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A New Approach to Automatic Mesh Generation Over Polygonal Domains Using All Quadrilaterals of Quintic Order to Decic Order Finite Elements of Complete Lagrange Family

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Abstract

This paper presents a novel mesh generation scheme of all quadrilateral elements over a linear polygonal domain. We first decompose the linear polygon into simple sub regions in the shape of quadrilaterals. These simple regions are then quadrangulated to generate first into a fine mesh of four node quadrilateral elements using bilinear transformations.We have already proposed this automatic quadrilateral mesh generation scheme in our recent paper[31]. In this scheme each four node quadrilateral is converted to higher order quadrilaterals by inserting the midside nodes appropriately. Examples were presented to illustrate the simplicity and efficiency of the new mesh generation method for standard and arbitrary shaped domains for linear to quartic order quadrilaterals.In this paper,we continue our study and generate meshes of quintic to decic order quadrilaterals for Complete Lagrange family having 24,32,40,49,60 and 72 nodes.They are actually the Serendipity famiy quadrilaterals with appropriate number of interior nodes to incorporate the complete monomial basis of 5^{th} to 10^{th} degree polynomials in bivariates

We have appended two important MATLAB programs which incorporate the mesh generation scheme for the 72-noded decic order complete Lagrange quadrilateral elements developed in this paper.Other MATLAB programs can be coded on similar lines. These programs provide valuable output on the nodal coordinates ,element connectivity and graphic display of the all quadrilateral meshes for application to finite element analysis.The typical domains include rectangles,arbitrary oriented rectangles,an equilateral triangle, arbitrary quadrilateral,convex and nonconvex polygons.

Keywords: Finite elements of Serendipity and Complete Lagrange families, quintic to decic order quadrilateral mesh generations, convex and nonconvex polygonal domains, uniform refinement, quadrangulation and triangulation.

1. Introduction

In our recent paper[31] we have already given details about the mesh generation process and its importance in finite element analysis.Hence,we are not repeating this topic.We are now continuing our presentation further directly to generate meshes for remaining higher order elements from quintic order to decic order

In this paper, we present a novel mesh generation scheme of all quadrilateral elements for linear polygonal domains. We have explained the scheme with more clarity and precision.This scheme converts the elements in background quadrilateral into quadrilaterals using bilinear transformation We first decompose the linear polygon into simple subregions in the shape of quadrilaterals. These simple subregions are then quadrangulated by using the one to one mapping concept between the original quadrilateral and the square. We propose then an automatic quadrilateral conversion scheme in which each background quadrilateral with n by m divisions into nm quadrilaterals according to the bilinear mapping scheme. Further, to preserve the mesh conformity a similar procedure is also applied to every quadrilateral of the domain and this fully discretizes the given linear polygonal domain into all quadrilaterals, thus propogating uniform refinement and quadrangulation. In section-2 of this paper,we explain the particular transformations required in generating the degenerate forms of the quadrilateral: rectangles,parallelograms,trapeziums etc shapes, In section-3 of this paper,we present a scheme to discretize the arbitrary quadrilateral into a fine mesh of quadrilateral elements.In section- 4,we explain the procedure to create higher order quadrilateral by inserting midside nodes in each four node element. In section-5,we have presented a method of piecing together of all quadrilateral subregions and eventually creating an all quadrilateral mesh for the given linear polygonal domain. In section-6,we present several examples to illustrate the simplicity and efficiency of the proposed mesh generation method for triangles, rectangles , arbitrary quadrilaterals, convex and non convex polygonal domains.

2. Linear Convex Quadrilateral and Isoparametric Coordinate Transformation with global vertices

Let us first consider an arbitrary four noded linear convex quadrilateral element Q_e in the Cartesian space (x,y) *with global vertices* (x_{n_k}, y_{n_k}) which is mapped into a 2-square in the local parametric space (ξ,η),where $(n_k, k=1,2,3,4)$ *are global nodenumbers*. This is shown in the following figures Fig.1a and Fig.1b

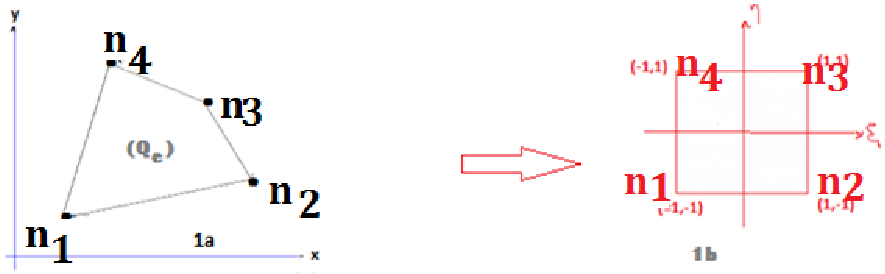


Fig.1a Linear: convex quadrilateral Q_e in (x,y) space, Fig.1b: Standard 2-square in in (ξ,η)space

The Isoparametric coordinate transformation from (x,y) space to the (ξ,η) space is given by $\begin{pmatrix} x^e \\ y^e \end{pmatrix} = \sum_{k=1}^{4} \frac{1}{4}(1+\xi\xi_k)(1+\eta\eta_k)\begin{pmatrix} x^e_{n_k} \\ y^e_{n_k} \end{pmatrix}$..(1)

Where ,

$\left((x^e_{n_k}, y^e_{n_k}), k=1,2,3,4\right)$ are the vertices of the linear convex quadrilateral element 'Q_e' in the Cartesian space (x,y)

and $(n_k(\xi_k,\eta_k), k=1,2,3,4) = \{n_1(-1,-1).n_2(1,-1), n_3(1,1), n_4(-1,1)\}$

are the vertices of the 2-square in (ξ,η) space.

For the sake of simplicity and without loss of generality,we may assume that $n_1=1, n_2$=2 ,

$n_3=3, n_4$=4 to demonstrate the various quadrilateral shapes described by eqn(1).

From Eqn(1), we obtain

$\begin{pmatrix} x^e \\ y^e \end{pmatrix} = \begin{pmatrix} a^e_0+a^e_1\xi+a^e_2\eta+a^e_3\xi\eta \\ b^e_0+b^e_1\xi+b^e_2\eta+b^e_3\xi\eta \end{pmatrix}$ --(2)

where

$$\begin{pmatrix} a_0^e \\ b_0^e \end{pmatrix} = \begin{pmatrix} \frac{1}{4}(x_1^e+x_2^e+x_3^e+x_4^e) \\ \frac{1}{4}(y_1^e+y_2^e+y_3^e+y_4^e) \end{pmatrix} \qquad \text{-----(3a)}$$

$$\begin{pmatrix} a_1^e \\ b_1^e \end{pmatrix} = \begin{pmatrix} \frac{1}{4}(-x_1^e+x_2^e+x_3^e-x_4^e) \\ \frac{1}{4}(-y_1^e+y_2^e+y_3^e-y_4^e) \end{pmatrix} \qquad \text{-----(3b)}$$

$$\begin{pmatrix} a_2^e \\ b_2^e \end{pmatrix} = \begin{pmatrix} \frac{1}{4}(-x_1^e-x_2^e+x_3^e+x_4^e) \\ \frac{1}{4}(-y_1^e-y_2^e+y_3^e+y_4^e) \end{pmatrix} \qquad \text{-----(3c)}$$

$$\begin{pmatrix} a_3^e \\ b_3^e \end{pmatrix} = \begin{pmatrix} \frac{1}{4}(x_1^e-x_2^e+x_3^e-x_4^e) \\ \frac{1}{4}(y_1^e-y_2^e+y_3^e-y_4^e) \end{pmatrix} \qquad \text{-----(3d)}$$

The nature of the constants $\left((a_i^e, b_i^e), i = 0,1,2,3\right)$ will determine the element geometry. We have briefly listed some of these element geometries.

Rectangular elements

When $a_1^e = a^e,\ \ a_2^e = 0,\ \ a_3^e = 0; b_1^e = 0,\ \ b_2^e = b^e,\ \ b_3^e = 0$

and $(a_0^e,\ b_0^e)$ as coordinate of the element centroids, we can generate rectangular elements whose sides are parallel to coordinate axes with half side length = a^e and half side width = b^e. This gives

$$x^e = a_0^e + a^e\xi, \quad y^e = b_0^e + b^e\eta \qquad \text{-----(4a)}$$

Example 1:We consider a domain with vertices $\{1(0,0),(\sqrt{5}, 0), (\sqrt{5}, 2\sqrt{5}), (0,2\sqrt{5})\}$ which is a rectangle of length=$2\sqrt{5}$ units and breadth=$\sqrt{5}$ .It is quite easy to show that the transformation

x^e=$\sqrt{5/4}$ (1+ξ),and y^e =$\sqrt{5}$ (1+η) maps the rectangle into a 2-square in $(\xi, \eta)space$

Parallelogram elements

(i)When $a_3^e = 0, \quad b_1^e = 0, \quad b_3^e = 0$, gives

$$x^e = a_0^e + a_1^e\boldsymbol{\xi} + a_2^e\eta, \quad y^e = b_0^e + b_2^e\eta \qquad \text{-----(4b)}$$

and this will generate a parallelogram whose two sides are parallel to y-axis.

(ii)When $a_2^e = 0, \quad a_3^e = 0, \quad b_3^e = 0$, this gives

$$x^e = a_0^e + a_1^e\boldsymbol{\xi}, \quad y^e = b_0^e + b_1^e\xi + b_2^e\eta \qquad \text{-----(4c)}$$

and this will generate parallelogram element whose two sides are parallel to x- axis

(iii)Arbitrary oriented rectangles and parallelograms are generated

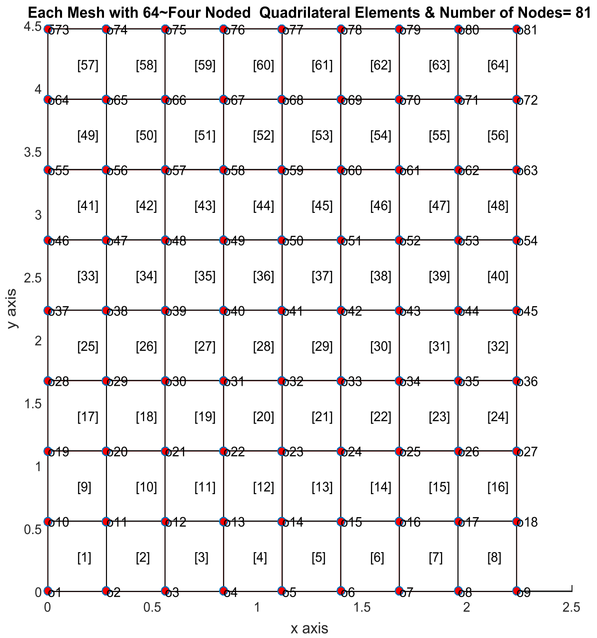
When $x^e = a_0^e + a_1^e\boldsymbol{\xi} + a_2^e\eta,, y^e = b_0^e + b_1^e\xi + b_2^e\eta$ ----------------(4d)

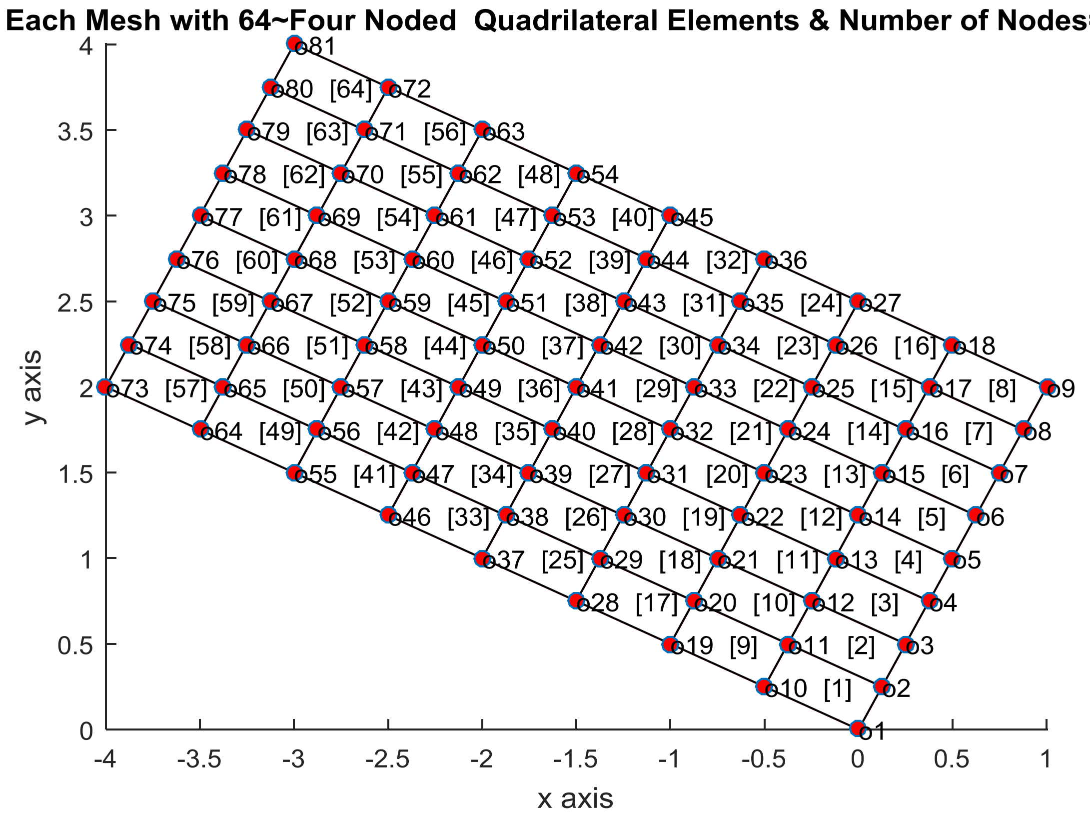
with $a_3^e = 0, \quad b_3^e = 0$

Example 2: We again consider a domain with vertices $\{1(0,0),(\sqrt{5}, 0), (\sqrt{5}, 2\sqrt{5}), (0,2\sqrt{5})\}$ which is a rectangle of length=$2\sqrt{5}$ units and breadth=$\sqrt{5}$ and rotate this rectangle about the origin by an angle arctan(2) in anticlockwise direction we obtain the rectangle {(0,0),(1,2),(-3,4),(-4,2)} and then use bilinear mapping to transform this to a 2-square are

x^e= -3/2+$\xi/2$-2η and y^e=2+$\xi + \eta$

We have included the mesh generation for example 1 and example 2 in the application section. using higher order elements.Now as an illustration,we present here the mesh generation for rectangles of Examples 1-2 for the four node element. :





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(iv)When all the parameters $((a_i^e,\ b_i^e), i = 0,1,2,3)$ are non-zero, arbitrary quadrilaterals are generated and this is the general case and we have

$x^e = a_0^e + a_1^e\xi + a_2^e\eta + a_3^e\xi\eta$

$y^e = b_0^e + b_1^e\xi + b_2^e\eta + b_3^e\xi\eta$ ---(4e)

This case also covers the trapezium elements when either x^e is linear or y^e is linear and one of these say when either x^e or y^e is nonlinear.That is:

$x^e = a_0^e + a_1^e\xi + a_2^e\eta$

$y^e = b_0^e + b_1^e\xi + b_2^e\eta + b_3^e\xi\eta$ ---(4f)

and

$x^e = a_0^e + a_1^e\xi + a_2^e\eta + a_3^e\xi\eta$

$y^e = b_0^e + b_1^e\xi + b_2^e\eta$ ---(4g)

This analysis as explained above covers all bilinear mappings for various shapes degenerated from the arbitrary linear convex quadrilateral.

3. Mesh Generation over a Linear Convex Quadrilaterals with global vertices

We can map an arbitrary quadrilateral Q_e with global vertices $((x_{n_i}^e, y_{n_i}^e), i = 1,2,3\ 4)$, where ($n_i$ i=1,2,3,4) are global node numbers in Cartesian space (x, y) into a unit square in the local space (u, v) with local node numbers (n_i i=1,2,3,4) in the parametric space. The mapping is shown in figs.1a and 1c The unit square in uv-space is a convenient choice for division into smaller squares or rectangles

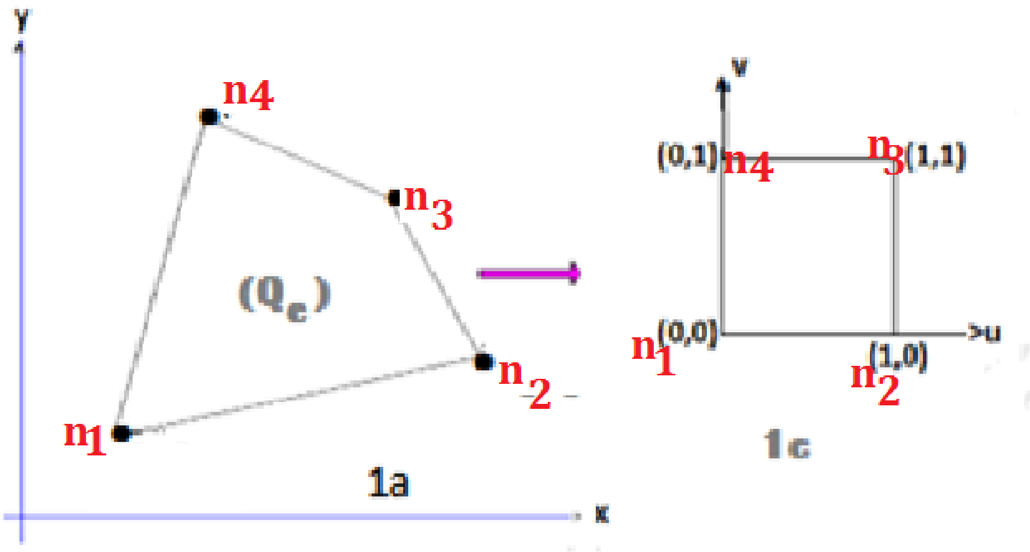

Fig.1a: a linear convex quadrilateral in (x,y) space,

Fig.1c:standard 1-square in (u,v) space,

Let us consider the bilinear mapping of an arbitrary quadrilateral Q_e as shown in Fig.1a and Fig.1c with vertices $((x_{n_i}^e, y_{n_i}^e), i = 1,2,3\ 4)$, into a standard unit square. The above mapping is defined as

$x^e(u,v) = x_{n_1}^e + u(x_{n_2}^e - x_{n_1}^e) + v(x_{n_4}^e - x_{n_1}^e) + uv(x_{n_1}^e - x_{n_2}^e + x_{n_3}^e - x_{n_4}^e)$ ....(5a)

$y^e(u,v) = y_{n_1}^e + u(y_{n_1}^e - y_{n_2}^e) + v(y_{n_4}^e - y_{n_1}^e) + uv(y_{n_1}^e - y_{n_2}^e + y_{n_3}^e - y_{n_4}^e)$ ....(5b)

We divide the unit square into $m \times n$ rectangles by making m divisions along u-axis and n-divisions along v-axis and this division (u,v) space has a one to one correspondence with a similar division of quadrilateral Q_e in (x,y) space.We now display the above mapping which shows divisions of a quadrilateral Q_e in (x, y) space and the corresponding divisions of a unit square in (u, v) space in Fig.2a and Fig.2b:

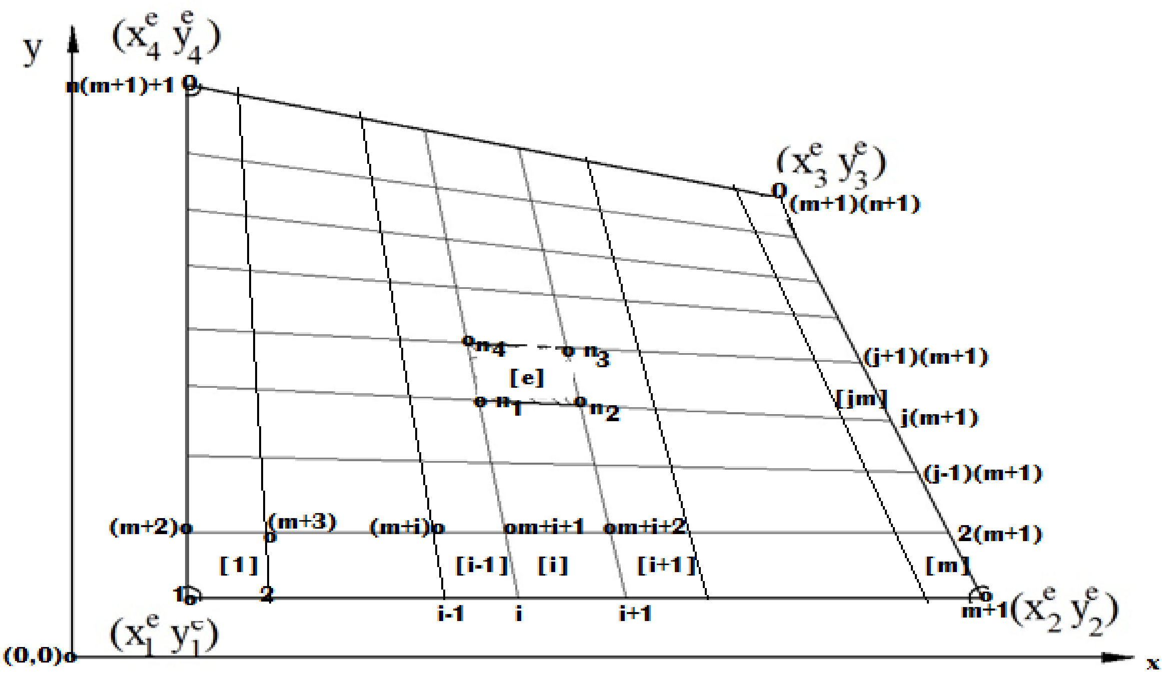

Fig.2a: mxn divisions of an arbitrary quadrilateral Q_e in (x, y) space and a typical quadrilateral element 'e' in the interior

[e]:quadrilateral with nodal vertices<n_1 ,n_2 ,n_3 ,n_4>,

n_1= (j-1)(m+1)+i, n_2= (j-1)(m+1)+i+1, n_3= j(m+1)+i+1 , n_4= j(m+1)+i.

