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Existence of $(\emptyset^{\alpha} \otimes \psi^{\alpha})$ bounded solutions of Kronecker product first order system of Differential equations

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Abstract: In this paper we shall be concerned with the existence of fuzzy bounded solutions of first order Kronecker product system involving two first order fuzzy differential equations of different orders. We show that kronecker product system is stable and asymptotically stable,if one of fuzzy systems is stable and asymptotically stable.

Keywords: First order linear differential systems, fuzzy sets and systems, Kronecker product of matrices, Ascoli's Lemma.

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1. Introduction

In this paper we shall be concerned with the question of existence of $(\emptyset^{\alpha} \otimes \psi^{\alpha})$ bounded solutions of two first order fuzzy linear system differential equations. In modelling real systems one can frequently encounter a first order linear system of differential equations of the form

$$(y^{\alpha})' = A(t)y^{\alpha} + f_1(t) \qquad 1.1$$

$$(x^{\alpha})' = B(t)x^{\alpha} + f_2(t) \qquad 1.2$$

Where, A(t) and B(t) are continuous (nxn) and (mxm) matrices, $y^{\alpha} = (y_1^{\alpha}, y_2^{\alpha}, \dots, y_n^{\alpha})^T$

and$x^{\alpha} = (x_1^{\alpha}, x_2^{\alpha}, \dots, x_m^{\alpha})^T$ and the structures of matrices is known, structures of the model parameters y^{α} and x^{α} are not known for any $\alpha \epsilon [0,1]$. One of the methods of treating this uncertainty is to use a fuzzy set theory for mutation of problems (1.1) and (1.2). The theory of fuzzy sets, fuzzy valued functions and necessary calculus for fuzzy functions have been investigated in recent monograph by Lakshmikantham and Mohapatra [7] and the references cited there in. Recently fuzzy differential equations have also been studied by Viswanadh et. al[2 – 8] in their recent papers. Their novel treatment helped us to study the Kronecker Product systems associated with two fuzzy differential systems of different orders. For recent novel results on fuzzy differential equations and inclusions, we refer to Kasi Viswanadh V, Kanuri., Murty K. N. and Sailaja et.al, [[2 - 8], 13].

2. Preliminaries

In this section we introduce the notations, definitions, preliminary results and definitions of fuzzy sets and systems that are needed for our discussion throughout the paper. Metrics that suit in this paper are taken from [6].

Definition 2.1: Let X be a non empty set. A fuzzy set A in X is characterized by its membership function $A: X \to [0,1]$ and A(x) is interpreted as degree of membership of element x in fuzzy set A for each $x \epsilon X$.

Note that the value of zero is used to represent non-membership, the value of one is used to represent membership and the values in between [0, 1] are used to represent intermediate degrees of membership. The mapping of the function $A: X \to [0,1]$ is called membership function.

The membership function close to one is represented by

$$A(t) = e^{(-\beta(t-1))^2} ,$$

where β > 0 and membership function close to zero is defined as

$$A(t) = \frac{1}{1+t^3} .$$

Using this function, we can determine the membership grade of each real number in the fuzzy set, which signifies the degree to which that membership is close to zero. For instance the number 1/3 is assigned a grade of 0.035 and for the number 1is a grade of 0.5 and for the number zero a grade of 1 is assigned. We denote the set of all non-empty compact, convex subsets of $R^n$by $\mathcal{C}(R^n)$ and

$$E^n = \{y: R^n \to [0,1]\} \text{ such that}$$

I. y is normal, that is there exists $x_0 \in R^n$ such that $y(x_0) = 1$.

II.y is fuzzy convex, that there exists $x \in R^n$ and $0 < \lambda < 1$,

$$y(\lambda x + (1-\lambda)z) \geq min\{y(x), y(z)\}$$

III.y is upper semi continuous and

IV .$[y]^{(0)} = \{x \in R^n | y(x) > 0\}$ is compact. For$0 \leq \alpha \leq 1$, we define

$[y]^{(\alpha)} = \{x \in R^n | y(x) \geq \alpha\}$. Then from (I) – (IV), it follows that the α level sets

