

# A Study on Weibull Distribution for Estimating the Reliability

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**Abstract:** *In this paper we estimate reliability of a system using Weibull distribution. Reliability is defined as when a product does not fail in an expected lifetime in specified conditions. In reliability applications Weibull Distribution is commonly used distribution. The Weibull distribution is a generalization of the exponential distribution. The two-parameter Weibull distribution can accommodate increasing, decreasing, or constant failure rates because of the great flexibility expressed through the model's parameter. This distribution is recognized today as industrial standard. This paper presents the maximum likelihood method for estimating the Weibull distribution parameters. Weibull distribution is widely used in engineering applications. Weibull distribution is analysis with the Weibull Data Plot depict failure analysis with extremely small samples. The Data plot is very helpful for engineers and managers.*

**Keywords:** Reliability, Weibull distribution, Parameter estimation, Maximum likelihood method, Weibull probability plot..

## 1. Introduction

With the development of new technologies and techniques the demand of bug free software increasing ,the software undergo extensive testing and debugging which is very costly and time consuming, and companies demand accurate information how software reliability grows. Thus Weibull distribution is widely used in theory of reliability. This distribution was named by Swedish scientist Waloddi Weibull (1887-1979), who developed it in 1937 and published it in 1951. Much type of failure rate behaviors can be represented by Weibull distribution. It is very flexible and reliable in many practical applications. Reliability can be estimated using two or three parameter (shape, scale and location) Weibull distributions. In this we estimate the reliability using two parameter Weibull distributions. Dolas et.al [1]describes Graphical methods and analytical methods for estimating the values of these parameters. Parameters estimating using Graphical methods like Weibull probability plotting and hazard plot are not very accurate but they are fast. But analytical methods like maximum likelihood method, least square method etc are considered more accurate. This distribution is popular among engineers because it has the ability to assume the characteristics of many distributions.

Ivana et.al [2]introduced the four methods for estimating the Weibull distribution parameters: the least square method (LSM), the weighted least square method (WLSM), the maximum likelihood method (MLM) and the method of

moments (MOM) and compare their performance using the Monte Carlo simulation. The maximum likelihood method is most commonly used method for estimating the parameters of a distribution and commonly used by researchers. Li et.al [7] used mean square error to compare the least square method, the

weighted least square method, the maximum likelihood method and the method of moments. Estimation method used Monte Carlo simulation to obtain their statistical properties. An Advanced algorithm for maximum likelihood estimation is given by A Yang et.al. [3].Wind Energy has been expressed by Weibull distribution function and two analytical methods for calculation of parameters are given by Paritosh [4]. Distribution of wind speeds of wind power stations is most commonly application of the Weibull distribution. In[8], three different models are used for calculating the shape and scale parameters of the Weibull distribution. To determine which model is best suited to design wind conversion machines, in the Nigerian locations. The method of moments is an appropriate method for estimating. The failure probability of certain components can be derived from the Weibull plot [5].

There are many engineering problems that can be solved with Weibull analysis. Examples of problem solved are: [6]to order spare parts and schedule maintenance labor, how many units will be returned to depot for overhaul for each failure mode month-by-month next year? The program manager wants to be 95% confident that he will have enough spare parts and labor available to support the overall program. Or a project engineer reports three failures of a component in service operations during a three-month period. The Program Manager asks, "How many failures will we have in the next quarter, six months, and year?" What will it cost? What is the best corrective action to reduce the risk and losses? Thus Weibull analysis proactively manage spare parts inventories, compare reliability or prediction about product life.

Kuang Chu et.al [10] calculate sample means and the sample root mean square errors for simulate three types of Weibull distribution constant failure rate, increasing failure rate and decreasing failure rate. The performance of the maximum likelihood method and least squares method is understand by comparison between these two computation approaches.

