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Trajectory Tracking Composite Control for a Quadrotor UAV

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Abstract:
The trajectory tracking composite control scheme is proposed for quadrotor unmanned aerial vehicle (UAV) with perturbation parameters and external disturbances. Firstly, the dynamic model of quadrotor UAV is decoupled into two subsystems, which are outer loop position control system and the inner loop angle control system, respectively. Secondly, by using the interval matrices to describe the perturbation parameters, the considered control problem is transformed into robust control with uncertainties. Based on these, an effective tracking composite control strategy is proposed by combining disturbance observer control (DOBC) with H_∞ control. Finally, the simulation results show the effectiveness of the proposed method.

Keywords: quadrotor UAV, trajectory tracking, interval matrix, robust H_∞ control, disturbance observer

I. Introduction:

UAV has many advantages of high flexibility, autonomous hovering, little demand for landing site, simple maintenance and low cost [1]. It is widely used in many fields such as: military reconnaissance, agricultural monitoring, civil aerial photography, etc [1]. However, it is still challenging issue of track trajectory because the parameters of UAV are uncertain and susceptible for external disturbances [2].

So far, a lot of methods for trajectory tracking control have been proposed, such as PID control, sliding mode control, back-stepping control and H_∞ control, etc [3,4,5,7,8]. It is noted that the air resistance is often ignored in many methods under low-speed state. For example, the proposed simplified model in [9] is only applicable to very low speed. However, the quadrotor UAV under medium speed state are easily affected by air resistance, blade rotation and other external disturbances, which can not be ignored. Therefore, the designed controller must be robust to these. In [10] the air resistance and blade rotation are described as the bounded uncertainties, which are further constructed via neural networks. Moreover, the perturbation parameters are described by using the interval matrix in [11]. Then, the interval matrix method is extended to the tracking control of UAV in [12], which improves the stability of the trajectory tracking. However, this method does not consider the effects of external disturbances.

Most of the time, UAV need to operate in harsh environments, which requires the UAV to be able to handle disturbance. Some works approached the disturbance problem by detailed modelling of aerodynamics and wind effects on UAV to estimate the disturbance [6]. In recent years, disturbance observer has been widely used due to its excellent anti-disturbance capability and robustness to uncertain [13]. The control based on disturbance observer is proposed in [14, 15], which basic idea is to estimate the disturbances by disturbance observer and to compensate it in the feedforward channel. In [16, 17], the anti-disturbance control based on disturbance observer is proposed, which significantly improve the control effect. For the tracking control of UAV [19, 20], there are few works to do by combining DOBC and H_∞ control.

Therefore, inspired by the above problems, in this paper, the trajectory tracking compound control scheme is proposed for UAV with perturbation parameters and external disturbances. Firstly, the dynamic model of quadrotor UAV is decoupled into two subsystems, which are outer loop position control system and the inner loop angle control system, respectively. Secondly, by using the interval matrices to describe the perturbation parameters, the considered control problem is transformed into robust control with uncertainties. Based on these, an effective tracking compound control strategy is proposed by combining DOBC with H_∞ control. Finally, the simulation results show the effectiveness of the proposed method.

Notation: In order to facilitate the description, the symmetric position of the symmetric matrix is represented by the" ∗ ". And the superscript "T ", " −1", " † " stand for matrix transposition, inverse, and pseudo inverse.

II. Description of Model:

A six-degree-of-freedom model of quadrotor UAV is described in this paper with following two coordinates:

Position coordinate: $\xi(t)=[x(t),y(t),z(t)]^T$

Angle coordinate: $\eta(t)=[\phi(t),\theta(t),\psi(t)]^T$

The quadrotor UAV model is considered as a symmetrical rigid body, ignoring the blade elastic deformation and vibration, which the dynamic equation is[18]:

$$\begin{cases} m\ddot{x}(t)=-k_1\dot{x}(t)+(\cos\phi(t)sin\theta(t)\cos\psi(t) \\ \qquad +\sin\phi(t)\sin\psi(t))u(t)+d_1(t) \\ m\ddot{y}(t)=-k_2\dot{y}(t)+(\cos\phi(t)(sin\theta(t)sin\psi(t) \\ \qquad +\sin\phi(t)\cos\psi(t)u(t)+d_2(t) \\ m\ddot{z}(t)=-k_3\dot{z}(t)-mg+(\cos\phi\cos\theta)u(t)+d_3(t) \\ J_1\ddot{\phi}(t)=-k_4l\dot{\phi}(t)+l\tau_1(t)+d_4(t) \\ J_2\ddot{\theta}(t)=-k_5l\dot{\theta}(t)+l\tau_2(t)+d_5(t) \\ J_3\ddot{\psi}(t)=-k_6\dot{\psi}(t)+c\tau_3(t)+d_6(t) \end{cases} \tag{1}$$

The meaning of each variable in the equation (1) is shown in table Ⅰ.