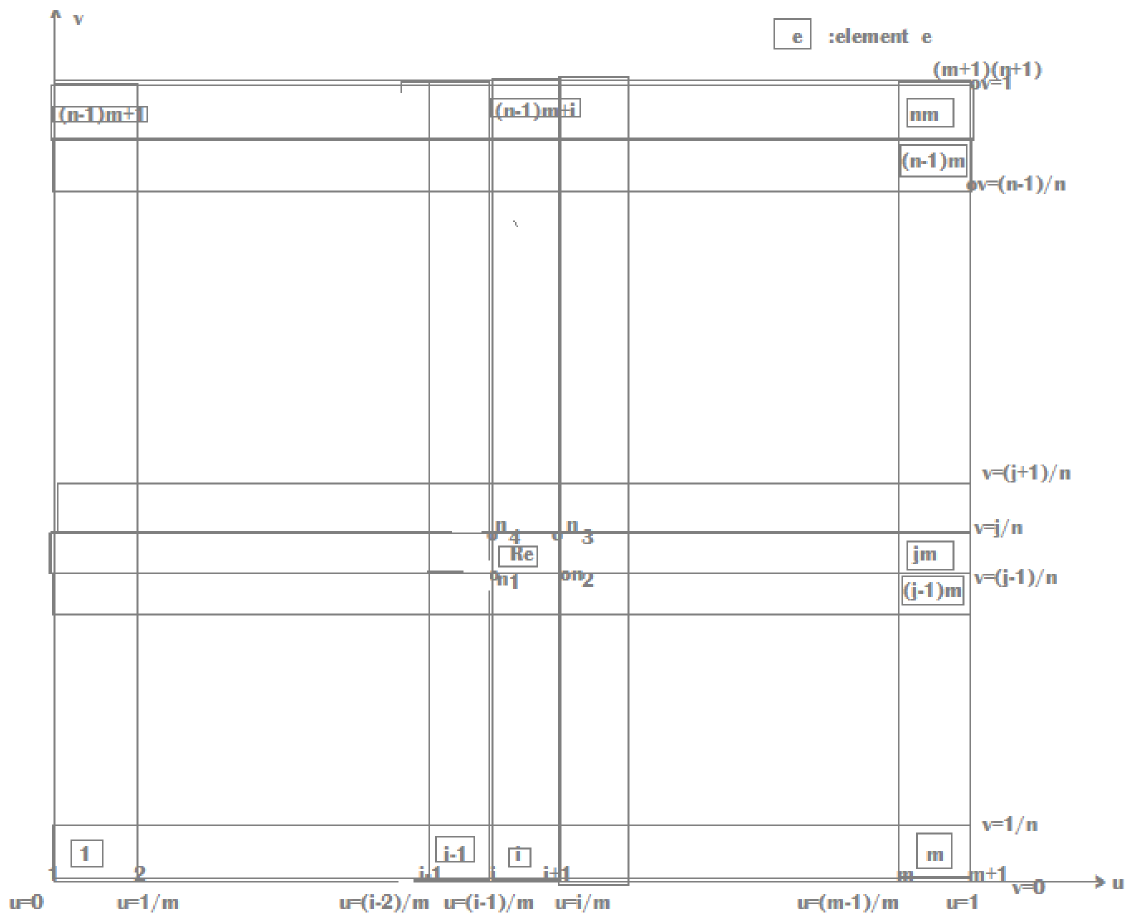

Fig.2b: Division of a unit square into 'mn' four node rectangles, in (u,v) space

$[e]$::element'e'

[Re]:rectangle with nodal vertices<n_1 ,n_2 ,n_3 ,n_4>,

(n_i ,i=1,2,3,4)::node numbers of element vertices

((u_k,v_l) ,k=1,2,3,….m+1),$l = 1,2,3, \ldots n + 1$)):local coordinates of unit square

n_1= (j-1)(m+1)+i, n_2= (j-1)(m+1)+i+1, n_3= j(m+1)+i+1 , n_4= j(m+1)+i.

The vertices of corner nodes for the element (e) has coordinates (in anticlockwise sense) are

$n_1\left(x^e(u_i, v_i), y^e(u_i, v_i)\right), \quad n_2\left(x^e\left(u_{i+1}, v_j\right), y^e\left(u_{i+1}, v_j\right)\right)$

$n_3\left(x^e(u_{i+1}, v_{i+1}), y^e(u_{i+1}, v_{i+1})\right)$; $\quad n_4\left(x^e\left(u_i, v_{j+1}\right), y^e\left(u_i, v_{j+1}\right)\right)$ ------------(6)

Where

$n_1\left(u_i, v_j\right) = n_1((i-1)/m, \ \ (j-1)/n)$,

$n_2\left(u_{i+1}, v_j\right) = n_2(i/m, \ \ (j-1)/n)$,

$n_3\left(u_{i+1}, v_{j+1}\right) = n_3(i/m, \ \ j/n)$,;

$n_4\left(u_i, v_{j+1}\right) = n_4((i-1)/m, \ \ j/n)$, --(7)

and the node numbers in both the spaces are

$n_1 = j(m+1) + i$; $\quad n_2 = j(m+1) + i + 1$;

$n_3 = (j+1)(m+1) + i + 1$; $n_4 = (j+1)(m+1) + i$ -----------------------------(8)

All the element nodes and coordinates can be obtained by varying i and j, $i = 1,2, \ldots, (m+1)$; $j = 1,2, \ldots, (n+1)$ and naturally over any typical element $(i+1) \leq (m+1)$ and $(j+1) \leq (n+1)$.

We have shown the division of an arbitrary quadrilateral Q_e and a unit square in Fig. 2a and Fig. 2b. respectively. We divide each side of the quadrilateral and unit square (in Cartesian space(x,y) and natural space(u,v)) into m equal division along x and u axes and n equal divisions along y and v axes. This creates (m+1)* $(n+1)$ nodes. These nodes are numbered from base line l_{12} (letting l_{ij} as the line joining the vertex $(x_i^e, y_i^e)$and(x_j^e, y_j^e)) $and\ move\ \ upwards\ upto\ the\ \ line\ \ l_{34}\ \ \ in\ quadrilateral\ Q_e$; now with respect to the unit squqre in Fig.2b,we move along the line $v = 0$ and upwards up to the line $v = 1$. The nodes along v=0 are 1, 2,…(m+1);and then on $v_1 = 1/n$ are (m+2),(m+3),….2(m+1);etc and finally on v=1 are n(m+1)+1,n(m+1)+2,….,(n+1)*(m+1) and they are numbered layer by layer.This is shown in the following matrix of node numbers $\underline{rr}$:

$$\underline{rr} = \begin{array}{c}
\begin{array}{ccccccc} u=0 & u=1/m & u=2/m & u=(i-1)/m & u=i/m & u=(m-1)/m & u=1 \end{array} \\
\left[\begin{array}{ccccccc}
1 & 2 & 3 & i & i+1 & m & (m+1) \\
(m+2) & (m+3) & (m+4) & (m+i+1) & (m+i+2) & (2m+1) & 2(m+1) \\
2(m+1)+1 & & & & & (3m+2) & 3(m+1) \\
 & & & & & & \\
(j-1)(m+1)+1 & & & (j-1)(m+1)+i+1 & (j-1)(m+1)+i+2 & & j(m+1) \\
j(m+1)+1 & & & j(m+1)+i+1 & j(m+1)+i+2 & & (j+1)(m+1) \\
 & & & & & & \\
(n-1)(m+1)+1 & & & (n-1)(m+1)+i+1 & (n-1)(m+1)+i+2 & & n(m+1) \\
n(m+1)+1 & & & n(m+1)+i+1 & n(m+1)+i+2 & & (n+1)(m+1)
\end{array}\right]
\begin{array}{l} \Rightarrow v=0 \\ \Rightarrow v=1/n \\ \Rightarrow v=2/n \\ \\ \Rightarrow v=(j-1)/n \\ \Rightarrow v=j/m \\ \\ \Rightarrow v=(n-1)/n \\ \Rightarrow v=1 \end{array}
\end{array}$$

Fig 2c. Matrix rr of node numbers for the division of a unit square

…………………………………………(9)

In the present algorithm the four corners of quadrilateral Q_e as well the unit square are defined as n_1 =1 , n_2 = m+1 , n_3 =(m+1)(n+1) and n_4 =n(m+1)+1,where m is the number of divisions along x or u axis n is the number of divisions along y or v axis in both the spaces viz: (x,y) and (u,v)

We may further note that when the present algorithm is applied to a single quadrilateral (m=1,n=1) then the boundary nodes are n_1 =1 , n_2 = 2 , n_3 =4 and n_4 =3.This will help us in the graphic display of quintic order to decic order complete Lagrange elements over the standard squares.

4. Mesh Generation Using Higher order Quadrilaterals

In finite element applications, we may have to generate higher order quadrilateral elements. They contain midside nodes. We can obtain quadratic elements by inserting additional nodes at the midpoints of the linear four node element boundaries which gives us Serendipity quadratic elements. In addition to this when a node is also inserted at the centroid of the quadrilateral, we obtain Lagrange quadratic elements.

We next consider cubic elements, they can be obtained by inserting nodes at the trisectional points of the four node element boundaries which gives us Serendipity cubic elements. In addition to this, if we insert nodes in the interior of the elements at trisectional points, we obtain the Lagrange cubic elements. Zienkiewicz[17] intended to define a Serendipity family so that polynomial completeness is realized with necessary minimum nodes and presented the few lower order elements viz.Linear,Quadratic and Cubic elements which have equal number of nodes along each side which are uniformly spaced.It is obvious that the Basis functions for Seredipity elements with nodes placed only along the edges cannot generate complete polynomials beyond cubic,for this reason,Zienkiewicz[17] has suggested a central node for the next Quartic member of this family, and remarks that progression to yet higher order members is difficult and requires some ingenuity. M.Okabe[29], H.T.Rathod and Sridevi. Kilari [30] determined the Basis functions of the Serendipity and Complete Lagrange family elements which allow uniform spacing of nodes over the element domain for orders 4-10.We have already demonstrated the generation of finite element meshes over polygonal domains upto Quartic order for Serendipity, Complete Lagrange and Lagrange family elements[31]. In this paper we intend to generate finite element meshes for remaining orders viz quintic order to decic order complete Lagrange elements.The present algorithm can also be applied to Lagrange family elements of quintic order to decic order,this presentation is excluded here because they contain a lage number of interior nodes.

These rectangular elements for Complete Lagrange family from Quintic order to Decic order in the local parametric space are depicted in the following figures.

(1) QUINTIC ORDER COMPLETE LAGRANGE ELEMENT OVER A 2-SQUARE

η- axis

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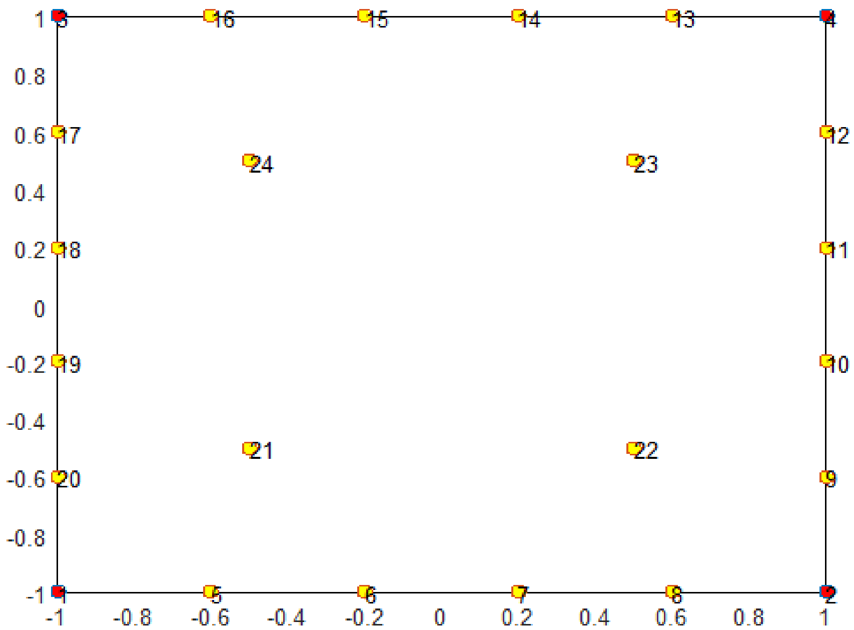

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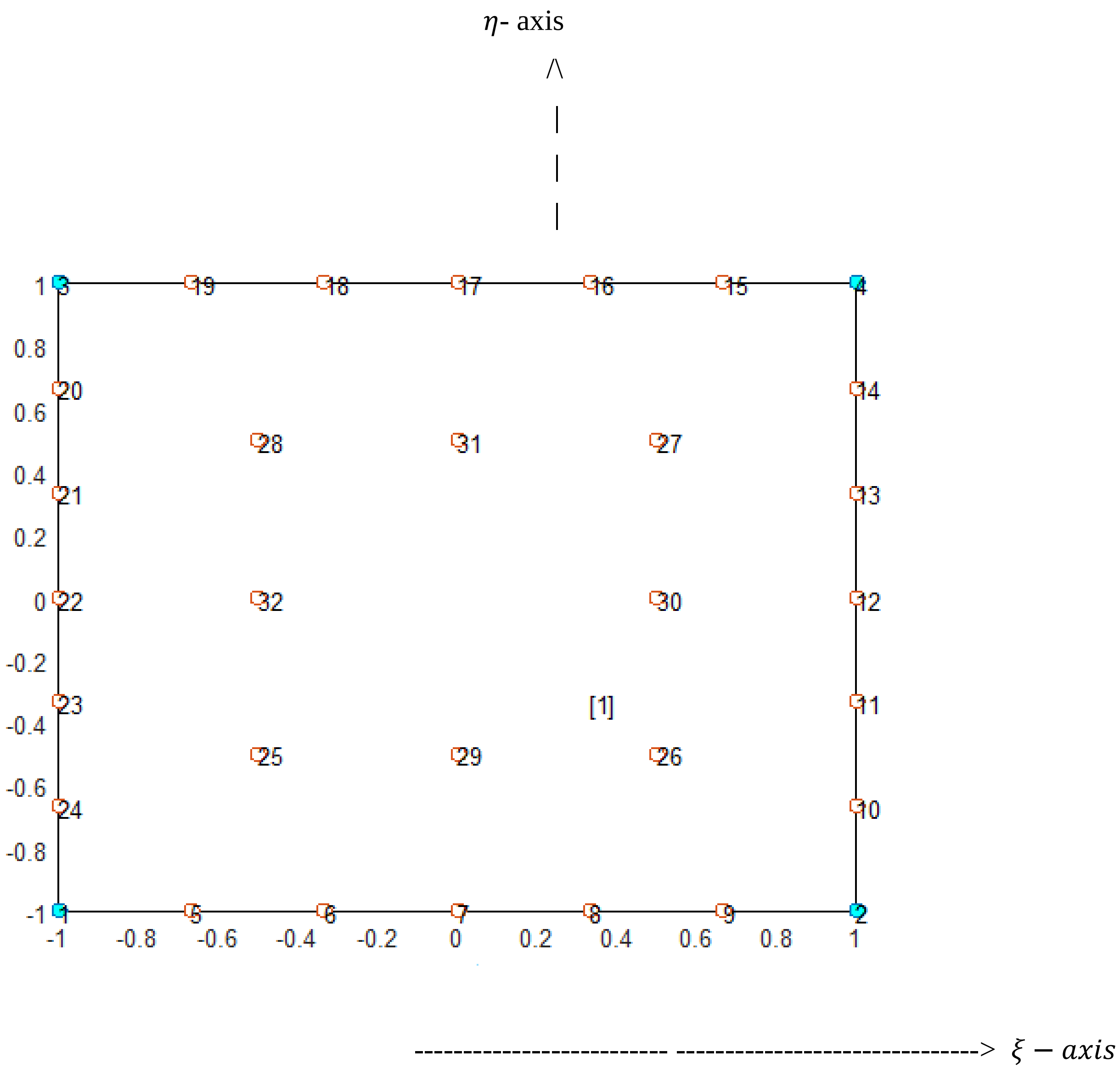
(2) SEXTIC ORDER COMPLETE LAGRANGE ELEMENT OVER A 2-SQUARE



(3) SEPTIC ORDER COMPLETE LAGRANGE ELEMENT OVER A 2-SQUARE

η- axis

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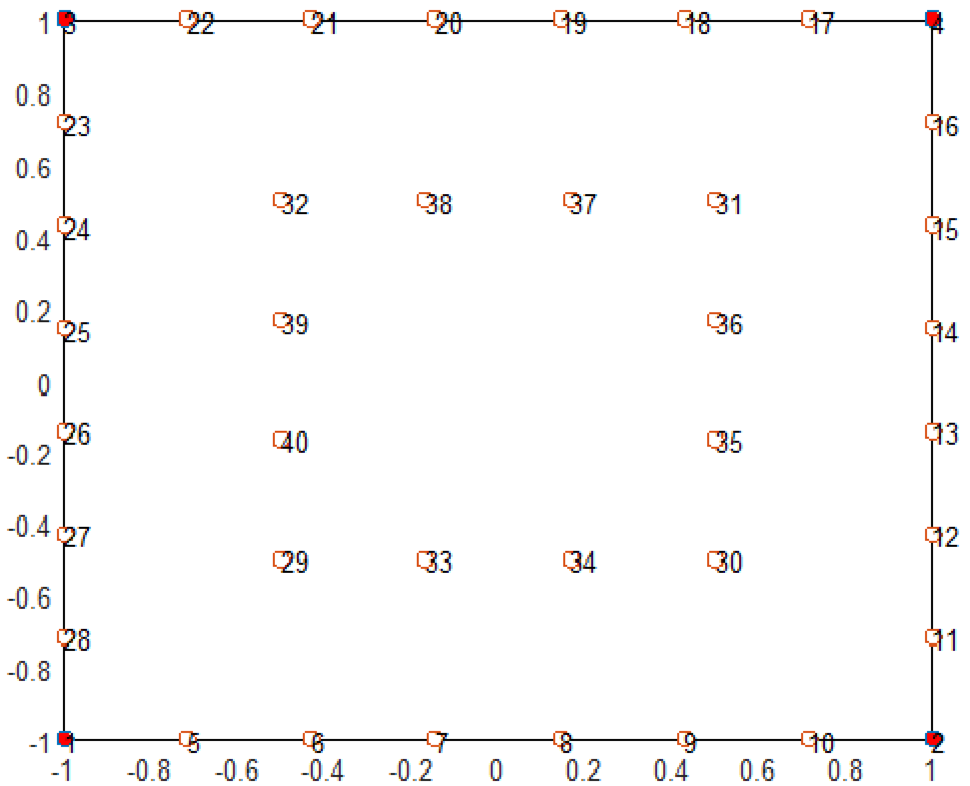

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$\xi - axis$

(4) OCTIC ORDER COMPLETE LAGRANGE ELEMENT OVER A 2-SQUARE

η- axis

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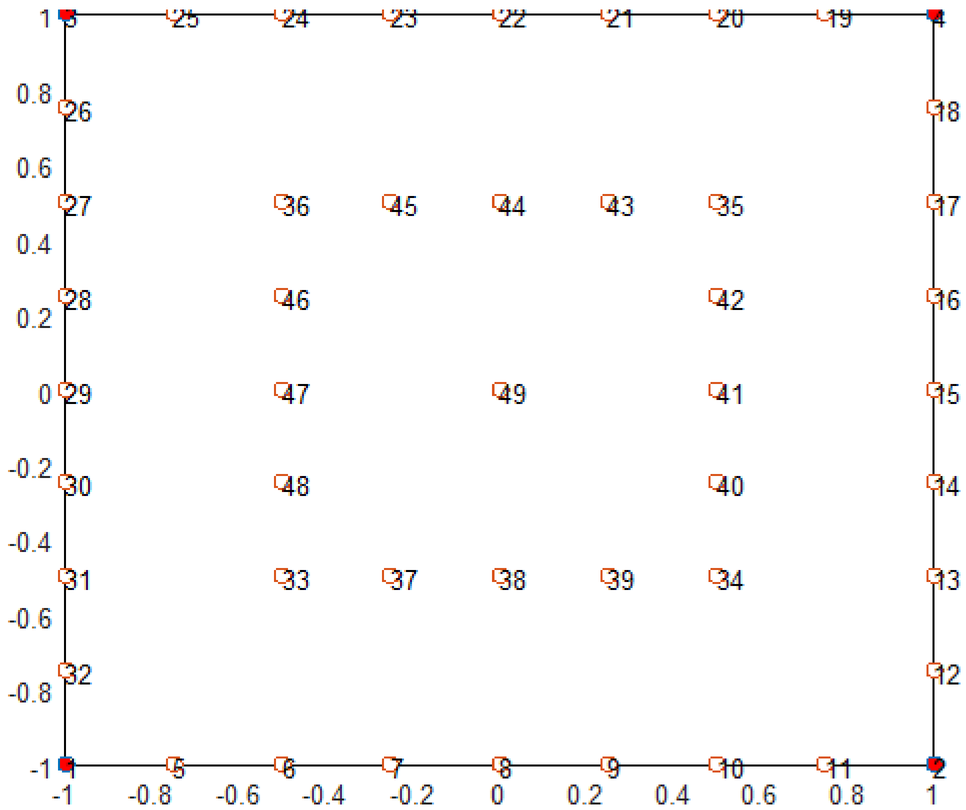