Muhammad et. al [11] proves how Weibull distribution is flexible to different distribution. They also study the life time of the electrical and mechanical components with the help of Weibull Quantile modeling. Reliability and Quantile analysis of the Weibull distribution is introduced. Quantile models are also presented graphically and mathematically. Existing in literature there are many computation approaches to estimate the values of the parameters. Also comparison between different approaches has been introduced. The main purpose of Weibull distribution is used to define reliability and unreliability. Weibull Plot is very useful in determining the life of the components or their failure rates. Weibull Plot allow the user to determine that the proposed distribution fits the data it do not give confidence limits

## 2. Weibull Distribution in Weibull Failure Class Models Model

A large number of models have been derived from the two-parameter Weibull distribution and Referred as Weibull Models. Weibull and gamma are two types of failure time classes models. In this we take the per-fault failure distribution to be the traditional Weibull and gamma distributions, respectively rather than the exponential distribution. In short Weibull model is statistical rather than exponential. Weibull distribution has great flexibility given for failure modeling because of the shape and scale parameters that define them. Hardware failure used this distribution. Weibull model is one of the most widely used models for hardware reliability modeling in the Weibull distribution. It can accommodate increasing, decreasing, or constant failure rates because of the great flexibility expressed through the model's parameter.

Weibull model is based on the following assumptions:

- the number of faults detected in each interval is independent for any finite collection of times.
- There is fixed number of faults(N) in the software at the beginning of the time in which the software is observed.
- The time to fault of failure is distributed as a Weibull distribution with parameters  $\alpha$  and  $\beta$ .
- A fault is corrected instantaneously without introducing new faults in the software.

The Data requirements to implement this fault count model are: (a) The number of fault on each testing interval i.e.,  $f_1, f_2, \dots, f_n$ . (b)The Completion time of each period that the software is under observation: i.e.,  $X_1, X_2, \dots, X_n$ . The Weibull distribution can be used in Weibull Model.

The probability density function  $f(x)$  of the Weibull distribution  $W(\beta, \alpha)$  with parameters  $\beta > 0$  and  $\alpha > 0$  is given as:

$$f(x) = \frac{\beta}{\alpha} \times \left(\frac{x}{\alpha}\right)^{\beta-1} \times e^{-\left(\frac{x}{\alpha}\right)^\beta} \quad (1)$$

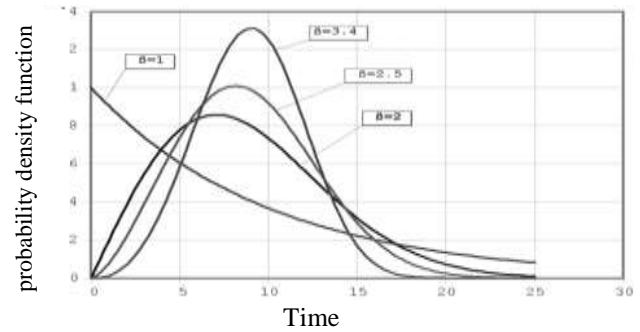


Figure 1: The Weibull probability density function

Where  $x > 0, \beta$  is the shape parameter, and  $\alpha$  is the scale parameter or the characteristic life at which 63.2% of the population can be expected to have failed. Probability density Function  $f(x)$  determines how many parts have failed or will fail up to a defined running time. Figure 1 shows the shape of Weibull probability density function with value of  $\alpha=10$  and  $\beta=(1, 2, 2.5, 3.4)$

The cumulative distribution function  $F(x)$  of the Weibull distribution is given as:

$$F(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^\beta} \quad (2)$$

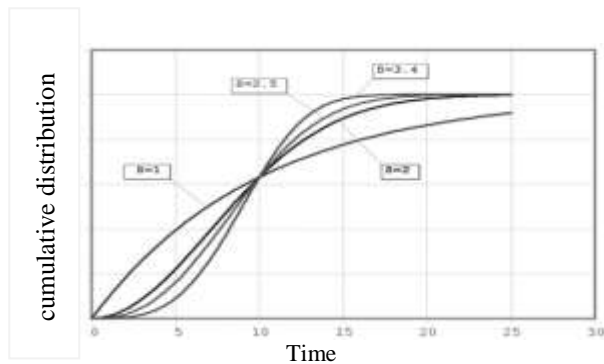


Figure 2: The Weibull cumulative distribution function

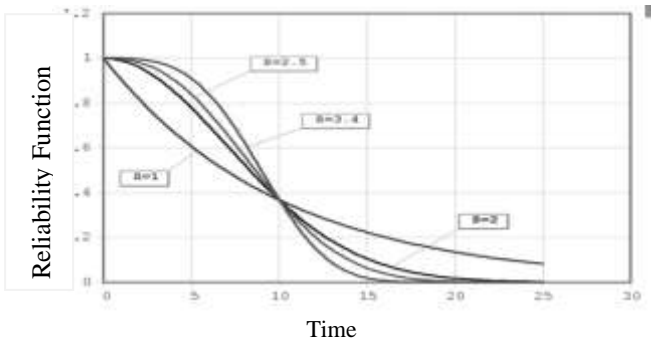
Where  $x > 0$ , the constant  $\alpha$  is called the scale parameter [9], because it scales the  $x$  variable and  $\beta$  is called the shape parameter, because it decides the shape of the rate function. The constant  $x$  is called lifetime variable.

