www.ijecs.in
International Journal Of Engineering And Computer Science ISSN:2319-7242
Volume 6 Issue 4 April 2017, Page No. 20957-20962
Index Copernicus value (2015): 58.10 DOI: 10.18535/ijecs/v6i4.29

An Overview of Theoretical Models of Exchange Bias Mechanism

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Abstract: *The research activities on the phenomenon of exchange bias (EB) has recently promulgated due to its versatile use in magnetic sensors and as stabilizers in magnetic reading heads. Since the discovery of EB in 1956, the attention on this matter was limited until such applications related to giant magnetoresistance were developed during late 90's. This study primarily focuses on the understanding of the phenomenon of EB by reviewing and critically discussing the various theoretical models developed and refined over the years. Relevant characteristics of the bias, coercivity fields, the sensitivity and dependence of the fields to interface structure and quality, long-term stability for the magnitude of the bias field and the orientation of the ferromagnet are discussed in the light of these theoretical models.*

Keywords: Exchange bias, Interfaces, Ferromagnetism, Unidirectional anisotropy

1. Introduction

Design of magnetic structures and its successful development for application depends on the manipulation and control mechanism of magnetic properties. The basic energies involved are exchange and anisotropy, where the former is responsible for magnetic ordering and the latter controls the preferred orientation. These are phenomenological descriptions of fundamental correlations and energies associated with the electronic and crystalline structure of a material. An effective technique for modifying and controlling magnetic characteristics is based on the use of magnetic heterostructures with properties governed by the interface region. The coupling of a ferromagnetic film to an antiferromagnetic material significantly changes some of the properties of ferromagnet. In such heterostructures, exchange coupling between the ferromagnet and antiferromagnet, in principle, can demonstrate a ferromagnetic behaviour with stable order and high anisotropy and the anisotropy may also behave as uni-directional, a feature not commonly found in ferromagnet. This phenomena is called Exchange Bias (EB) because the hysteresis loop associated with the ferromagnet/antiferromagnet structure can be centred about a non-zero magnetic field. A complete theoretical understanding of the EB phenomenon has posed a formidable challenge for over four decades since its discovery half a century ago by Meiklejohn and Bean [1]. EB phenomenon has recently received renewed attention due to the many supplementary phenomena that are involved and its important technological applications. This study focuses on the status of theoretical understanding of the EB phenomenon by reviewing the established theoretical approaches in deriving the magnitude of exchange anisotropy field and coercivity assuming different relevant interface characteristics such as interface structure and interface magnetic coupling direction.

2. Theoretical Models

Since the discovery of EB phenomenon, several theoretical models have been developed to analytically describe the mechanism. This review paper covers the fundamental theories along with the underlying assumptions of EB mechanism.

2.1 Meiklejohn-Bean Model

In their early and intuitive model, Meiklejohn and Bean assumed coherent rotation of two coupled macro spins describing the F layer and AF uncompensated layer. As suggested by them, the shift in the hysteresis loop is due to the large anisotropy in the antiferromagnet and weaker exchange energy coupling the ferromagnet and antiferromagnet. In this simple model the total free energy per unit area can be written as, [2]

$$E = -\mu_0 H M_F t_F \cos(\theta-\beta) + K_F t_F \sin^2\beta + K_{AF} t_{AF} \sin^2\alpha - J_{int}\cos(\beta-\alpha) \quad (1)$$

Where H is the applied external field, M_F is the saturation magnetization of the ferromagnetic layer, t_F and t_{AF} are the thicknesses of the ferromagnetic and antiferromagnetic layers respectively, K_F and K_{AF} are the anisotropy constants of the ferromagnetic and antiferromagnetic layers respectively and J_{int} is the interface coupling constant. α is the angle between the magnetization and the easy axis of the ferromagnet, β is the angle between the antiferromagnetic sub-lattice magnetization and the antiferromagnetic anisotropy axis and θ is the angle between the external field and the ferromagnetic anisotropy axis. Neglecting the F anisotropy, which in general is considerably smaller than K_{AF} ($K_F \ll K_{AF}$), the AF spins remain fixed, i.e., $\alpha \approx 0$ and $\sin\alpha \approx 0$. Minimizing with respect to α and β, the hysteresis loop shift that Meiklejohn obtained is [2]