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$\xi - axis$

(5) NONIC ORDER COMPLETE LAGRANGE ELEMENT OVER A 2-SQUARE

η- axis

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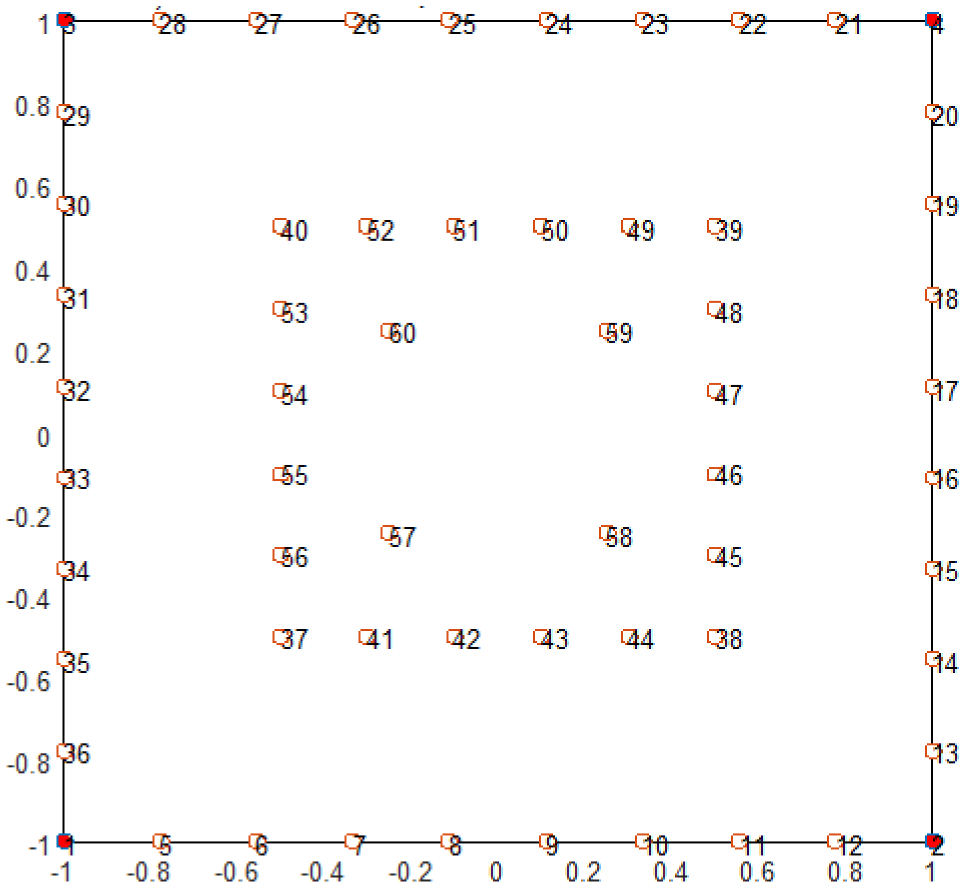

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$\xi - axis$

(6) DECIC ORDER COMPLETE LAGRANGE ELEMENT OVER A 2-SQUARE

η- axis

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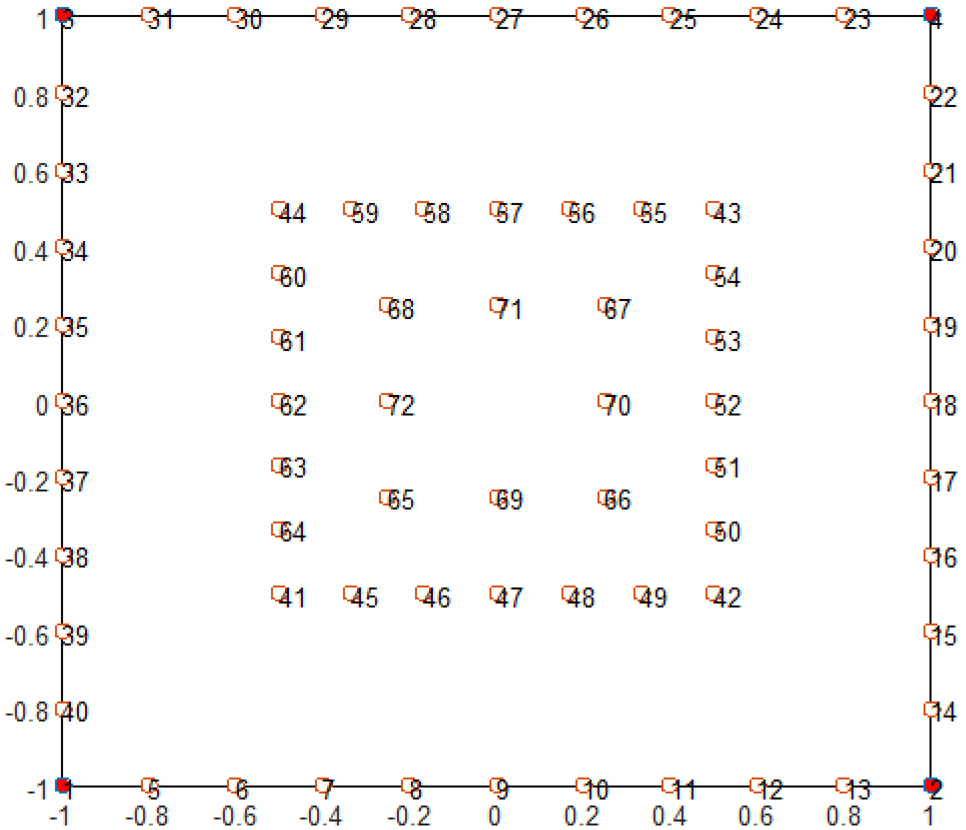

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$\xi - axis$

5. Mesh Generation Algorithm over a linear convex Quadrilateral

We briefly present here the mesh generation algorithm over a Quadrilateral in Cartesian space

(i)Divide the unit square into 'mn' rectangles or squares of uniform size,this is done by making m divisions of equal spacing along u-axis and v-axis.This requires '(m+1)*(n+1) nodes.

(ii)Use the bilinear transformation to obtain (m+1)*(n+1) Cartesian coordinates over the original quadrilateral $Q_e\ in\ xy-space$ corresponding to the (m+1)*(n+1) nodes in uv-space.This generates 'mn' rectangles in local uv-space and 'mn' quadrilaterals in the Cartesian xy-space.

(iii)Now insert midside nodes using pigeon hole principle which is elaborated in the MATLAB CODE and then insert the interior nodes if any. This discritises the 'mn' quadrilaterals into higher order quadrilaterals as per the requirements. Please note that the insertion of midsidenodes has to be done on all the four sides of the quadrilateral 'e' in $Q_{e.}$,where e=1,2,3,…mn.

(iv) Using bilinear mapping Cartesian coordinates for all the nodes can be computed

(v)The nodal coordinates and element nodal connectivity data is passed on to the main program.

Then the main program generates the desired mesh for the element type.

6. Quad angulation of an Arbitrary Polygon

Finite element applications to physical problem require mesh generation over polygonal domains. We divide this domain into a coarse mesh of triangles or quadrilaterals or both. Our aim now is to generate a mesh of all quadrilaterals. This is first done by generating quadrilateral meshes over each coarse shape (triangles or quadrilateral) and then piecing together, we obtain an all quadrilateral mesh for the polygonal domains. We have presented in our recent paper[31] the mesh generation of polygonal domains using the 4, 8, 9 12, 16,17,25 noded quadrilaterals elements of Serendipity, Lagrange and Complete Lagrange elements upto Quartic order. In the present paper,we have generated meshes from Quintic order to Decic order elements,the details on the monomial basis and the element geometry are presented in author's paper[30].

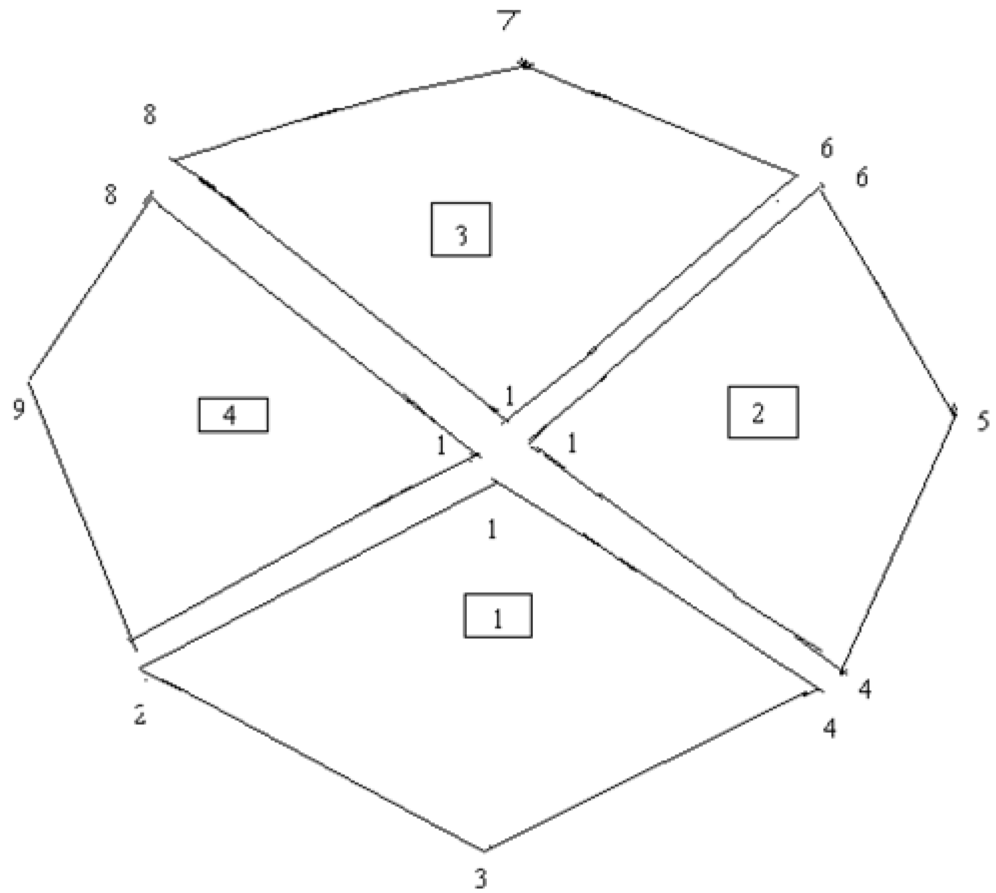

Fig.4

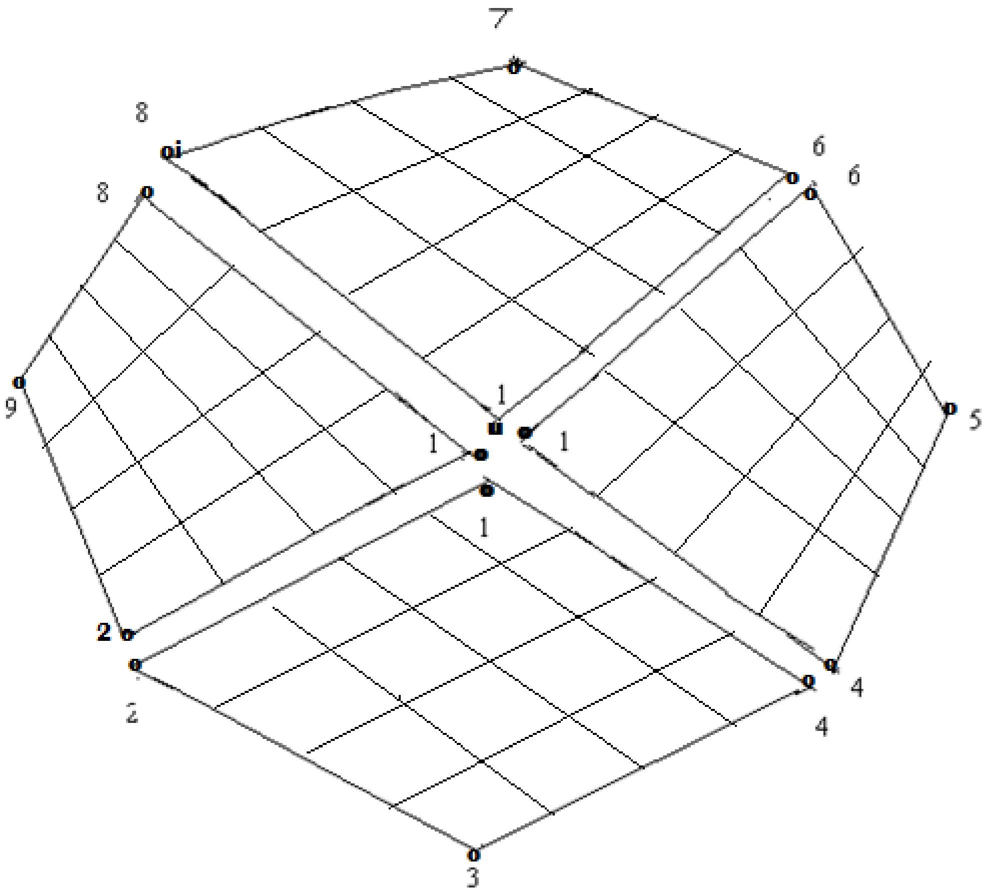

Fig.5

Figs.4-5 Piecing together of four quadrilaterals

7. Application Examples

In applications to boundary value problems, we may have to discretize an arbitrary polygonal domain using linear,quadratic,cubic and quartic finite elements. Our purpose is to have codes which automatically generates elements with linear convex quadrilaterals over the domain by assuming the input as coordinates of the vertices. We have choosen four typical examples:

(i)Rectangles described in section 2(Examples 1-2)

(ii)An Arbitrary Quadrilateral

(iii)An Equilateral Triangle

(iv)A Convex Polygon

(v)A Nonconvex Polygon

We may note that the rectangles and parallelograms of any orientation will be discritsed into finite element meshes of rectangles(or squares) and parallelograms.Two codes written in MATLAB programming and based on the proposed scheme of this paper to generate meshes using Decic order Complete Lagrange elements with 72-nodes are appended. Codes for Linear,Quadratic, Cubic,Quartic,Quintic,Sextic,Septic,Octic,Nonic order finite elements were developed on similar lines and the schemes explained in this paper but they are not included here. Several Figures on Finite Element mesh generation using Quintic to Decic order elements(i.e24,32,40,49,60,72-noded each) are presented immediately after **References.**

8 Conclusions

Zienkiewicz[17] intended to define a Serendipity family so that polynomial completeness is realized with necessary minimum nodes and presented the few lower order elements viz.Linear,Quadratic and Cubic elements which have equal number of nodes along each side which are uniformly spaced.It is obvious that the Basis functions for Seredipity elements with nodes placed only along the edges cannot generate complete polynomials beyond cubic,for this reason,Zienkiewicz[17] has suggested a central node for the next Quartic member of this family, and remarks that progression to yet higher order members is difficult and requires some ingenuity

This paper presents a novel mesh generation scheme of all quadrilateral elements over a linear polygonal domain. We first decompose the linear polygon into simple sub regions in the shape of quadrilaterals. These simple regions are then quadrangulated to generate first into a fine mesh of four node quadrilateral elements using bilinear transformations.We have already proposed this automatic quadrilateral mesh generation scheme in our recent paper. In this scheme each four node quadrilateral is converted to higher order quadrilaterals by inserting the midside nodes appropriately. Examples were presented to illustrate the simplicity and efficiency of the new mesh generation method for standard and arbitrary shaped domains for linear to quartic order quadrilaterals.In this paper,we continue our study and generate meshes of quintic to decic order quadrilaterals for Complete Lagrange family having 24,32,40,49,60 and 72 nodes.They are actually the Serendipity famiy quadrilaterals with appropriate number of interior nodes to incorporate the complete monomial basis of quintic to decic degree polynomials in bivariates

We have appended two important MATLAB programs which incorporate the mesh generation scheme for the 72-noded decic order complete Lagrange quadrilateral elements developed in this paper see references [29,30]. Other MATLAB programs for lower order elements can be coded on similar lines. These programs provide valuable output on the nodal coordinates ,element connectivity and graphic display of the all quadrilateral meshes for application to finite element analysis.The typical domains include rectangles,arbitrary oriented rectangles,an equilateral triangle, arbitrary quadrilateral,convex and nonconvex polygons.These programs provide valuable output on the nodal coordinates ,element connectivity and graphic display of the all quadrilateral meshes for application to finite element analysis.

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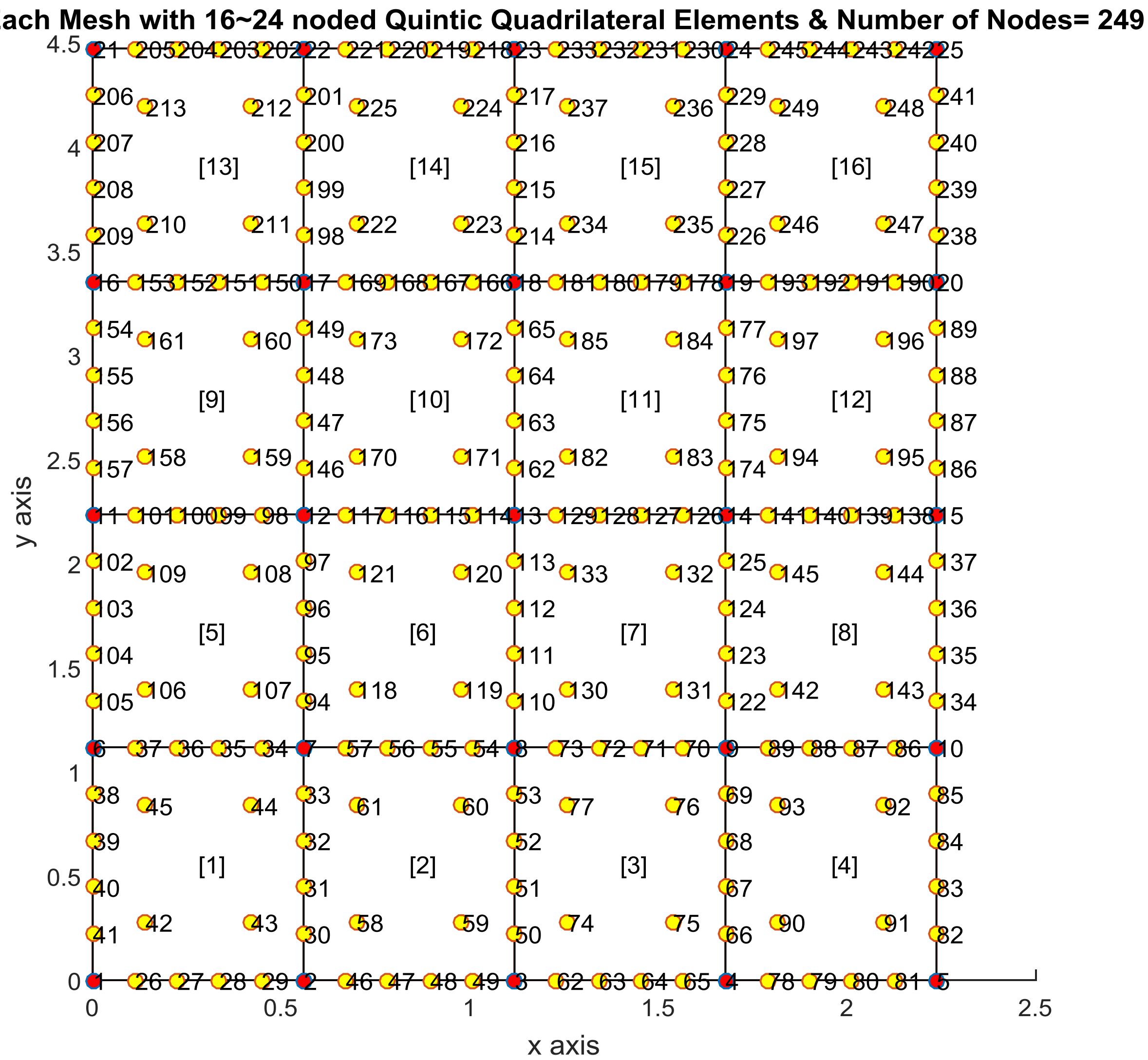
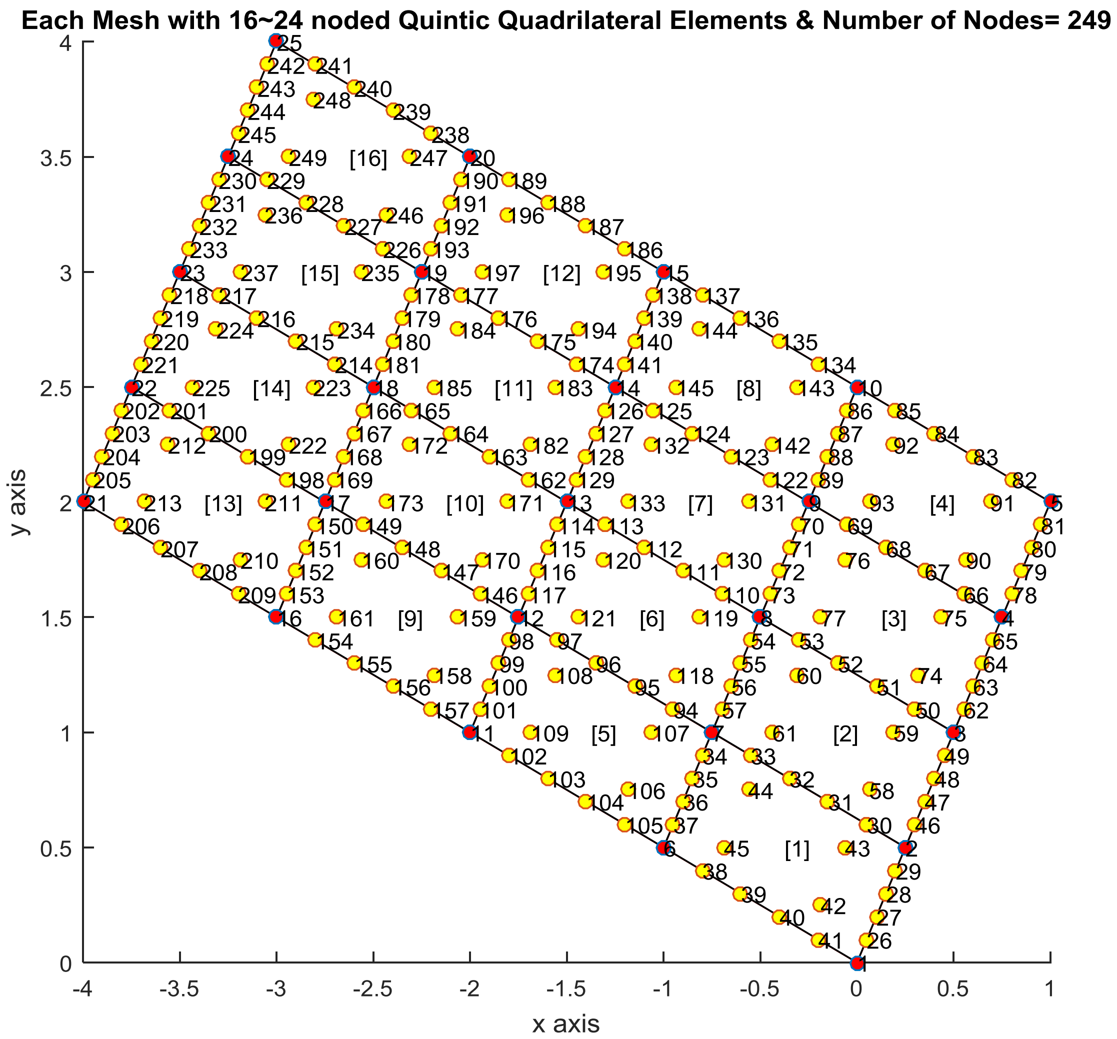
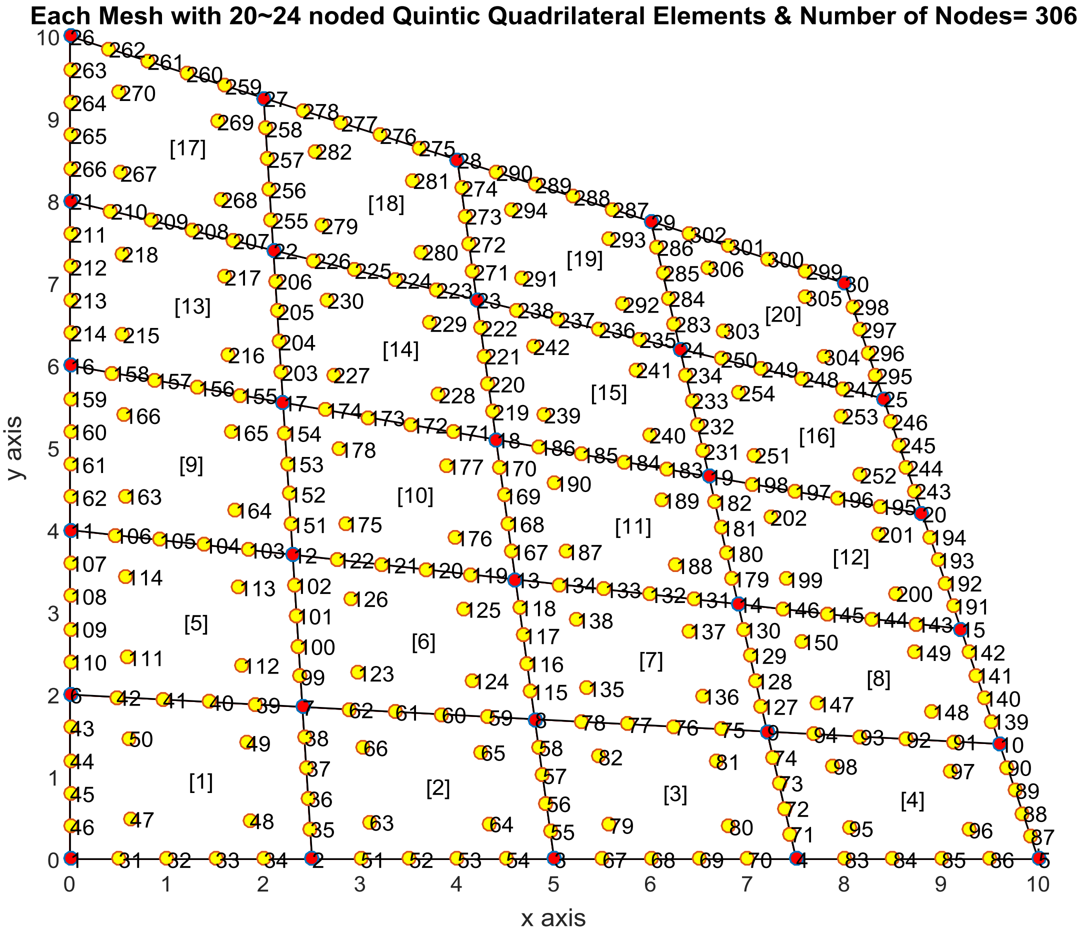
FIGURES