Table Ⅰ: The meaning of each variable in the equation(1)

Variable	***The meaning of the variable***
m	Weight of the UAV $m\in R^+$
k_i	Resistance coefficient($i=1,2,\cdots,6$)
g	Coefficient of gravity $g\in R^+$
J_i	Moment of inertia($i=1,2,3$)
c	Force and torque scale factor $c\in R^+$
$l\in R^+$	The distance from the center of the UAV body to the axis of the rotor $l\in R^+$
$d_i(t)$	Unknown external disturbance($i=1,2,\cdots,6$)
$u(t)$	Total lift
$\tau(t)$	$\tau_1(t)$, $\tau_2(t)$, $\tau_3(t)$ Respectively, rolling force, pitching force, yaw force

The quadrotor UAV model is simplified as follows:

$$\begin{cases} m\ddot{\xi}(t)=-k_\xi\dot{\xi}(t)-mge+u(t)_B^ER(\eta)e+d_\xi(t) \\ m\ddot{\eta}(t)=-k_\eta\dot{\eta}(t)-\tau(t)+d_\eta(t) \end{cases} \tag{2}$$

The meaning of each variable in the equation (2) is shown in table Ⅱ.

TableⅡ: The meaning of each variable in the equation (1)

Variable	***The meaning of the variable***
k_ε	Resistance coefficient diag$[k_1,k_2,k_3]$
e	$[0,0,1]^T$
$_B^ER(\eta)$	The rotation matrix between the position coordinates and the angle coordinates
$d_\varepsilon(t)$	The external disturbance $[d_1,d_2,d_3]^T$
J	Moment of inertia diag$[J_1/l,J_2/l,J_3/c]$
k_η	Resistance coefficient diag$[k_4,k_5,k_6/c]$
$d_\eta(t)$	The external disturbance $[d_4/l,d_5/l,d_6/c]^T$

The expected position and angle are described by $\xi_d(t)=[x_d(t),y_d(t),z_d(t)]^T$, $\eta_d(t)=[\phi_d(t),\theta_d(t),\psi_d(t)]^T$ respectively. Then the tracking error can be obtained as:

$$x_w(t)=\begin{bmatrix}\xi(t)-\xi_d(t)\\ \dot{\xi}(t)-\dot{\xi}_d(t)\end{bmatrix}, x_j(t)=\begin{bmatrix}\theta(t)-\theta_d(t)\\ \dot{\theta}(t)-\dot{\theta}_d(t)\end{bmatrix}.$$

According to the [12], without considering the nonlinear coupling term $_B^ER(\eta)$, the error model can be decoupled and described as:

$$\begin{cases} \dot{x}_w(t)=A_1x_w(t)+B_1u(t)+B_1f_w(t)+B_1\omega_1(t) \\ \dot{x}_j(t)=A_2x_j(t)+B_2\tau(t)+B_2f_j(t)+B_{22}\omega_2(t) \end{cases} \tag{3}$$

Where

$$A_1=\begin{bmatrix}0 & I\\ 0 & -\frac{k_\xi}{m}\end{bmatrix}, A_2=\begin{bmatrix}0 & I\\ 0 & -J^{-1}k_\eta\end{bmatrix},$$

$$B_1=B_{22}=\begin{bmatrix}0 & 0\\ 0 & I\end{bmatrix}, B_2=\begin{bmatrix}0 & 0\\ 0 & J^{-1}\end{bmatrix},$$

$$f_w(t)=-\ddot{\xi}_d(t)-\frac{k_\xi}{m}\dot{\xi}_d(t),\ f_j(t)=-K_\eta\dot{\eta}_d(t)-J_{22}\ddot{\eta}_d(t),$$

$$\omega_1(t)=\begin{bmatrix}0\\ \frac{d_\xi(t)}{m}\end{bmatrix},\omega_2(t)=\begin{bmatrix}0\\ J^{-1}d_\eta(t)\end{bmatrix}.$$

When the quadrotor UAV flies, the perturbation of its parameters (such as: k_i, k_ξ, k_η) are within a certain range, so the related system matrices A_1, A_2, B_2 can be regarded as the interval matrices.

$$A_1^m \le A_1 \le A_1^M,$$
$$A_2^m \le A_2 \le A_2^M.$$

For any interval matrix $A\in[A^m, A^M]$, we can use the following description in [12].

$$A = A_0 + E_a\Sigma_a F_a, \Sigma_a \in \Sigma_a^*,$$

Where

$$A_0 = (A^m + A^M)/2;$$
$$E_a = [\sqrt{\chi_{11}}e_1,\cdots,\sqrt{\chi_{1j}}e_1,\cdots,\sqrt{\chi_{n1}}e_n,\cdots,\sqrt{\chi_{nn}}e_n];$$
$$F_a = [\sqrt{\chi_{11}}e_1,\cdots,\sqrt{\chi_{1j}}e_n,\cdots,\sqrt{\chi_{n1}}e_1,\cdots,\sqrt{\chi_{nn}}e_n]^T;$$
$$\Sigma_a^* = diag[\lambda_{11},\cdots,\lambda_{1n},\cdots,\lambda_{n1},\cdots,\lambda_{nn}];$$
$$|\lambda_{ij}|\le 1, i, j = 1,\cdots,n;$$
$$\chi_{ij} = (A_{ij}^m + A_{ij}^M)/2, i, j = 1,\cdots,n;$$

$e_i (i = 1,\cdots,n)$ is the column i of the unit matrix.

