An Enhanced Level Set Segmentation for gray Images Using Fuzzy Clustering and Lattice Boltzmann Method

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Abstract— *In the last decades, image segmentation has proved its applicability in various areas like satellite image processing, medical image processing and many more. In the present scenario the researchers tries to develop hybrid image segmentation techniques to generates efficient segmentation. Due to the development of the parallel programming, the lattice Boltzmann method (LBM) has attracted much attention as a fast alternative approach for solving partial differential equations. In this paper, first designed an energy functional based on the fuzzy c-means objective function which incorporates the bias field that accounts for the intensity in homogeneity of the real-world image. Using the gradient descent method, corresponding level set equations are obtained from which we deduce a fuzzy external force for the LBM solver based on the model by Zhao. This is a fast, robust method for denoising, independent to the position of the initial contour, effective in the presence of intensity in homogeneity, highly parallelizable and can segment objects with or without edges. assessment on medical and real-world images manifest the performance of the proposed method in terms of speed and efficiency. The work proposed in this paper concentrates on the gray level images.*

Key words: Fuzzy c-means (FCM), image segmentation, intensity inhomogeneity, lattice Boltzmann method (LBM), level set equation (LSE), partial differential equation (PDE).

I. INTRODUCTION

In computer vision, image segmentation [38]–[40] is a major and nontrivial task which aims to partition a given image into several regions or to detect an object of interest from the background. This task is more challenging that most of the actual imaging devices produce images corrupted by intensity inhomogeneity. The level set method (LSM) is a part of the whole family of active contour methods (ACMs). The key idea that started the level set fanfare was the Hamilton–Jacobi approach, i.e., a time-dependent equation for a moving surface. This was first done in the seminal work of Osher and Sethian [1]. In 2-D space, the LSM represents a closed curve in the

plane as the zero level set of a 3-D function Ø. For instance, starting with a curve around the object to be detected, the curve moves toward its inner normal and has to terminate on the boundary of the image. Two approaches are usually used to stop the evolving curve on the boundary of the desired object; the first one uses an edge indicator depending on the gradient of the image like in classical snakes and ACMs [2]–[5], [21], [31], and the second one uses some regional attributes to stop

the evolving curve on the actual boundary [22], [23], [32] where the authors extend the representative region-based level set from scalar to tensors by simultaneously taking into account the pixel's gray level and some local statistics such as gradient and orientation. The latter is more robust against noise and can detect objects without edges.

In addition, the Chan Vese (CV) method is not suitable for parallel programming because, at each iteration, the average intensities inside and outside the contour should be computed, which increases drastically the CPU time by increasing communications between processors. For this purpose, we propose a new method which tries to overcome the aforementioned drawbacks. Our method is based on a new idea which aims to stop the evolving curve according to the membership degree of the current pixel to be inside or outside of the active contour. This is done with the help of the modified fuzzy C-means (FCM) objective function obtained in [19] which also takes into consideration the shading image due to the intensity inhomogeneity.

In the LSM, the movement of the zero level set is actually driven by the level set equation (LSE), which is a partial differential equation (PDE). For solving the LSE, most classical methods such as the upwind scheme are based on some finite difference, finite volume or finite element approximations and an explicit computation of the curvature [20]. Unfortunately, these methods cost a lot of CPU time.

Recently, the lattice Boltzmann method (LBM) has been used as an alternative approach for solving LSE [12],

[14], [29], [36]. It can better handle the problem of time consuming because the curvature is implicitly computed and the algorithm is simple and highly parallelizable.

In this paper, the LBM is used to solve the LSE. The proposed method is based on the approach of the LBM PDE solver defined in [14]. In our proposed method, using a modified FCM objective function, we design a new fuzzy external force (FEF). The method is fast, robust against noise, and efficient whatever the position or the shape of the initial contour and can detect efficiently objects with or without edges. It has, first, the advantage of the FCM which gives it the latitude to stop the evolving curve according to the membership degree of the current pixel, second, the advantages of the LSM which allow it to handle complex shapes, topological changes, and different constraints on the contour smoothness, speed, size, and shape which are easily specified, and, third, the advantages of the LBM which make it very suitable for parallel programming due to its local and explicit nature.