(1) Quintic order complete Lagrange Elements




Each Mesh with 16~24 noded Quintic Quadrilateral Elements & Number of Nodes= 249
y axis
x axis



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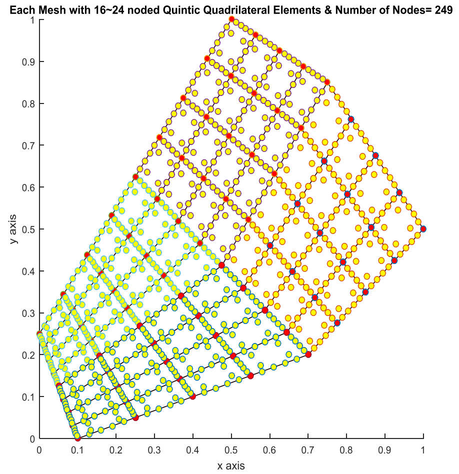
y axis
x axis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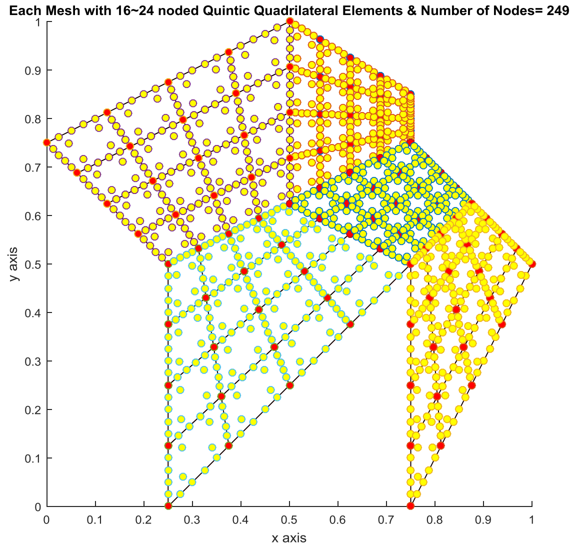
x axis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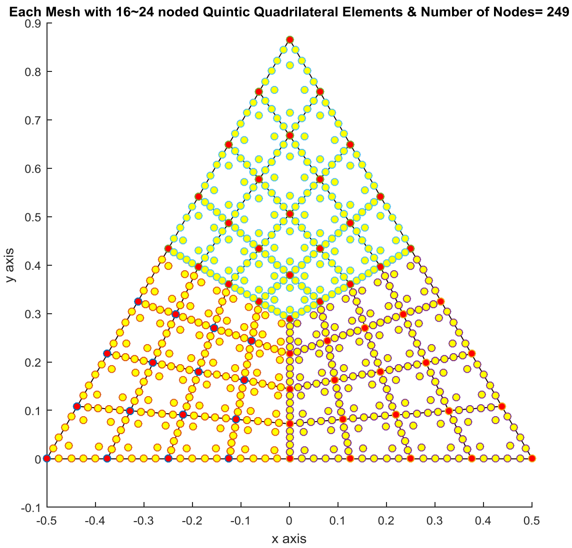
y axis
x axis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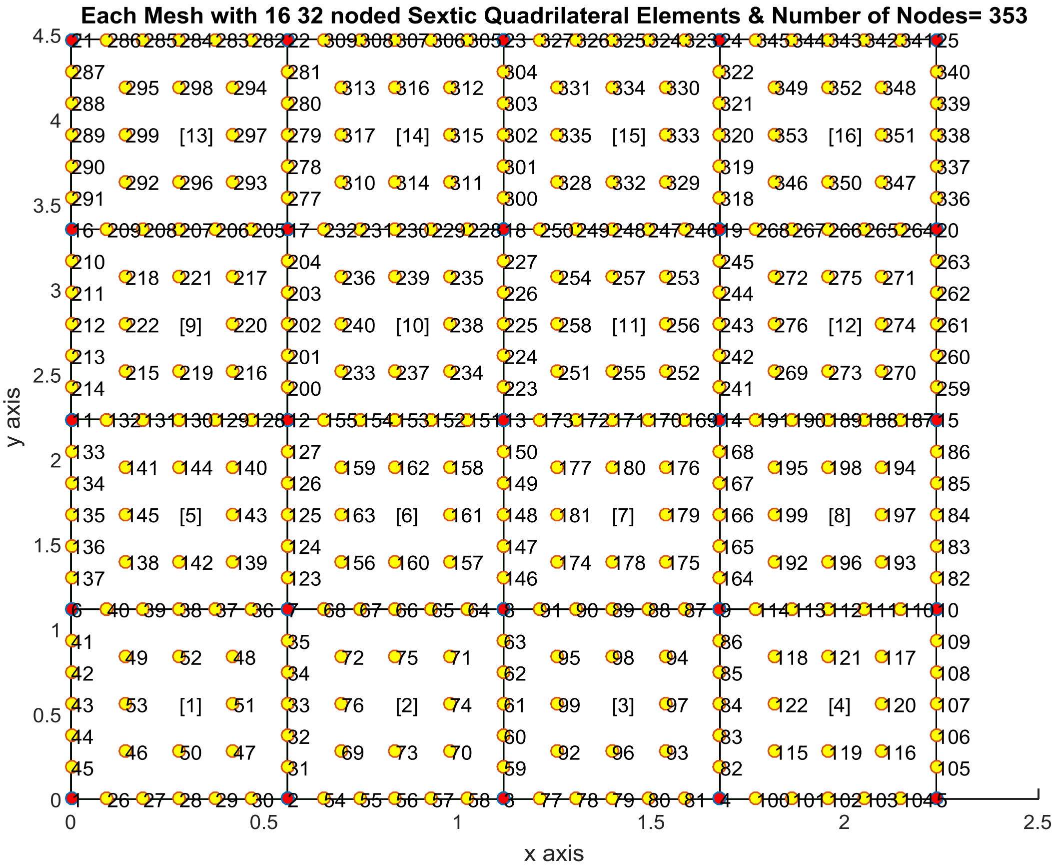
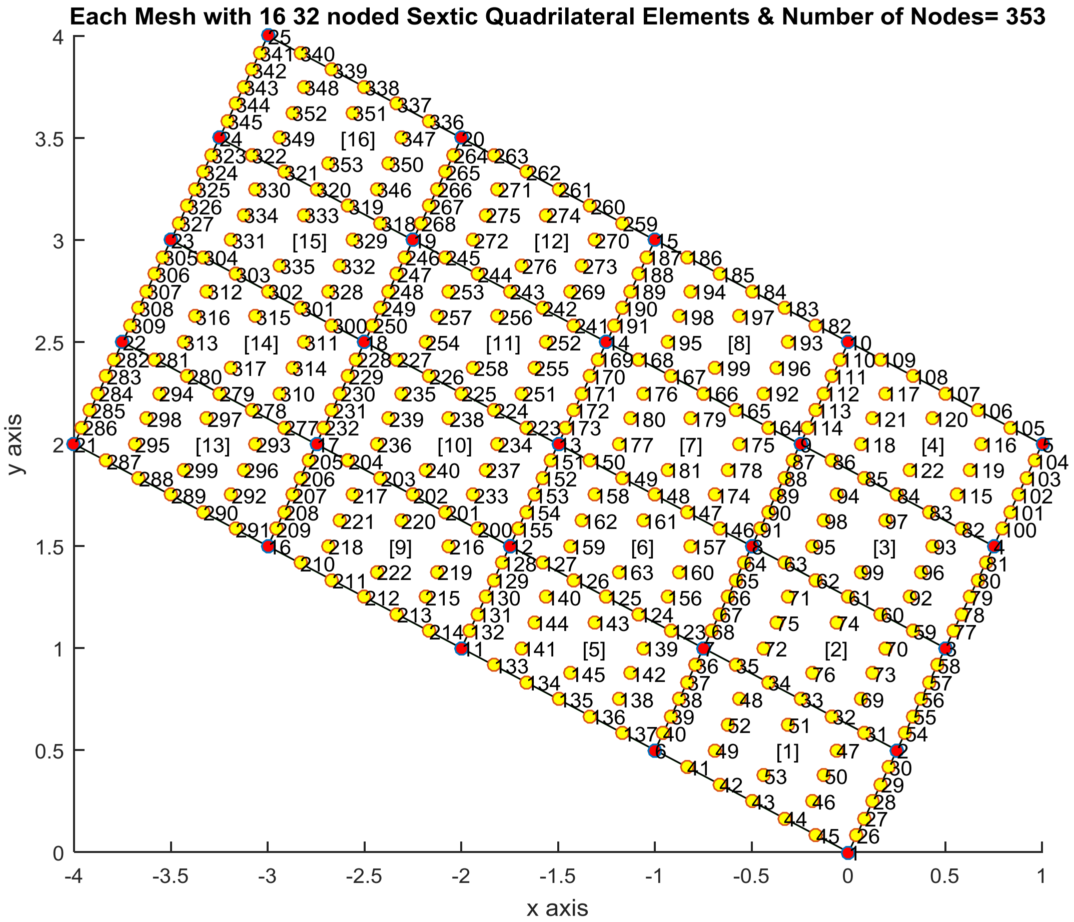
y axis
x axis

(2) Sextic order complete Lagrange Elements

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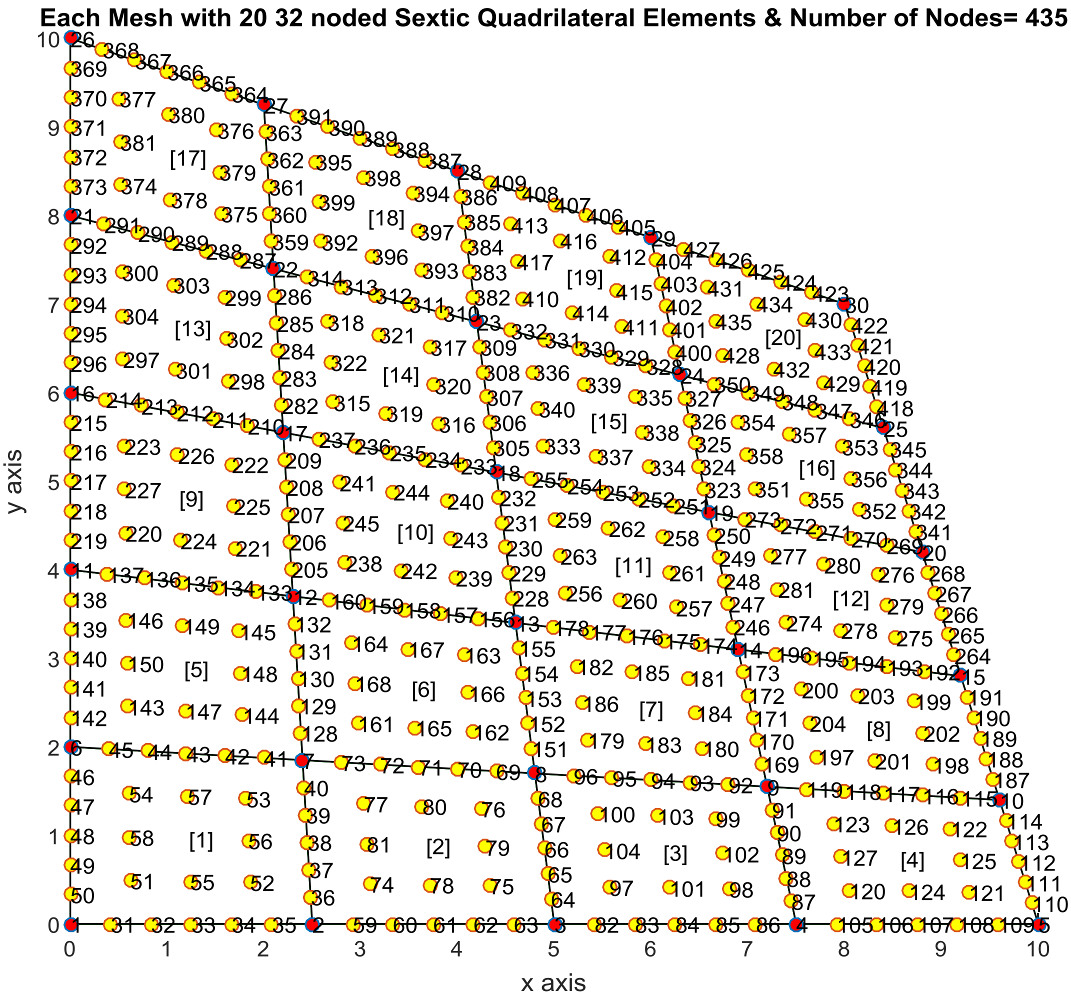
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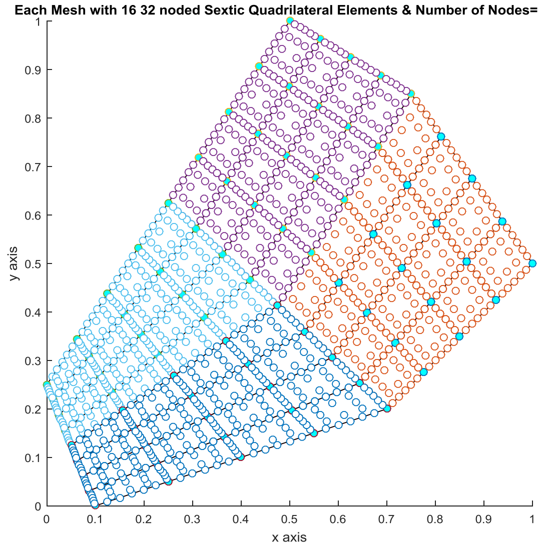
x axis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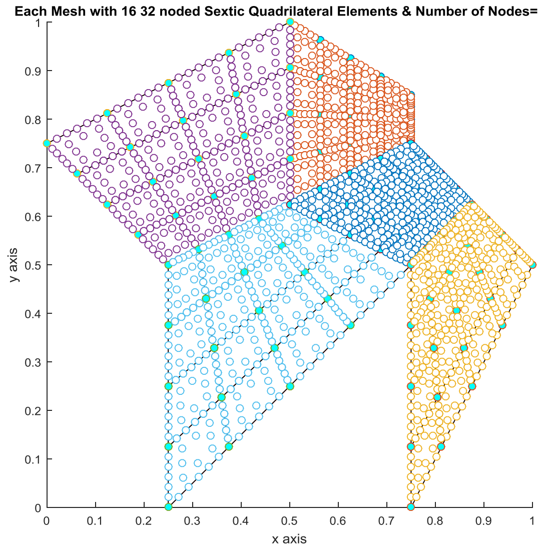
y axis
x axis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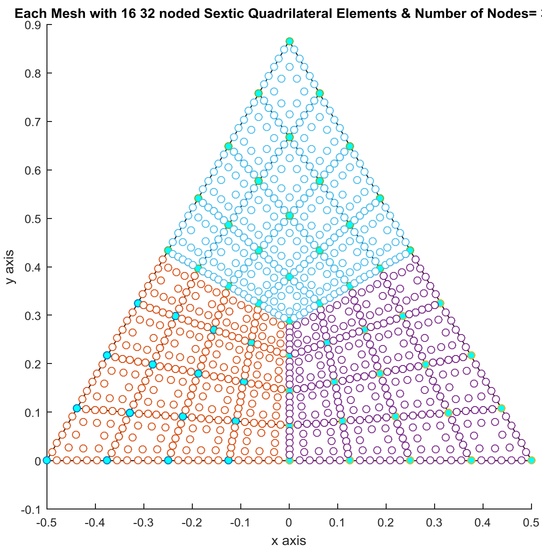
y axis
x axis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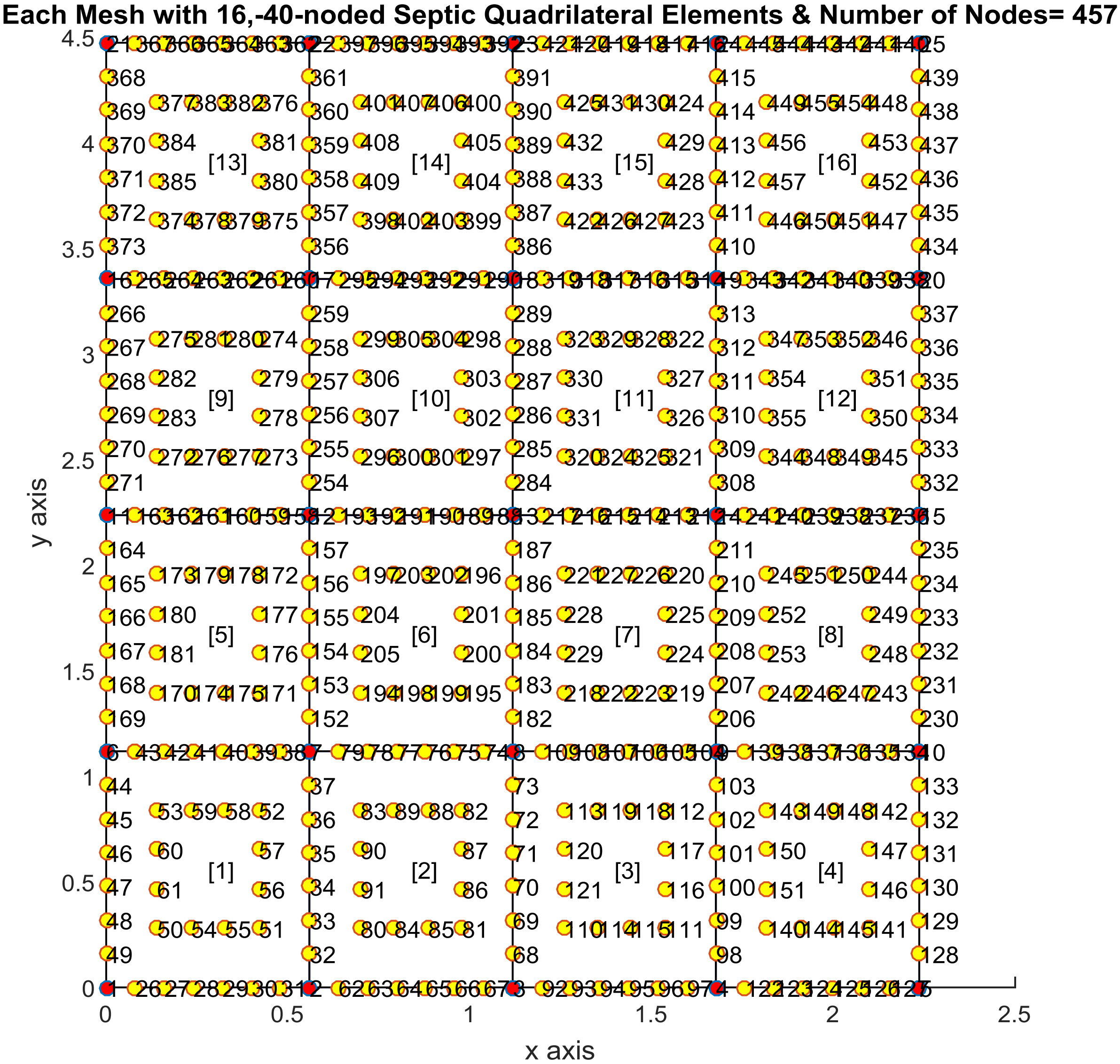
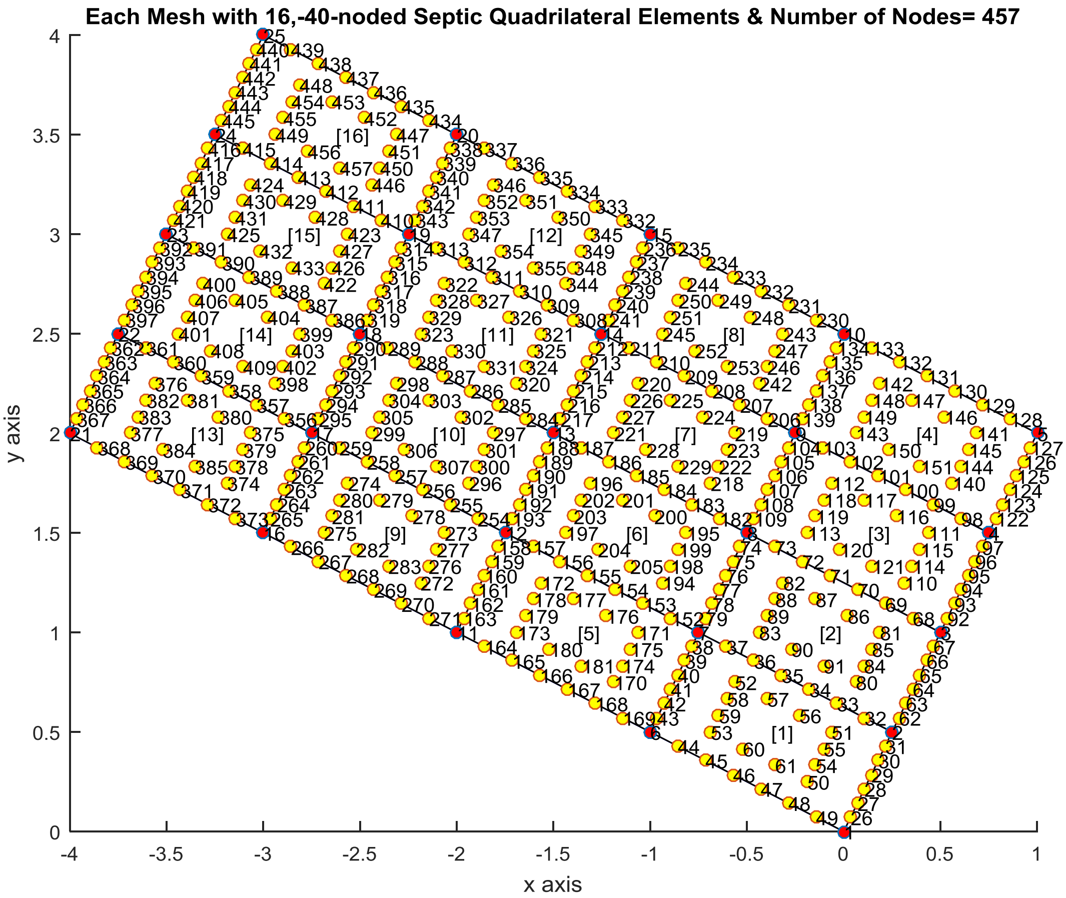
y axis
x axis