Obviously, $\forall\Sigma_a \in \Sigma_a^*, \Sigma_a^T\Sigma_a^* \le 1$

Based the above interval matrix description, the interval matrix A_1, A_2, B_2 are described as:

$$A_1 = A_{10} + E_w\Sigma_w F_w,$$
$$\Sigma_w \in \Sigma_w^*, A_2 = A_{20} + E_j\Sigma_j F_j, \Sigma_j \in \Sigma_j^*.$$

Thus, the error of position model and the error of angle model can be written as:

$$\dot{x}_w(t) = (A_{10} + E_w\Sigma_w F_w)x_w(t) + B_1(u(t) + f_w(t) + \omega_1(t)) \quad (4)$$
$$\dot{x}_j(t) = (A_{20} + E_j\Sigma_j F_j)x_j(t) + B_2(\tau(t) + f_j(t)) + B_{22}\omega_2(t) \quad (5)$$

Our object in this paper is to design the composite controllers $u(t)$ and $\tau(t)$ such that the error systems (4) and (5) are robust stable with the parameter perturbations and external disturbances. The control block diagram is shown in Figure 1.

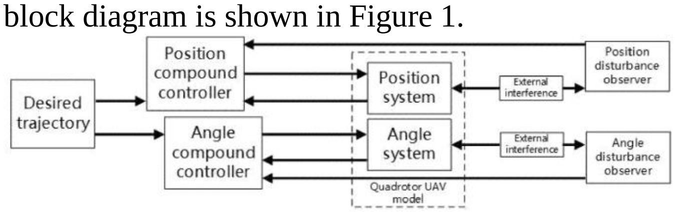

Figure 1: composite controller structure diagram II.

III. Composite Controller for Position Model*:*

A. Design of disturbance observer

For the disturbance $\omega_1(t)$, the following disturbance observer is designed:

$$\begin{cases}\dot{R}_1(t) = -N_1B_1[R_1(t) + N_1x_w(t)] - N_1[(A_{10} + E_w\Sigma_w \\ \quad F_w)x_w(t) + B_1(u(t) + f_w(t))] \\ \hat{\omega}_1(t) = R_1(t) + N_1x_w(t)\end{cases} \quad (6)$$

Where R_1 is state variable of disturbance observer, and N_1 represent the gain of disturbance observer, and $\hat{\omega}_1(t)$ is the estimated value of $\omega_1(t)$.

Defining the error of the disturbance and its estimated value as $e_1(t) = \hat{\omega}_1(t) - \omega_1(t)$, then the following error system can be obtained as:

$$\dot{e}_1(t) = -N_1B_1e_1(t) + \dot{\omega}_1(t) \quad (7)$$

It can be seen that the estimation error is related to $\dot{\omega}_1(t)$. If $\dot{\omega}_1(t)$ is too large, the errors will become larger. Then it will lose the aim of disturbance attenuation, therefore the disturbance observers only apply to the slow time-varying situations of $\omega_1(t)$.

B. Design of composite controller

Based on the above disturbance observer, the position composite controller is designed as

$$u(t) = K_1x_w(t) - \hat{\omega}_1(t) \quad (8)$$

Where K_1 is the gain of controller?

With the composite controller $u(t)$, we can combine equation (4) with equation (7) to obtain the following position system:

$$\dot{x}_g(t) = A_gx_g(t) + B_g\omega_g(t) \quad (9)$$

Where

$$x_g(t) = \begin{bmatrix}x_w(t)\\ e_1(t)\end{bmatrix}, \omega_g(t) = \begin{bmatrix}f_w(t)\\ \dot{\omega}_1(t)\end{bmatrix},$$
$$B_g = \begin{bmatrix}B_1 & 0\\ 0 & I\end{bmatrix}, A_g = \begin{bmatrix}(A_{10} + E_w\Sigma_w F_w) + B_1K_1 & B_1\\ 0 & -N_1B_1\end{bmatrix}$$

The reference output of the position system is

$$z_g(t) = C_gx_g(t) \quad (10)$$

Where, $C_g = [C_1 \quad C_2]$.

Then, the position system with equation (9) and equation (10) can be obtained as follows:

$$\begin{cases}\dot{x}_g(t) = A_gx_g(t) + B_g\omega_g(t)\\ z_g(t) = C_gx_g(t)\end{cases} \quad (11)$$

The following Theorem can be obtained for the position system.