Figure 2 shows the Weibull cumulative distribution function with value of  $\alpha=10$  and  $\beta=(1, 2, 2.5, 3.4)$ . All curves intersect at one point called the characteristic point.

Reliability  $R(x)$  determines how many parts are still in use after a certain running time. The reliability function is probability in which system performs its intended function over a specific period of time.

The two-parameter Weibull distribution reliability is given as:

$$R(x) = e^{-\left(\frac{x}{\alpha}\right)^\beta} \quad (3)$$



**Figure 3:** The Weibull Reliability Function

Figure 3 shows the Weibull Reliability function with value of  $\alpha=10$  and  $\beta=(1, 2, 2.5, 3, 4)$ .

The Weibull failure rate function  $\lambda(x)$  is defined as the number of failures per unit time that can be expected to occur for the product. It is given as:

$$\lambda(x) = \frac{\beta}{\alpha} \times \left(\frac{x}{\alpha}\right)^{\beta-1} \quad (4)$$

The mean life is the average time of failure-free operation up to a failure event. The MTTF of the Weibull Probability density Function is given as:

$$MTTF = \Gamma\left(\frac{1}{\beta} + 1\right) \quad (5)$$

### 3. Estimation of Parameter

The most commonly method used method to estimate the parameters are the least square method (LSM), the weighted least square method (WLSM), the maximum likelihood method (MLM) and the method of moments (MOM). All these methods are the analytical methods. In this paper we used the maximum likelihood method which is used by researchers to estimate parameters of Weibull distribution. The estimation of parameters can be obtained only numerically [2]. Consider Weibull random variables  $X_1, X_2, \dots, X_n$  are independent and identically distributed. Probability density function is given in equation (1). The likelihood function of  $X_1, X_2, \dots, X_n$  can be constructed from equation (1) as:

$$P(\beta, \alpha) = \prod_{i=1}^n f(x_i, \beta, \alpha) = \prod_{i=1}^n \frac{\beta}{\alpha^\beta} x_i^{\beta-1} e^{-\left(\frac{x_i}{\alpha}\right)^\beta} \quad (6)$$

Now take natural logarithmic on both the sides, we

$$\ln P(\beta, \alpha) = n \ln \beta - n\beta \ln \alpha - \frac{1}{\alpha^\beta} \sum_{i=1}^n x_i^\beta + (\beta - 1) \sum_{i=1}^n \ln x_i \quad (7)$$

Now differentiating equation (7) with respect to  $\beta$  and  $\alpha$

Equating to zero we obtain following equations (8) and (9):

$$\frac{\partial \ln P(\beta, \alpha)}{\partial \alpha} = -\frac{n\beta}{\alpha} + \frac{\beta}{\alpha^{\beta+1}} \sum_{i=1}^n x_i^\beta = 0 \quad (8)$$

$$\frac{\partial \ln P(\beta, \alpha)}{\partial \beta} = \frac{n}{\beta} \ln \alpha - \frac{\sum_{i=1}^n x_i^\beta \ln x_i - \ln \alpha \sum_{i=1}^n x_i^\beta}{\alpha^\beta} + \sum_{i=1}^n \ln x_i = 0 \quad (9)$$

On simplifying the above equations (8) and (9) we obtain the equations:

$$\alpha = \left(\frac{1}{n} \sum_{i=1}^n \ln x_i\right)^{\frac{1}{\beta}} \quad (10)$$

$$\frac{1}{\beta} - \frac{\sum_{i=1}^n x_i^\beta \ln x_i}{\sum_{i=1}^n x_i^\beta} + \frac{1}{n} \sum_{i=1}^n \ln x_i = 0 \quad (11)$$

The estimate of the parameter  $\alpha$  is obtained in equation (10). The estimate of the parameter  $\beta$  can be obtained by solving the equation (11) with respect to  $\beta$ . The equation (11) must be solved numerically not analytical. This parameter is estimated using Newton method.

