

## Error-based Adaptive Update Rate in multi-function Phased-array Radar Using IMM Target-Tracking Algorithm

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**Abstract**— An adaptive-update-rate target-tracking algorithm is indeed a developed version of constant-update-rate tracking algorithms. The update rate is closely related with the level of target maneuvering and the required quality of tracking performance—the shorter the time intervals between two successive updates, the better the radar resources are made use of. Considering the needs of the phased-array radar system and multi-function tracking, we here propose an algorithm which makes a better use of radar resources; therefore, many targets can be tracked with an improved accuracy, and more regions can be detected. The proposed algorithm is based upon the adaptive interactive tracking algorithm (AIMM); the revisit time has been calculated using Van Keuk method, and then the fast AIMM has been amended such that the tracking error is no more than that in Van Keuk method. Our proposed algorithm which has a revisit time different than Van Keuk’s results in 62% saving of the radar resources.

### I. INTRODUCTION

A tracking algorithm suitable for optimal management of radar resources has the lowest tracking load—such an algorithm requires longer revisit times for target tracking. An algorithm with constant coefficients of  $\alpha\beta$  has a higher tracking load of about 0.8 seconds as compared to the 1.-1.2 seconds of Kalman filter and to the 1.5 seconds of the IMM filter [9]. As opposed to a constant revisit time, an adaptive revisit time in a tracking algorithm allows for the resource management optimization. Kalman filters used in interactive multiple models (IMM) are optimal when the revisit time (T) is constant and the noise is Gaussian; therefore, when the target maneuvers, the covariance matrix (Q) of the measurement noise and the transition matrix ( $\phi$ ) need to be updated at each revisit time in order for the algorithm to stay optimal. The lower the revisit time is, the more accurately the tracking will be done, and the more stable the algorithm is against errors. More observations results in a higher cumulative probability of detection and a lower noise variance or uncertainty in position [7]. Overall, the stability and accuracy of the tracking method must be met simultaneously; if the tracks are not updated adaptively—their update rates are not specific to their nature, then radar resources will be wasted when the target stops maneuvering but the update rate is still constantly high. The target trajectory is generally composed of segments corresponding to different motions; therefore, only an algorithm like the IMM which contains a filter bank of different motion models can realistically model the motion of the target [5-8]. Here, we have made use of the IMM algorithm as the basis for resource management optimization through improving the tracking method; the IMM enjoys a bank of filters appropriate for particular kinematics of different targets, which reduces the tracking load; therefore, if we adaptively make use of such a filter bank, we can greatly minimize the use

of radar resources. Choosing a revisit time according to the maneuvering of the target while keeping tracking error under a given threshold, we have proposed an adaptive version of the IMM algorithm, which has been shown through computer simulations to considerably reduce the use of radar resources.

### II. THE IMM ALGORITHM

The IMM algorithm is most commonly used for estimating the overall state (position, velocity, and acceleration) of a moving target; this algorithm employs many sub-filters run in parallel, each of which is connected to a particular model assumed for the motion of the target. The output of each of these sub-filters is indeed the target’s state estimated by that sub-filter using its corresponding model of motion. The accuracy of the state estimation of each sub-filter is found through using its residual and Bayes’ rule. A weight is assigned to each sub-filter, which is the probability of the model of that sub-filter; therefore, the overall estimated state of the maneuvering target, the output of the IMM algorithm, is a weighted sum of the states estimated by all the sub-filters running in parallel; that is, the output of the IMM algorithm is a weighted sum of the outputs of the relevant sub-filters. The covariance matrix of the different models used in the IMM algorithm is based on Markov’s model for transitioning of the maneuvering target from one model state to another. These different models of motion of the target, the transition probabilities between different states, and the parameters of the models, specifically their noise levels, all have to be appropriately chosen in order that the IMM algorithm works correctly. This algorithm consists of four steps:

#### A. Interaction mixing

the initial values for each model are determined from the weighted sum of the estimated values of all the models in the previous time step. If  $\hat{X}_i(k|k)$  is the estimation obtained by model i at time k, then the initial values for model j is given by the following equation:

$$\hat{X}_{0j}(k|k) = \sum_{i=1}^r \mu_{ij}(k|k) \hat{X}_i(k|k)$$

$$\hat{P}_{0j}(k|k) = \sum_{i=1}^r \mu_{ij}(k|k) \left\{ \hat{P}_i(k|k) + [\hat{X}_i(k|k) - \hat{X}_{0j}(k|k)] \times [\hat{X}_i(k|k) - \hat{X}_{0j}(k|k)]^T \right\} \quad 2-2$$

