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Analysis of New-Fangled Mandelbrot Sets Controlled by TAN Function

Suraj Singh**, **Panwar

Faculty of Technology,
Computer Science and Engineering Department,
Uttarakhand Technical University, Dehradun, INDIA
surajpanwar_usic@rediffmail.com

Abstract — ***In this paper we have presented the complex dynamics of new-fangled fractal controlled by transcendental function i.e. tan function using iterative procedure known as Ishikawa Method. Fractals are generated and analyzed for integer and non-integer values. New Mandelbrot sets are generated for different values of parameters defined by Ishikawa iteration i.e. s, s' and transcendental controlled function. Here different ovoid's or lobe are analyzed and our study relies on the generation of pattern defined by mini Mandelbrot sets.***

Keywords — Mandelbrot Set, Complex Fractals, Ishikawa iteration, Transcendental function, Ovoids.

1. Introduction

Complex Graphics of nonlinear dynamical systems have been a focus of research nowadays. These graphics of complex plane is studied under Fractal Theory. A Fractal is a statistical shape that is difficult and detailed at every level of magnification, as fit as self-similar. Fractal is defined as a set, which is self–similar under magnification [1]. Self-similarity means looking the same structure over all ranges of scale, i.e. a small section of a fractal can be viewed as a part of the larger fractal. Fractal Theory is an exciting branch of applicable Mathematics and Computer Science. Benoit Mandelbrot (1924-2010) is known as the father of fractal geometry. He coined the word fractal in the late 1970s. Fractal geometry provided by Mandelbrot's gives an explanation and mathematical model for numerous such complex forms found in nature.

The word Fractal is derived from Latin word 'fractus', which describes the appearance of broken stone: irregular and fragmented. He explain geometric fractals as "a rough or fragmented geometric shape that can be divided into parts, every one of which is a reduced-size duplicate of the whole".[2] There are many variety of fractals found in nature in the form of many usual objects such as mountains, coastlines, trees ferns and clouds[3,4]. They all are fractals in nature and can be represented on a computer by a recursive algorithm of computer graphics.

The Julia sets and the Mandelbrot sets are two most important images under various researches in the field of fractal theory [5]. In 1918, French Mathematician Gaston Julia (1893-1978) [6] investigated the iteration process of a complex function and attained a Julia set, whereas the Mandelbrot set was given by Benoit B. Mandelbrot [2] in 1979.

2. Preliminaries

Here we have used the transformation function $Z \rightarrow (Z^n+C)$, $n>=2.0$ and $C=\tan(1/\#pixel^p)$, $p>=1.0$ for generating fractal images with respect to Ishikawa iterates, where z and c are the complex quantities and n, p are real numbers. Each of these fractal images is constructed as a two-dimensional array of pixels. Each pixel is represented by a pair of (x, y) coordinates. The complex quantities z and c can be represented as:

$Z=Z_x+iZ_y$
$C=C_x+iC_y$

where $i=\sqrt{(-1)}$ and Z_x , C_x are the real parts and Z_y & C_y are the imaginary parts of Z and C , respectively. The pixel coordinates (x, y) may be associated with (C_x , C_y) or (Z_x , Z_y). Based on this concept, the fractal images can be classified as follows:

z-Plane fractals, where in (x,y) is a function of (z_x,z_y).
c-Plane fractals, where in (x,y) is a function of (c_x,c_y).

In the literature, the fractals for n=2 in z plane are termed as the Mandelbrot set while the fractals for n=2 in c plane are known as Julia sets [7].

2.1 Mann's Iteration : One Step Iteration

Mann's iteration technique is a one-step iteration technique given by William Robert Mann (1920-2006), a mathematician from Chapel Hill, North Carolina. The iteration technique involves one step for iteration, and is given as [8,9]:

$x_{n+1} = s.f(x_n) + (1\text{-}s)x_n$, where $n \geq 0$ and $0 < s < 1$

2.2 Ishikawa Iteration : Two Step Iteration

Ishikawa iteration [11] technique is a two-step iteration method known after Ishikawa. Let X be a subset of complex number and $f : X \rightarrow X$ for all $x_0 \epsilon X$, we have the sequence numbers for $\{x_n\}$ and $\{y_n\}$ in X according to following way. [12] :

$$y_n = S'_n\, f(x_n) + (1 - S'_n)x_n$$

$$x_{n+1} = S_n\, f(y_n) + (1 - S_n)x_n$$

Where, $0 \leq S'_n \leq 1$, $0 \leq S_n \leq 1$ and S'_n & S_n are both convergent to non-zero number.