$$H_{eb} = -\frac{J_{int}}{\mu_0 M_F t_F} \quad (2)$$

This equation developed in the MB model exhibits the relation of dependency of $\mu_0 H_{eb}$ on the F layer thickness, on the magnetization of the F layer and on the interface magnetic moments represented in the phenomenological interface coupling constant, J_{int}; though no information about the origin of interface magnetic moments is provided in the MB model. Meiklejohn later considered a finite AF anisotropy desiring to explain the rotational hysteresis observed in torque measurements. He described that in case of $K_{AF}t_{AF}/J_{int} \geq 1$, the AF remains still rigid when the F is cycled and hence exchange bias is observed. Otherwise, when $K_{AF}t_{AF}/J_{int} < 1$, the AF spin

DOI: 10.18535/ijecs/v6i4.29

follows the F magnetization reversal and the loop shift becomes zero with an increase of coercivity.

C. Binek et al. [3] derived an analytical expression from the MB model by adding a Zeeman term involving M_{AF}.

$$E = -\mu_0 H M_F t_F \cos(\theta-\beta) - \mu_0 H M_{AF} t_{AF} \cos(\theta-\alpha) + K_F t_F \sin^2\beta + K_{AF} t_{AF} \sin^2\alpha - J_{int}\cos(\beta-\alpha) \quad (3)$$

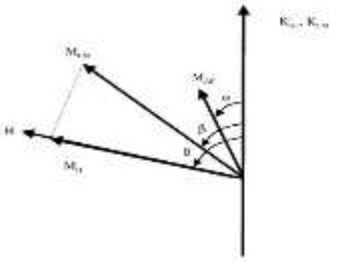

Figure 1: Vector diagram involving the angles α, β and θ related to the orientation of the net AF magnetization M_{AF}, the magnetization of the ferromagnet M_{FM} and the applied field H with respect to the easy axis of the antiferromagnet and ferromagnet designated by the corresponding anisotropy constants K_{AF} and K_{FM}, respectively.

In the case of infinite anisotropy K_{AF}, the minimization of free energy yields α=0. In case of strong finite anisotropy, the series expansion of Equation (3) with respect to α=0 reads as,

$$E \approx -J_{int}\cos\beta - \mu_0 H M_F t_F \cos(\theta-\beta) - \mu_0 H M_{AF} t_{AF}\cos\theta + K_F t_F \cos\theta + K_F t_F \sin^2\beta + \alpha(-J_{int}\sin\beta - \mu_0 H M_{AF} t_{AF}\sin\theta) + \alpha^2\left(K_{AF}t_{AF} + \frac{1}{2}J_{int}\cos\beta + \frac{1}{2}\mu_0 H M_{AF} t_{AF}\cos\theta\right) \quad (4)$$

This expression is minimized with respect to β and α, to determine the equilibrium angles β_{eq} and α_{eq} of vanishing torque. $\partial E/\partial\alpha$=0 yields,

$$\alpha_{eq} = \frac{J_{int}\sin\beta + \mu_0 H M_{AF} t_{AF}\sin\theta}{2K_{AF}t_{AF} + J_{int}\cos\beta + \mu_0 H M_{AF} t_{AF}\cos\theta} \quad (5)$$

From $\partial E/\partial\beta$=0, magnetic fields H_{c1} and H_{c2} can be calculated fulfilling the condition $M_H(H_{c1})=M_H(H_{c2})=0$, where $M_H=M_{FM}cos(\theta-\beta)$, the magnetization component of M_{FM} pointing parallel to the applied magnetic field. Putting $\alpha=\alpha_{eq}$, $\beta_1(M_H=0)=\theta-\pi/2$ and β_2 $(M_H=0)=\theta-3\pi/2$ into $\partial E/\partial\beta$=0 and expanding $\partial E/\partial\beta$ to the first order with respect to $M_{AF}\approx0$ yields two corresponding linear equations of H, which provides H_{c1} and H_{c2}, respectively. The exchange bias field is then calculated by

$$H_{eb} = (H_{c1}+H_{c2})/2 \quad (6)$$

Expanding Eq. 4 in to a Taylor series with respect to $M_{AF}\approx0$ and assuming strong and infinite anisotropy, i.e., $1/K_{AF}\approx0$ up to first and second order, a θ-dependant expression can be obtained.

$$\mu_0 H_{eb} = \frac{J_{int}\cos\theta}{M_{FM}t_{FM}} \quad (7)$$

Which has already been derived in Ref. [4]. This equation provides the basic MB expression in the case θ=0, which implies parallel orientation of the applied filed with the easy axis.