, where r is the number of the interacting models, and the mixing probability  $\mu_i(k|k)$  is given by:

$$\mu_{ij}(k|k) = \frac{1}{\sum_{j=1}^r \mu_j(k+1|k)} \rho_{ij} \mu_i(k|k) \quad 3-2$$

, where  $\rho_{ij}$  is the transition probability from sub-filter i, connected with model i, to sub-filter j which is matched to model j. Like the covariance matrix, the value of  $\rho_{ij}$  is fixed by the designer of the algorithm. The following relation predicts the probability of model j for the next time step:

$$\mu_{ij}(k+1|k) = \frac{1}{\sum_{j=1}^r \mu_j(k+1|k)} \rho_{ij} \mu_i(k|k) \quad 4-2$$

### B. The filtering algorithm

The values obtained above are applied to the corresponding filter as the initial values; finally, through using the measured values at time k+1, we can have the values obtained from the jth filter as  $\hat{X}_j(k+1|k+1)$  and  $\hat{P}_j(k+1|k+1)$ . Depending on the type of the filter, all the relations given in the previous step of the algorithm can now be used. Moreover, the likelihood function of model j is as follows:

$$\Lambda_j = \frac{1}{(2\pi)^{0.5n} \sqrt{|S_j|}} \exp \left\{ -0.5 v_j^T S_j^{-1} v_j \right\} \quad 5-2$$

where  $v_j$  and  $S_j$  are respectively innovation vector of dimension n, and the innovation covariance matrix of model j.

### C. Calculation of the probability of the model

after the measurements at time step k+1 are done, the probability of model j at the time step k+1 can be updated with the use of the likelihood function  $\Lambda_j$  and the probability of the model predicted in the previous step,  $\mu_j(k+1|k)$ , as follows:

$$\mu_j(k+1|k+1) = \frac{1}{C} \mu_j(k+1|k) \Lambda_j \quad 6-2$$

$$C = \sum_{i=1}^r \mu_i(k+1|k) \Lambda_i \quad 7-2$$

, where C is the normalization factor.

### D. Mixing the sub-filters' estimates and calculating the overall estimate

finally, using the updated probability of model j,  $\mu_j(k+1|k+1)$ , we can mix the estimated states and the covariance errors of the sub-filters as follows:

$$\hat{X}(k+1|k+1) = \sum_{j=1}^r \mu_j(k+1|k+1) \hat{X}_j(k+1|k+1) \quad 8-2$$

$$\hat{P}(k+1|k+1) = \sum_{j=1}^r \mu_j(k+1|k+1) \left\{ \hat{P}_j(k+1|k+1) + [\hat{X}_j(k+1|k+1) - \hat{X}(k+1|k+1)] \times [\hat{X}_j(k+1|k+1) - \hat{X}(k+1|k+1)]^T \right\} \quad [1]$$

9-2

## III. THE MOTIN MODEL OF THE TARGET

To build an algorithm which can track both highly and lowly maneuvering targets, we have made use of the constant-velocity and constant-acceleration motion models in the IMM algorithm. Either of the models is expressed as follows:

$$x(k+1) = Fx(k) + \Gamma v(k) \quad 1-3$$

, where F is the transition matrix of the state of the system,  $\Gamma$  is the noise efficiency, and vector  $v(k)$  is a zero-mean white tail with the variance of  $\delta_v^2$ .

## IV. CALCULATION OF THE REVISIT TIME

It should be noted that a maneuvering target increases the uncertainty in the estimates, which then results in an increase in the value of the error covariance. Therefore, the revisit time should be selected accordingly. We here review the methods used for calculating an appropriate revisit time.

### A. The IMM-controlled revisit time

Three motion models, the constant-velocity (CV), exponentially-increasing acceleration (EIA), and three-dimensional turning rate (3DTR), are used in the IMM algorithm for tracking a maneuvering target. The revisit time is calculated such that the predicted error covariance in the position of the target exceeds a given threshold [8].