II. BACKGROUND

The proposed method uses mainly two techniques belonging to different frameworks: the LSM and the LBM.

A. LSM

The LSM is a numerical technique for tracking interfaces and shapes. Using an implicit representation of active contours, it has the advantage of handling automatically topological changes of the tracked shape. In 2-D image segmentation, the LSM represents a closed curve as the zero level set of φ, called the level set function. The evolution of the curve starts from an arbitrary starting contour and evolves itself driven by the LSE which can be seen as a convection–diffusion equation

$$\frac{\partial\theta}{\partial t} + \vec{v}.\nabla\emptyset = b\Delta\emptyset \qquad \text{.....(1)}$$

Where $\nabla\emptyset$ and $\Delta\emptyset$ are the gradient and the Laplacian of $\emptyset$, respectively. The term $b\Delta\emptyset$ is called artificial viscosity (Sethian suggested replacing it with $bk|\nabla\emptyset|$ which is better for handling the evolution of lower dimensional interfaces [12]), and k is the curvature of the distance function $\emptyset$. The LSE can therefore be written as

$$\frac{\partial\theta}{\partial t} + \vec{v}.\nabla\emptyset = bk|\Delta\emptyset| \qquad \text{… (2)}$$

Being an alternative method for solving PDE, the LBM has several advantages, such as parallelizability and simplicity. In this paper, we use the D2Q9 LBM model to resolve the LSE in 2-D space.

B. LBM

The LBM is a numerical framework for modeling Boltzmann particle dynamics on a 2-D or 3-D lattice [13]. It was first designed to solve macroscopic fluid dynamics problems [14]. The method is second order accurate both in time and in Fig. 1. Spatial structure of the D2Q9 LBM lattice. space, and in the

limit of zero time step and lattice spacing, it yields the Navier Stokes equations for an incompressible fluid [15].
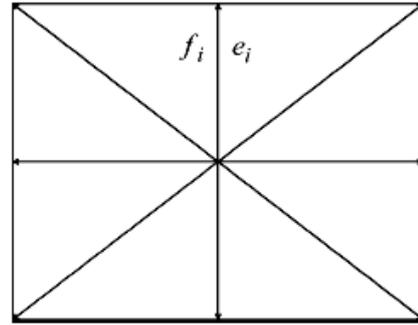

Fig. (1) Spatial structure of the D2Q9 LBM lattice.

The proposed method uses the D2Q9 (2-D with eight links with its neighbors and one link for the cell itself) LBM lattice structure. Fig. 1 shows a typical D2Q9 model. Each link has its velocity vector $e_i(\vec{r}, t)$ and the particle distribution $f_i(\vec{r}, t)$ that moves along this link, where $\vec{r}$ is the position of the cell, and t is the time. The LBM evolution equation can be written as follows using the Bhatnagar, Gross, and Krook collision model [7].

$$f_i(\vec{r} + \vec{e}_i, t + 1) = f_i(\vec{r}, t) + \frac{1}{\tau}\left[f_i^{\text{eq}}(\vec{r}, t) - f_i(\vec{r}, t)\right] \qquad \text{…(3)}$$

where τ represents the relaxation time determining the kinematic viscosity ϑ of the fluid by

$$\vartheta = \frac{1}{3}\left(\tau - \frac{1}{2}\right) \qquad \text{…(4)}$$

and $f$eq i is the equilibrium particle distribution defined as $f$eq

$$f_i^{\text{eq}}(\rho, \vec{u}) = \rho\left(A_i + B_i(\vec{e}_i \cdot \vec{u}) + C_i(\vec{e}_i \cdot \vec{u})^2 + D_i(\vec{u})^2\right) \qquad \text{…(5)}$$