Theorem 1. For a given scalar $r_1 > 0$, if there are a scalar $\alpha_1 > 0$, positive definite matrices $Q_1 > 0$, $P_2 > 0$, and any matrices M_1, M_2, such that the following LMI is hold:

$$\begin{bmatrix} \lambda_g & B_1 & B_1 & 0 & Q_1C_1^T & 0 & Q_1F_w^T \\ * & \nu_g & 0 & P_2 & 0 & C_2^T & 0 \\ * & * & -r_1^2 I & 0 & 0 & 0 & 0 \\ * & * & * & -r_1^2 I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -\alpha_1 I \end{bmatrix} < 0 \qquad (12)$$

Then, the position system (11) is asymptotically stable with r_1 -disturbance attenuation under the controller gain $K_1 = M_1 Q_1^{-1}$ and the observer gain $N_1 = P_2^{-1} M_2$.

Where

$\lambda_g = Q_1 A_{10}^T + A_{10} Q_1 + M_1^T B_1^T + B_1 M_1 + \alpha_1 E_w E_w^T$,

$\nu_g = -B_1^T M_2^T - M_2 B_1$.

Proof: For system (11), the following Lyapunov function is chosen

$$V(x_g(t)) = x_g^T(t) P_g x_g(t) \qquad (13)$$

Where $P_g = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$ with $P_1 > 0$ and $P_2 > 0$. By differentiating $V(x_g(t))$ along the trajectory of the system (11), we can obtain

$$\dot{V}(x_g(t)) = x_g^T(t) \Pi_1 x_g(t) \qquad (14)$$

Where

$$\Pi_1 = \begin{bmatrix} \lambda_1 & P_1B_1 \\ * & -(N_1B_1)^T P_2 - P_2 N_1 B_1 \end{bmatrix},$$

$$\lambda_1 = A_{10}^T P_1 + P_1 A_{10} + (E_w \Sigma_w F_w)^T P_1 + P_1 (E_w \Sigma_w F_w) + (B_1K_1)^T P_1 + P_1 B_1 K_1$$

For the uncertain terms, we can use the method in [12]. Then, the following inequality is hold

$$(E_w \Sigma_w F_w)^T P_1 + P_1 (E_w \Sigma_w F_w) \le \alpha_1 P_1 E_w E_w^T P_1 + \alpha_1^{-1} F_w^T F_w$$

Then, we can obtain that

$$\dot{V}(x_g(t)) \le x_g^T(t) \Pi_2 x_g(t) \qquad (15)$$

Where

$$\Pi_2 = \begin{bmatrix} \lambda_2 & P_1B_1 \\ * & -(N_1B_1)^T P_2 - P_2 N_1 B_1 \end{bmatrix},$$

$$\lambda_2 = A_{10}^T P_1 + P_1 A_{10} + (B_1K_1)^T P_1 + P_1 B_1 K_1 + \alpha_1 P_1 E_w E_w^T P_1 + \alpha_1^{-1} F_w^T F_w$$

If the inequality (12) holds, which means $\Pi_2 < 0$, Therefore, the position system (11) is asymptotically stable when $\omega_g(t) = 0$.

When $\omega_g(t) \ne 0$, we consider the following H_∞ performance index

$$J_w = \int_0^{+\infty} [x_g(s)^T x_g(s) - r_1^2 \omega_g^T(s) \omega_g(s)] d(t) \le \int_0^{+\infty} \begin{bmatrix} x_g(s) \\ \omega_g(s) \end{bmatrix}^T \Pi_3 \begin{bmatrix} x_g(s) \\ \omega_g(s) \end{bmatrix} d(t) \qquad (16)$$

Where

$$\Pi_3 = \begin{bmatrix} C_g^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} C_g & 0 & 0 \end{bmatrix} + \begin{bmatrix} \Pi_2 & P_g B_g \\ * & -r_1^2 I \end{bmatrix}$$

Defining $Q_1 = P_1^{-1}$ and combined with Schur complement theory, we can obtain the position system (11) is asymptotically stable with disturbance attenuation r_1 under the composite control algorithm. The proof is completed. II.

IV. Composite Controller for Angle Model

A. Design of disturbance observer

For the disturbance $\omega_2(t)$, the following disturbance observer is designed:

$$\begin{cases} \dot{R}_2(t) = -N_2 B_{22}[R_2(t) + N_2 x_j(t)] - N_2[(A_{20} + E_j \Sigma_j F_j) x_j(t) + B_2(\tau(t) + f_j(t))] \\ \hat{\omega}_2(t) = R_2(t) + N_2 x_j(t) \end{cases} \qquad (17)$$

Where R_2 is state variable of disturbance observer, and N_2 represent the gain of disturbance observer, and $\hat{\omega}_2(t)$ is the estimated value of $\omega_2(t)$.

Defining the error of the disturbance and its estimated value as $e_2(t) = \hat{\omega}_2(t) - \omega_2(t)$, then the following error system can be obtained as:

$$\dot{e}_2(t) = -N_2 B_{22} e_2(t) + \dot{\omega}_2(t) \qquad (18)$$

B. Design of composite controller

Based on the above disturbance observer, the position composite controller is designed as

$$\tau(t) = K_2 x_j(t) - B_2^{\dagger} \hat{\omega}_2(t) \qquad (19)$$

Where K_2 is the gain of controller.