### B. The adaptive-tracking-IMM revisit time from Van Keuk's method

Van Keuk's criterion assumes that the next revisit time is chosen such that the predicted error variance in the position of the target is always kept below a given threshold [7].

$$T = 0.4 P_D \left( \frac{\sigma_0 \sqrt{\tau_m}}{\sigma_m/R} \right)^{0.4} \frac{v_0^2}{1+0.5 v_0^2} \quad 1-4, \quad v_0^2 = \frac{\sigma_p}{\sigma_v}$$

is a constant,  $\sigma_v$  is the covariance of the  $v_0$ , where  $\tau_m$  is the maneuver correlation time, and  $\tau_m$  measurement error,  $\sigma_m$  is the covariance of the acceleration of the target. Ref [8]  $\sigma_m$  gives the relation between the variance of the estimation error of the trajectory and the revisit time (T). Here,  $\sigma$  is the standard deviation of radar's observations,  $\sigma_m$  is the standard deviation of the maneuvers (meter per square second), and R is the range of the target in meters. A closed formula has also been suggested for the revisit time as [8]:

$$\sigma_m^y = \sum_{j=1}^r \mu_j(k) \sigma_{mj}^y \quad 2-4$$

, where  $\mu_j(k)$  is the probability of model  $j$  at time  $k$ ,  $r$  is the number of the models used in the IMM algorithm, and  $\sigma_{mj}^y$  is the acceleration covariance for model  $j$ . Here,  $\sigma_m^y$  in Van Keuk's formula has been replaced with an estimated parameter.

#### C. Adaptive update rate in the Fast Adaptive Interacting Multiple Models (FAIMM) algorithm

In this method, the IMM algorithm is used with the constant-velocity and constant-acceleration motion models. The update time at scan  $k$  is obtained from[5]:

$$T = \sum_{j=1}^r \mu_j(k) T_j \quad 3-4$$

, where  $\mu_j(k)$  is the probability of model  $j$  at time  $k$ ;  $r=2$ ;  $T_1=T_{\min}$ ;  $T_2=T_{\max}$  with  $T_{\min}$  and  $T_{\max}$  being for maneuvering and non-maneuvering targets, respectively.

#### D. Revisit time in the error-based adaptive IMM algorithm (EAIMM)

We now propose an improved method which is based on the AIMM tracking algorithm with Van Keuk's method of revisit time calculation. Here, the revisit time is calculated such that the error standard deviation in the stable state of the target in the highest maneuver is the same for both of the IMM and AIMM algorithms. The revisit time ( $T$ ) in the highest maneuver is then placed as the revisit time in the IMM tracking algorithm.

$$T_{\text{IMM}} = T_{\text{MIN\_AIMM}} \quad 4-4$$

The minimum revisit rate of the AIMM algorithm in high maneuverings is used for the IMM algorithm. Therefore, the error standard deviations of these two algorithms will always be the same. A low maneuvering target with a constant velocity will be tracked by the IMM algorithm with the same revisit time of  $T_{\text{MIN\_AIMM}}$ , but by the AIMM algorithm with a lower update rate; therefore, the error standard deviation will remain relatively constant at different conditions.

Using the minimum and maximum revisit times, the FAIMM method calculates a new revisit time for the next revisit on the basis of the probability of the model at the present revisit. The criterion for choosing the minimum revisit time in the FAIMM is the same as that explained above; that is,  $T_{\text{MIN\_EAIMM}} = \mu T_{\text{IMM}}$ , where  $\mu < 1$  is obtained with the consideration of other parameters and the required accuracies. A  $T_{\text{MAX\_EAIMM}}$  selected on the basis of the FAIMM algorithm, where  $T_{\text{max}}=5$  sec, will results in a delay in the algorithm, which will dramatically increase the error standard deviation. Therefore, the revisit time should be optimized with respect to the maneuvering conditions of the target and an acceptable state transition error. Considering the maximum maneuver of 25 g, for example in the cross maneuvering of a tactical air-defense missile, the value of  $T_{\text{max}}=2$  sec is the choice for computer simulations; moreover, we increased the transition probability in Markov's matrix from 0.05 to 0.2 to control the transition error and to improve the maximum error at the condition of highest maneuvering.

## V. SIMULATIONS

The simulations evaluated the IMM, AIMM, and EAIMM algorithms. A rather complicated scenario consisting of an aerodynamic target, a fighter, and an anti-aircraft tactical missile were evaluated in a flight trajectory. In the first part of this flight scenario, the target moves with a constant velocity; thereafter, it immediately receives a high acceleration in both X and Y directions; such a two-dimensional acceleration can be an appropriate test for the performance of the tracking algorithm. The one-sigma measurement error of the radar sensor was assumed to be 100 meters in all x, y, and z directions. In the implementation of the AIMM algorithm, the predicted error was always kept below the measurement error in the input of the tracking algorithm. The errors of the tracking algorithms were evaluated using Monte Carlo method based on Blackman's rule [1] with 1000 runs.