$y^\alpha \in \mathcal{C}(R^n)$. If $f: R^n X R^n \to R^n$ is a function, then according to Zadeh's extension principle, we can extend $f: E^n X E^n \to E^n$ by the function defined by

$$f(y, \bar{y})(z) = \sup min(y(z), \bar{y}\ (z)) .$$

It is well known that $g[(y, \bar{y})]^\alpha = g[[y]^\alpha, [\bar{y}]^\alpha]$, for all $y, \bar{y} \in E^n, 0 \leq \alpha \leq 1$ and g is continuous. Especially for addition and scalar multiplication, we have

$$[y + \bar{y}]^{(\alpha)} = [y]^\alpha + [\bar{y}]^\alpha \text{and} [cy]^{(\alpha)} = c[y]^{(\alpha)}$$

where $y, \bar{y} \in E^n, c \in R\ and\ 0 \leq \alpha \leq 1$.

Definition 2.2: A mapping $f: [0,1] \to E^n$ is said to be level wise continuous at$t_0 \in [0,1]$, if the multi valued map $f_\alpha(t) = [f(t)]^\alpha$ is continuous at t = t_0 with respect to the Housdorff metric d is continuous for all $\alpha \in [0,1]$.

A map $f: [0,1] \to E^n$ is said to be integrably bounded, if there exists an integrable function g such that

$$\|y\| \leq g(t) \text{for all } y \in f_0(t) .$$

Definition 2.3: A map $f: [0,1] \to E^n$ is said to be differentiable at $t_0 \in [0,1]$ if there exists a $f'(t_0) \in E^n$ such that $\lim_{h\to 0^+} \frac{f(t_0+h)-f(t_0)}{h}$ and $\lim_{h\to 0^-} \frac{f(t_0)-f(t_0-h)}{h}$ exist and is equal to $f'(t_0)$. Here limit is taken over the metric space(E^n, Hd). At the end points of [0, 1], we can consider only one side derivative.

If $f: [0,1] \to E^n$ is differentiable at$t_0 \in [0,1]$, then we say that $f'(t_0)$ is the fuzzy derivative of f(t) at the point $t_0$or Hukuhara derivative of f(t) at t_0 and is denoted by $D_H f(t_0)$. For the concepts of fuzzy measurability and fuzzy continuity, we refer [2].

If $f: E^n \to E^n$ is differentiable at t_0 then it is continuous at t_0. If $f, g: E^n \to E^n$ are differentiable and λ is any scalar then, $(f^\alpha + g^\alpha)'(t) = [f^\alpha(t)]' + [g^\alpha(t)]'$ for all $\alpha \in [0,1]$ with the above notation on derivatives of fuzzy sets and systems. We now consider the fuzzy differentiable first order system of the form,

$$[y^\alpha(t)]' = A(t) y^\alpha(t) \tag{2.1}$$

$$[x^\alpha(t)]' = B(t) x^\alpha(t) \tag{2.2}$$

Where A(t) and B(t) are continuous (nxn) and (mxm) matrices respectively. Let $Y^\alpha(t)$ be a fundamental matrix of (2.1) and $X^\alpha(t)$ be a fundamental matrix of (2.2). Then we first mention some of the basic properties of Kronecker product of matrices. Let $A \in R^{mxn}$ and $B \in R^{pxq}$matrices, then their Kronecker product or tensor product is defined as

$(A \otimes B)(t) = A(t) \otimes B(t) = a_{ij}B for\ i = 1, 2, \ldots m\ and\ j = 1, 2, \ldots, n$ and is of order mpxnq. This Kronecker product defined above has the following properties:

1. $(A \otimes B)^* = A^* \otimes B^*$ (* refers to complex conjugate)
2. $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$ (provided A and B are invertible)
3. $(A \otimes B)(C \otimes D) = (AC \otimes BD)$
4. $(A + B) \otimes C = (A \otimes C) + (B \otimes C)$ (provided A and B are of same order)
5. $\|A \otimes B\| = \|A\|\|B\|$
6. $(A \otimes B)' = (A' \otimes B + A \otimes B')$ (‘ indicates derivative) ,