With the composite controller $\tau(t)$, we can combine equation (5) with equation (18) to obtain the following position system:

$$\dot{x}_h(t) = A_h x_h(t) + B_h \omega_h(t) \qquad (20)$$

Where

$$x_h(t)=\begin{bmatrix} x_j(t) \\ e_2(t) \end{bmatrix}, \omega_h(t)=\begin{bmatrix} f_j(t) \\ \dot{\omega}_2(t) \end{bmatrix},$$

$$B_h=\begin{bmatrix} B_2 & 0 \\ 0 & I \end{bmatrix}, A_h=\begin{bmatrix} (A_{20}+E_j\Sigma_j F_j)+B_2K_2 & B_{22} \\ 0 & -N_2B_{22} \end{bmatrix}.$$

The reference output of the angle system is

$$z_h(t)=C_h x_h(t) \tag{21}$$

Where, $C_h=\begin{bmatrix} C_3 & C_4 \end{bmatrix}$.

The angle composite system is derived as follows:

$$\begin{cases} \dot{x}_h(t)=A_h x_h(t)+B_h\omega_h(t) \\ z_h(t)=C_h x_h(t) \end{cases} \tag{22}$$

The following Theorem can be obtained for the position system.

Theorem 2. For a given scalar $r_2>0$, if there are a scalar $\alpha_2>0$, positive definite matrices $P_3>0$, $P_4>0$, and any matrices M_3, M_4, such that the following LMI is hold:

$$\begin{bmatrix} \lambda_h & B_{22} & B_2 & 0 & Q_2C_3^T & 0 & Q_2F_j^T \\ * & \nu_h & 0 & P_4 & 0 & C_4^T & 0 \\ * & * & -r_2^2 I & 0 & 0 & 0 & 0 \\ * & * & * & -r_2^2 I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -\alpha_2 I \end{bmatrix}<0 \tag{23}$$

Then, the angle system (22) is asymptotically stable with r_2-disturbance attention under the controller gain $K_2=M_3Q_2^{-1}$ and the disturbance observer gain $N_2=P_4^{-1}M_4$.

Where

$$\lambda_h=Q_2A_{20}^T+A_{20}Q_2+M_3^TB_2^T+B_2M_3+\alpha_2E_jE_j^T,$$

$$\nu_h=-B_{22}^TM_4^T-M_4B_{22}.$$

Proof: For system (22), the following Lyapunov function is chosen

$$V(x_h(t))=x_h^T(t)P_hx_h(t) \tag{24}$$

where $P_h=\begin{bmatrix} P_3 & 0 \\ 0 & P_4 \end{bmatrix}$ with $P_3>0$ and $P_4>0$. By differentiating $V(x_h(t))$ along the trajectory of the system (22), we can obtain

$$\dot{V}(x_h(t))=x_h^T(t)\Pi_4x_h(t) \tag{25}$$

Where

$$\Pi_4=\begin{bmatrix} \lambda_3 & P_3B_{22} \\ * & -(N_2B_{22})^TP_4-P_4N_2B_{22} \end{bmatrix},$$

$$\lambda_3=A_{20}^TP_3+P_3A_{20}+(E_j\Sigma_jF_j)^TP_3+P_3(E_j\Sigma_jF_j)$$
$$+(B_2K_2)^TP_3+P_3B_2K_2$$

Similar to the method in inequality (15), the inequality can be obtained as

$$(E_j\Sigma_jF_j)^TP_3+P_3(E_j\Sigma_jF_j)\le\alpha_2P_3E_jE_j^TP_3+\alpha_2^{-1}F_j^TF_j$$

Then, we can obtain that

$$\dot{V}(x_h(t))\le x_h^T(t)\Pi_5x_h(t) \tag{26}$$

Where

$$\Pi_5=\begin{bmatrix} \lambda_4 & P_3B_{22} \\ * & -(N_2B_{22})^TP_4-P_4N_2B_{22} \end{bmatrix},$$

$$\lambda_4=A_{20}^TP_3+P_3A_{20}+(B_2K_2)^TP_3+P_3B_2K_2+\alpha_2P_3E_jE_j^TP_3$$
$$+\alpha_2^{-1}F_j^TF_j$$

From (23), it can be seen that $\Pi_5<0$, According to Lyapunov stability theory, the angle system (22) is asymptotically stable when $\omega_h(t)=0$.

