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Image Segmentation Quality Assessment For "Truncated Compound Normal With Gamma Mixture Model"

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ABSTRACT
In this paper, we present image segmentation quality assessment study for Truncated Compound Normal with Gamma Mixture(TCNGM) under Expectation Maximization(EM) framework. Segmentation quality metrics such as Global Consistency Error(GCE), Probabilistic Rand Index(PRI), and Variation of Information(VoI) are applied to the clusters of image pixel labels produced by the model in comparison to the other models used for image segmentation. We show that our model is a competing one with other models.

GENERAL TERMS
Probabilistic Framework, Mixture Density Estimation, Expectation Maximization, Segmentation Quality Metrics

KEYWORDS
CNGM, TCNGM, EM, GCE, PRI, VoI

1. INTRODUCTION
Early stages of image analysis include edge detection and image segmentation whose solutions are mostly driven by intensity value discontinuity and similarity in the local neighborhood. Edge detection techniques look for intensity value discontinuity or abrupt changes in intensity value, based on which the image is partitioned into a set of edges. It is based on the use of some threshold, around which intensity value discontinuity occurs **[1]**,**[2]**,**[3].**

Segmentation methods use similarity property to detect distinct objects, each of which having similar properties for attributes like intensity value, texture, and others in a given image. In other words, the goal of image segmentation is to detect region homogeneity or similarity in the local neighborhood and extend this to the entire image based on different methods like thresholding, region growing, probabilistic distribution models, and other approaches. Thus, it results in a segment whose said attributes take similar values in it. A segmentation method, given a user specified number of segments, generates distinct segments, each of which satisfying the above mentioned goal. Region segmentation is considered more useful than edge detection since regions contain more information than edges. Thus, it has been still actively pursued by research community in general **[4],[5],[6],[7]**.

Further, to measure the segmentation quality produced by any proposed method, certain performance metrics are used. In this paper, our main goal is to use three such metrics, viz., Global Consistency Error(GCE), Probabilistic Rand Index(PRI), and Variation of Information(VoI) **[8]** for segmentation quality comparison with an established benchmark method.

In **[9]**,**[10]**, image segmentation problem as mixture density estimation problem has been formally described for the compound normal with gamma mixture model(CNGM) and its truncated form(TCNGM). Mathematical expressions for maximum likelihood estimates of model parameters α_l, μ_l, c_l, and v_l have been derived. In this paper, we present our work carried out in **[10]** for image segmentation using EM framework for Truncated Compound Normal with Gamma Mixture(TCNGM) that gives input to perform segmentation quality assessment **[8],[16]**.

This paper is organized into five sections.In Section 2, probabilistic model driven approach for mixture density estimation is outlined. In Section 3, the analytical expressions for the model parameters which were earlier derived in **[10]** are given for readers' understanding. EM algorithm for the proposed model is also outlined here. Implementation of EM algorithm for image segmentation and results produced are presented. In Section 4, quality metrics are explained and the segmentation quality as produced by the proposed model is compared with that of Normal/Gaussian mixture(NM), Compound Normal with Gamma Mixture models, and K-means clustering using these error metrics, viz., GCE, PRI, and VoI **[8]**.

2. PROBABILISTIC MODEL DRIVEN APPROACH FOR IMAGE SEGMENTATION
In this section, we briefly introduce the probabilistic model driven approach for image segmentation and the particular models we have used in our work**[9],[10]**.

2.1 Compound Normal With Gamma Distribution

As given in **[11]** by Normal L. Johnson et al, a compound normal with gamma distribution or $Normal(\mu,\sigma^2) \underset{\sigma^{-2}}{\Lambda} Gamma(c\chi_v^2)$ is formed by ascribing a distribution to σ^2 i.e., variance by considering it as a random variable and fitting a new distribution. The corresponding distribution is defined to have a density function given as

$$f(x) = \frac{1}{c^{1/2}B(1/2\,,v/2)}\left[1+\frac{(x-\mu)^2}{c}\right]^{-(v+1)/2} \qquad (1)$$

where $B(1/2\ ,v/2)$ is the beta function.