With these properties, we can embed systems (2.1) and (2.2) as

$$\left(y^\alpha(t) \otimes x^\alpha(t)\right)' = \left(\left(y^\alpha(t)\right)' \otimes x^\alpha(t)\right) + \left(y^\alpha(t) \otimes \left(x^\alpha(t)\right)'\right)$$

$$= A(t)y^\alpha(t) \otimes x^\alpha(t) + y^\alpha(t) \otimes B(t)x^\alpha(t)$$

$$= [A(t) \otimes I_m + I_n \otimes B(t)]\left(y^\alpha(t) \otimes x^\alpha(t)\right) \qquad (2.3)$$

Theorem 2.1: For any fixed$\alpha \in [0, 1]$, let $Y^\alpha(t)$ and $X^\alpha(t)$ be fundamental matrices of (2.1) and (2.2) respectively. Then $\left(Y^\alpha(t) \otimes X^\alpha(t)\right)$ is a fundamental matrix of (2.3) if and only if $Y^\alpha(t)$ and $X^\alpha(t)$ be fundamental matrices of (2.1) and (2.2) respectively.

Proof : Let $Y^\alpha(t)$ and $X^\alpha(t)$ be fundamental matrices of (2.1) and (2.2) respectively then

$[Y^\alpha(t)]' = A(t)Y^\alpha(t)$and

$[X^\alpha(t)]' = B(t)X^\alpha(t)$, for any fixed $\alpha \in [0, 1]$. Then consider

$$\left(Y^\alpha(t) \otimes X^\alpha(t)\right)' = \left(\left(Y^\alpha(t)\right)' \otimes X^\alpha(t)\right) + \left(Y^\alpha(t) \otimes \left(X^\alpha(t)\right)'\right)$$

$$= A(t)Y^\alpha(t) \otimes X^\alpha(t) + Y^\alpha(t) \otimes B(t)X^\alpha(t)$$

$$= [A(t) \otimes I_m + I_n \otimes B(t)]\left(Y^\alpha(t) \otimes X^\alpha(t)\right).$$

Hence $\left(Y^\alpha(t) \otimes X^\alpha(t)\right)$ is the fundamental matrix of (2.3).

Conversely, suppose $\left(Y^\alpha(t) \otimes X^\alpha(t)\right)$ be a fundamental matrix of (2.3), then it is claimed for any fixed$\alpha \in [0, 1]$, $Y^\alpha(t)$ and $X^\alpha(t)$ are fundamental matrices of (2.1) and (2.2) respectively . For,

$$\left(Y^\alpha(t) \otimes X^\alpha(t)\right)' = [A(t) \otimes I_m + I_n \otimes B(t)]\left(Y^\alpha(t) \otimes X^\alpha(t)\right),$$

$$\left(\left(Y^\alpha(t)\right)' \otimes X^\alpha(t)\right) + \left(Y^\alpha(t) \otimes \left(X^\alpha(t)\right)'\right) = [A(t) \otimes I_m + I_n \otimes B(t)]\left(Y^\alpha(t) \otimes X^\alpha(t)\right)$$

$$\left(Y^{\alpha'}(t) - A(t)Y^\alpha(t)\right) \otimes X^\alpha(t) = Y^\alpha(t) \otimes \left(B(t)X^\alpha(t) - X^{\alpha'}(t)\right)$$

Multiplying both sides with $(Y^\alpha)^{-1} \otimes (X^\alpha)^{-1}$, we get

$$(Y^\alpha)^{-1}\left(Y^{\alpha'}(t) - A(t)Y^\alpha(t)\right) \otimes I_m = I_n \otimes (X^\alpha)^{-1}\left(B(t)X^\alpha(t) - X^{\alpha'}(t)\right)$$

The above relation is true if and only if each of the bracket relations is either an identity matrix or zero matrix. If

$(Y^\alpha)^{-1}\left(Y^{\alpha'}(t) - A(t)Y^\alpha(t)\right) = I_n$ or

$(X^\alpha)^{-1}\left(B(t)X^\alpha(t) - X^{\alpha'}(t)\right) = I_m$.

