie et al (1977), Osaki et al (1980), Klein and Moeschberger (1986), Joshi and Dharmadhikari(1989), Goel and Shrivastava(1991). All these authors have evaluated reliability parameters for two-unit standby repairable system by taking different set of distributions. In the present paper, we have considered a two-unit series standby system under the assumption that whenever any working fails it is immediately taken over by standby unit. There is a single repair facility so that the repair work is started as soon as any unit fails.

We define the reliability of any system as R (t) $=p_0$ (t) $+p_1$ (t) and we are interested in calculating the values of p_0 (t) and p_1 (t) for which we employ the techniques of obtaining differential equations satisfied by

$p_i(t)$, i= 1,2 and making the use of Laplace transform and Cramer's rule and using the formula for MTBF as

$$MTBF=\int_0^\infty R(t)\,dt$$

System Description and Assumptions :

1. The system consists of a two-units in series in which one unit is operational whereas the other units is in standby mode.
2. Whenever a working unit fails, it is immediately taken over by standby unit and the failed unit is symultaneously sent for repair.
3. There is a single repair facility which repairs the units on first come first serve basis.
4. After repair any unit works as a new unit.
5. Switching system is perfect and switchover is perfect.
6. Failure times are identically distributed exponential random variable whereas the repair time is exponentially distributed
7. In State one both the components are operable.
8. In State two one component is in failed mode whereas the other component is operable.
9. In State three both the components are in the failed state

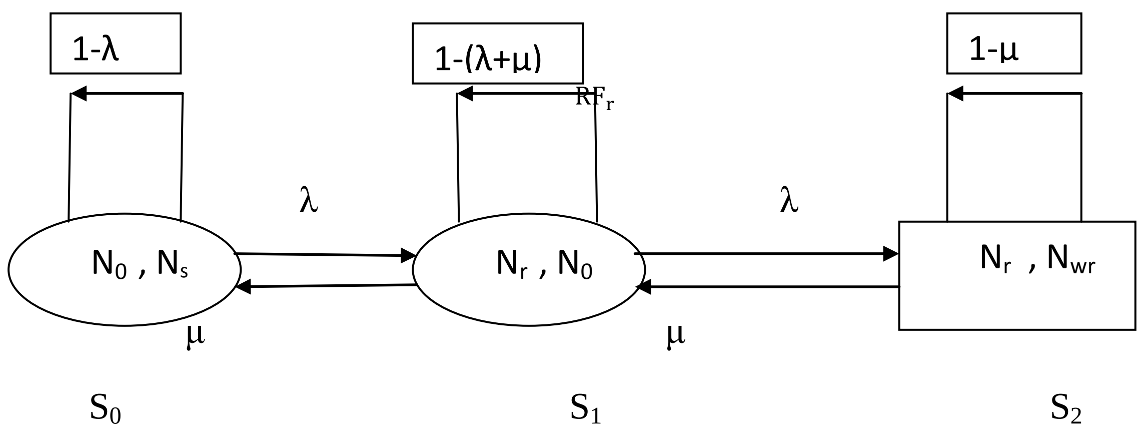
Notations and Symbols

λ= Failure rate of Operative unit ; μ= repair rate of failed unit

N_0= Unit in Operative mode ; N_s= Unit in Standby mode

N_r= Unit in repair mode ; N_{wr}=Unit waiting for repair

Upstate : [S_0, S_1] **Downstate** :[S_2]



State Transition Diagram

Transition Probabilities and mean sojourn times

Simple probabilistic considerations give the following expressions for the non-zero elements:

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)\,dt$$

With respect to the above model, we have the following results

$$Q_{00}(t) = \int_0^t e^{-\lambda t}(1-\lambda)e^{-(1-\lambda)t}dt \quad ; \quad Q_{01}(t) = \int_0^t \lambda e^{-\lambda t} e^{-(1-\lambda)t}dt$$

$$Q_{10}(t) = \int_0^t \mu e^{-\mu t} e^{-\lambda t} e^{-[1-(\lambda+\mu)]t}dt \quad ; \quad Q_{11}(t) = \int_0^t [1-(\lambda+\mu)]e^{-[1-(\lambda+\mu)t]} e^{-(\lambda+\mu)t}dt$$

$$Q_{12}(t) = \int_0^t \lambda e^{-\lambda t} e^{-[1-(\lambda+\mu)]t} e^{-\mu t}dt \quad ; \quad Q_{21}(t) = \int_0^t \mu e^{-\mu t} e^{-(1-\mu)t}dt$$

$$Q_{22}(t) = \int_0^t (1-\mu)e^{-(1-\mu)t} e^{-\mu t}dt$$

Taking lim $t \to \infty$, we get

$$p_{00} = 1-\lambda \; ; \quad p_{01} = p_{12} = \lambda \; ; \quad p_{10} = p_{21} = \mu \; ; \quad p_{11} = 1-(\lambda+\mu) \; ; \quad p_{22} = 1-\mu$$

MTBF Analysis :

We have

$$p_i(t) = \Pr\{X(t) = i \,\vdots\, X(0) = 0\}, i = 0,1,2$$

$$p_0(t+\Delta t) = \Pr\{X(t+\Delta t) = 0 \,\vdots\, X(0) = 0\}$$
$$= \sum_{j=0}^{2} \Pr\{X(t+\Delta t) = 0 \,, X(t) = j \,\vdots\, X(0) = 0\}$$

Using $P(A \cap B) = P(A/B)\,P(B)$, we get

$$p_0(t+\Delta t) = \sum_{j=0}^{2} \Pr\{X(t+\Delta t) = 0 \,\vdots\, X(t) = j \,,\, X(0) = 0\}\{\Pr$$
$$= \sum_{j=0}^{2} \Pr\{X(t+\Delta t) = 0 \,\vdots\, X(t) = j\}\, p_j(t)$$

$$= \Pr\{X(t+\Delta t) = 0 \vdots X(t) = 0\}\, p_0(t) + \Pr\{X(t +$$
$$+ \Pr\{X(t+\Delta t) = 0 \vdots X(t) = 2\}\, p_2(t)$$

$$p_0(t+\Delta t) = (1-\lambda\Delta t)\,p_0(t) + \mu\Delta t\, p_1(t) + O(\Delta t)$$

$$p_0(t+\Delta t) - p_0(t) = -\lambda\Delta t\, p_0(t) + \mu\Delta t\, p_1(t) + O(\Delta t)$$

Dividing by Δt and taking $\underset{\Delta t \to 0}{Lim}$ both the sides, we get

$$\underset{\Delta t \to 0}{Lim} \frac{p_0(t+\Delta t) - p_0(t)}{\Delta t} = -\lambda\, p_0(t) + \mu\, p_1(t) + \underset{\Delta t \to 0}{Lim} \frac{O(\Delta t)}{\Delta t}$$

, we get

$$p_0^{/}(t) = -\lambda\, p_0(t) + \mu\; p_1(t) \qquad (1)$$

Similarily, developing the other relation for $p_1^{/}(t)$, we get

$$p_1^{/}(t) = \lambda\, p_0(t) - (\lambda+\mu)\; p_1(t) \qquad (2)$$

Taking Laplace transforms of equations (1) and (2) and using Cramer's rule and again taking Inverse Laplace transform and using $\lambda = k_1+k_3$ and $\mu = k_2+k_3$, we get the following results :

$$p_0(t) = \frac{(k_1+k_2+k_3+s_1)e^{s_1 t} - (k_1+k_2+k_3+s_2)e^{s_2 t}}{s_1 - s_2}$$

and

$$p_1(t) = \frac{(k_1+k_3)(e^{s_1 t} - e^{s_2 t})}{s_1 - s_2}$$

Using R (t) =p_0 (t) +p_1 (t), we get the following result for the Reliability

$$R(t) = \frac{(s_1 e^{s_2 t} - s_2 e^{s_1 t})}{s_1 - s_2}$$

$$And\, MTBF = \int_0^\infty R(t)\, dt = -\frac{s_1 + s_2}{s_1 s_2} = \frac{2k_1 + k_2 + 2k_3}{k_1(k_1 + k_3)}$$