Where Ai to Di are constant coefficients depending on the geometry of the lattice links and ρ and $\vec{u}$ are the macroscopic fluid density and velocity, respectively, computed from the particle distributions as

$$\rho = \sum_i f_i \quad \vec{u} = \frac{1}{\rho}\sum_i f_i\vec{e}_i. \qquad \text{… (6)}$$

For modeling typical diffusion computations, the equilibrium function can be simplified as follows [14]: $f$eq

$$f_i^{\text{eq}}(\rho, \vec{u}) = \rho A_i. \qquad \text{… (7)}$$

In the case of D2Q9 model, $Ai = 4/9$ for the zero link, $Ai = 1/9$ for the axial links, and $Ai = 1/36$ for the diagonal links. Now, the relaxation time τ is determined by the diffusion coefficient γ defined as

$$\gamma = \frac{2}{9}(2\tau - 1) \qquad \text{… (8)}$$

As shown in [14], LBM can be used to solve the parabolic diffusion equation which can be recovered by the Chapman–Enskog expansion

$$\frac{\partial\rho}{\partial t} = \gamma\nabla \cdot \nabla\rho. \qquad \text{… (9)}$$

In this case, the external force can be included as follows:

$$f_i \leftarrow f_i + \frac{2\tau - 1}{2\tau}B_i(\vec{F} \cdot \vec{e}_i) \qquad \text{… (10)}$$

Moreover, thus, (9) becomes

$$\frac{\partial \rho}{\partial t} = \gamma \nabla \cdot \nabla \rho + F.$$

… (11)

Replacing ρ by the signed distance function $\emptyset$, the LSE can be formed.

III. PROPOSED METHODOLOGY

This section details first the conception of the FCM-based energy function from which we deduce the corresponding LSE.

We then set the FEF. Moreover, finally, we implement the proposed method.

A. Energy Function Design

In the image segmentation context, the standard FCM algorithm is an optimization problem for partitioning an image of N pixels, X = {xi}$^N_{i=1}$,intoc classes. It aims to minimize a clustering criterion as [7]

$$J(U,V,X) = \sum_{k=1}^{c} \sum_{i=1}^{N} u_{ki}^p \|x_i - v_k\|^2$$

$$\text{s.t.} \sum_{k=1}^{c} u_{ki} = 1 \,\forall i \qquad 0 \le u_{ki} \le 1 \,\forall k, i$$

…(12)

Where U is the partition matrix whose element u_{ki} is the membership of the ith voxel for kth class. V is the centroid vector whose element v_k is the centroid (or prototype) of k$_{th}$ class. The parameter p, is the index for fuzzy, is a exponent for weighting on membership in each fuzzy set and determines the amount of "fuzziness" of the resulting segmentation. The norm operator $\|.\|$ represents the standardized Euclidean distance. The objective function J is decreased when high membership values are assigned to the pixels whose intensities are close to the centroid of its particular class and low membership values are assigned to the pixels whose intensities are far from the centroid. As done in [7], the bias field is incorporated into the FCM framework by modeling the observed image as follows:

$$Y_i = X_i G_i \,\forall i \in \{1, 2, \dots, N\}$$

…(13)

Where Yi, Xi, and Gi are the observed intensity, true intensity, and gain field at the ith pixel, respectively. N is the total number of pixels in the magnetic resonance image. The artifact can be modeled as an additive bias field by applying a logarithmic transformation to both sides of (13) [7], [8]

$$y_i = x_i + \beta_i \,\forall i \in \{1, 2, \dots, N\}$$

…(14)

Where yi and xi are the observed and true log-transformed intensities at the ith voxel, respectively, and ß is the bias field at the ith voxel. By incorporating the bias field model into an FCM framework, we will be able to iteratively estimate both the true intensity and the bias field from the observed intensity. By substituting (14) into (12), the clustering criterion to minimize in the presence of bias field becomes a constrained optimization problem.