(3) Septic order complete Lagrange Elements

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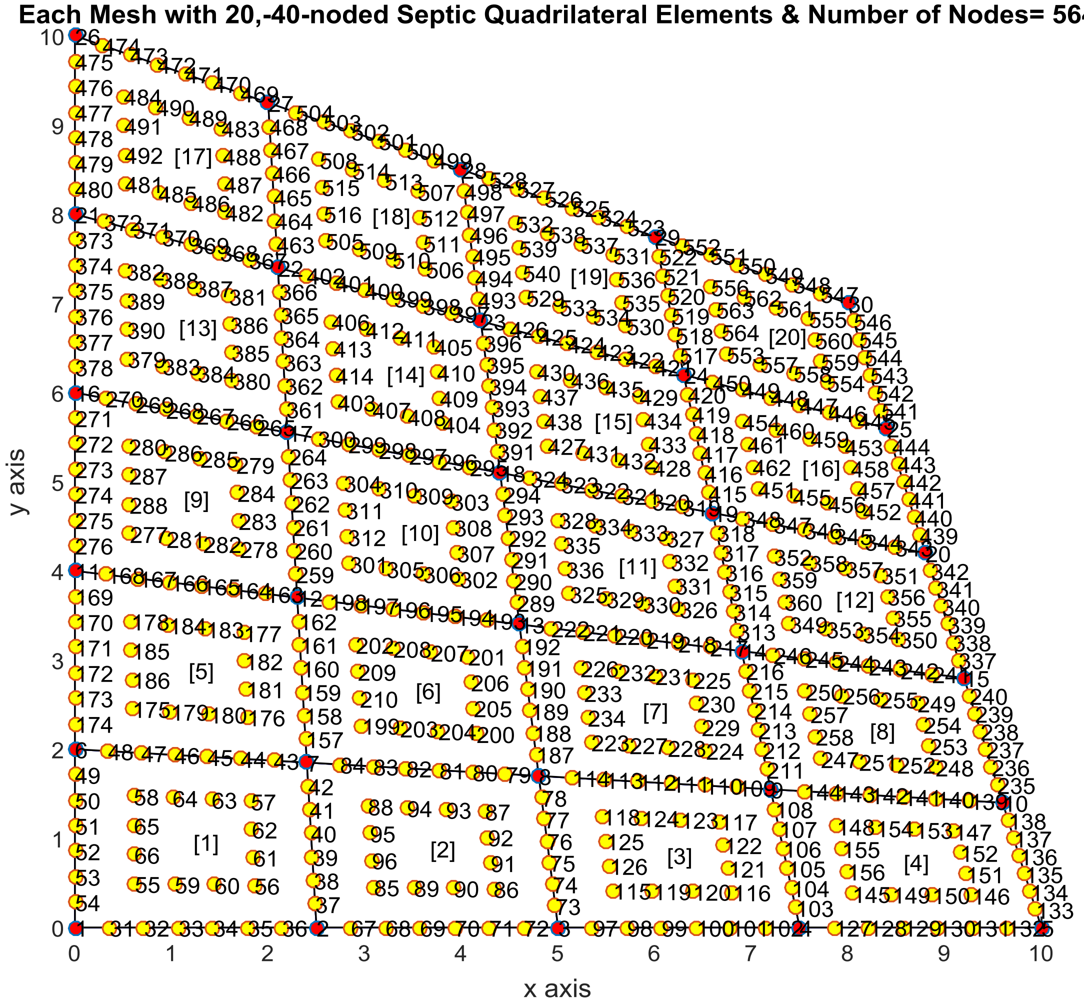
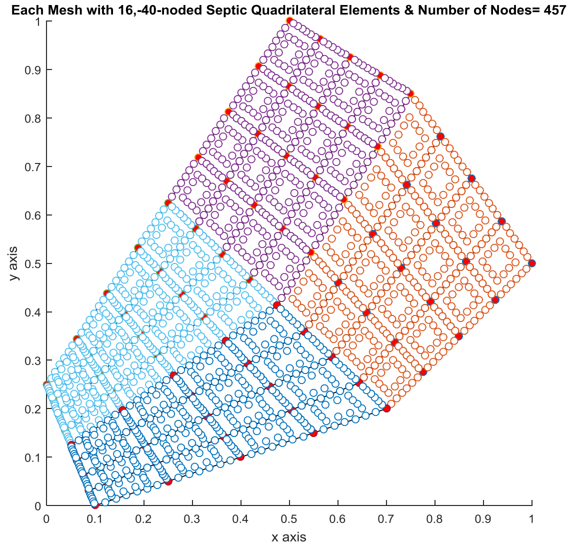
y axis
x axis

Each Mesh with 20,-40-noded Septic Quadrilateral Elements & Number of Nodes= 564

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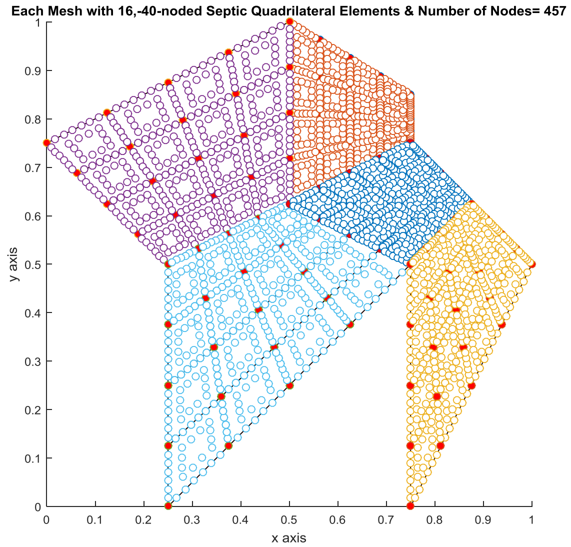
y axis
x axis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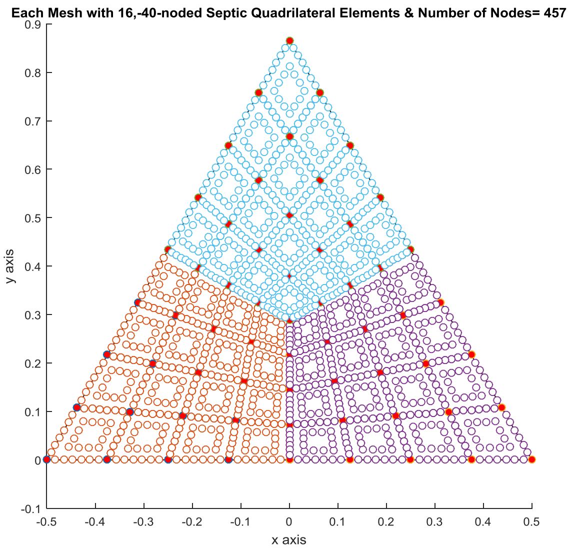
y axis
x axis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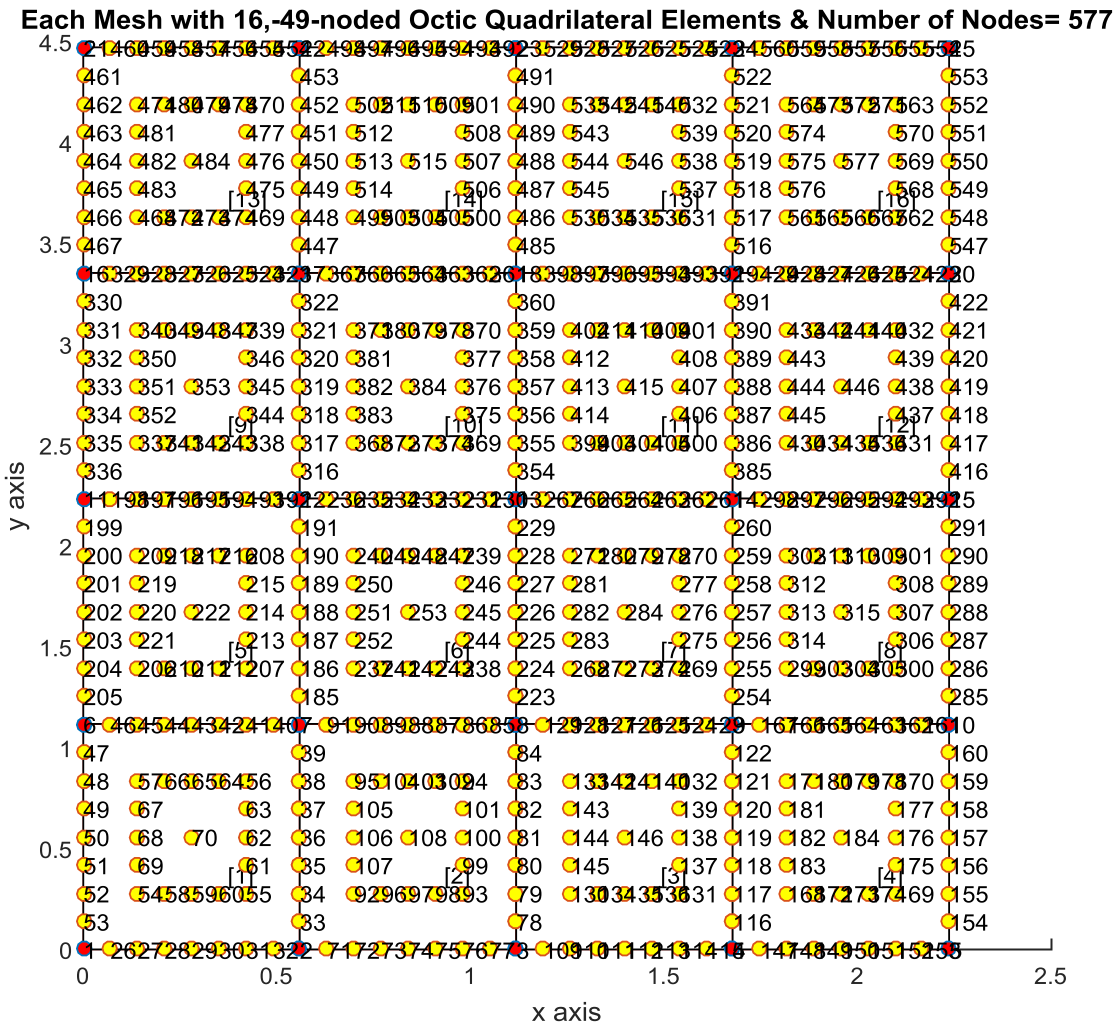
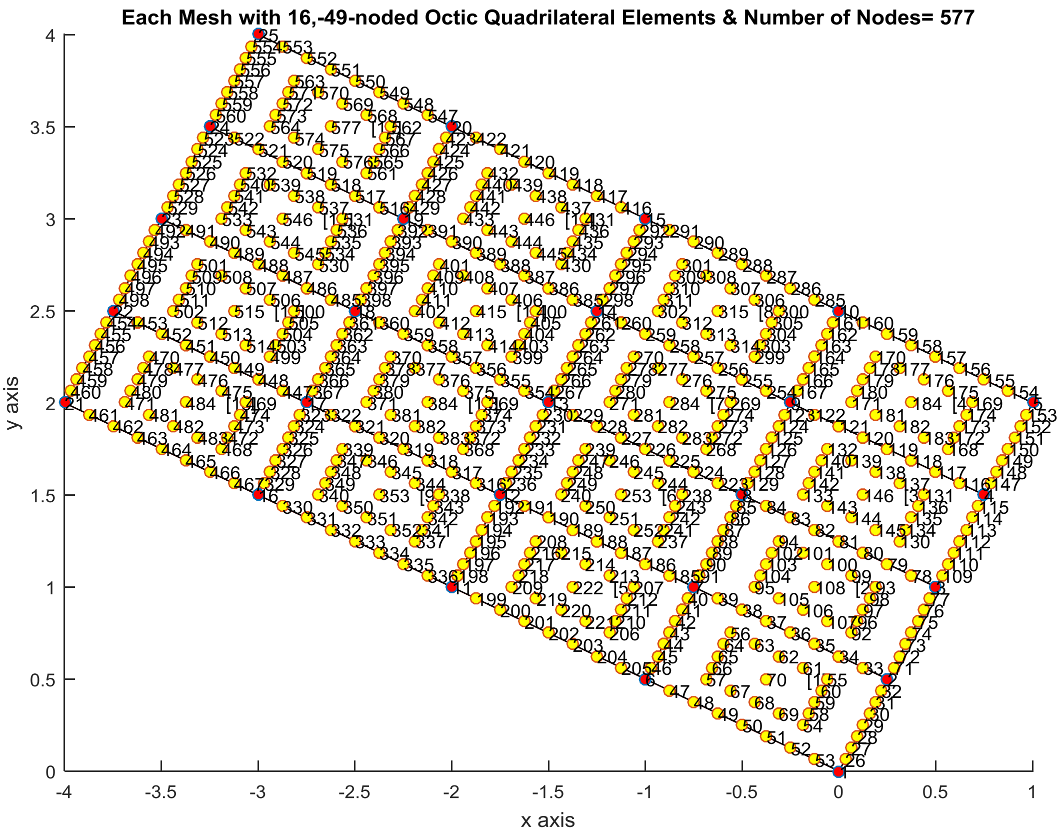
y axis
x axis

(4) Octic order complete Lagrange Elements





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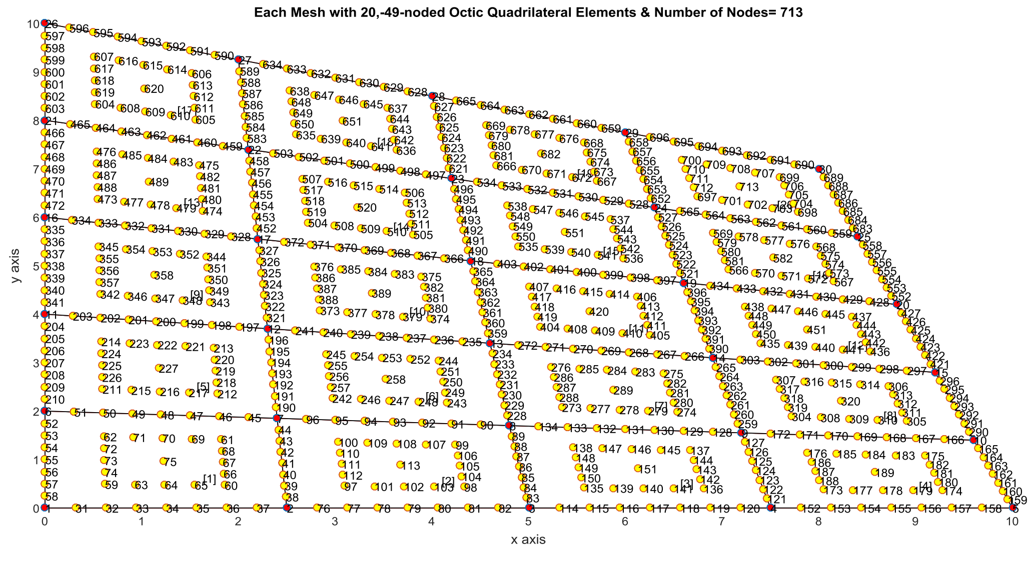
y axis
x axis



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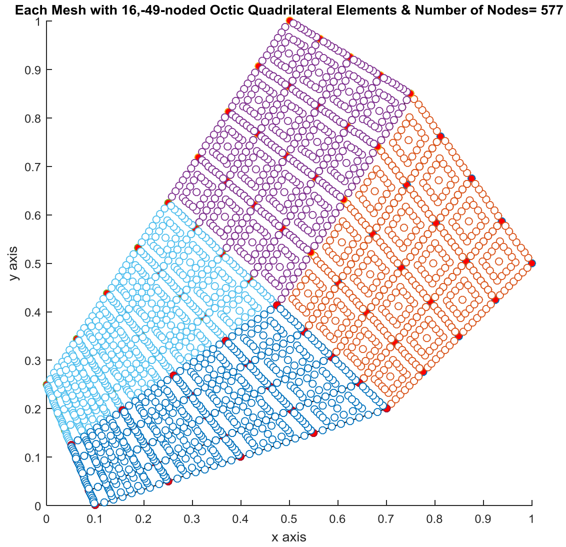
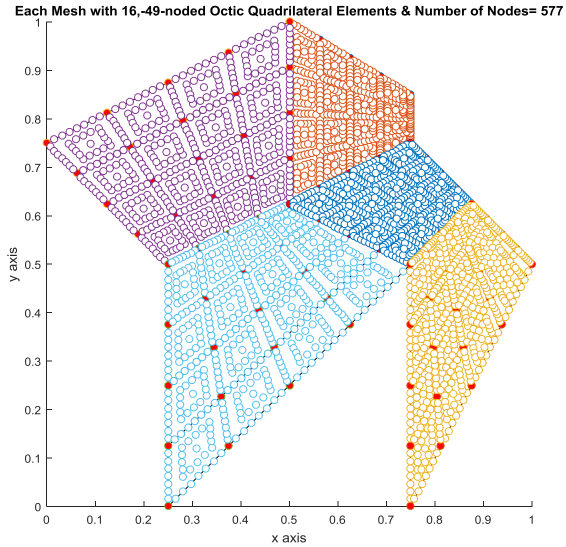
y axis
x axis



Each Mesh with 16,-49-noded Octic Quadrilateral Elements & Number of Nodes= 577
y axis
x axis