## 4. Weibull Analysis and Results

Weibull analysis helps to determine life of software by fitting a statistical distribution. Weibull analysis is useful in graphical plot of the failure data. The data plot is extremely important to the engineer and to the manager. The Weibull data plot is particularly informative as Weibull pointed out in his 1951 paper [6]. Figure 6 is a typical Weibull plot. Weibull probability plot is generated to see how well random Weibull data points are actually fit by a Weibull distribution. The Weibull Plot is a plot of  $F(x)$  (cumulative distribution function) vs. time. Note log scale used is base 10. Reliability function  $R(x)$  is also known as survivor function [11].

Weibull parameter  $\beta$  is the slope of the Weibull plot determines which member of the family of Weibull failure distributions best fits or describes the data. [6] The slope,  $\beta$ , also indicates which class of failures is present. Shape parameter  $\alpha$  which is nothing else than the slope of the line on the linearized Weibull graph

### 4.1 Effects of the Shape Parameter $\beta$

#### 4.1.1 Weibull Distribution with $\beta$ Less Than 1

In early stage of software failures occur frequently called infant mortality (premature failures). The data that fit this distribution have initial high number of failures, which decrease over time as the defective items are eliminated from the sample. The probability decreases exponentially from infinity.

#### 4.2.2 Weibull Distribution with $\beta$ Equal to 1

Failure rate over the time remain consistent called random failures. This shape model the useful life of products. Figure 1 shows the distribution decreases exponentially from  $1/\alpha$  scale parameter.

#### 4.2.3 Weibull Distribution with $\beta$ between 1 and 2

Failure Rate shows when  $\beta$  increase quickly, and then decrease with time. This shape indicates early wear-out failures.

#### 4.2.4 Weibull Distribution with $\beta$ Equal to 2

When Risk of wear-out failure increases steadily over the product lifetime this form of distribution is also called Rayleigh distribution. Figure 1 shows  $\beta = 2$ .

#### 4.2.5 Weibull Distribution with $\beta$ between 3 and 4

When software is in final period of product life most failure happen. The bell-shaped is formed like normal curve.

### 4.2 Case Study

We calculate now probability density function  $f(x)$ , cumulative distribution function  $F(x)$  and reliability  $R(x)$  of the component for an operating time of 1000 hours. The failure time of certain Component has a Weibull distribution with Weibull parameters are:

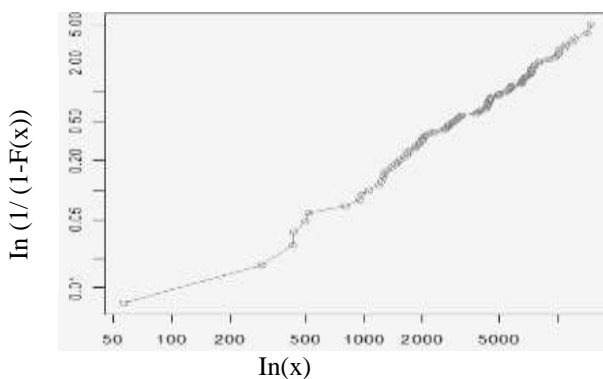
$$x = 1000 \quad \beta = 1.5 \quad \alpha = 5000$$

$$\text{Reliability} = R(x) = e^{-\left(\frac{x}{\alpha}\right)^\beta} = e^{-\left(\frac{1000}{5000}\right)^{1.5}} = 0.91444$$

$$\begin{aligned} \text{Cumulative density function} = F(x) &= 1 - e^{-\left(\frac{x}{\alpha}\right)^\beta} \\ &= 1 - e^{-\left(\frac{1000}{5000}\right)^{1.5}} = 0.08556 \end{aligned}$$

$$\begin{aligned} \text{Failure rate} = \lambda(x) &= \frac{\beta}{\alpha} \times \left(\frac{x}{\alpha}\right)^{\beta-1} = \frac{1.5}{5000} \times \left(\frac{1000}{5000}\right)^{1.5-1} \\ &= 0.0001345086 \end{aligned}$$

$$\begin{aligned} \text{Probability density function} &= R(x) \times \lambda(x) \\ &= 0.91444 \times 0.0001345086 = 0.000123 \end{aligned}$$