The compound normal with gamma distribution model that has been introduced has formed the basis for our work**[9],[10]** and a mixture model for this is used to solve the image segmentation problem.

2.2 Truncated Distributions

In statistics, a truncated distribution is a conditional distribution that results from restricting the domain of some other probability distribution. Truncated distributions arise in practical statistics in cases where the ability to record, or even to know about, occurrences is limited to values which lie above or below a given threshold or within a specified range **[12]**.

In general, if *X* is a random variable with density $f_x(.)$ and cumulative distribution $F_x(.)$, then the density of *X* truncated on the left at *a* and on the right at *b* is given by

$$\frac{f_x(x)}{F_x(b)-F_x(a)} \qquad (2)$$

For example, image segmentation problem may be viewed as mixture density estimation problem and since gray level images are spatially represented using an eight bit intensity or pixel value, the pixels only take values ranging between 0 and 255, each representing a particular gray value ranging between black and white. This strongly suggests to define a truncated mixture model, with $0 \le x \le 255$ in place of the more general case of $-\infty < x < +\infty$ for the random variable *x* that represents intensity value, for image segmentation because truncated distributions model finite range data well in comparison to the more general model.

2.3 Truncated Compound Normal With Gamma Distribution

We know that, for compound normal with gamma distribution, the equality that is given below holds.

$$f(x) = \frac{1}{c^{1/2}B(v/2\,,1/2)}\int_{-\infty}^{+\infty}\left[1+\frac{(x-\mu)^2}{c}\right]^{-\frac{(v+1)}{2}} dx = 1 \qquad (3)$$

For the above, the cumulative distribution may be obtained **[11]**, given a value for $z \ge 0$ or $(x-\mu) \ge 0$ (by choosing a value for *x*) as derived in **[10]**

$$\Rightarrow 1-\frac{1}{2}\, I_t\left(\frac{v}{2}\,,\frac{1}{2}\right) \qquad (4)$$

where $I_t\left(\frac{v}{2}\,,\frac{1}{2}\right)$ is incomplete beta function ratio defined as

$$I_t\left(\frac{v}{2}\,,\frac{1}{2}\right) = \frac{1}{B(v/2,1/2)}\int_0^t w^{\left(\frac{v}{2}-1\right)}(1-w)^{\left(\frac{1}{2}-1\right)}\,dw \qquad (5)$$

The cumulative distribution for $z \le 0$ or $(x-\mu) \le 0$ is

$$1-\left(1-\frac{1}{2}\, I_t\left(\frac{v}{2}\,,\frac{1}{2}\right)\right) = \frac{1}{2}\, I_t\left(\frac{v}{2}\,,\frac{1}{2}\right) \qquad (6)$$

Therefore, the probability density function of Truncated Compound Normal with Gamma Mixture(TCNGM) distribution after choosing left and right truncating points as *a* and *b* is defined as in Equation(2) where $f(x) = \frac{1}{c^{1/2}B(v/2\,,1/2)}\left[1+\frac{(x-\mu)^2}{c}\right]^{-\frac{(v+1)}{2}}$ is the density function defined for the compound normal with gamma distribution,

$$F(b) = 1-\frac{1}{2}\, I_{b_1}\left(\frac{v}{2}\,,\frac{1}{2}\right) \qquad (7)$$

is the cumulative distribution function for some *x* taking value *b* such that $x \ge \mu$, and

$$F(a) = \frac{1}{2}\, I_{a_1}\left(\frac{v}{2}\,,\frac{1}{2}\right) \qquad (8)$$

is the cumulative distribution function for some *x* taking value *a* such that $x \le \mu$ results in the the new density function for the truncated compound normal with gamma distribution with *a* and *b* as left and right truncation point and is given as

$$f(x) = \frac{2}{c^{1/2}\left[2B(v/2\,,1/2)-\left[B_{a_1}\left(\frac{v}{2},\frac{1}{2}\right)+B_{b_1}\left(\frac{v}{2},\frac{1}{2}\right)\right]\right]}\left[1+\frac{(x-\mu)^2}{c}\right]^{-\frac{(v+1)}{2}} \qquad (9)$$

where $B_{a_1}\left(\frac{v}{2}\,,\frac{1}{2}\right)$ *and* $B_{b_1}\left(\frac{v}{2}\,,\frac{1}{2}\right)$ are incomplete beta functions. The $f(x)$ in Equation (9) is the new density function for the truncated compound normal with gamma distribution with *a* and *b* as left and right truncation points.