3. GENERATION OF RELATIVE SUPERIOR MANDELBROT SETS

We present here some Relative Superior Mandelbrot sets[13,14] for the function $Z \rightarrow (Z^n+C)$, $n>=2.0$ and $C=\tan(1/\#pixel^p)$, $p>=1.0$ for integer and some non-integer values of n and p. Process of generating fractal images is similar to self-squared function [15]. Fractals analysis is evaluated in Ultra Fractal Software having many features i.e. creating fractals, changing formula parameters, layer properties, coloring algorithm, zooming the Image, creating animation, compiler message, fractal mode, etc. Fractal images are created by repeatedly calculating a fractal formula. Although these formulas are purely mathematical, the resulting pictures are often very beautiful and complex.

The Fractals have been generated by the iterative procedure starting with initial value z_0, where z and c are both complex quantities. The parameter s and s' also changes the structure and beauty of fractals.

3.1 Relative Superior Mandelbrot sets for Quadratic function:

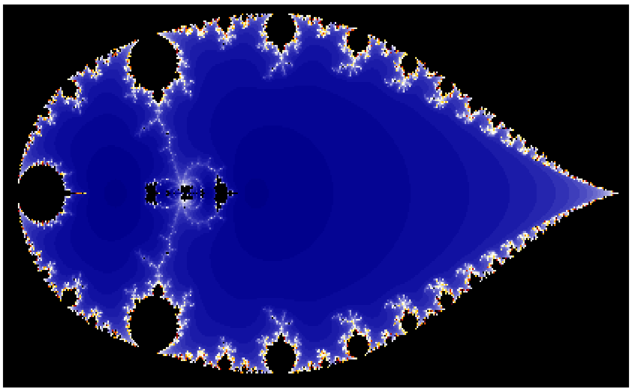

Figure 1: Relative Superior Mandelbrot set for s=s'=1,p=1,n=2.

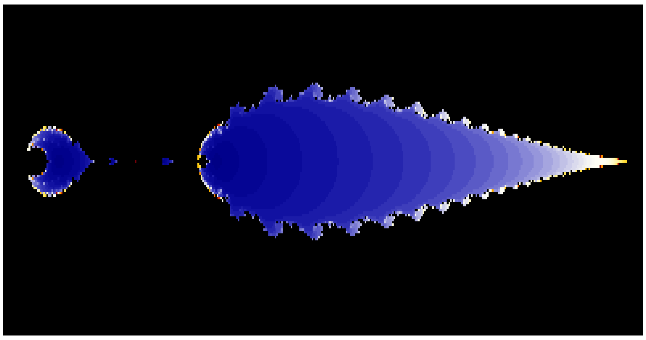

Figure 2: Relative Superior Mandelbrot set for s=s'=0.5,p=1,n=2.

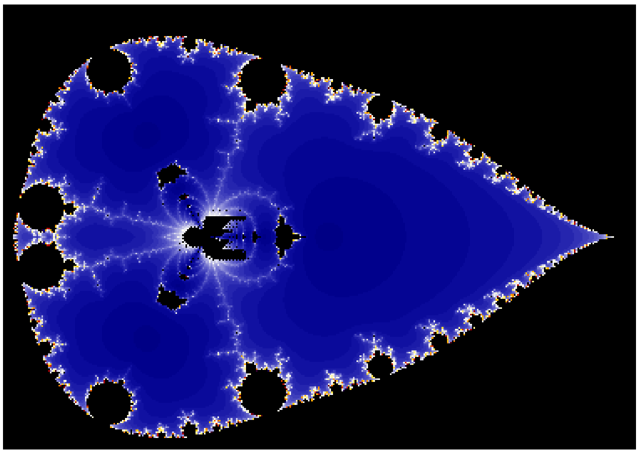

Figure 3: Relative Superior Mandelbrot set for s=s'=1,p=1.5,n=2.