The MB model gives an intuitive insight into the exchange bias phenomenon despite lacking mesoscopic and microscopic level interpretation. Specifically, it predicts an exchange bias field several orders of magnitude larger than all experimentally measured values. Moreover, exchange bias was found experimentally for uncompensated as well as for fully compensated interfaces [5, 6].

2.2 The Néel Model

Néel [7] developed a model that applied to a system which consists of a weakly anisotropic uncompensated AF layer ferromagnetically coupled across the interface to an F slab. Néel adopted the theoretical approach of MB Model by introducing the concept of planar domain wall forming during the magnetization reversal. Néel's principle was that, the AF domain wall will store partial portion of the exchange coupling energy and hence the exchange bias field would be reduced. He assumed that the magnetization $\overrightarrow{m_i}$ of layer *i*, both in the F and the AF, is uniform within the layer and parallel to the interface. The condition for $\overrightarrow{m_i}$ to be in equilibrium, as described by Néel is,

$$J_{int}S^2\left[\sin\frac{1}{2}(\theta_{i+1}-\theta_i) + \sin\frac{1}{2}(\theta_{i-1}-\theta_i)\right] - 2K\sin\theta_i = 0 \quad (8)$$

Where, $\frac{1}{2}\theta_i$ is the angle between $\overrightarrow{m_i}$ and the easy magnetization axis and J_{int} and K are the interfacial exchange constant or coupling constant and anisotropy energy respectively. In the continuum approximation, Néel obtained the following differential equation depicting the magnetization profile in the AF.

$$J_{int}S^2\frac{d^2\theta}{di^2} - 4K\sin\theta = 0 \quad (9)$$

Under appropriate conditions, domains develop both in the F and in the AF. However, the continuum approximation requires a minimum width of the F and AF slabs to be valid; for example, a ferromagnetic iron slab in excess of 1000A° is needed [8]. The application of the Néel's model to the better characterized and well-controlled thin film EB systems is hence quite restricted.

2.3 Mauri Domain Wall Model

Mauri et al. [9] proposed a mechanism of formation of a planar domain wall at the interface as the ferromagnet rotates. Depending on the domain wall energies, the domain wall could be formed in the ferromagnet or in the antiferromagnet. Mauri et al. considered the case that the domain wall energy in the antiferromagnet is much lower than in the ferromagnet. Hence, the domain wall formed in the antiferromagnet. The main assumptions of this model are: (i) F interface coupling across a perfect flat interface; (ii) parallel magnetization of the F and AF sublattices in the absence of an external field and (iii) the AF is infinitely thick and F slab thickness is much smaller than the F domain wall width and (iv) the spins are restricted to planes perpendicular to the z-axis and the spins of one sublattice are parallel.

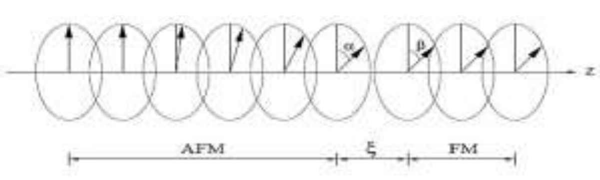

Figure 2: In the antiferromagnet only the magnetization of one sublattice is shown. α and β are the angles between the interfacial spins and the easy axes of the antiferromagnet and ferromagnet respectively, ξ denotes the distance between the two layers.

The total free energy per unit area [10],

DOI: 10.18535/ijecs/v6i4.29

$$E = 2\sqrt{A_{AF}K_{AF}}(1-\cos\alpha) + \frac{A_{AF-F}}{\xi}[1-\cos(\alpha-\beta)] + K_F t_F \cos^2\beta + \mu_0 H M_F t_F \cos(\theta-\beta) \quad (10)$$

Where K_{AF} and A_{AF} denote the anisotropy constant and the exchange stiffness of the antiferromagnet, respectively and K_F is the anisotropy constant of the ferromagnet. A_{AF-F} denotes the interfacial coupling constant, ξ is the distance between the two layers, H is the external field and θ is the angle of H with respect to the easy axis of ferromagnet. The first term of Equation (10) is the domain wall energy in the antiferromagnet and the second is the interface energy. The third and last terms denote the anisotropy energy and the Zeeman energy of the ferromagnet, respectively. Minimizing with respect to α and β for a given external field yields,