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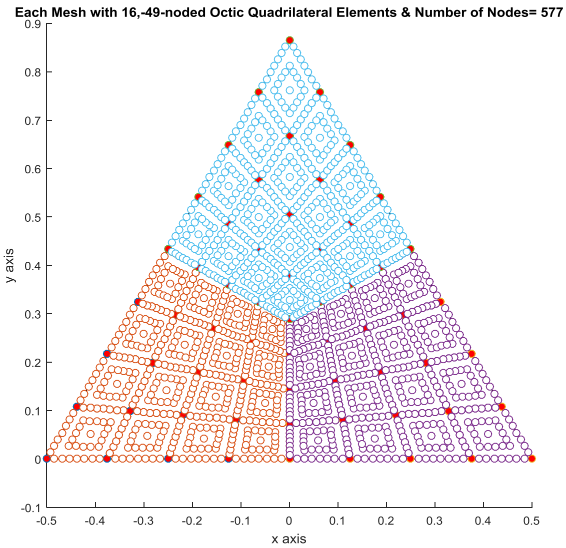
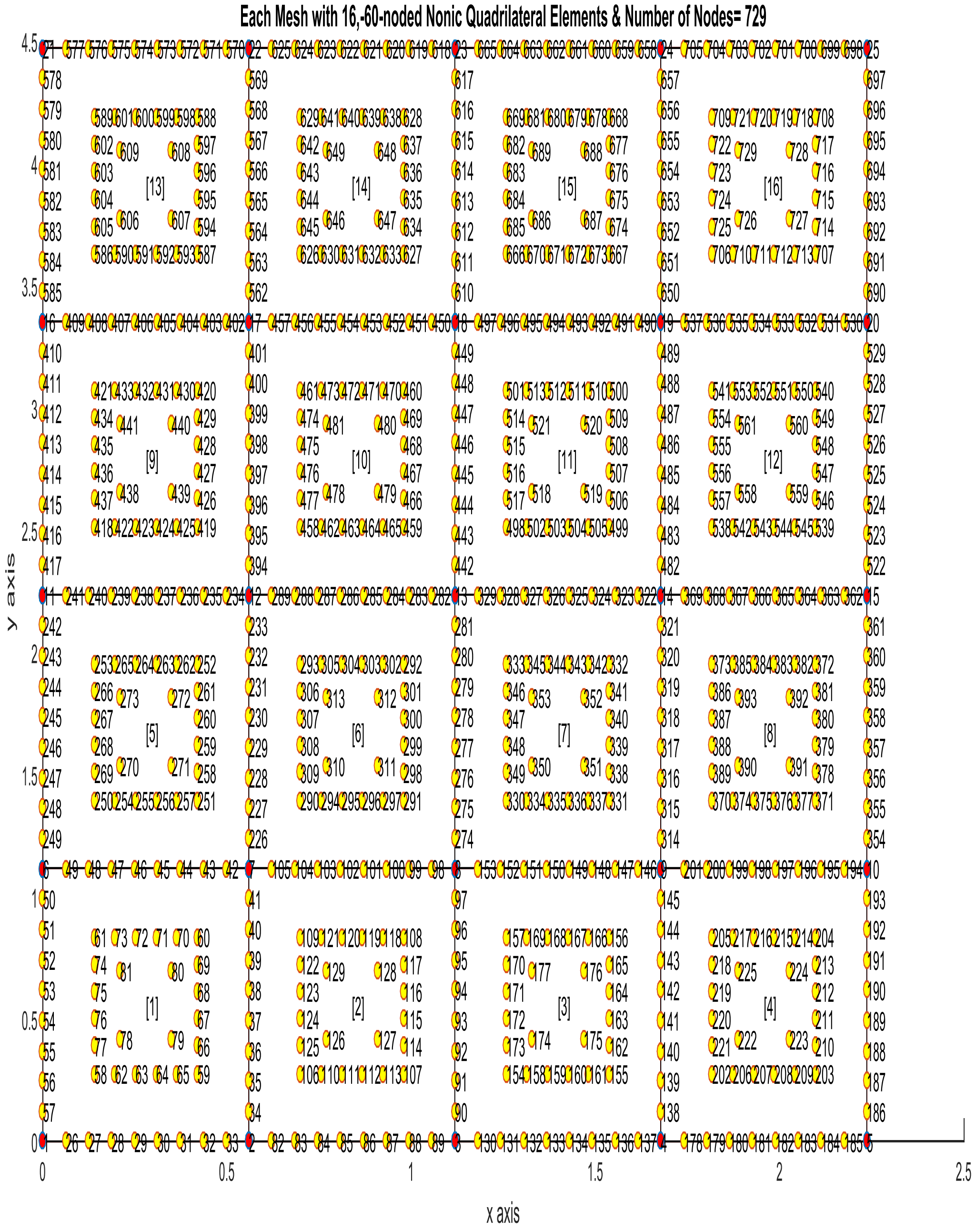
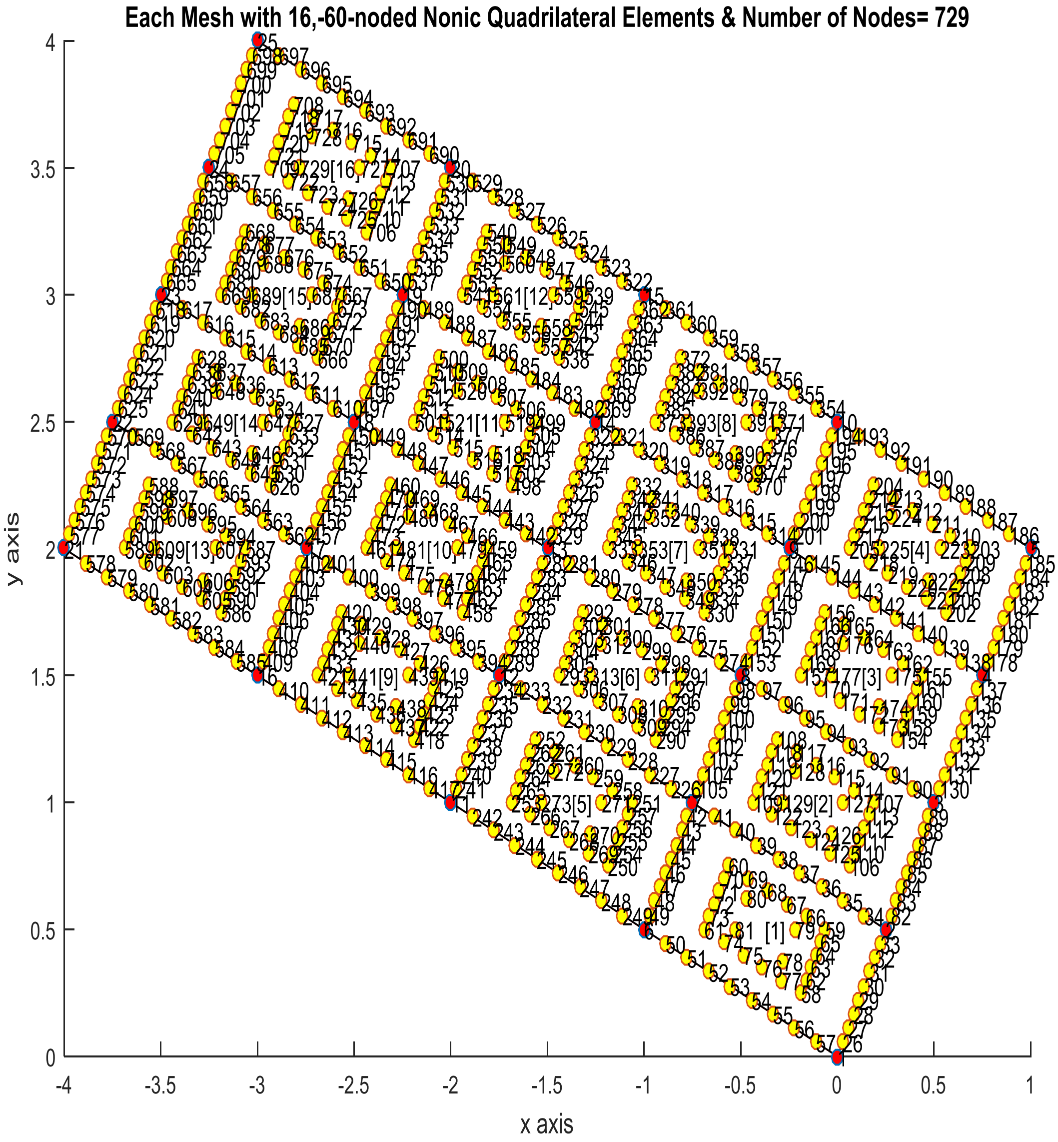
y axis
x axis



Each Mesh with 16,-49-noded Octic Quadrilateral Elements & Number of Nodes= 577
y axis
x axis

(5) Nonic order complete Lagrange Elements

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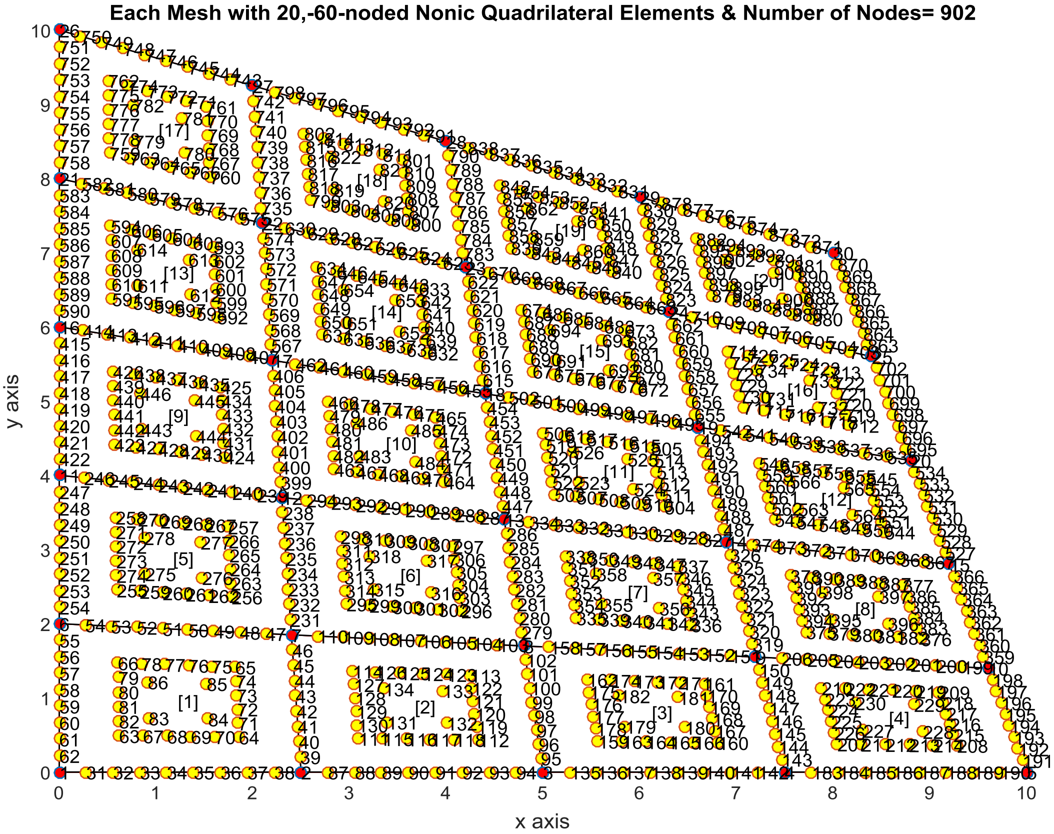
y axis
x axis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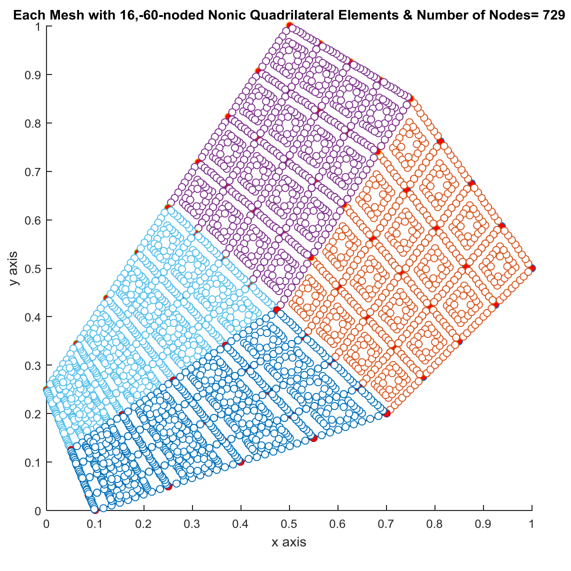
x axis
y axis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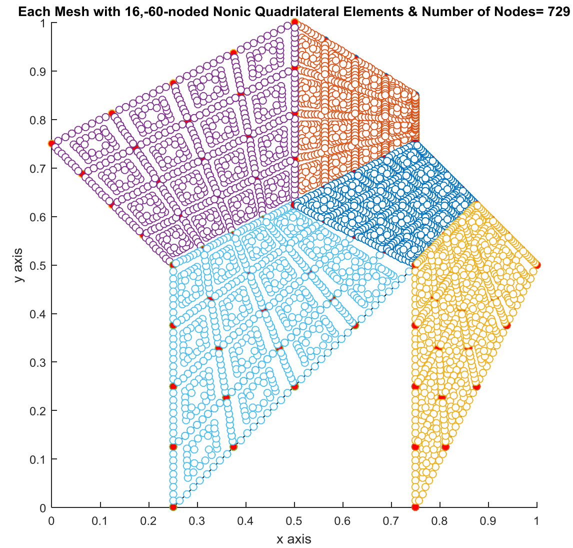
x axis
y axis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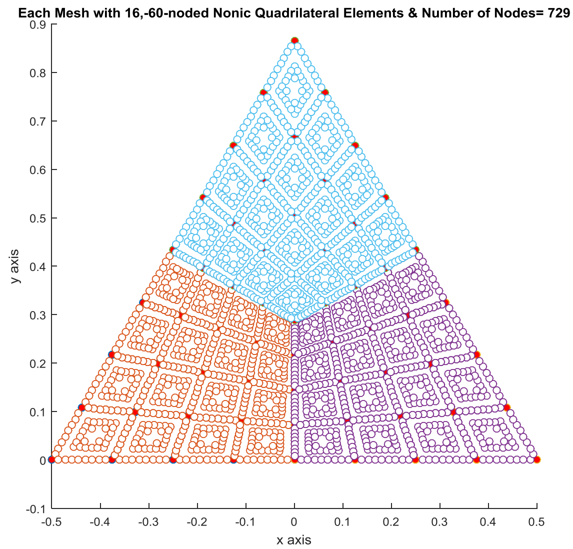
y axis
x axis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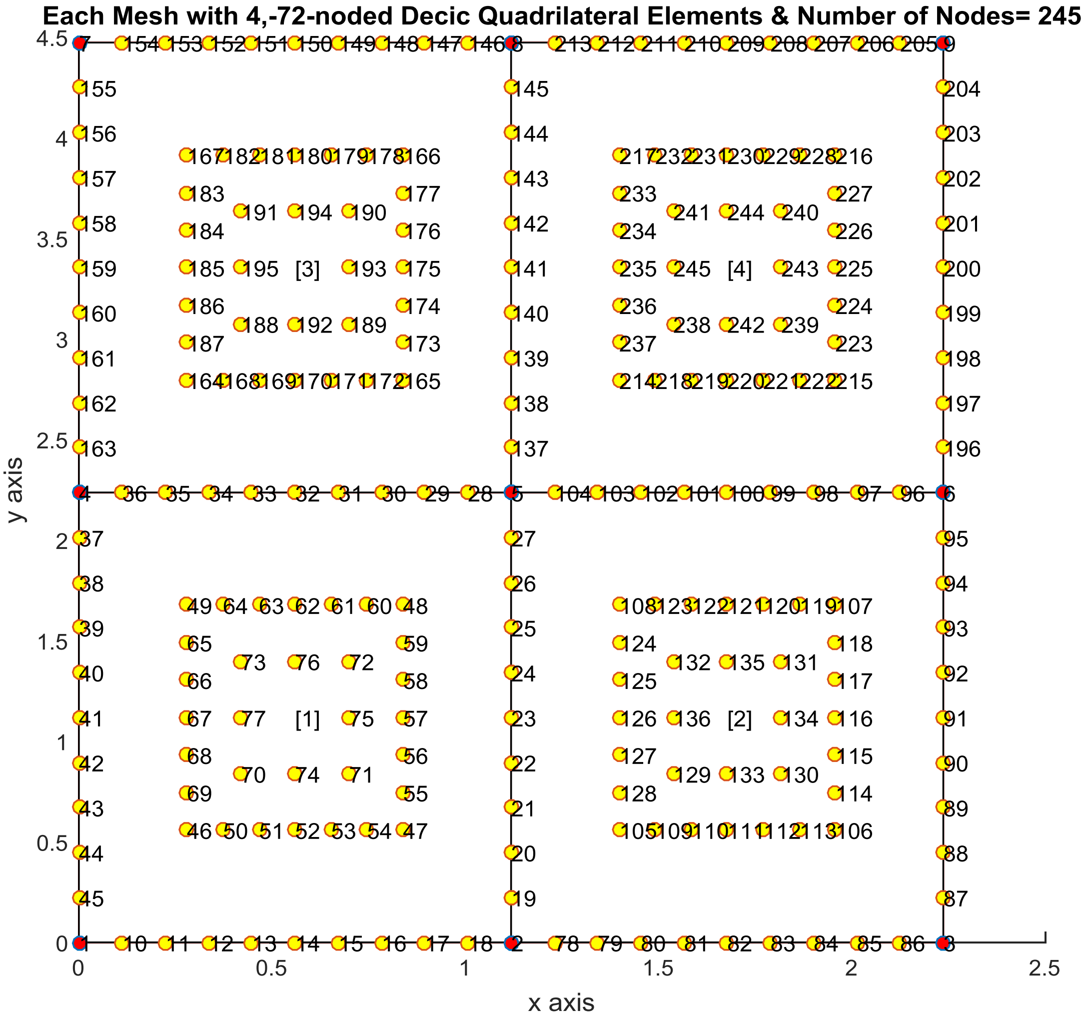
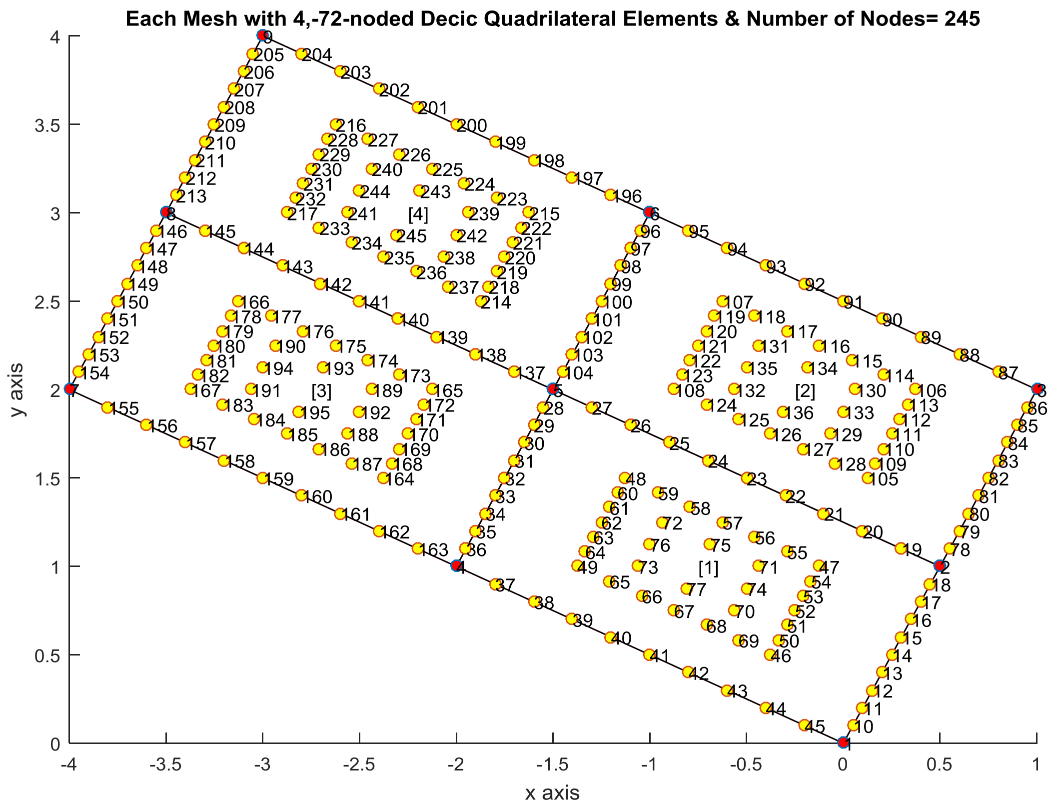
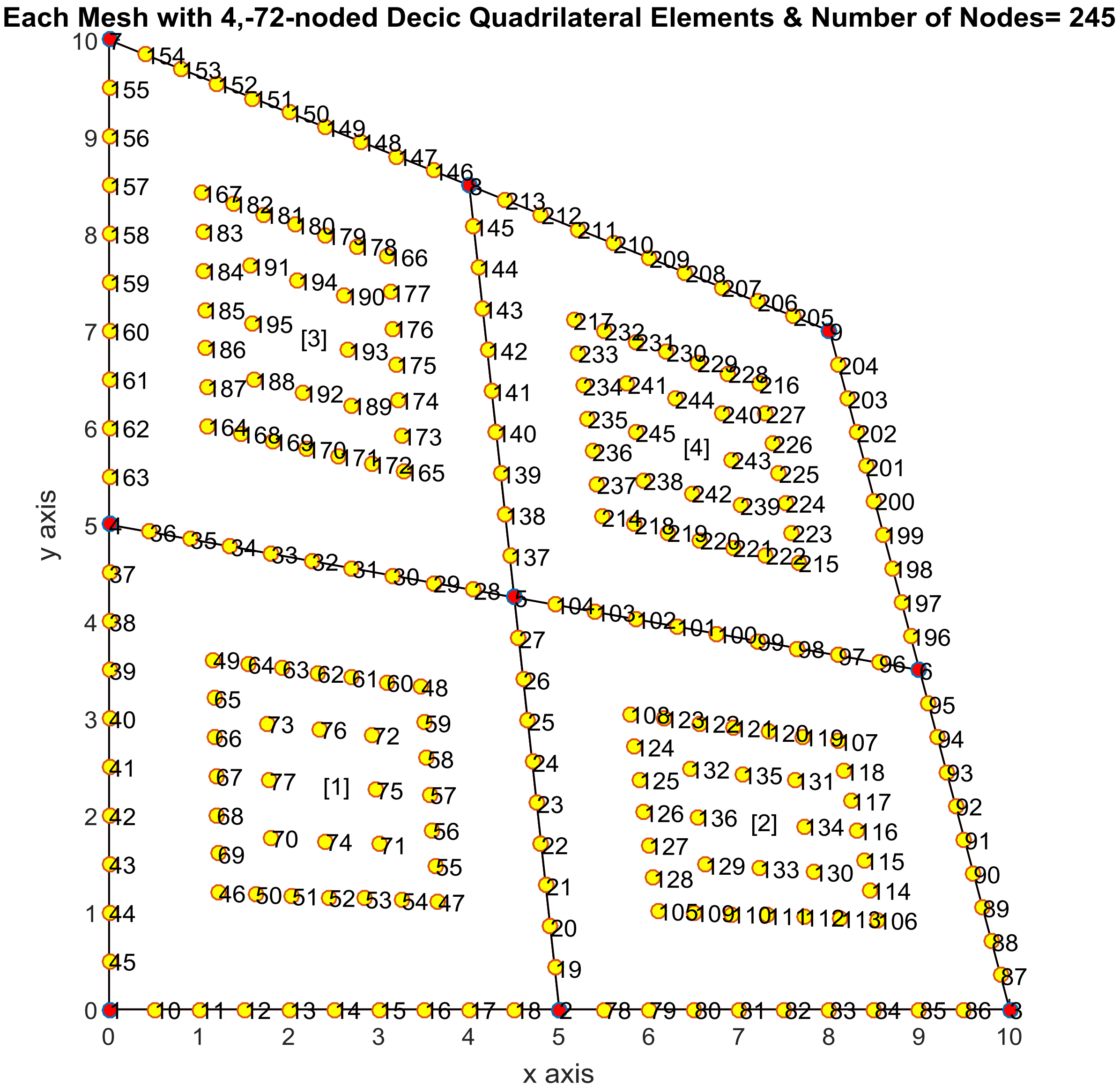
y axis
x axis