#### A. The flight scenario

The target first moves with the constant velocity of 300 m/s along the x-axis, and then moves along the y-axis with the velocity of 100 m/s. After travelling a distance from the 20th second for 20 seconds, the target will maneuver with the acceleration of 25g along the x-axis and 10g along the y-axis, and then continues moving with its constant velocity. Then, it will maneuver at the 30th second with the acceleration of -10g along the x-axis.

#### B. The simulation results

The simulations results of the IMM algorithm show that the radar visits the target 257 times within 180 seconds with a constant update rate. The AIMM algorithm with the Van Keuk's revisit time tracks the target 224 times within the 180 seconds, which means that the 13% of the radar resources have been saved. Moreover, the EAIMM algorithm needed only 109 revisits to track the target, which obviously means a considerable saving of 62% in the radar resources has now been achieved. The tracking error standard deviation was also controlled in all conditions, particularly at the transition moments. Regulating the coefficients of CV and CA filters, and increasing the transition probability in Markov's matrix, we managed to produce a better response from the IMM and AIMM filters. Therefore, we have shown that our improving the IMM tracking algorithm has considerably lowered the use of the radar resources.

## VI. CONCLUSIONS

We have shown that our improving the IMM tracking algorithm has considerably lowered the use of the radar resources. This allows multi-functional radar to either extend its scanning region or track multiple targets simultaneously. Our proposed algorithm can be further improved through incorporating more complete and diverse motion models into the filter bank of the tracking algorithm. Furthermore, separation of targets on the basis of their maximum accelerations can also help us further develop more efficient tracking algorithms.

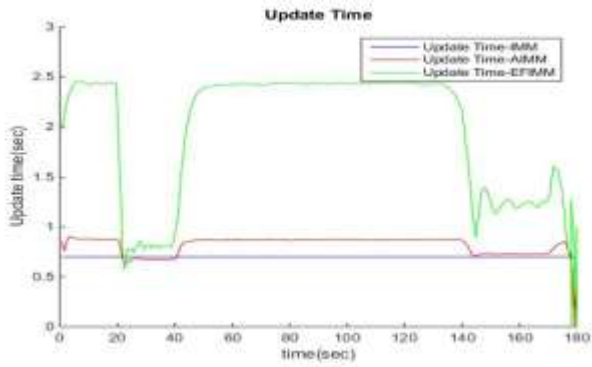


Fig.1 . The update diagram in the algorithms evaluated by the computer simulations.

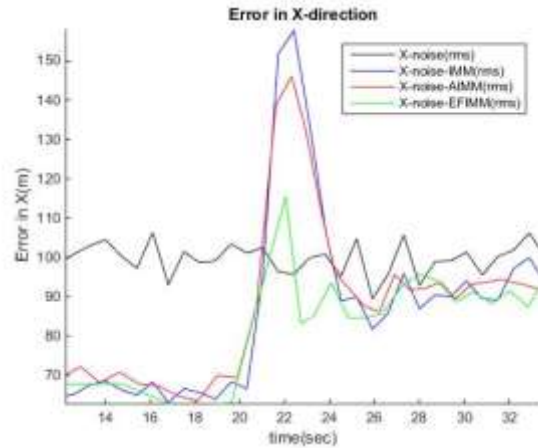


Fig.3 . Error calculation at transition from the CV motion to the high-maneuvering condition

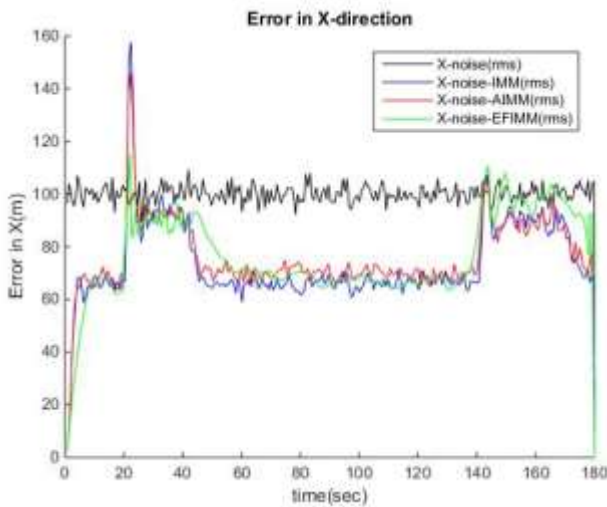


Fig. 2: error in position along the x-axis

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