$\boldsymbol{H_{eb}} = -2\frac{\sqrt{A_{AF}K_{AF}}}{\mu_0 M_F t_F}$, when the interface energy is much larger than the domain wall energy and $\boldsymbol{H_{eb}} = -\frac{A_{AF-F}}{\xi\mu_0 M_F t_F}$, when the interface energy is much smaller than the domain wall energy where putting $J_{int}=A_{AF-F}/\xi$ gives the basic expression of Meiklejohn and Bean. The concept of Mauri predicts more reasonable values of exchange bias when the AF layer is thick enough. However, it does not provide information to understand the fact that the compensated interfaces can yield values of He as large as, or even larger than, uncompensated one [6]. Furthermore, in the magnetic ground state configuration, the F magnetic moments are orthogonal to the bulk AF easy axis [11-13]. Another drawback of Mauri's domain wall model is the fact that in order to develop a domain wall in the AF, the anisotropy constant K_{AF} needs to be quite small, otherwise it is energetically favourable for the domain wall to form in the F side as inferred experimentally in Refs [14-19] and argued theoretically in Refs. [20-22]. The key point to note is that the Néel/Mauri domain wall model introduced the concept of AF reconfiguration over F magnetization reversal.

2.4 Malozemoff's Random Field Model

Malozemoff [23] proposed a model of exchange anisotropy based on the assumption of rough F/AF compensated and uncompensated interfaces, as shown in fig. 3. He discarded the assumption of a perfectly uncompensated and smooth interface and considered an imbalance of the interfacial antiferromagnetic moments as a result of roughness and structural defects. The domain walls in this model are perpendicular to the interface, in contrast to Mauri's model. Random interface roughness gives rise to the random magnetic field that acts on the interface spins, yielding unidirectional anisotropy. The latter causes the asymmetric offset of the hysteresis loop. A large number of antiferromagnetic domains would lower the interface energy but enhance the domain wall energy.

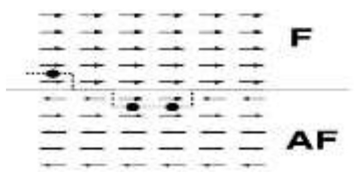

Figure 3: AF rough interface with frustated interactions marked by full dots. The dashed line denotes the boundary between the F and AF.

The expression given by Malozemoff for the shift H_E of the hysteresis loop is,

$$H_E = \frac{2}{M_F t_F}\sqrt{\frac{J_{int}K_{AF}}{a}} \quad (11)$$

Where, 'a' is the lattice parameter. In spite of its success in obtaining a reasonable estimate for H_E and explaining the training effect due to annihilation of domains during a hysteresis cycle, this model has a severe drawback: it essentially depends on a defect concentration at the interface which is not consistent with experiments.

2.5 Koon's Spin Flop Coupling at Compensated Interfaces

Koon [11] attempted to explain the exchange bias in thin films by proposing his spin flop model at fully compensated interfaces [11] by means of a micromagnetic calculation. Koon's main result was to establish that the ground state configuration corresponds to perpendicular orientation of the bulk moments relative to the AF magnetic easy axes direction.

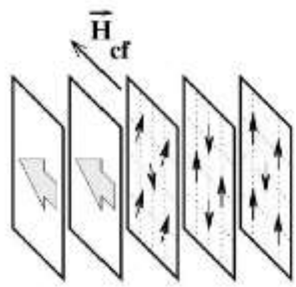

Figure 4: Illustration of the perpendicular F and AF magnetic interface configuration, with spin canting in the first AF layer.

For absolutely antiparallel spins the antiferromagnet would provide no net moment. Obviously, all orientations of the ferromagnetization would therefore result in the same interface energy. Koon considered the case that the interfacial antiferromagnetic spins are not fully antiparallel but canted by a small angle θ because of the coupling of the ferromagnet. The ferromagnet produces a small net moment in the antiferromagnet parallel to the direction of the magnetization in the ferromagnetic layer and perpendicular to the easy axis in the antiferromagnet (fig. 5). The antiferromagnetic spins at the interface thus align perpendicular to the ferromagnetic moment.