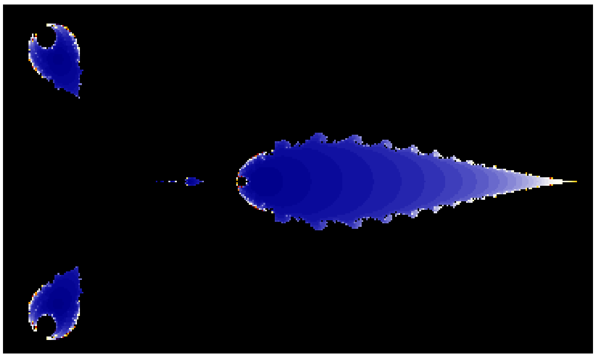

Figure 4: Relative Superior Mandelbrot set for s=s'=0.5,p=1.5,n=2

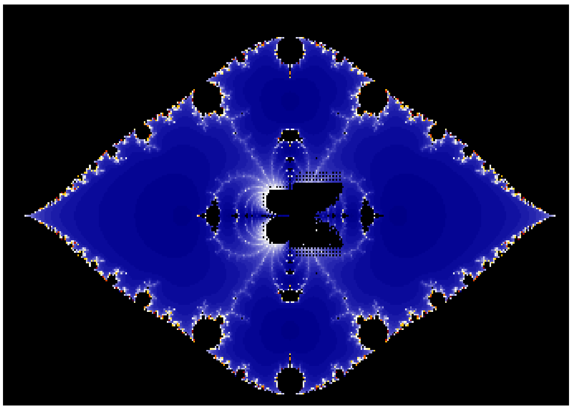

Figure 5: Relative Superior Mandelbrot set for s=s'=1,p=2,n=2

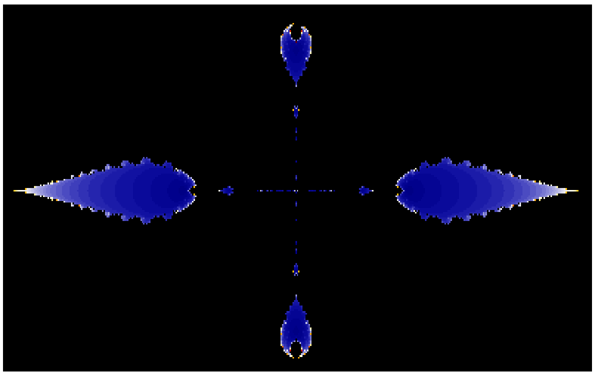

Figure 6: Relative Superior Mandelbrot set for s=s'=0.5,p=2,n=2

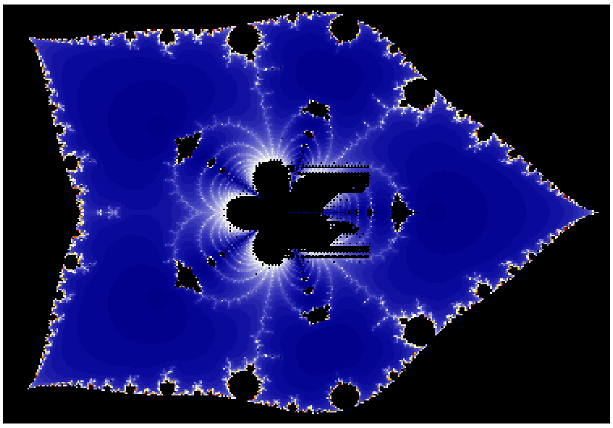

Figure 7: Relative Superior Mandelbrot set for s=s'=1,p=2.5,n=2

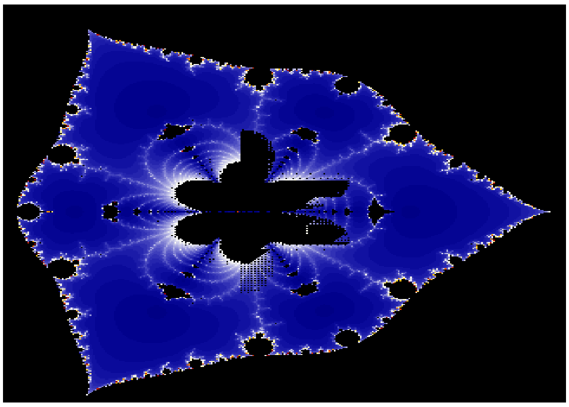

Figure 8: Relative Superior Mandelbrot set for s=s'=1,p=3,n=2