When $\omega_h(t)\neq 0$,the following H_∞ performance index is considered

$$J_j=\int_0^{+\infty}[x_h(s)^Tx_h(s)-r_2^2\omega_h^T(s)\omega_h(s)]d(t)$$
$$\le\int_0^{+\infty}\begin{bmatrix} x_h(s) \\ \omega_h(s) \end{bmatrix}^T\Pi_6\begin{bmatrix} x_h(s) \\ \omega_h(s) \end{bmatrix}d(t) \tag{13}$$

Where

$$\Pi_6=\begin{bmatrix} C_h^T \\ 0 \\ 0 \end{bmatrix}[C_h \quad 0 \quad 0]+\begin{bmatrix} \Pi_5 & P_hB_h \\ * & -r_2^2I \end{bmatrix}$$

Defining $Q_2=P_3^{-1}$ and combined with Schur complement theory, we can obtain the position system (22) is asymptotically stable with disturbance attenuation r_2 under the composite control algorithm.

V. Simulation And Analysis:

The model parameters of the quadrotor UAV are shown in table Ⅲ.

TableⅢ: UAV model parameters

Parameter	***Parameter range***
m	$2.5kg$
l	$0.245m$
g	$9.8m/s$
c	0.11
J_1, J_2	$0.24kg\cdot m^2$
J_3	$0.01kg\cdot m^2$
k_i	$[0.01, 0.12]Ns/m$

Assuming that the starting position of the quadrotor UAV is the origin of the coordinates, the spiral is flying upwards and the simulation lasts for 50s. The external disturbance acting on the quadrotor UAV is white noise. UAV in the XY plane do a diameter of 8m circular motion, uniform 1m/s rise, the flight speed of 5m/s. In order to verify the effectiveness of the proposed method, the simulation study of the quadrotor UAV system is carried out in the Matlab environment. The system with and without the observer are simulated on the basis of the perturbation. The flight trajectory is shown in Figure 2, 3.

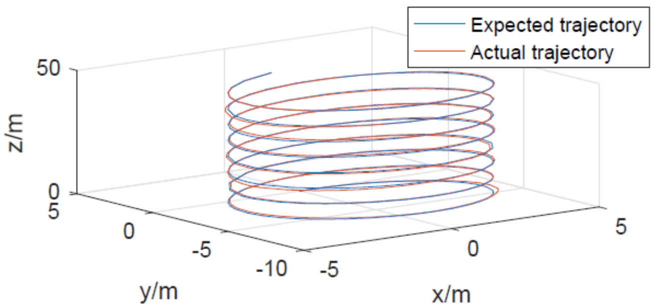

Figure 2: Trajectory diagram without disturbance observer

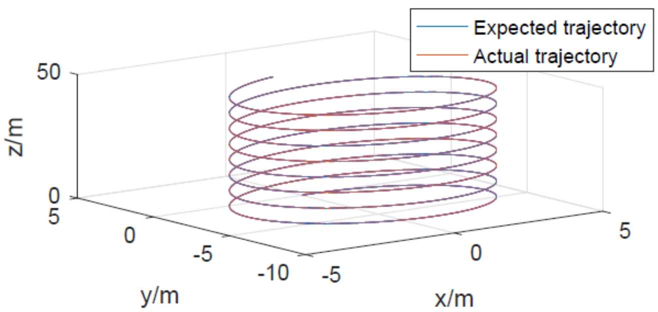

Figure 3: Trajectory diagram with disturbance observer

From the comparison of Figure2 and Figure3, it can be seen that the disturbance observer can effectively estimate the external bounded disturbance. The composite control method can track the desired trajectory well. The following comparison is from the various parameters. Position system with and without disturbance observer are shown in figure 4-7. The angle system with and without disturbance observer are shown in Figure 8-11.