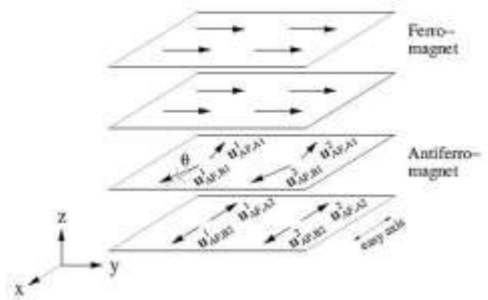

Figure 5: Spin flop coupling of a ferromagnet/antiferromagnet bilayer system. $u^2_{AF,A1}$ denotes the unit vector of the spin direction of sublattice A and monolayer 1 in the antiferromagnetic grain 2.

Koon also showed that the minimum energy is achieved with the AF spins adopting a relatively small canting angle (θ<10°) relative to the AF bulk easy axis, with a component opposite to the cooling field direction. Moreover, Koon assumed strong interface coupling and that the antiferromagnetic spins are restricted to planes parallel to the interface. Suess et al. [24] investigated the dependence of θ on the distance from the interface. They found out that θ decreases from one antiferromagnetic monolayer to the next by approximately a factor of 10. The spin flop coupling is strongly localized at the interface and the canted spin structure at the interface relaxes within a few monolayers to the totally antiparallel alignment. Despite Koon's model's ability to explain exchange bias, even a positive shift of the hysteresis loop, Schulthess and Butler [25] pointed out that the partial domain wall essential for Koon's model are not stable due to out of plane rotation.

2.6 Random Interface Field Model

Schulthess and Butler [25, 26] showed that Malozemoff's random interface field and Koon's orthogonal magnetic arrangement, rather than being in conflict, could be combined to provide an explanation of EB. In their model they added to the usual exchange, Zeeman and anisotropy energies, the dipolar interaction term E_D

$$E_D = \sum_{i\neq j} \frac{[\overrightarrow{\mu_i}.\overrightarrow{\mu_j} - 3(\overrightarrow{\mu_i}.\widehat{n_{ij}})(\overrightarrow{\mu_j}.\widehat{n_{ij}})]}{|\overrightarrow{R_i} - \overrightarrow{R_j}|^3} \qquad (12)$$

Where $\{\mu_i\}$is the magnetic moment configuration and $\hat{n}_{ij}$ is a unit vector parallel to $\vec{R}_i - \vec{R}_j$.Magnetic properties were obtained using a classical micromagnetic approach, [27-29] solving the Landau-Lifshitz equations of motion, including a Gilbert-Kelley damping term, in order to attain stable or metastable equilibrium. When this model is applied to the Koon orthogonal interface configuration, for flat interfaces the coupling that results does not yield unidirectional anisotropy, but rather irreversible magnetization curves with finite coercivity is observed. Thus, additional elements are required to generate exchange bias. Following the principle of Malozemoff's model, in Refs. [25, 26] surface defects were introduced by assuming a 4x4 2D interface unit cell, with one interfacial F site occupied by an AF magnetic moment. This way values of H_E; and of the coercivity H_c; of comparable magnitude to experimental observation [30] for the CoO/F system (F :Co and permalloy) are obtained, when exchange and anisotropy parameters of reasonable magnitude, and a canting angle of 10°; are adopted. However, there is a limitation that the model hinges qualitatively on the assumption of a rough interface, and the quantitative results depend on the nature and concentration of the interface defects that are incorporated.

2.7 Polycrystalline Antiferromagnets of Stiles and McMichael

In the model proposed by Stiles and McMichael, the ferromagnetic layer interacts with independent antiferromagnetic grains [31]. The external field is assumed to be high enough so that the ferromagnetic magnetization can be considered to be uniform. Since the antiferromagnetic grains are presumed to be small enough they do not break up into domains. However, partial domain walls parallel to the interface as a result of the coupling to the ferromagnet are allowed to occur. The energy for each grain with the interfacial area N/a^2 can be written as,

$$\frac{E}{Na^2} = -\frac{J_{net}}{a^2}\left(\widehat{M_{FM}}.\hat{m}(0)\right) + \frac{J_{sf}}{a^2}\left(\widehat{M_{FM}}.\hat{m}(0)\right)^2 + \frac{\sigma}{2}\left(1 - \hat{m}(0).(\pm\hat{u})\right) \qquad (13)$$