The above relations imply that

$$Y^{\alpha'}(t) = \left(I_n + A(t)\right)Y^\alpha(t) \qquad (2.4)$$

$X^{\alpha'}(t) = (B(t) + I_m)X^\alpha(t)$ (2.5)

(2.4) and (2.5) clearly show that $Y^\alpha(t)$ and $X^\alpha(t)$ are fundamental matrices of

$y^{\alpha'}(t) = \big(I_n + A(t)\big)y^\alpha(t)$ and $x^{\alpha'}(t) = (B(t) + I_m)x^\alpha(t)$, which is contradiction.

If each of the bracket relations is a null matrix, then the result follows. This is true for any fixed $\alpha \in [0, 1]$.

Existence of Ψ bounded solutions of linear system of differential equations are established by Kasi Viswanath, R. Suryanarayana and K. N. Murty [3] in the year 2020. We make use of these results to establish our main results in next section. Metrics that suit for dichotomy and well conditioning of object oriented design in measure chains are established in [6]. We also make use of the other results established by Kasi Viswanathet.al., [9] are used as a tool to establish our main result.

3. Main results

In this section we establish our main result namely the $\big(\emptyset^\alpha(t) \otimes \Psi^\alpha(t)\big)$ bounded solution of the Kronecker product system (2.3).

Definition 3.1: A function $y: R^+ \to R^n$ is said to be Ψ integrable on R^+ if, $\Psi(t)$ is continuous and $\Psi(t)y(t)$ is Lebesgue integrable on R^+ .

Definition 3.2: A function $y^\alpha: R^+ \to R^n$ for each $\alpha \in [0, 1]$is said to be ψ^α Lebesgue ψ^α integrable on R^+ if, $\psi^\alpha(t)$ is measurable function for each $\alpha \in [0, 1]$ and $\psi^\alpha(t)y^\alpha(t)$ is Lebesgue integrable on R^+ .

Definition 3.3: A function $y^\alpha: R^+ \to R^n$for each$\alpha \in [0, 1]$is said to be ψ^α bounded on R^+ if, $\psi^\alpha(t)y^\alpha(t)$ is bounded on R^+.

By a solution of Kronecker product system (2.3), we mean an absolutely continuous function $\big(y^\alpha(t) \otimes x^\alpha(t)\big)$ for each $\alpha \in [0, 1]$ and it satisfies the system (2.3) for all $t \geq 0$. Let $Y^\alpha(t)$ and $X^\alpha(t)$ be fundamental matrices of (2.1) and (2.2) respectively, satisfying $Y^\alpha(0) = I_m$and$X^\alpha(0) = I_n$. Let the vector space R^{mn} be represented as a direct sum of three subspaces X_-, X_0, X_+such that a solution$Y^\alpha(t)$ of (2.1) and$X^\alpha(t)$ of (2.2) is Ψ bounded solution on R^{mn} if, and only if $y^\alpha(0) \in X_0$ and $\emptyset^\alpha(t)$ is bounded on $R^+ \in [0, \infty)$

If, and only if $y^\alpha(0) \in X_- \otimes X_0$ also$x^\alpha(0) \in X_0 \otimes X_+$. Also let P_-, P_0, P_+denote the corresponding projections of R^{mn} onto X_-, X_0, X_+ respectively. We assume throughout the paper $(\emptyset^\alpha \otimes \Psi^\alpha)(t) = \chi^\alpha(t)$ and $(y^\alpha \otimes x^\alpha)(t) = z^\alpha(t)$ for all $t \geq 0$and $\alpha \in [0, 1]$. We have the following result .