Availability Analysis:

Developing the similar expressions for $p_0^/(t)$, $p_1^/(t)$ & $p_2^/(t)$, we get

$$p_0^/(t) = -\lambda\, p_0(t) + \mu\;\; p_1(t)$$

$$p_1^/(t) = \lambda\, p_0(t) - (\lambda + \mu)\;\; p_1(t)$$

$$p_2^/(t) = \lambda\, p_1(t) - \mu\;\; p_2(t)$$

Similarly, taking Laplace transform of the above three equations and using Cramer's rule and taking λ= k_1+k_3 and μ= k_2+k_3, we get the following results for point wise availability of the system as

A (t) = p_0 (t) +p_1 (t) = 1-p_2 (t)

$$= s_1 s_2 - \frac{k_1(k_1 + k_3)}{s_1 - s_2}[s_1(1 - e^{s_2 t}) - s_2(1 - e^{s_1 t})]$$

And the steady-state availability of the system is given by the expression:

$$A_\infty = \underset{t \to \infty}{Lim}\, A(t) = 1 - \frac{k_1(k_1 + k_3)}{s_1 s_2} = \frac{(k_2 + 2k_3)(k_1 + k_2 + 2k_3)}{(k_2 + 2k_3)^2 + k_1(k_1 + k_2 + 3k_3)}$$

The mean down time of the system is given by the expression :

$$MDT = \frac{MTBF\,(1 - A_\infty)}{A_\infty}$$ Where

$$MTBF = \frac{(2k_1 + k_2 + 2k_3)}{k_1(k_1 + k_3)} \quad and$$

$$A_\infty = \frac{(k_2 + 2k_3)(k_1 + k_2 + 2k_3)}{(k_2 + 2k_3)^2 + k_1(k_1 + k_2 + 3k_3)}$$

References :

1. Gray, H.L. and Lewis, T.O.[1967] A confidential interval for the availability ratio. Technometrics, 9, pp 465-469.
2. Harris, R.[1968] Reliability applications of a bivariate exponential distribution. Operation Research , 16, pp18-27.
3. Butterworth, R.W. and Nikolaisen, T.[1973] Bounds on the availabilty function, Naval Research Logistics Quarterly, 20, pp 289-296.
4. Lie, C.H. Hwang, C.L. and Tillman, F.A.[1977] Availability of maintained systems; Astate-of-the-art-survey, AIIE rans. 9, no.3, pp 247-259.
5. Osaki, S.[1980] A two-unit parallel redundant system with bivariate exponential life times. Microelectron & Reliab., 20, pp 521-523.
6. Klien , J.P. and Moeschberger, M.L.[1986] The independence assumption for series or parallel systems when component lifetimes are exponential;IEEE Trans Reliab., R-35,No.3, 330-335.

7. Joshi, B.H. and Dharmadhikari, A.D.[1989] Maximization of avaialbility of 1-out-of-2: G repairable dependent system. Adv appl. Prob. , 21, pp 717-720.
8. Lu,Jye-Chyi[1992] Effects of dependence on modelling system reliability and mean life via a multivariate Weibull Distribution. IAPQR Trans. 17, pp1-22.
9. Pathak, V.K. Singh S.K. and Ram R.C.[2002] Stochastic analysis of a system having three types of dissimilar units. Journal of National Academy of Mathematics, Vol.22 , pp85-96.
10. Pathak, V.K. Mehta, Kamal Sahu, Seema and Chaturvedi, Riteshwari[2013] Profit analysis of a system having main unit and two supporting units. International Journal of Engineering and Computer Science Vol.2, pp 1-13.