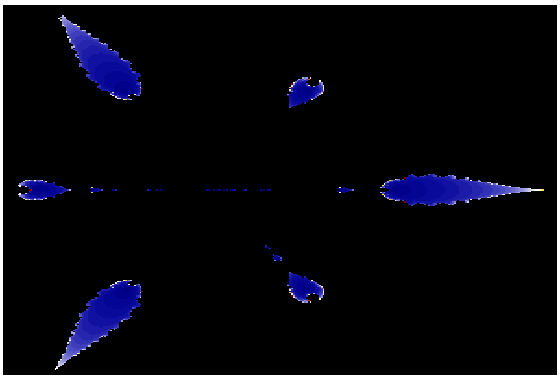

Figure 9: Relative Superior Mandelbrot set for s=s'=0.5,p=3,n=2

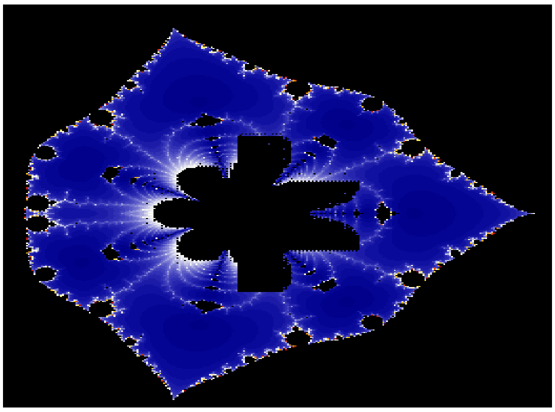

Figure 10: Relative Superior Mandelbrot set for s=s'=1,p=3.5,n=2

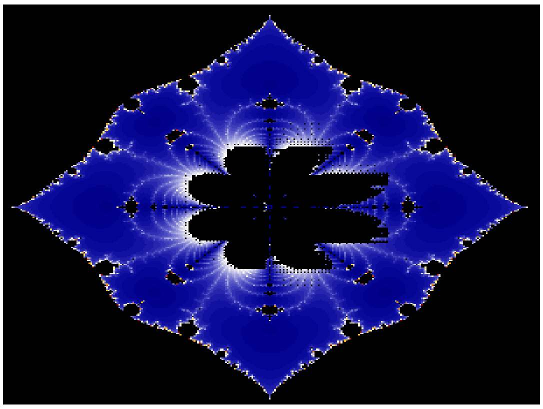

Figure 11: Relative Superior Mandelbrot set for s=s'=1,p=4,n=2

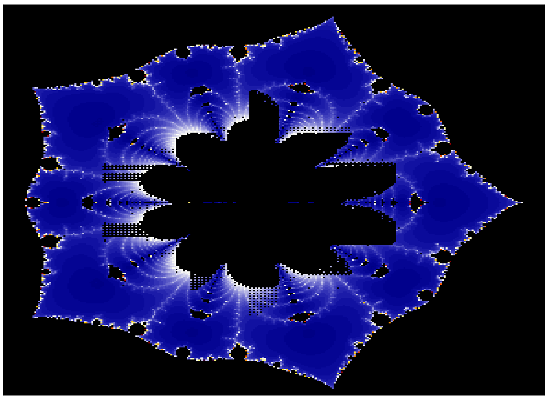

Figure 12: Relative Superior Mandelbrot set for s=s'=1,p=5,n=2

3.2 Relative Superior Mandelbrot set for Cubic function.