$$J(U,V,B,Y) = \sum_{k=1}^{c} \sum_{i=1}^{N} u_{ki}^p \|y_i - \beta_i - v_k\|^2$$

$$\text{s.t.} \sum_{k=1}^{c} u_{ki} = 1 \,\forall i \qquad 0 \le u_{ki} \le 1 \,\forall k, i$$

…(15)

Where $Y = \{y_i\}_{i=1}^{N}$ is the observed image and $B = \{B_i\}_{i=1}^{N}$ is the bias field image. In a continuous form, the aforementioned criterion can be written as

$$J(U,V,B,Y)$$
$$= \sum_{k=1}^{c} \int_{\Omega_k} U_k^p(x,y) \|Y(x,y) - B(x,y) - v_k\|^2 \, dx$$

$$\text{s.t.} \sum_{k=1}^{c} U_k(x,y)$$
$$= 1 \,\forall x, y \qquad 0 \le U_k(x,y) \le 1 \,\forall k, x, y.$$

…(16)

Consider the two-phase level set although the method can be easily extended to more than two phases. The image domain Ω is segmented into two disjoint regions Ω1 and Ω2, i.e., c =2.In this case, we can introduce a level set function as follows:

$$J(U,V,B,Y,\phi)$$
$$= \int_{\Omega} U_1^p(x,y) \|Y(x,y) - B(x,y) - v_1\|^2 H(\phi) dx dy$$
$$+ \int_{\Omega} U_2^p(x,y) \|Y(x,y) - B(x,y) - v_2\|^2 (1 - H(\phi)) \, dx dy$$

$$\text{s.t.} \, U_1(x,y) + U_2(x,y)$$
$$= 1 \,\forall x, y \qquad 0 \le U_k(x,y) \le 1 \,\forall k, x, y$$

…(17)

Where $\emptyset$ is a signed distant function. The aforementioned term J(U, V, B, Y, $\emptyset$) is used as the data link in our energy functional which is defined as follows:

$$E(U,V,B,Y,\phi) = J(U,V,B,Y,\phi) + \nu|C|$$

…(18)

where v|C| is a regularization term with v>0 being a fixed parameter and C being a given curve which is represented implicitly as the zero level of $\emptyset$ and |C| is the length of C and can be expressed by the following equation [9]

$$|C| = \int_{\Omega} |\nabla H(\phi)| \, dx dy.$$

…(19)

B. LSE

As done in [10], to obtain the LSE, we minimize E(U, V, B, Y, $\emptyset$) with respect to f. For fixed U, V, and B, we use the gradient descent method

$$\frac{\partial \phi}{\partial t} = -\frac{\partial E}{\partial \phi}$$

…(20)

Where $\partial E/\partial f$ is the Gateaux derivative [11] of E. We obtain the following LSE:

$$\frac{\partial \phi}{\partial t} = \delta(\phi) \left(U_1^p(x,y) \|Y(x,y) - B(x,y) - v_1\|^2 \right.$$
$$\left. - U_2^p(x,y) \|Y(x,y) - B(x,y) - v_2\|^2 \right)$$
$$+ \nu \delta(\phi) \text{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right)$$

$$\text{s.t. } U_1(x,y) + U_2(x,y) = 1 \; \forall x,y$$

$$0 \le U_k(x,y) \le 1 \; \forall k,x,y. \qquad \dots(21)$$

However, for solving the minimization problem of E(U, V, B, Y, Ø), we should also compute the first derivatives of E(U, V, B, Y, Ø) with respect to u_{ki}, v_k, and β_i and set them equal to zero. We thus obtain three necessary conditions