(6) Decic order complete Lagrange Elements




Each Mesh with 4,-72-noded Decic Quadrilateral Elements & Number of Nodes= 245
x axis
y axis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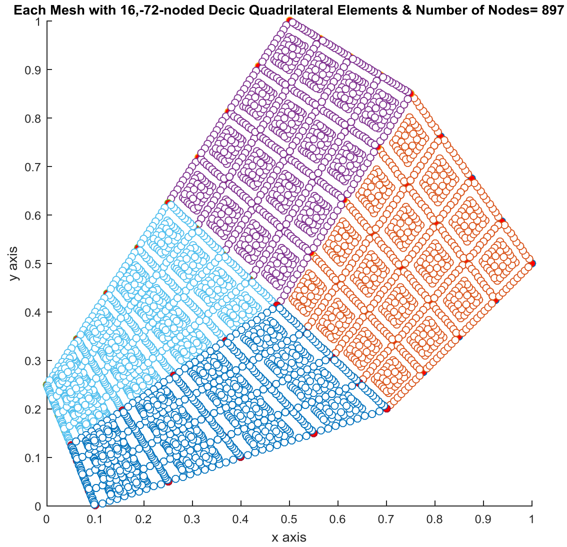
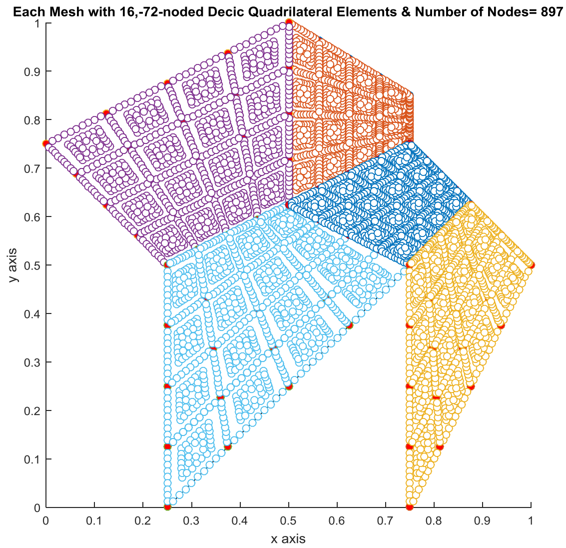
y axis
x axis
[1]
[2]
[3]
[4]



Each Mesh with 16,-72-noded Decic Quadrilateral Elements & Number of Nodes= 897
y axis
x axis
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



Each Mesh with 16,-72-noded Decic Quadrilateral Elements & Number of Nodes= 897
y axis
x axis
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

MATLAB CODES

Code (1)

```
function[]=FEMmeshExample4triangleNquadrilateral72node(gdata)
%This example generates NE elements for a rectangular structure of
    %length = Ly units and width = Lx units with Nx divisions on the x
   % Ny=NE/(2*Nx);   %Divisions on y axis
   % cla
   N=0;
    switch gdata
    case 1
    Lx=1;
    Ly=1;
    Nx=8;
    NE=144;
    X=[0;10;8; 0]
    Y=[0; 0;7;10]
    hdata=gdata
    case 2
    Lx=1;
    Ly=1;
    Nx=4;
    NE=40;
    X=[0;10;8; 0]
    Y=[0; 0;7;10]
    hdata=gdata
    case 3
    Lx=1;
    Ly=1;
    Nx=2;
    NE=8;
    X=[0;10;8; 0]
    Y=[0; 0;7;10]
    hdata=gdata
    case 4
    Lx=1;
    Ly=1;
    Nx=16;
    NE=288;
    X=[0;10;8; 0]
    Y=[0; 0;7;10]
    hdata=gdata
     case 5
    Lx=1;
    Ly=1;
    Nx=16;
    NE=288;
    X=[0;1;1;0]
    Y=[0;0;1;1]
    hdata=gdata
    case 6
    Lx=1;
    Ly=1;
    Nx=8;
    NE=144;
    X=[0;1;1;0]
    Y=[0;0;1;1]
    hdata=gdata
   case 7
    Lx=1;
    Ly=1;
```

```
    Nx=4;
    NE=40;
    X=[0;1;1;0]
    Y=[0;0;1;1]
    hdata=gdata
    case 8
    Lx=1;
    Ly=1;
    Nx=2;
    NE=8;
    X=[0;1;1;0]
    Y=[0;0;1;1]
    hdata=gdata
        case 9%beginning-Q1
    Lx=1;
    Ly=1;
    Nx=10;
    NE=160;
    X=[0;10;5;0]
    Y=[0;0;10;10]
    hdata=9
         case 10%beginning-Q2
    Lx=1;
    Ly=1;
    Nx=10;
    NE=160;
    X=[-10;0;0;-5]
    Y=[  0;0;10;10]
    hdata=9
          case 11%beginning-Q3
    Lx=1;
    Ly=1;
    Nx=10;
    NE=160;
    X=[0;-10; -5;  0]
    Y=[0;  0;-10;-10]
    hdata=9
           case 12%beginning-Q4
    Lx=1;
    Ly=1;
    Nx=10;
    NE=160;
    X=[10;0;  0;  5]
    Y=[ 0;0;-10;-10]
    hdata=9
        case 13%6-node convex polygonQ1
     Lx=1;    Ly=1;    Nx=2;    NE=8;
     Lx=1;    Ly=1;    Nx=2;    NE=8;N=2;
Lx=1;    Ly=1;    Nx=4;    NE=32;N=4
     A1=   0;A2=  .05;A3=0.1;A4=0.7;A5=  1;  A6=0.75;A7=0.5;A8= 0.25;A9=0.95/2 ;
     B1=0.25;B2=0.125;B3=  0;B4=0.2;B5=0.5;  B6=0.85;B7=  1;B8=0.625;  ;B9=0.825/2 ;
    X=[A9;A4;A5;A6]
    Y=[B9;B4;B5;B6]
    hdata=10
       case 14%6-node convex polygonQ2
     Lx=1;    Ly=1;    Nx=2;    NE=8;
     Lx=1;    Ly=1;    Nx=2;    NE=8;N=2;
Lx=1;    Ly=1;    Nx=4;    NE=32;N=4
     A1=   0;A2=  .05;A3=0.1;A4=0.7;A5=  1;  A6=0.75;A7=0.5;A8= 0.25;A9=0.95/2 ;
     B1=0.25;B2=0.125;B3=  0;B4=0.2;B5=0.5;  B6=0.85;B7=  1;B8=0.625;  ;B9=0.825/2 ;
    X=[A8;A9;A6;A7]
    Y=[B8;B9;B6;B7]
    hdata=10
       case 15%6-node convex polygonQ3
     Lx=1;    Ly=1;    Nx=2;    NE=8;
```

```
     Lx=1;    Ly=1;    Nx=2;    NE=8;N=2;
Lx=1;    Ly=1;    Nx=4;    NE=32;N=4
     A1=   0;A2=  .05;A3=0.1;A4=0.7;A5=  1;  A6=0.75;A7=0.5;A8= 0.25;A9=0.95/2 ;
     B1=0.25;B2=0.125;B3=  0;B4=0.2;B5=0.5;  B6=0.85;B7=  1;B8=0.625;  ;B9=0.825/2 ;
    X=[A9;A8;A1;A2]
    Y=[B9;B8;B1;B2]
    hdata=10
        case 16%6-node convex polygonQ4
     Lx=1;    Ly=1;    Nx=2;    NE=8;
     Lx=1;    Ly=1;    Nx=2;    NE=8;N=2;
Lx=1;    Ly=1;    Nx=4;    NE=32;N=4
     A1=   0;A2=  .05;A3=0.1;A4=0.7;A5=  1;  A6=0.75;A7=0.5;A8= 0.25;A9=0.95/2 ;
     B1=0.25;B2=0.125;B3=  0;B4=0.2;B5=0.5;  B6=0.85;B7=  1;B8=0.625;  ;B9=0.825/2 ;
    X=[A4;A9;A2;A3]
    Y=[B4;B9;B2;B3]
    hdata=10
  %=========6-node convex polygon discritised into a coarse mesh of four quadrilateral
with no subdivisions==============
        case 17%6-node convex polygonQ1
     Lx=1;    Ly=1;    Nx=1;    NE=2;
     A1=   0;A2=  .05;A3=0.1;A4=0.7;A5=  1;  A6=0.75;A7=0.5;A8= 0.25;A9=0.95/2 ;
     B1=0.25;B2=0.125;B3=  0;B4=0.2;B5=0.5;  B6=0.85;B7=  1;B8=0.625;  ;B9=0.825/2 ;
    X=[A9;A4;A5;A6]
    Y=[B9;B4;B5;B6]
    hdata=11
       case 18%6-node convex polygonQ2
     Lx=1;    Ly=1;    Nx=1;    NE=2;
     A1=   0;A2=  .05;A3=0.1;A4=0.7;A5=  1;  A6=0.75;A7=0.5;A8= 0.25;A9=0.95/2 ;
     B1=0.25;B2=0.125;B3=  0;B4=0.2;B5=0.5;  B6=0.85;B7=  1;B8=0.625;  ;B9=0.825/2 ;
    X=[A8;A9;A6;A7]
    Y=[B8;B9;B6;B7]
    hdata=11
       case 19%6-node convex polygonQ3
     Lx=1;    Ly=1;    Nx=1;    NE=2;
     A1=   0;A2=  .05;A3=0.1;A4=0.7;A5=  1;  A6=0.75;A7=0.5;A8= 0.25;A9=0.95/2 ;
     B1=0.25;B2=0.125;B3=  0;B4=0.2;B5=0.5;  B6=0.85;B7=  1;B8=0.625;  ;B9=0.825/2 ;
    X=[A9;A8;A1;A2]
    Y=[B9;B8;B1;B2]
    hdata=11
        case 20%6-node convex polygonQ4
     Lx=1;    Ly=1;    Nx=1;    NE=2;
     A1=   0;A2=  .05;A3=0.1;A4=0.7;A5=  1;  A6=0.75;A7=0.5;A8= 0.25;A9=0.95/2 ;
     B1=0.25;B2=0.125;B3=  0;B4=0.2;B5=0.5;  B6=0.85;B7=  1;B8=0.625;  ;B9=0.825/2 ;
    X=[A4;A9;A2;A3]
    Y=[B4;B9;B2;B3]
    hdata=11
   %=====================================================
        case 21%standard square
    Lx=1;    Ly=1;    Nx=1;    NE=2;
   X=[-1; 1;1; -1]
   Y=[-1;-1;1;  1]
   hdata=12
        case 22%arbitrary quadrilateral
    Lx=1;    Ly=1;    Nx=1;    NE=2;
   X=[-1;2;3;1]
   Y=[ 2;1;3;4]
   hdata=13
   %++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
   case 23%Q1<11,5,6,7 >==>FIRST QUADRILATERAL OF NINE NODE NONCONVEX POLYGON
Lx=1;    Ly=1;    Nx=2;    NE=8;N=2;
Lx=1;    Ly=1;    Nx=4;    NE=32;N=4;
%Lx=1;    Ly=1;    Nx=8;    NE=256;N=8;
```

```
 A1=0.25;A2=0.75;A3=0.75;A4=  1;A5=0.75;A6=0.75;A7=0.5;A8=   0;A9=0.25;A10=1.75/2;A11=
0.5;
 B1=   0;B2= 0.5;B3=   0;B4=0.5;B5=0.75;B6=0.85;B7=  1;B8=0.75;B9=
0.5;B10=1.25/2;B11=1.25/2;
   X=[A11; A5; A6; A7];
   Y=[B11; B5; B6; B7];
hdata=14

case 24%Q2<9,11,7,8>==>SECOND QUADRILATERAL OF NINE NODE NONCONVEX POLYGON
    Lx=1;    Ly=1;    Nx=2;    NE=8;
    Lx=1;    Ly=1;    Nx=2;    NE=8;N=2;
Lx=1;    Ly=1;    Nx=4;    NE=32;N=4 ;
%Lx=1;    Ly=1;    Nx=8;    NE=256;N=8;
 A1=0.25;A2=0.75;A3=0.75;A4=  1;A5=0.75;A6=0.75;A7=0.5;A8=   0;A9=0.25;A10=1.75/2;A11=
0.5;
 B1=   0;B2= 0.5;B3=   0;B4=0.5;B5=0.75;B6=0.85;B7=  1;B8=0.75;B9=
0.5;B10=1.25/2;B11=1.25/2;

    X=[A9; A11; A7; A8];
    Y=[B9; B11; B7; B8];
hdata=14

case 25%Q3<11,9,1,2>==>THIRD QUADRILATERAL OF NINE NODE NONCONVEX POLYGON
    Lx=1;    Ly=1;    Nx=2;    NE=8;
    Lx=1;    Ly=1;    Nx=2;    NE=8;N=2;
Lx=1;    Ly=1;    Nx=4;    NE=32;N=4
    A1=0.25;A2=0.75;A3=0.75;A4=  1;A5=0.75;A6=0.75;A7=0.5;A8=
0;A9=0.25;A10=1.75/2;A11=   0.5;
    B1=   0;B2= 0.5;B3=   0;B4=0.5;B5=0.75;B6=0.85;B7=  1;B8=0.75;B9=
0.5;B10=1.25/2;B11=1.25/2;
   X=[A11; A9; A1; A2];
   Y=[B11; B9; B1; B2] ;
hdata=14
case 26%Q4<5;11;2;10>==>FOURTH QUADRILATERAL OF NINE NODE NONCONVEX POLYGON
    Lx=1;    Ly=1;    Nx=2;    NE=8;
    Lx=1;    Ly=1;    Nx=2;    NE=8;N=2;
Lx=1;    Ly=1;    Nx=4;    NE=32;N=4 ;
%Lx=1;    Ly=1;    Nx=4;    NE=256;N=8;
  A1=0.25;A2=0.75;A3=0.75;A4=  1;A5=0.75;A6=0.75;A7=0.5;A8=   0;A9=0.25;A10=1.75/2;A11=
0.5;
  B1=   0;B2= 0.5;B3=   0;B4=0.5;B5=0.75;B6=0.85;B7=  1;B8=0.75;B9=
0.5;B10=1.25/2;B11=1.25/2;
   X=[A5; A11; A2; A10];
   Y=[B5; B11; B2; B10];
   hdata=14

case 27%Q5<10;2;3;4>==>FIFTH QUADRILATERAL OF NINE NODE NONCONVEX POLYGON
    Lx=1;    Ly=1;    Nx=2;    NE=8;
    Lx=1;    Ly=1;    Nx=2;    NE=8;N=2;
Lx=1;    Ly=1;    Nx=4;    NE=32;N=4 ;
%Lx=1;    Ly=1;    Nx=4;    NE=256;N=8;
  A1=0.25;A2=0.75;A3=0.75;A4=  1;A5=0.75;A6=0.75;A7=0.5;A8=   0;A9=0.25;A10=1.75/2;A11=
0.5;
  B1=   0;B2= 0.5;B3=   0;B4=0.5;B5=0.75;B6=0.85;B7=  1;B8=0.75;B9=
0.5;B10=1.25/2;B11=1.25/2;
    X=[A10; A2; A3; A4];
    Y=[B10; B2; B3; B4];
    hdata=14

   %+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
       case 28%arbitrary quadrilateral
    Lx=1;    Ly=1;    Nx=4;    NE=40;
   X=[-1;2;3;1]
   Y=[ 2;1;3;4]
```

```
   hdata=15
   case 29
    Lx=1;
    Ly=1;
    Nx=4;
    NE=32;
    X=[0;1;1;0]
    Y=[0;0;1;1]
    hdata=16
    case 30%Q1<7;6;1;4>==>FIRST QUADRILATERAL OF EQUILATERAL  TRIANGLE
    Lx=1;    Ly=1;    Nx=2;    NE=8;
    Lx=1;    Ly=1;    Nx=2;    NE=8;N=2;
    Lx=1;    Ly=1;    Nx=4;    NE=32;N=4 ;
%Lx=1;    Ly=1;    Nx=4;    NE=256;N=8;
  A1=-0.5;A2=0.5;A3=        0;A4=0;A5=    0.25;A6=    -0.25;A7=0;
  B1=   0;B2=  0;B3=sqrt(3)/2;B4=0;B5=sqrt(3)/4;B6=sqrt(3)/4;B7=sqrt(3)/6;
    X=[A7; A6; A1; A4];
    Y=[B7; B6; B1; B4];
    hdata=17
     case 31%Q2<7;4;2;5>==>SECOND QUADRILATERAL OF EQUILATERAL  TRIANGLE
    Lx=1;    Ly=1;    Nx=2;    NE=8;
    Lx=1;    Ly=1;    Nx=2;    NE=8;N=2;
    Lx=1;    Ly=1;    Nx=4;    NE=32;N=4 ;
%Lx=1;    Ly=1;    Nx=4;    NE=256;N=8;
  A1=-0.5;A2=0.5;A3=        0;A4=0;A5=    0.25;A6=    -0.25;A7=0;
  B1=   0;B2=  0;B3=sqrt(3)/2;B4=0;B5=sqrt(3)/4;B6=sqrt(3)/4;B7=sqrt(3)/6;
    X=[A7; A4; A2; A5];
    Y=[B7; B4; B2; B5];
    hdata=17
      case 32%Q3<7;5;3;6>==>THIRD QUADRILATERAL OF EQUILATERAL  TRIANGLE
    Lx=1;    Ly=1;    Nx=2;    NE=8;
    Lx=1;    Ly=1;    Nx=2;    NE=8;N=2;
    Lx=1;    Ly=1;    Nx=4;    NE=32;N=4 ;
%Lx=1;    Ly=1;    Nx=4;    NE=256;N=8;
  A1=-0.5;A2=0.5;A3=        0;A4=0;A5=    0.25;A6=    -0.25;A7=0;
  B1=   0;B2=  0;B3=sqrt(3)/2;B4=0;B5=sqrt(3)/4;B6=sqrt(3)/4;B7=sqrt(3)/6;
    X=[A7; A5; A3; A6];
    Y=[B7; B5; B3; B6];
    hdata=17
    case 33%Decic element over a 2-square
        Lx=1;    Ly=1;    Nx=1;    NE=2;
    X=[-1; 1;1;-1]
    Y=[-1;-1;1; 1]
    hdata=18
    case 34
         Lx=1;    Ly=1;    Nx=1;    NE=2;
         Lx=1;    Ly=1;    Nx=2;    NE=8;N=2;
         Lx=1;    Ly=1;    Nx=4;    NE=32;N=4 ;
         Lx=1;    Ly=1;    Nx=4;    NE=32 ;N=0;
        % Lx=1;    Ly=1;    Nx=8;    NE=128;N=0;
          Lx=1;    Ly=1;    Nx=2;    NE=8;N=0;
    X=[0;sqrt(5);  sqrt(5);        0]
    Y=[0;      0;2*sqrt(5);2*sqrt(5)]
    hdata=19
        case 35
         Lx=1;    Ly=1;    Nx=1;    NE=2;
         Lx=1;    Ly=1;    Nx=2;    NE=8;N=2;
          Lx=1;    Ly=1;    Nx=4;    NE=32;N=4 ;
         Lx=1;    Ly=1;    Nx=4;    NE=32;N=0 ;
         % Lx=1;    Ly=1;    Nx=8;    NE=128;N=0;
         Lx=1;    Ly=1;    Nx=2;    NE=8;N=0;
    X=[0;1;-3;-4]
    Y=[0;2; 4; 2]
    hdata=20
```

```
    end

[xygcoord,xycoords,xycoordrgqd,xycoordrgqdm,cT,qT,nNodes]=femTriangularMeshGenerator4tr
iangleNquadrilateral72node(Lx,Ly,Nx,NE,X,Y)
   % return
    [nnode,dimension]=size(xygcoord)
    disp(['Number of nodes =  ',num2str(nnode)])
    disp('Connectivity Table')
    disp(cT)
    disp(qT)
    axis square
   %axis equal
    z=1;
    for i=1:NE
        figure(2*hdata-
1),patch('Vertices',xycoords(z:z+2,:),'Faces',[1,2,3],'FaceColor','none','EdgeColor','g
')
        if Nx<9
        midx=mean(xycoords(z:z+2,1));
        midy=mean(xycoords(z:z+2,2));
        text(midx,midy,['[',num2str(i),']']);
        end
        hold on
        z=z+3;
    end
    %*************************************
    hold on
xlabel('x axis')
ylabel('y axis')
st1='FEM MESH WITH  ';
st2=num2str(NE);
st3='; 3-node Linear ';
st4='Triangular';
st5=' Elements'
st6='& Nodes='
st7=num2str(nNodes);
title([st1,st2,st3,st4,st5,st6,st7])

    %*********************************************
    figure(2*hdata-1),scatter(xycoords(:,1),xycoords(:,2),'MarkerFaceColor','r')

    hold on
    %put node numbers
if Nx<9
for jj=1:nNodes
text(xygcoord(jj,1),xygcoord(jj,2),['.',num2str(jj)]);
end
end
%*****************************************************************************************
******
disp('nodal  connectivity  for 60 noded decic convex quadrilaterals ')
disp(qT) %

  z=1;
    for i=1:NE/2

figure(2*hdata),patch('Vertices',xycoordrgqd(z:z+3,:),'Faces',[1,2,3,4],'FaceColor','no
ne','EdgeColor','r')
        xx=xycoordrgqd(z:z+3,1); yy=xycoordrgqd(z:z+3,2);
        hold on
```

```
        patch(xx,yy,'w')
        if (Nx<9)&(Nx~=N)
        midx=mean(xycoordrgqd(z:z+3,1));
        midy=mean(xycoordrgqd(z:z+3,2));

        text(midx,midy,['[',num2str(i),']']);
        end
        hold on
        z=z+4;
    end


    figure(2*hdata),scatter(xycoordrgqd(:,1),xycoordrgqd(:,2),'MarkerFaceColor','r')
    figure(2*hdata),scatter(xycoordrgqdm(:,1),xycoordrgqdm(:,2),'MarkerFaceColor','y')
    hold on
    %put node numbers


if (Nx<9)&(Nx~=N)
% figure(2*hdata),scatter(xygcoord(:,1),xygcoord(:,2),'MarkerFaceColor','w')
for jj=1:nnode
text(xygcoord(jj,1),xygcoord(jj,2),num2str(jj));
end
end
% %
hold on
xlabel('x axis')
ylabel('y axis')
st1='Each Mesh with ';
st2=num2str(NE/2);
st3=',-72-noded Decic ';
st4='Quadrilateral';
st5=' Elements ';
st6='& Number of Nodes= ';
st7=num2str(nnode);
title([st1,st2,st3,st4,st5,st6,st7])
   end
```