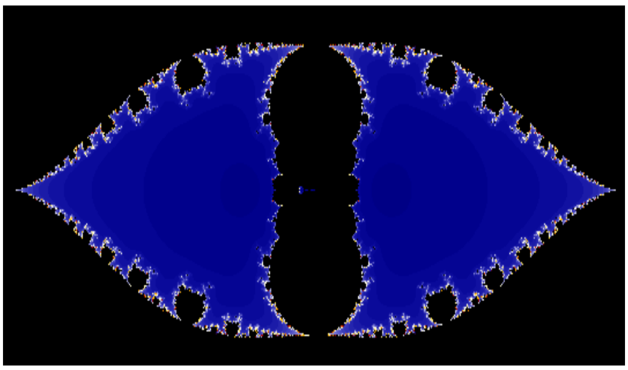

Figure 13: Relative Superior Mandelbrot set for s=s'=1,p=1,n=3.

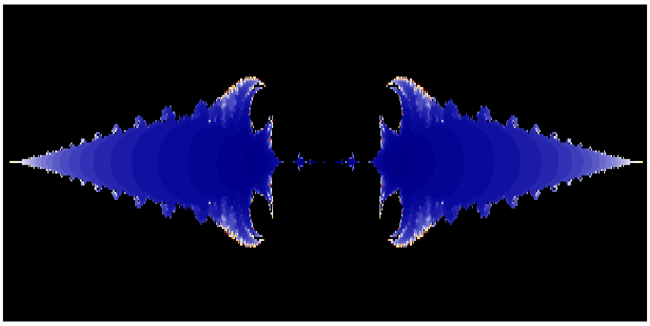

Figure 14: Relative Superior Mandelbrot set for s=s'=0.5,p=1,n=3.

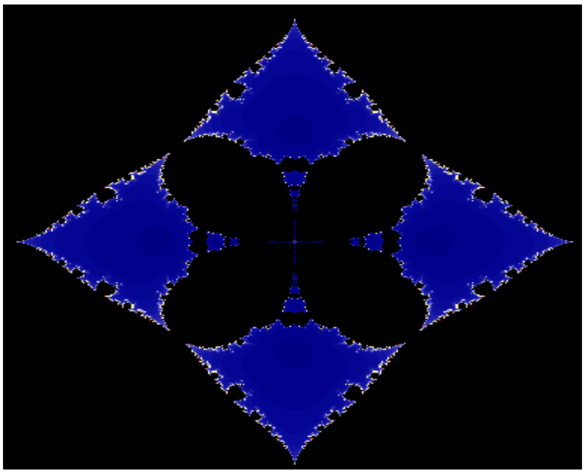

Figure 15: Relative Superior Mandelbrot set for s=s'=1,p=2,n=3.

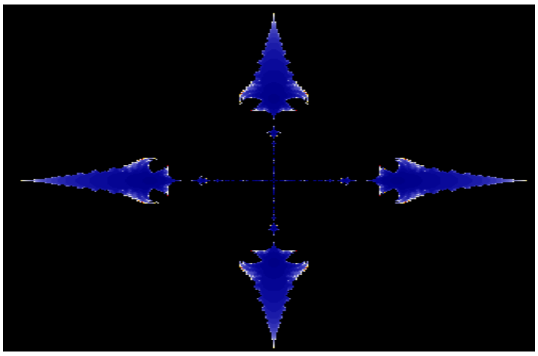

Figure 16: Relative Superior Mandelbrot set for s=s'=0.5,p=2,n=3.

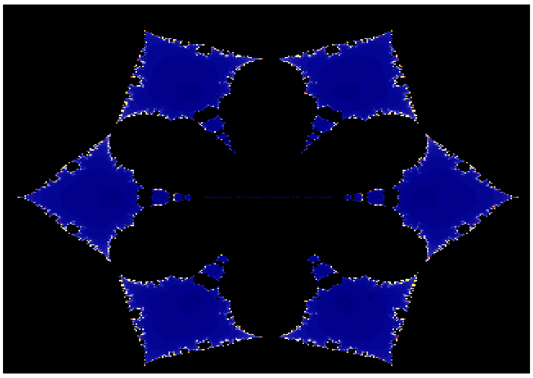

Figure 17: Relative Superior Mandelbrot set for s=s'=1,p=3,n=3.