Theorem 3.1: Let A be an (mxm) continuous matrix and B be an (nxn) continuous matrix on R, then the system (2.3) has at least one $\chi^\alpha(t)$bounded solution on R for every continuous $\chi^\alpha(t)$ bounded function $(f_1 \otimes f_2): R \to R^{mn}$, if, and only if there exists a positive constant K such that

$$\int_{-\infty}^{t} \|\chi^\alpha(t)z^\alpha(t)P_-(z^\alpha)^{-1}(s)(\chi^\alpha)^{-1}(s)\|ds + \int_{t_0}^{0} \|\chi^\alpha(t)z^\alpha(t)(P_0 + P_+)(z^\alpha)^{-1}(s)(\chi^\alpha)^{-1}(s)\|ds + \int_{0}^{\infty} \|\chi^\alpha(t)z^\alpha(t)P_+(z^\alpha)^{-1}(s)(\chi^\alpha)^{-1}(s)\|ds \leq K$$ for all $t \geq 0$ and $\alpha \in [0, 1]$ and

$$\int_{-\infty}^{0} \|\chi^\alpha(t)z^\alpha(t)P_-(z^\alpha)^{-1}(s)(\chi^\alpha)^{-1}(s)\|ds + \int_{0}^{t} \|\chi^\alpha(t)z^\alpha(t)(P_0 + P_-)(z^\alpha)^{-1}(s)(\chi^\alpha)^{-1}(s)\|ds + \int_{t}^{\infty} \|\chi^\alpha(t)z^\alpha(t)P_+(z^\alpha)^{-1}(s)(\chi^\alpha)^{-1}(s)\|ds \leq K$$ for all $t \geq 0$and $\alpha \in [0, 1]$.

Proof :Let us first suppose that the Kronecker product system (2.3) has at least one

$\chi^\alpha(t)$bounded solution for each $\alpha \in [0, 1]$ and for every continuous $\chi^\alpha(t)$ bounded function $f = f_1 \otimes f_2: R \to R^{mn}$.

Let b be a Banach space of all $\chi^\alpha(t)$ bounded and continuous functions

$(y \otimes x): R \to R^{mn}$with norm defined by$\|y \otimes x\| = \sup_{t \in R}\|\chi^\alpha(t)(y \otimes x)(t)\|$.

Let D denote the set of all $\chi^\alpha(t)$ bounded and continuously differentiable functions such that $(y \otimes x)(0): X_- \otimes X_0$ and $(y^\alpha)' - A(t)y^\alpha \in B$and $(x^\alpha)' - B(t)x^\alpha \in B$. Clearly D is a vector space and

$\|y^\alpha \otimes x^\alpha\|_D = \|y^\alpha \otimes x^\alpha\|_D + \|(y^\alpha \otimes x^\alpha)' - (A(t) \otimes I_n + I_m \otimes B(t))\|_D$. Further $\lim_{n\to\infty}\|\chi_n^\alpha(t)(y_n \otimes x_n)(t)\| = \chi^\alpha(t)(y^\alpha \otimes x^\alpha)(t)$ for each $\alpha \in [0,1]$. Note that the above convergence is uniform.

We first prove that $(D, \|.\|_D)$ is a Banach space. Let $\{(y_n^\alpha \otimes x_n^\alpha)\}_{n\in N}$ be a fundamental sequence of elements of D. Then $\{(y_n^\alpha \otimes x_n^\alpha)\}_{n\in N}$ is also a fundamental sequence in B. Therefore, there exists a continuous $\chi_n^\alpha(t)$ bounded function on R such that $\lim_{n\to\infty}\chi_n^\alpha(t)(y_n^\alpha \otimes x_n^\alpha)(t)$ converges uniformly to continuous function say

$\chi^\alpha(y^\alpha \otimes x^\alpha)$. From the inequality,

$\|Z_n^\alpha(t) - Z^\alpha(t)\| \le \|(\chi^\alpha)^{-1}(t)\|\|(y^\alpha \otimes x^\alpha)(t)\|$.