$$U_k^*(x,y) = \frac{1}{\sum_{l=1}^{c} \left(\frac{\|Y(x,y) - B(x,y) - v_k\|}{\|Y(x,y) - B(x,y) - v_l\|} \right)^{\frac{2}{(p-1)}}} \qquad \dots(22)$$

$$v_k^* = \frac{\int_\Omega U_k^p(x,y)\,(Y(x,y) - B(x,y))\,dxdy}{\int_\Omega U_k^p(x,y)dxdy} \qquad \dots(23)$$

$$B^*(x,y) = Y(x,y) - \frac{\sum_{k=1}^{c} U_k^p(x,y)v_k}{\sum_{k=1}^{c} U_k^p(x,y)}. \qquad \dots(24)$$

C. Lattice Boltzmann Solver for LSE

By using the gradient projection method of Rosen [17], we can replace d(Ø) by $|\nabla \emptyset|$ in the proposed LSE, and as Ø is a distance function, we have $|\nabla \emptyset| = 1$[16], [20] and will stay at each step since an adaptive approach is not used and the distant field is valid in the whole domain [25]. Thus, the proposed LSE becomes

$$\frac{\partial \phi}{\partial t} = U_1^p(x,y)\|Y(x,y) - B(x,y) - v_1\|^2$$
$$- U_2^p(x,y)\|Y(x,y) - B(x,y) - v_2\|^2 + \nu \operatorname{div}(\nabla\phi)$$

$$\text{s.t. } U_1(x,y) + U_2(x,y) = 1 \; \forall x,y$$

$$0 \le U_k(x,y) \le 1 \; \forall k,x,y. \qquad \dots (25)$$

Replacing ρ by the signed distance function Ø, (11) becomes

$$\frac{\partial \phi}{\partial t} = \gamma \operatorname{div}(\nabla\phi) + F. \qquad \dots (26)$$

By setting the external force

$$F = \lambda \left(U_1^p(x,y)\|Y(x,y) - B(x,y) - v_1\|^2 \right.$$
$$\left. - U_2^p(x,y)\|Y(x,y) - B(x,y) - v_2\|^2 \right) \qquad \dots (27)$$

Where λ is a positive parameter; we can see that (25) is only a variational formula of (26) and, thus, can be solved by the LBM with the above-defined FEF. The choice of parameter p is at great importance for the segmentation result. Distinct values for p will result in the divergent results, as following.

i If p>2, then the exponent 2/(p - 1) in (22) decreases the membership value of the pixels that are closed to the centroid. The segmentation result will therefore be wrong since it is intuitively better that the membership value be high for those pixels who are closed to the centroid.

ii If p →∞, all the membership values tend to 1/c. This implies that the

$$\text{FEF} \to \lambda \left(\left(\frac{1}{c}\right)^p \|Y(x,y) - B(x,y) - v_1\|^2 \right.$$
$$\left. - \left(\frac{1}{c}\right)^p \|Y(x,y) - B(x,y) - v_2\|^2 \right) \to 0.$$

There is, therefore, no link with the image data in the LSM process. Therefore, segmentation is impossible.

iii If p → 1, the exponent 2/(p - 1) increases the membership values of the pixels who are closed to the centroid. As p → 1, the membership tends to one for the closest pixels and tends to zero for all the other pixels. This case is equivalent to the use of the k-means objective function instead of the FCM one. The segmentation is therefore rigid, and we lose the advantage of FCM over k-means. For all these reasons, a suitable choice of the parameter p can be the value of two, which is therefore used in all our experiments.