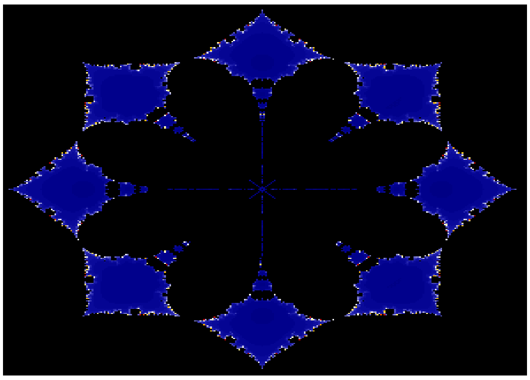

Figure 18: Relative Superior Mandelbrot set for s=s'=1,p=4,n=3.

3.3 Relative Superior Mandelbrot set for Bi-Quadratic function.

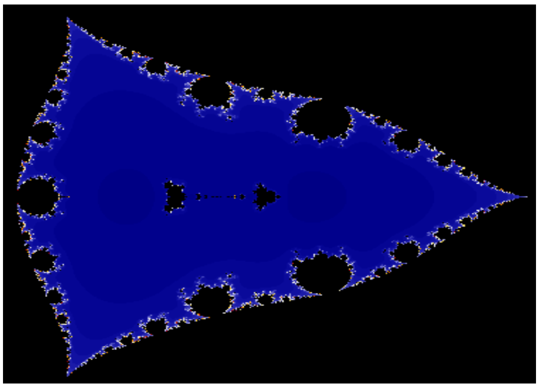

Figure 19: Relative Superior Mandelbrot set for s=s'=1,p=1,n=4

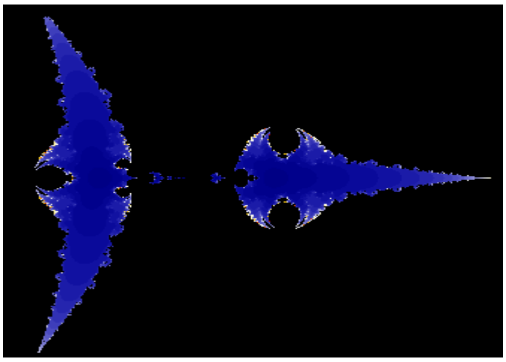

Figure 20: Relative Superior Mandelbrot set for s=s'=0.5,p=1,n=4

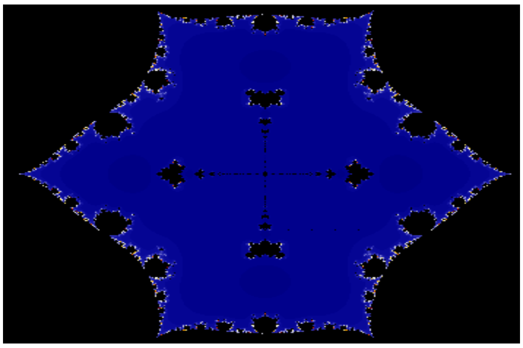

Figure 21: Relative Superior Mandelbrot set for s=s'=1,p=2,n=4

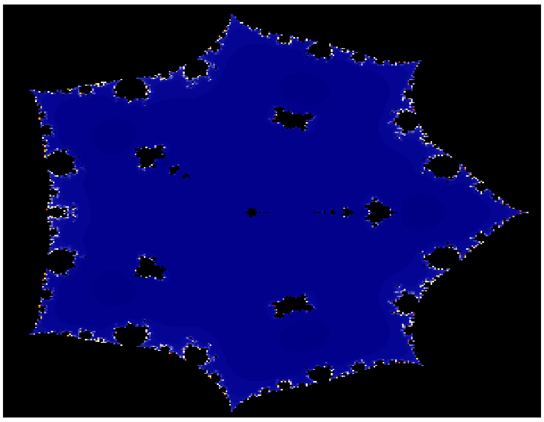

Figure 22: Relative Superior Mandelbrot set for s=s'=1,p=2.5,n=4

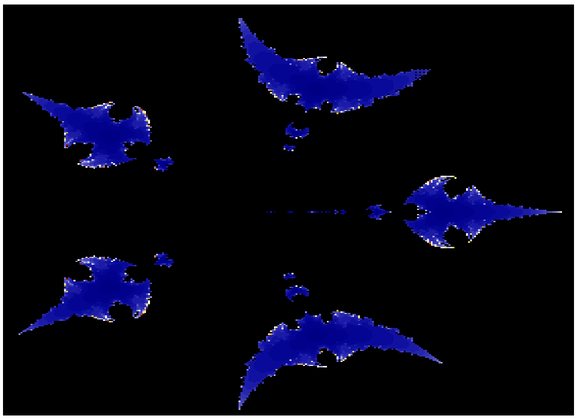

Figure 23: Relative Superior Mandelbrot set for s=s'=0.5,p=2.5,n=4

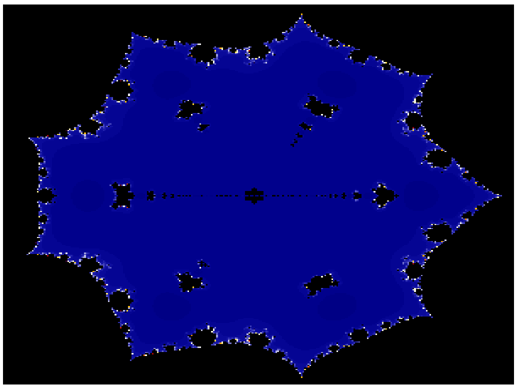

Figure 24: Relative Superior Mandelbrot set for s=s'=1,p=3,n=4

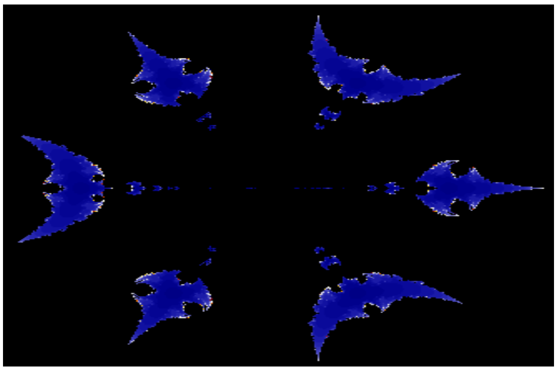

Figure 25: Relative Superior Mandelbrot set for s=s'=0.5,p=3,n=4

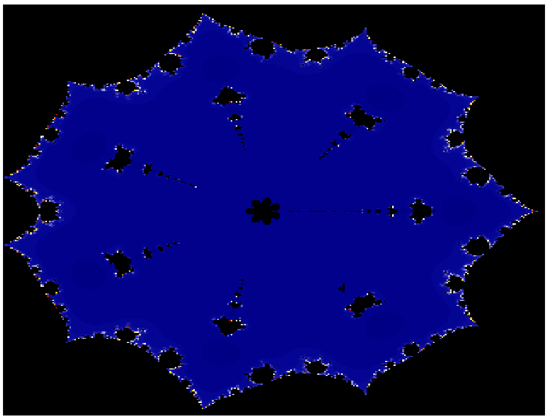