code(2)

```
function
[xygcoord,xycoords,xycoordrgqd,xycoordrgqdm,cT,qT,nNodes]=femTriangularMeshGenerator4triangleNquadrilateral72
node(Lx,Ly,Nx,NE,X,Y)
    %   This function generates triangular mesh for a rectangular
    %   shape structure for finite element analysis
    %   coords  =   x and y coordinates of each element nodes
    %   cT      =   nodal connectivity for triangles
    %   qT      =   nodal connectivity for quadrilaterals
    %   nNodes  =   Number of nodes
    %   Lx      =   width of the rectangular structure
    %   Ly      =   Height of the rectangular structure
    %   Nx      =   Number of divisions on x- axis
    %   NE      =   Number of elements
    %


    x1=X(1,1);
    x2=X(2,1);
    x3=X(3,1);
    x4=X(4,1);
    y1=Y(1,1);
    y2=Y(2,1);
    y3=Y(3,1);
```

```
    y4=Y(4,1);

    if mod((NE/Nx),2)~=0
        errordlg('The No of divisions on X axis must divide No of Elements twice')

    end

    Ny=NE/(2*Nx);   %Divisions on y axis

    nNodes =(Nx+1)*(Ny+1);  %No of nodes

     m=1;
    j=(1:Nx);
    k=linspace(Nx*2,NE,Ny);

    for i=1:Ny
        cT(m:2:k(i),1)= j';  %node 1 of 1st element
        cT(m+1:2:k(i),1)= j';  %node 1 of 2nd element
        cT(m:2:k(i),2)=(j+1)'; %%node 2 of 1st element
        cT(m+1:2:k(i),2)=(j+Nx+2)';%node 2 of 2nd element
        cT(m:2:k(i),3)= (j+Nx+2)';  %node 3 of 1st element
        cT(m+1:2:k(i),3)=(j+1+Nx)';    %%node 3 of 1st element

        m=k(i)+1;
        j=j+Nx+1;
    end
    for m=1:NE/2
        qT(m,1)=cT(2*m-1,1);
        qT(m,2)=cT(2*m-1,2);
        qT(m,3)=cT(2*m-1,3);
        qT(m,4)=cT(2*m,3);
    end
     qT


    ax=linspace(0,Lx,Nx+1); %%%x coordinates
    by=linspace(0,Ly,Ny+1); %%%y coordinates
    X1=[];
    Y1=[];
    for i1=1:Ny+1
        %    General Nodal Coordinates layer by layer
        by1(1:Nx+1)=by(i1);
        X1=[X1 ax];
        Y1=[Y1 by1];
    end

   gcoord(:,1)=X1';
   gcoord(:,2)=Y1';
   NN=(1:nNodes)';
   [NN gcoord]




    j=1:3;
    %each element coordinates for triangles
    for n=1:NE
        X(j,1) = X1(cT(n,:));
        Y(j,1)=Y1(cT(n,:));
        j=j+3;
    end
    coords=[X Y];   %x and y coordinates for triangles
      j=1:4;
    %each element coordinates for quadrilaterals
    for n=1:NE/2
        XX(j,1) = X1(qT(n,:));
        YY(j,1)=Y1(qT(n,:));
        j=j+4;
    end
    coord=[XX YY];   %x and y coordinates for quadrilaterals
  %^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
 % mesh generation of 16-node special quadrilaterals
nnd=nNodes;
for inum=1:nnd
    for jnum=1:nnd
        trisect(inum,jnum,1)=0;
        trisect(inum,jnum,2)=0;
        trisect(inum,jnum,3)=0;
        trisect(inum,jnum,4)=0;
```

```
        trisect(inum,jnum,5)=0;
    end
end
nd=nnd;mm=NE/2;
for mmm=1:mm
mmm1=qT(mmm,1);
mmm2=qT(mmm,2);
mmm3=qT(mmm,3);
mmm4=qT(mmm,4);
%trisectional points and midpoints: side-1 of 4-node  quadrilateral
if((trisect(mmm1,mmm2,1)==0))
    nd=nd+1;
   trisect(mmm1,mmm2,1)=nd;
end

if(trisect(mmm1,mmm2,2)==0)
    nd=nd+1;
   trisect(mmm1,mmm2,2)=nd;
end
if((trisect(mmm1,mmm2,3)==0))
    nd=nd+1;
   trisect(mmm1,mmm2,3)=nd;
end
if(trisect(mmm1,mmm2,4)==0)
    nd=nd+1;
   trisect(mmm1,mmm2,4)=nd;
end
if((trisect(mmm1,mmm2,5)==0)&&(trisect(mmm2,mmm1,5)==0))
    nd=nd+1;
   trisect(mmm1,mmm2,5)=nd;
   trisect(mmm2,mmm1,5)=nd;
end
if((trisect(mmm1,mmm2,4)~=0)&&(trisect(mmm2,mmm1,4)==0))
    nd=nd+1;
   trisect(mmm2,mmm1,4)=nd;
end

if((trisect(mmm1,mmm2,3)~=0)&&(trisect(mmm2,mmm1,3)==0))
    nd=nd+1;
   trisect(mmm2,mmm1,3)=nd;
end
if((trisect(mmm1,mmm2,2)~=0)&&(trisect(mmm2,mmm1,2)==0))
    nd=nd+1;
   trisect(mmm2,mmm1,2)=nd;
end
if((trisect(mmm1,mmm2,1)~=0)&&(trisect(mmm2,mmm1,1)==0))
    nd=nd+1;
   trisect(mmm2,mmm1,1)=nd;
end
% trisectional points and midpoints: side-2 of 4-node  quadrilateral

if((trisect(mmm2,mmm3,1)==0))
    nd=nd+1;
   trisect(mmm2,mmm3,1)=nd;
end
 if(trisect(mmm2,mmm3,2)==0)
   nd=nd+1;
   trisect(mmm2,mmm3,2)=nd;
 end
 if(trisect(mmm2,mmm3,3)==0)
   nd=nd+1;
   trisect(mmm2,mmm3,3)=nd;
 end

 if(trisect(mmm2,mmm3,4)==0)
    nd=nd+1;
   trisect(mmm2,mmm3,4)=nd;
 end
 if((trisect(mmm2,mmm3,5)==0)&&(trisect(mmm3,mmm2,5)==0))
    nd=nd+1;
   trisect(mmm2,mmm3,5)=nd;
   trisect(mmm3,mmm2,5)=nd;
end
if((trisect(mmm2,mmm3,4)~=0)&&(trisect(mmm3,mmm2,4)==0))
    nd=nd+1;
   trisect(mmm3,mmm2,4)=nd;
end

 if((trisect(mmm2,mmm3,3)~=0)&&(trisect(mmm3,mmm2,3)==0))
    nd=nd+1;
```

```
   trisect(mmm3,mmm2,3)=nd;
end
if((trisect(mmm2,mmm3,2)~=0)&&(trisect(mmm3,mmm2,2)==0))
    nd=nd+1;
   trisect(mmm3,mmm2,2)=nd;
end
if((trisect(mmm2,mmm3,1)~=0)&&(trisect(mmm3,mmm2,1)==0))
    nd=nd+1;
   trisect(mmm3,mmm2,1)=nd;
end
% trisectional points  and midpoints: side-3 of 4-node  quadrilateral

if((trisect(mmm3,mmm4,1)==0))
    nd=nd+1;
   trisect(mmm3,mmm4,1)=nd;
end
if(trisect(mmm3,mmm4,2)==0)
    nd=nd+1;
   trisect(mmm3,mmm4,2)=nd;
end
if(trisect(mmm3,mmm4,3)==0)
    nd=nd+1;
    trisect(mmm3,mmm4,3)=nd;
end

if(trisect(mmm3,mmm4,4)==0)
    nd=nd+1;
   trisect(mmm3,mmm4,4)=nd;
end
if((trisect(mmm3,mmm4,5)==0)&&(trisect(mmm4,mmm3,5)==0))
    nd=nd+1;
   trisect(mmm3,mmm4,5)=nd;
   trisect(mmm4,mmm3,5)=nd;
end

if((trisect(mmm3,mmm4,4)~=0)&&(trisect(mmm4,mmm3,4)==0))
    nd=nd+1;
   trisect(mmm4,mmm3,4)=nd;
end

if((trisect(mmm3,mmm4,3)~=0)&&(trisect(mmm4,mmm3,3)==0))
    nd=nd+1;
   trisect(mmm4,mmm3,3)=nd;
end
if((trisect(mmm3,mmm4,2)~=0)&&(trisect(mmm4,mmm3,2)==0))
    nd=nd+1;
   trisect(mmm4,mmm3,2)=nd;
end
if((trisect(mmm3,mmm4,1)~=0)&&(trisect(mmm4,mmm3,1)==0))
    nd=nd+1;
   trisect(mmm4,mmm3,1)=nd;
end
% trisectional points  and  midpoints: side-4 of 4-node  quadrilateral

if((trisect(mmm4,mmm1,1)==0))
    nd=nd+1;
   trisect(mmm4,mmm1,1)=nd;
end
if(trisect(mmm4,mmm1,2)==0)
    nd=nd+1;
   trisect(mmm4,mmm1,2)=nd;
end

if(trisect(mmm4,mmm1,3)==0)
    nd=nd+1;
   trisect(mmm4,mmm1,3)=nd;
end

if(trisect(mmm4,mmm1,4)==0)
    nd=nd+1;
   trisect(mmm4,mmm1,4)=nd;
end
if((trisect(mmm4,mmm1,5)==0)&&(trisect(mmm1,mmm4,5)==0))
    nd=nd+1;
   trisect(mmm4,mmm1,5)=nd;
   trisect(mmm1,mmm4,5)=nd;
end

if((trisect(mmm4,mmm1,4)~=0)&&(trisect(mmm1,mmm4,4)==0))
```

```
    nd=nd+1;
   trisect(mmm1,mmm4,4)=nd;
end

if((trisect(mmm4,mmm1,3)~=0)&&(trisect(mmm1,mmm4,3)==0))
    nd=nd+1;
   trisect(mmm1,mmm4,3)=nd;
end

if((trisect(mmm4,mmm1,2)~=0)&&(trisect(mmm1,mmm4,2)==0))
    nd=nd+1;
   trisect(mmm1,mmm4,2)=nd;
end
if((trisect(mmm4,mmm1,1)~=0)&&(trisect(mmm1,mmm4,1)==0))
    nd=nd+1;
   trisect(mmm1,mmm4,1)=nd;
end
%assign matrices trisect  to  mid-side nodes of element
   qT(mmm,5)=trisect(mmm1,mmm2,1);
   qT(mmm,6)=trisect(mmm1,mmm2,2);
   qT(mmm,7)=trisect(mmm1,mmm2,3);
   qT(mmm,8)=trisect(mmm1,mmm2,4);
   qT(mmm,9)=trisect(mmm1,mmm2,5);
   qT(mmm,10)=trisect(mmm2,mmm1,4);
   qT(mmm,11)=trisect(mmm2,mmm1,3);
   qT(mmm,12)=trisect(mmm2,mmm1,2);
   qT(mmm,13)=trisect(mmm2,mmm1,1);
   %
   qT(mmm,14)=trisect(mmm2,mmm3,1);
   qT(mmm,15)=trisect(mmm2,mmm3,2);
   qT(mmm,16)=trisect(mmm2,mmm3,3);
   qT(mmm,17)=trisect(mmm2,mmm3,4);
   qT(mmm,18)=trisect(mmm2,mmm3,5);
   qT(mmm,19)=trisect(mmm3,mmm2,4);
   qT(mmm,20)=trisect(mmm3,mmm2,3);
   qT(mmm,21)=trisect(mmm3,mmm2,2);
   qT(mmm,22)=trisect(mmm3,mmm2,1);
   %
   qT(mmm,23)=trisect(mmm3,mmm4,1);
   qT(mmm,24)=trisect(mmm3,mmm4,2);
   qT(mmm,25)=trisect(mmm3,mmm4,3);
   qT(mmm,26)=trisect(mmm3,mmm4,4);
   qT(mmm,27)=trisect(mmm3,mmm4,5);
   qT(mmm,28)=trisect(mmm4,mmm3,4);
   qT(mmm,29)=trisect(mmm4,mmm3,3);
   qT(mmm,30)=trisect(mmm4,mmm3,2);
   qT(mmm,31)=trisect(mmm4,mmm3,1);
   %
   qT(mmm,32)=trisect(mmm4,mmm1,1);
   qT(mmm,33)=trisect(mmm4,mmm1,2);
   qT(mmm,34)=trisect(mmm4,mmm1,3);
   qT(mmm,35)=trisect(mmm4,mmm1,4);
   qT(mmm,36)=trisect(mmm4,mmm1,5);
   qT(mmm,37)=trisect(mmm1,mmm4,4);
   qT(mmm,38)=trisect(mmm1,mmm4,3);
   qT(mmm,39)=trisect(mmm1,mmm4,2);
   qT(mmm,40)=trisect(mmm1,mmm4,1);
   %
   for icc=41:72
   nd=nd+1;
   qT(mmm,icc)=nd;
   end

end%for

nnode=nd;
nel=mm;
% % spqd=nodes;
 MM=(1:mm)';
  disp([MM qT])
  [nel,nnel]=size(qT)
 % return
for mmm=1:nel
   mmm1=qT(mmm,1);
   mmm2=qT(mmm,2);
   mmm3=qT(mmm,3);
   mmm4=qT(mmm,4);
   mmm5=qT(mmm,5) ;
   mmm6=qT(mmm,6);
   mmm7=qT(mmm,7);
```

```
    mmm8=qT(mmm,8);
    mmm9=qT(mmm,9);
    mmm10=qT(mmm,10);
    mmm11=qT(mmm,11);
    mmm12=qT(mmm,12);    %
    mmm13=qT(mmm,13);
    mmm14=qT(mmm,14);
    mmm15=qT(mmm,15);
    mmm16=qT(mmm,16);
    mmm17=qT(mmm,17);
    mmm18=qT(mmm,18);
    mmm19=qT(mmm,19);
    mmm20=qT(mmm,20);
    mmm21=qT(mmm,21);
    mmm22=qT(mmm,22);
    mmm23=qT(mmm,23);
    mmm24=qT(mmm,24);
    mmm25=qT(mmm,25);
    mmm26=qT(mmm,26);
    mmm27=qT(mmm,27);
    mmm28=qT(mmm,28);
    mmm29=qT(mmm,29);
    mmm30=qT(mmm,30);
    mmm31=qT(mmm,31);
    mmm32=qT(mmm,32);
    mmm33=qT(mmm,33);
    mmm34=qT(mmm,34);
    mmm35=qT(mmm,35);
    mmm36=qT(mmm,36);
    mmm37=qT(mmm,37);
    mmm38=qT(mmm,38);
    mmm39=qT(mmm,39);
    mmm40=qT(mmm,40);
    mmm41=qT(mmm,41);
    mmm42=qT(mmm,42);
    mmm43=qT(mmm,43);
    mmm44=qT(mmm,44);
    mmm45=qT(mmm,45);
    mmm46=qT(mmm,46);
    mmm47=qT(mmm,47);
    mmm48=qT(mmm,48);
    mmm49=qT(mmm,49);
    mmm50=qT(mmm,50);
    mmm51=qT(mmm,51);
    mmm52=qT(mmm,52);
    mmm53=qT(mmm,53);
    mmm54=qT(mmm,54);
    mmm55=qT(mmm,55);
    mmm56=qT(mmm,56);
    mmm57=qT(mmm,57);
    mmm58=qT(mmm,58);
    mmm59=qT(mmm,59);
    mmm60=qT(mmm,60);
    mmm61=qT(mmm,61);
    mmm62=qT(mmm,62);
    mmm63=qT(mmm,63);
    mmm64=qT(mmm,64);
    mmm65=qT(mmm,65);
    mmm66=qT(mmm,66);
    mmm67=qT(mmm,67);
    mmm68=qT(mmm,68);
    mmm69=qT(mmm,69);
    mmm70=qT(mmm,70);
    mmm71=qT(mmm,71);
    mmm72=qT(mmm,72);
    %

    xi1=gcoord(mmm1,1);
    xi2=gcoord(mmm2,1);
    xi3=gcoord(mmm3,1);
    xi4=gcoord(mmm4,1);
    %
    yi1=gcoord(mmm1,2);
    yi2=gcoord(mmm2,2);
    yi3=gcoord(mmm3,2);
    yi4=gcoord(mmm4,2);
    %

%compute element local coordinates for midside and interior  nodes
%      1  2 3 4   5    6   7   8   9   10  11  12  13  14  15  16  17   18  19  20  21
```

```
%                                                                                           22  23  24  25  26
27  28  29  30  31  32  33  34  35   36 37  38 39   40  41 42   43  44   45  46   47   48  49   50    51  52
53  54   55   56   57  58    59    60   61  62   63   64  65  66 67  68  69  70  71  72
    ui=[0;1;1;0; 0.1; 0.2; 0.3;0.4;0.5;0.6;0.7;0.8;0.9;  1;  1;  1;  1;  1;  1;  1;  1;
1;0.9;0.8;0.7;0.6;0.5;0.4;0.3;0.2;0.1;  0;  0;  0;  0;  0;  0;  0;  0;
0;1/4;3/4;3/4;1/4;4/12;5/12;1/2;7/12;8/12; 3/4; 3/4;3/4; 3/4; 3/4;8/12;7/12;6/12;5/12;4/12; 1/4; 1/4;1/4;
1/4; 1/4;3/8;5/8;5/8;3/8;1/2;5/8;1/2;3/8];
    vi=[0;0;1;1;   0;   0;   0;  0;  0;  0;  0;  0;  0;0.1;0.2;0.3;0.4;0.5;0.6;0.7;0.8;0.9;  1;  1;  1;  1;
1;  1;  1;  1;  1;0.9;0.8;0.7;0.6;0.5;0.4;0.3;0.2;0.1;1/4;1/4;3/4;3/4; 1/4; 1/4;1/4; 1/4;
1/4;4/12;5/12;1/2;7/12;8/12; 3/4; 3/4; 3/4; 3/4;
3/4;8/12;7/12;1/2;5/12;4/12;3/8;3/8;5/8;5/8;3/8;1/2;5/8;1/2];

mi=[mmm1;mmm2;mmm3;mmm4;mmm5;mmm6;mmm7;mmm8;mmm9;mmm10;mmm11;mmm12;mmm13;mmm14;mmm15;mmm16;mmm17;mmm18;mmm19;
mmm20;mmm21;mmm22;mmm23;mmm24;mmm25;mmm26;mmm27;mmm28;mmm29;mmm30;mmm31;mmm32;mmm33;mmm34;mmm35;mmm36;mmm37;m
mm38;mmm39;mmm40;mmm41;mmm42;mmm43;mmm44;mmm45;mmm46;mmm47;mmm48;mmm49;mmm50;mmm51;mmm52;mmm53;mmm54;mmm55;mm
m56;mmm57;mmm58;mmm59;mmm60;mmm61;mmm62;mmm63;mmm64;mmm65;mmm66;mmm67;mmm68;mmm69;mmm70;mmm71;mmm72];
    for ii=5:72
        mii=mi(ii,1)
        uii=ui(ii,1);
        vii=vi(ii,1);
        gcoord(mii,1)=xi1+uii*(xi2-xi1)+vii*(xi4-xi1)+uii*vii*(xi1-xi2+xi3-xi4);
        gcoord(mii,2)=yi1+uii*(yi2-yi1)+vii*(yi4-yi1)+uii*vii*(yi1-yi2+yi3-yi4);
    end

end%for nel
disp(gcoord)
[nnode,dimension]=size(gcoord)
%===============================================================================================
  j=1:68;
    %each element coordinates for quadrilaterals at midside  and interior nodes
    for n=1:NE/2
        XM(j,1)=gcoord(qT(n,5:72),1);
        YM(j,1)=gcoord(qT(n,5:72),2);
        j=j+68;
    end
    coordm=[XM YM];   %x and y coordinates for quadrilaterals  at midside  nodes

%=============================================================================================
 %*************************************************************************************************
    for ii=1:nnode
        r=gcoord(ii,1);s=gcoord(ii,2);
        xygcoord(ii,1)=x1+r*(x2-x1)+s*(x4-x1)+r*s*(x1-x2+x3-x4);
        xygcoord(ii,2)=y1+r*(y2-y1)+s*(y4-y1)+r*s*(y1-y2+y3-y4);

    end
   kk=0;
    for n=1:NE
        for jj=1:3
            kk=kk+1
            rr=coords(kk,1);ss=coords(kk,2);
            xycoords(kk,1)=x1+rr*(x2-x1)+ss*(x4-x1)+rr*ss*(x1-x2+x3-x4);
            xycoords(kk,2)=y1+rr*(y2-y1)+ss*(y4-y1)+rr*ss*(y1-y2+y3-y4);
        end

    end

    kk=0;
    for n=1:NE/2
        for jj=1:4
            kk=kk+1
            rr=coord(kk,1);ss=coord(kk,2);
            xycoordrgqd(kk,1)=x1+rr*(x2-x1)+ss*(x4-x1)+rr*ss*(x1-x2+x3-x4);
            xycoordrgqd(kk,2)=y1+rr*(y2-y1)+ss*(y4-y1)+rr*ss*(y1-y2+y3-y4);
        end

    end
    %coordinates for the element midside and interior nodes
   kk=0;
    for n=1:NE/2
        for jj=1:68
            kk=kk+1;
            rr=coordm(kk,1);ss=coordm(kk,2);
            xycoordrgqdm(kk,1)=x1+rr*(x2-x1)+ss*(x4-x1)+rr*ss*(x1-x2+x3-x4);
            xycoordrgqdm(kk,2)=y1+rr*(y2-y1)+ss*(y4-y1)+rr*ss*(y1-y2+y3-y4);
        end

    end
```

```
end
```