The first, second and third terms refers to direct coupling, spin flop coupling and domain wall energy respectively. *N* denotes the number of spins at the interface of the grains, '*a*' is the lattice constant. The directions are $\widehat{M}_{FM}$, the ferromagnetic magnetization, $\hat{m}(0)$ the direction of the net sublattice magnetization at the interface and $\pm\hat{u}$ the two easy directions of the uniaxial anisotropy in the antiferromagnet. J_{net} is the average direct coupling to the net moment of the antiferromagnetic grain, J_{sf} designates the spin flop coupling, and σ is the energy of a 180° domain wall in the antiferromagnet. In the model of Stiles and McMichael the process of biased state by field cooling the system is stimulated by choosing the antiferromagnetic state for each grain with the lowest energy with respect to the fixed ferromagnetic magnetization with the exchange bias fields given by [10]

$$H_{eb} = \frac{J_{net}}{2a^2\mu_0 M_{FM} t_{FM}}, \text{ for } \frac{2J_{net}}{\sigma a^2} < 1 \qquad (14)$$

And $H_{eb} = \frac{\sigma}{4\mu_0 M_{FM} t_{FM}}$, for $\frac{2J_{net}}{\sigma a^2} > 1$ (15)

Stiles and McMichael postulated that some grains have a critical angle α_{crit}. A partial domain wall wound up above this angle becomes unstable and grain makes an irreversible transition to a new state with reversed order far from the interface (fig. 6) owing to the results of high field rotational hysteresis measurements and isotropic ferromagnetic resonance.

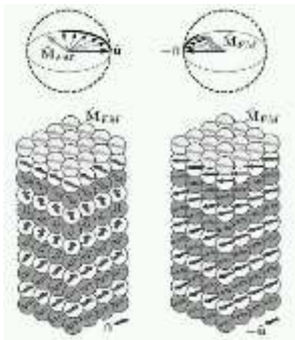

Figure 6: The white and grey spheres denote are the antiferromagnetic and ferromagnetic atoms of the sublattice, respectively. A partial domain wall is wound up due to the coupling of the two layers [31].

Stiles and McMichael found that grains with easy axes close enough to the interface normally maintain their antiferromagnetic order far from the interface when the ferromagnet switches in-plane. In addition, they concluded that the spin flop coupling for typical grain sizes and for comparable values of the interfacial constant and the antiferromagnetic exchange constant is much stronger than the direct coupling.

3. Conclusion

In order to completely understand the phenomenon of exchange bias it is required to be acquainted with some of the most difficult issues basic to magnetism. These issues include questions of magnetic ordering in frustrated systems, exchange interactions and correlations, disorder and impurity effects at the interface. The theme of this article was to study the different theoretical models explaining the phenomenon of exchange bias and understand the progression of refinement of assumptions and analytical expressions. All the theories actually at some point make a crucial assumption about the interface crystallographic and magnetic structure. Moreover, it has been argued that the magnetic and thermal stability of magnetic configurations formed on either side of the interface control the appearance of the exchange bias shifts in magnetic measurements. The magnetic configuration within a domain wall length of the interface determines the bias field and coercivity in the micromagnetic mechanisms. Exchange bias in small particle and grains composed of ferromagnetic and antiferromagnetic materials are particularly sensitive to geometry. Film thickness, grain sizes and particle dimensions of the order of the domain wall lengths can prohibit or destabilize partial wall formation and increase sensitivity to thermal fluctuations. The exchange bias is an interface effect, as clearly proved by the $1/t_F$ dependence. Deviations from this premise were observed in the literature, but fundamentally this expression is clearly well established. The AF anisotropies in the bulk of the AF layer and at the interface to a soft ferromagnetic layer give rise to an impressively rich behaviour of the magnetic properties: the hysteresis loops can be shifted along both field and magnetization axis and in both positive and negative directions, the azimuthal dependence of exchange bias exhibits non-intuitive behaviour such as a shift of its maximum with respect to the field cooling orientation, the hysteresis loops are asymmetrically shaped, etc. The property of azimuthal dependence of exchange bias could help to distinguish between two ideal mechanisms for exchange bias: M-B model and domain wall model of Mauri. In conclusion, the abundant new experimental information and the refined theories put forward during the last few years have allowed researchers to make significant headway in the description, understanding and technological use of the exchange bias phenomenon, but it is also clear that many important issues still remain open to be investigated and explored further.

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