IV. IMPLEMENTATION

When using LBM to resolve the convection–diffusion equation, the particle density is set as ϕ which is a signed distance function. Since the particle number of the cell cannot be negative, we modify the distance function as $\emptyset' = \emptyset - min(\emptyset)$. The contour is those pixels which satisfy $\emptyset' = -min(\emptyset)$. The steps for the computation are outlined as follows.

i Initialize the distance function ϕ and class centroid values $v1$ and $v2$. Initialize B with zeros.

ii Compute $U_1^p(x,y)$ and $U_2^p(x,y)$ with (22).

iii Compute $v1$ and $v2$ with (23).

iv Compute B with (24).

v Compute the external force with (27).

vi Include the external force based on (10).

vii Resolve the convection–diffusion equation with LBM with (3).

viii Accumulate the $fi(\vec{r},t)$ values at each grid point by (6), which generates an updated distance value at each point.

ix Find the contour.

x If the segmentation is not done, increase the value of λ and go back to step 5).

We should notice that the B obtained from (24) is a "residual" image but not necessarily the bias field image [7].

V. EVALUATION OF THE PROPOSED METHOD

In order to objectively measure the quality of the segmentations produced; three evaluation measures are considered in this paper. The first one is the Probabilistic Rand Index (PRI, [21]). This index compares results obtained from the tested algorithm to a set of manually segmented images. Since there is not a single correct output, considering multiple results allows to enhance the comparison and to take into account the variability of human perception.

The PRI is based on a soft non uniform weighting of pixel pairs as a function of the variability in the ground-truth. The ground-truth set is defined as $\{G_1, G_2, \cdots, G_L\}$ where L is the number of manually segmented images. Let S be the segmentation provided by the tested algorithm, l_i^{Gk} the label of pixel $\mathbf{x}_i$ in the k-th manually segmented image and l_i^S the label of pixel $\mathbf{x}i$ in the tested segmentation. Then, PRI is defined by

$$PR(S, G_k) = \frac{2}{N(N-1)} \sum_{i,j,i<j} \left(p_{ij}^{c_{ij}} (1 - p_{ij})^{1-c_{ij}} \right)$$

Where N is the number of pixels, c_{ij} is a Boolean function denoting if l_i^S is equal to l_j^{Gk}, and p_{ij} is the expected value of a Bernoulli distribution for the pixel pair. The PRI metric is in the range [0, 1], where high values indicate a large similarity between the segmented images and the ground-truth.

The second one is the Variation of Information (VOI, [15]). The VOI metric measures the sum of information loss and gain between two clustering belonging to the lattice of possible partitions. It is defined by

$$VOI(S, G_k) = H(S) + H(G_k) - 2I(S, G_k)$$

Where H is the entropy$-\sum_{i=1}^{c} \frac{n_i}{n} log \frac{n_i}{n}$, n_i being the number of points belonging to the ith cluster. The term I is the mutual information between two clustering, and it is defined by

$$I(S, G_k) = \sum_{i=1}^{c} \sum_{j=1}^{c} \frac{n_{i,j}}{n} \log \frac{n_{i,j}}{n} \frac{n_i}{n} \frac{n_j}{n}$$

Where $n_{i,j}$ is the number of points in the intersection of cluster i of S and j of G_k. The VOI measure is a distance, therefore the smaller it is, the closer the segmentation obtained and the ground-truth are.

The Global Consistency Error (GCE [14]) evaluates to what extent a segmentation can be viewed as the refinement of the other. A measure of error at each pixel $\mathbf{x}_i$ is defined by

$$E(S, G_k, \mathbf{x}_i) = \frac{|R(S, \mathbf{x}_i) \backslash R(G_k, \mathbf{x}_i)|}{|R(S, \mathbf{x}_i)|}$$

Where $|.|$ is the cardinality, $\backslash$ is the set difference, and $R(S, \mathbf{x}i)$ is the set of pixels corresponding to the region in segmentation S that contains the pixel $\mathbf{x}i$. The GCE measure, which forces all local refinements to be in the same direction, is then defined by

$$GCE(S, G_k) = \frac{1}{n} \min \left(\sum_{i=1}^{n} E(S, G_k, \mathbf{x}_i), \sum_{i=1}^{n} E(G_k, S, \mathbf{x}_i) \right)$$

The closer GCE is to zero, the better the segmentation S with respect to the ground-truth G_k.