Hence $\lim_{n\to\infty} z_n^\alpha(t) = z^\alpha(t)$ uniformly on every compact subset of R. Thus

$z^\alpha(0) \in X_- \otimes X_0$. Similarly the sequence $\{(y_n^\alpha)' - A(t)y_n^\alpha\}$ and $\{(x_n^\alpha)' - B(t)x_n^\alpha\}$ is a fundamental sequence in B. Therefore, there exists a continuous $\{\chi_n^\alpha(t)\}$ bounded functions such that

$\lim_{n\to\infty}\{\chi_n^\alpha(t) - (A(t) \otimes I_n) + (I_m \otimes B(t))\}$convergesuniformly to

$\{\chi^\alpha(t) - (A(t) \otimes I_n) + (I_m \otimes B(t))\}$. Furtherlim$_{n\to\infty}\{\chi_n^\alpha(t) - z_n^\alpha(t)\} = 0$. This proves the claim. It can easily be proved that there exists a compact K> 0 such that $\sup_{t\in D}\|\chi^\alpha(t) - z^\alpha(t)\| \le \mathrm{K}\sup_{t\in\mathrm{R}}\|\chi^\alpha(t) - (f_1 \otimes f_2)\|$t

Suppose $\theta_1 < 0 < \theta_2$ be two fixed points (arbitrary and let $f: R \to R^{mn}$ be a continuous $\chi^\alpha(t)$ bounded function on $(-\infty, \theta_1] \cup [\theta_2, \infty)$ and define

$$z^\alpha(t) = \begin{cases} \int_0^{\theta_1} \chi^\alpha(t)P_0\,(z^\alpha)^{-1}(s)(f_1 \otimes f_2)ds + \int_{\theta_2}^{\theta_1} \chi^\alpha(t)P_+\,(z^\alpha)^{-1}(s)(f_1 \otimes f_2)ds \end{cases}$$

$$+ \int_{\theta_1}^t \chi^\alpha(t)P_-\,(z^\alpha)^{-1}(s)(f_1 \otimes f_2)ds + \int_0^t \chi^\alpha(t)P_0\,(z^\alpha)^{-1}(s)(f_1 \otimes f_2)ds\,, \qquad \theta_1 < t < \theta_2.$$

And for t > T$_2$, we have

$$Z^\alpha(t) = \begin{cases} \int_{\theta_2}^{t} \chi^\alpha(t)P_+\,(Z^\alpha)^{-1}(s)(f_1 \otimes f_2)ds + \int_{\theta_1}^{\theta_2} \chi^\alpha(t)P_-\,(Z^\alpha)^{-1}(s)(f_1 \otimes f_2)ds \end{cases}$$

$$+ \int_0^{\theta_2} \chi^\alpha(t)P_0\,(zZ^\alpha)^{-1}(s)(f_1 \otimes f_2)ds\,,$$

Is the solution of the Kronecker product system (2.3) on R [9]. The other part fallows from the fact that if $\left(Y^\alpha(t) \otimes Z^\alpha(t)\right)$ is a solution of the Kronecker product system (2.3) if, and only if $Y^\alpha(t)$and $Z^\alpha(t)$ are solutions of (2.1) and (2.2) respectively.

As a particular case of the above theorem, we haven the following result.

Result 2.2 : If the homogenous systems (2.1) and (2.2) have non trivial χ^α- bounded solution on R, then the system (2.3) has a unique χ^α- bounded solution on R for every continuous and χ^αbounded function $(f_1 \otimes f_2)$: R→ R^{mn} if, and only if

$\int_{-\infty}^t\|\chi^\alpha(t)z^\alpha(t)P_-(Z^\alpha)^{-1}(s)(\chi^\alpha)^{-1}(s)(f_1 \otimes f_2)\|ds \quad + \quad \int_t^\infty\|\chi^\alpha(t)Z^\alpha(t)P_+(z^\alpha)^{-1}(s)(\chi^\alpha)^{-1}(s)(f_1 \otimes f_2)\|ds \le$ K.

Note that if P$_0$=0, the main theorem reduces to the above integral.

Results established on stability, controllability and observabilities for state space dynamical system on measure chains by Yan Wu, Sailaja.P and K.N.Murty[11] can be extended to fuzzy linear Kronecker product systems and work in this directionis in progress.

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