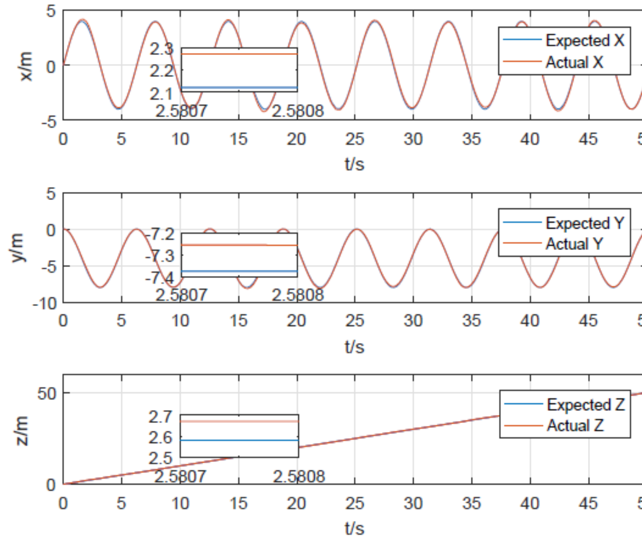

Figure 4: Position diagram without disturbance observer

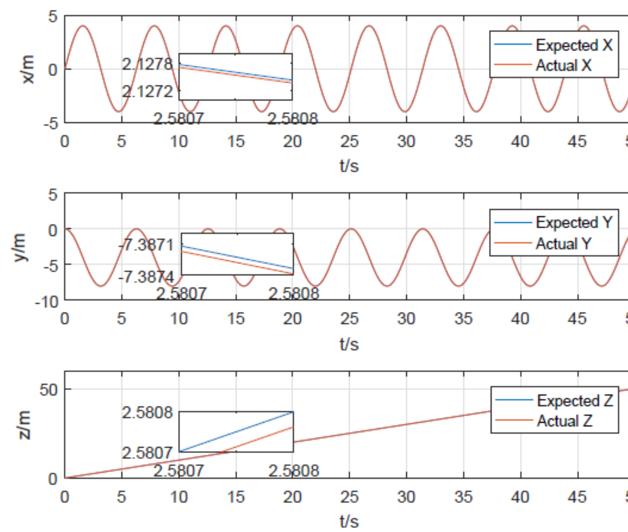

Figure 5: Position diagram with disturbance observer

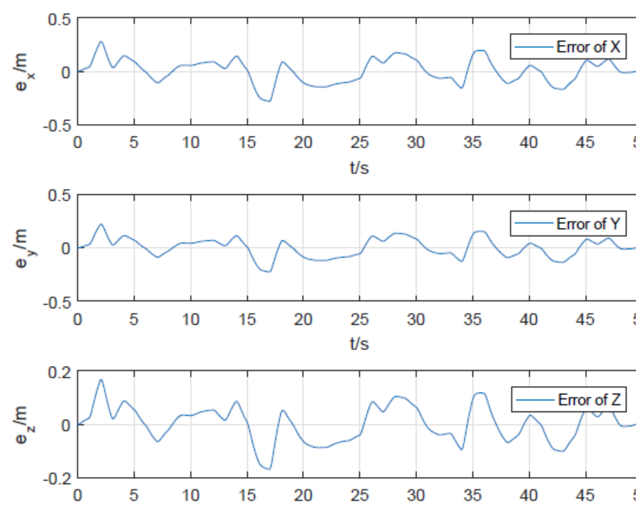

Figure 6: Position error of the observerwithout disturbance

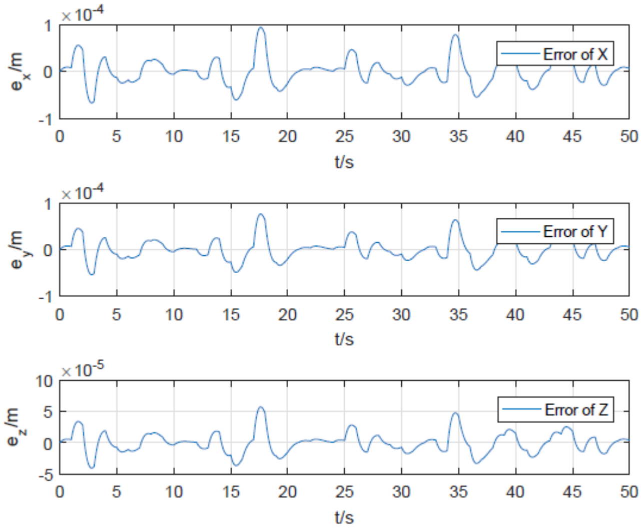

Figure 7: Position error of the observer with disturbance

It can be seen from Figure 4-7 that the position system with disturbance observer can estimate disturbance and compensate the error. Compared with the partial enlargement area, the disturbance observer significantly improves the tracking accuracy. Since the disturbance of the position system is generated by the same white noise, the waveform looks similar. The system error without disturbance observer can reach 0.25m, and the system error with interference observer is down to 0.09cm. Therefore, the combination of control and disturbance observer (DOBC) control can track the position of the quadrotor UAV better.

In this paper, the quadrotor UAV is decoupled into two independent control systems of position and angle. Therefore, it is necessary to consider the tracking effect of the angle.