**Figure 4:** Weibull probability plot[10]

The cumulative distribution function or reliability function of the two parameter Weibull distribution is :

$$F(x) = Q(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^\beta}$$

linearize this function into  $y = mt + c$

$$\begin{aligned} Q(x) &= 1 - e^{-\left(\frac{x}{\alpha}\right)^\beta} \\ \ln(1 - Q(x)) &= \ln\left(e^{-\left(\frac{x}{\alpha}\right)^\beta}\right) \\ \ln(1 - Q(x)) &= -\left(\frac{x}{\alpha}\right)^\beta \end{aligned}$$

$$\ln(-\ln(1 - Q(x))) = \beta \ln\left(\frac{x}{\alpha}\right)$$

$$\ln\left(\ln\left(\frac{1}{1-Q(x)}\right)\right) = \beta \ln(x) - \beta \ln(\alpha)$$

$$y = \ln\left(\ln\left(\frac{1}{1-Q(x)}\right)\right)$$

$$\text{or, } y = \beta t - \beta \ln(\alpha)$$

The X-axis simply shows  $\ln(x)$ . The Y-axis is  $\beta t - \beta \ln(\alpha)$  which is complicated. Now X-axis and Y-axis of the Weibull Probability Plot shown in Figure.4 is constructed. Weibull probability is one of the method to calculating the parameters of the Weibull distribution. Probability plot shows how well data fit by a Weibull distribution. Figure 4 show probability plot time to failure data. Today there are many software programs that are available in the market to analyze reliability data. If points makes a straight line then data follows Weibull distribution. Otherwise, we consider data is following another lifetime distribution.

## 5. Conclusion

This work has proposed a method of estimating the reliability using Weibull distribution. We have also study different distributions fit into Weibull distribution when shape parameter changes. The reliability of the component is 0.91444. Estimation of parameter is done by the maximum likelihood method which is mostly used by researchers. The approximation of functions was taken with Weibull parameter are  $\beta = 1.5$  and  $\alpha = 5000$ . Weibull analysis tells which specific failure distribution is following by the failure data. As the software component increases day by day complexity of the system also increases which leads an increase in failure mechanism. This analysis also help us to determine how items are failing.

## References

- [1] D.R.Dolas, M.D. Jaybhaye, S.D. Deshmukh, "Estimation the System Reliability using Weibull Distribution," IPEDR. 2014. V75. 29.
- [2] Ivana Pobočiková and Zuzana Sedláčková, "Comparison of Four Methods for Estimating the Weibull Distribution Parameters," Applied Mathematical Sciences, Vol. 8, 2014, no. 83, 4137 - 4149 HIKARI Ltd, www.m-hikari.com <http://dx.doi.org/10.12988/ams.2014.45389>.
- [3] J. Dai, A. Yang, and Z. Yang, "An Advanced Algorithm for Equations of Maximum Likelihood Estimation," Applied Mathematics A Journal of Chinese Universities (Ser. A), 03, 2009
- [4] Paritosh Bhattacharya, "A Study on Weibull Distribution for Estimating the Parameters," Journal applied quantitative methods, 2010, Vol.5 No.2, pp-234-241.
- [5] K. Weibull © Curt Ronniger 2012, "Reliability analysis with Weibull" [www.crgaph.com](http://www.crgaph.com).
- [6] Dr. Robert B. Abernethy, "The New Weibull Handbook, Chapter 1 An Overview Of Weibull Analysis," • 536 Oyster Road, North Palm Beach, FL 33408-4328 • 561-842-4082.

- [7] D. Wu, J. Zhou, Y. Li., "Methods for estimating Weibull parameters for brittle materials," Journal of Material Science, 41(2006), 5630-5638.
- [8] F.C.Odo and G.U.Akubue, "Comparative Assessment of Three Models for Estimating Weibull Parameters for Wind Energy Applications in a Nigerian Location," International Journal of Energy Science 2012 Vol.2 No.1, PP.22-25.
- [9] Yunn-Kuang Chu and Jau-Chuan Ke., "Computation Approaches For Parameter Estimation Of Weibull Distribution," Mathematical and Computational Applications, Vol. 17, No. 1, pp. 39-47, 2012
- [10] [Online]. Available: <http://www.itl.nist.gov/div898/handbook/apr/section1/apr162.htm>
- [11] Muhammad Sahib Khan, Ghulam Rasul Pasha and Ahmed Hesham Pasha., "Reliability and Quantile Analysis of the Weibull distribution" ISSN 1684-8403 Journal of Statistics Volume 14, 2007, pp. 32-52