Now in the remaining part of this section we will present the results obtained from the developed work. For the proper evaluation of the developed method three images have been used for example figure (5.1) and figure (5.2) shows the first input image and its segmentation using proposed work. Table 1 contains the three parameter values obtained after segmentation using proposed method.
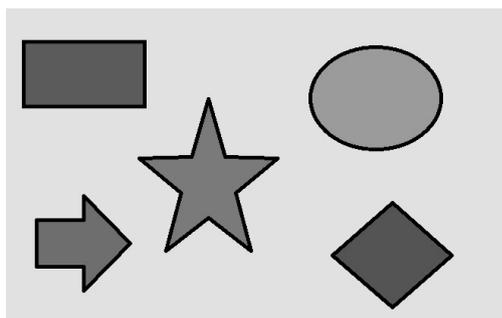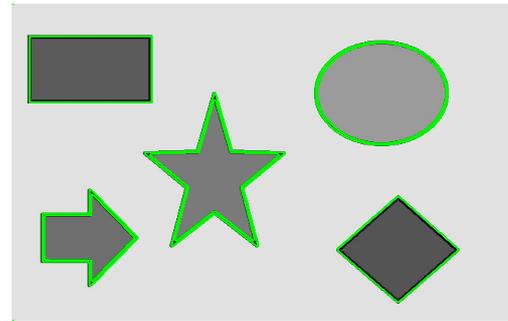

Figure (5.1) First Input Image

Figure (5.2) Segmented Image
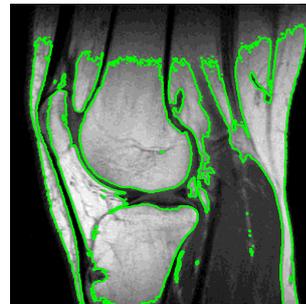

Figure (6.1) Second Input Image

Figure (6.2) Segmented Image
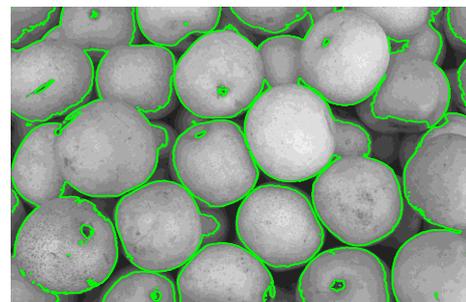

Figure (7.1) Third Input Image

Figure (7.2) Segmented Image

Table (1)

S. No.	Input Image	PRI	VOI	GCE

1	First	0.857307	0.584679	0.092926
2	Second	0.878131	0.567029	0.109219
3	Third	0.79065	0.779141	0.142078

VI. EXPECTED OUTCOME

This project work will brought forward a new hybrid frame work for multichannel image object segmentation. On the basis of nature of segmentation parameters we expects this proposed frame work will leads to an efficient solution for object segmentation for color images.

VII. CONCLUSION AND FUTURE WORK

This paper presents a level set image segmentation method based on the idea of stopping the evolving contour according to the degree of membership of the active pixel to be inside or outside of this evolving contour. It is done with the help of the FCM partition matrix. The LSE is solved by using the powerful, simple, and highly parallelizable LBM which allows the method to be a good candidate for GPU implementation. The method gives promising results. Experimental results on medical and real-world images have demonstrated the good performance of the proposed method in terms of PRI,VOI and GCE. It presents a fast and efficient comprehensive implementation for gray image segmentation . It aims to aims maximize PRI value where as to minimize VOI and GCE value. According to an extensive comparison with state of the art segmentation methods, this approach gives satisfactory results. The PRI, VOI, GCE value calculated for different images is found to be in a specific desired range which shows the successful implementation of the method. Future works can be an implementation of the proposed method for color images in order to fully take advantage of the LBM. GCE and VOI can be reduced more to segment the image more accurately.

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