Figure 26: Relative Superior Mandelbrot set for s=s'=1,p=3.5,n=4

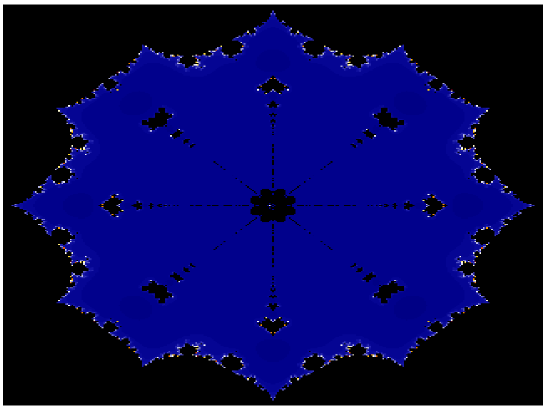

Figure 27: Relative Superior Mandelbrot set for s=s'=1,p=4,n=4

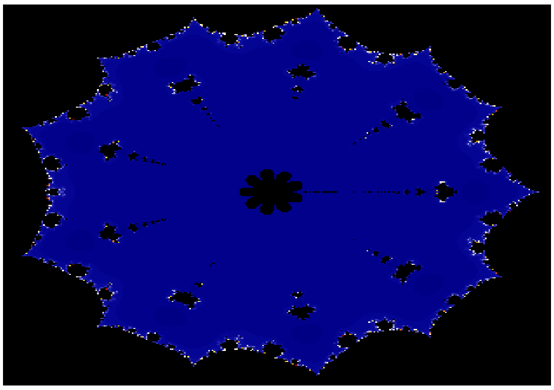

Figure 28: Relative Superior Mandelbrot set for s=s'=1,p=4.5,n=4

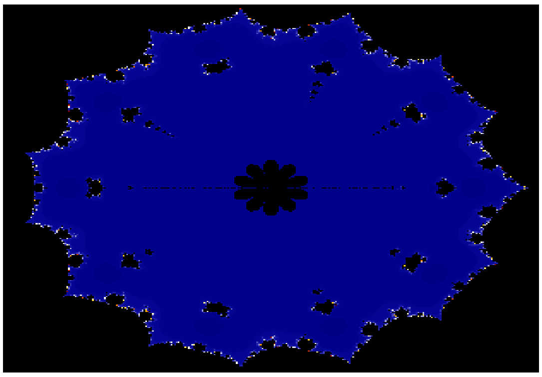

Figure 29: Relative Superior Mandelbrot set for s=s'=1,p=5,n=4

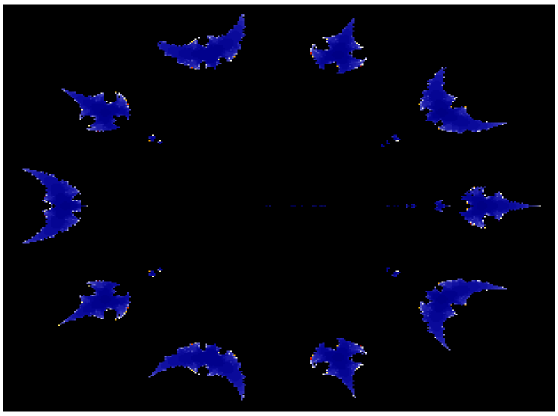

Figure 30: Relative Superior Mandelbrot set for s=s'=0.5,p=5,n=4

4. CONCLUSION

In the complex dynamics polynomial function for $z \rightarrow (z^n+c)$, where n>=2.0, control transcendental function is $c=\tan(1/\#pixel^P)$, p>=1.0. The fractals generated with power p and n are found as rotationally symmetric. We have analysis superior iterates at different power of n and p as shown in fig. There are ovoids or bulbs attached internally within the main body of Mandelbrot sets.

The controlling function tan for value c exhibits new characteristics for the generating fractals. Here we have presented the geometric properties of fractals along different axis. The fractals generated depend on the parameter p. From above observation and analysis 2*p image of mini Mandelbrot is generated for above tan controlling function. The number of corners follows the pattern (n-1)*p and the ovoid in the centre follows 2*p sequence. On changing the value of s, s' from 1 to 0.5 fractal images are changed and corners become filled-arrow shaped. For non-integer value of p new images are created from left side step by step till the upper integer value is met.

For all values of p and n, Mandelbrot Sets are symmetrical along real axis. The generated superior Mandelbrot sets using tan function resemble to beautiful images, full of colorful, arrows, symmetrical and other shapes. Further by using different iterative methods and more different functions we can create fractals and analyze their properties.

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