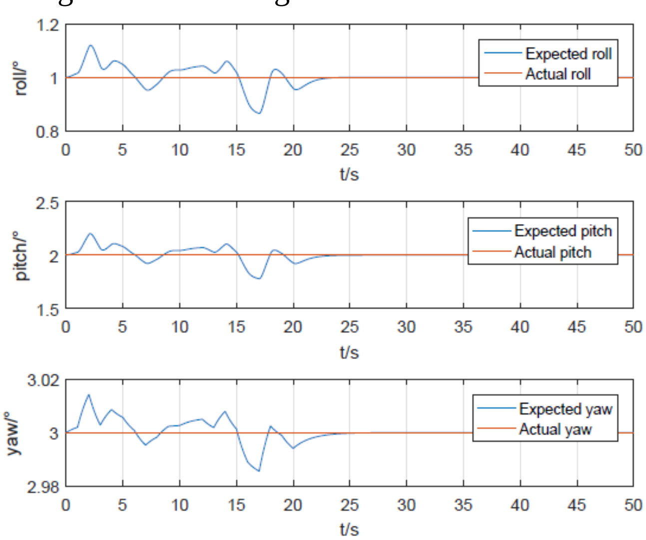

Figure 8: Angle diagram without disturbance observer

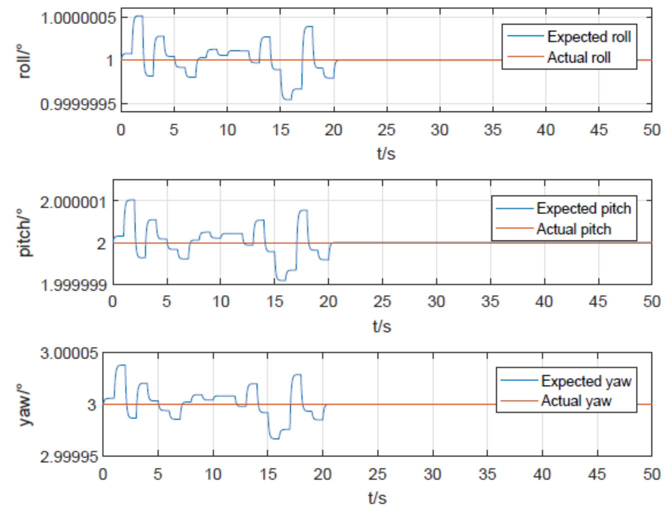

Figure 9: Angle diagram with disturbance observer

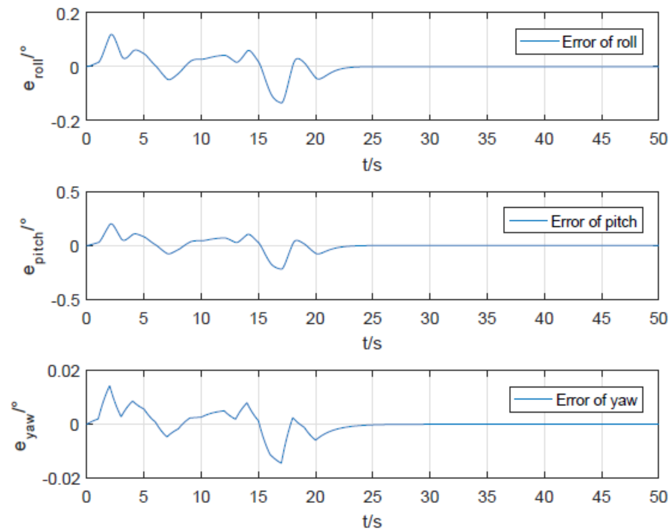

Figure 10: Error of angle without disturbance observer

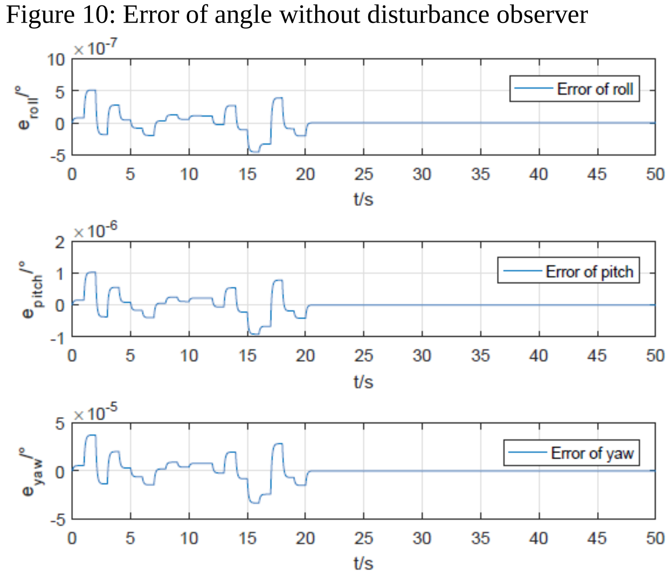

Figure 11: Error of angle with disturbance observer

It can be seen from Figure 8-11 that the error can be estimated accurately by disturbance observer and compensated by composite control. As the position module, the disturbance is also generated by white noise. The external disturbances occur at 0s-20s. The revocation of the disturbance after 20s. The angular error without disturbance observer is 0.2rad, and the system error with disturbance observer is down to 0.00005°. Therefore, the combination of robust H_{∞} control and disturbance observer control (DOBC) can track the angle of quadrotor UAV better.

VI. Conclusion:

The composite tracking control scheme is proposed for quadrotor unmanned aerial vehicle (UAV) with perturbation parameters and external disturbances. Firstly, the dynamic model of quadrotor UAV is decoupled into two subsystems, which are outer loop position control system and the inner loop angle control system, respectively. Secondly, by combining disturbance observer control (DOBC) with H_{∞} control, a composite tracking control is proposed. Finally, the simulation results show that the disturbance observer can be estimated the disturbances and the composite control scheme can compensated the error better than the case without disturbance observer. Thus, the accuracy of the trajectory tracking of the quadrotor UAV is improved effectively.

VII. References:

The heading of the References section must not be numbered. All reference items must be in 8 pt font. Please use Regular and Italic styles to distinguish different fields as shown in the References section. Number the reference items consecutively in square brackets (e.g. [1]).

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