2.4 Mixture Distribution

The probability density function of the mixture model **[13]** is

$$p(x|\theta) = \sum_{l=1}^{M}\alpha_l\, p_l(x|\theta_l) \qquad (10)$$

where the parameters are $\Theta = (\alpha_1,\dots,\alpha_M\,,\theta_1,\dots,\theta_M)$ such that $\alpha_l = P(\theta=\theta_l)$ with $0<\alpha_l<1$ such that $\sum_{l=1}^{M}\alpha_l = 1$. And each p_l is probability density function parameterized by θ_l where $\theta_l = (\mu_l, c_l, v_l)$. In other words, we assume we have *M* component densities mixed together with *M* mixing coefficients or weights α_l .

The probability density function p_l , for a given component in compound normal with gamma distribution , is defined, according to Equation (1), as

$$p_l(x|\theta_l) = \frac{1}{c_l^{1/2}B(1/2\,,v_l/2)}\left[1+\frac{(x-\mu_l)^2}{c_l}\right]^{-(v_l+1)/2} \qquad (11)$$

The probability density function p_l , for a given component in truncated compound normal with gamma distribution , is defined, according to Equations (1), (2), (7), (8) as

$$p_l(x_i|\theta_l) = \frac{2}{c_l^{1/2} B(v_l/2\,,1/2)\left[2-\left[I_{a_1}\left(\frac{v_l}{2},\frac{1}{2}\right)+I_{b_1}\left(\frac{v_l}{2},\frac{1}{2}\right)\right]\right]}\left[1+\frac{(x_i-\mu_l)^2}{c_l}\right]^{-\frac{(v_l+1)}{2}} \quad (12)$$

where x refers to each observation(individual pixel intensity value in the context of gray level images), μ_l, c_l, and v_l are, respectively, location, scale, and shape parameters of lth component of the mixture and $B(1/2\,, v_l/2)$ is the beta function.

3. ANALYTICAL EXPRESSIONS FOR MODEL PARAMETERS, $\theta_l(\mu_l, c_l, v_l)$, FOR CNGM and TCNGM

In **[9],[10]**, the steps involved in the maximum likelihood estimation **[12]** of the model parameters under Expectation Maximization framework **[13]** for a mixture density problem have been formally treated in the context of compound normal with gamma mixture model and its truncated version.

For example, the current literature on statistical image segmentation techniques mostly assumes the data describing the image as a mixture of component distributions, as shown in Fig. 1 **[4],[5],[6],[7]**.

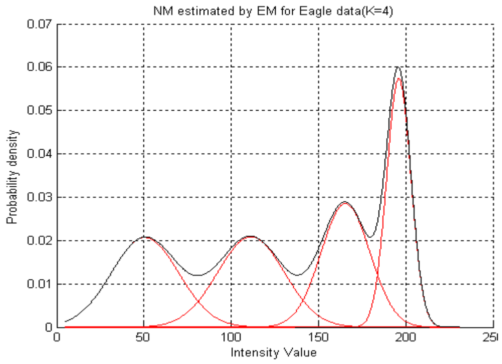

Figure 1 An Example Mixture Distribution

The solution in respect of Θ for CNGM is given as

$$\alpha_l = \frac{1}{N}\sum_{i=1}^{N} p(l|x_i, \Theta^g) \quad (13)$$

$$\mu_l = \frac{\sum_{i=1}^{N} x_i p(l|x_i,\Theta^g)}{N\alpha_l} \quad (14)$$

$$c_l = \frac{(v_l+1)}{N\alpha_l} \sum_{i=1}^{N}(x_i-\mu_l)^2 p(l|x_i,\Theta^g) \quad (15)$$

$$v_l = \frac{N\alpha_l}{\sum_{i=1}^{N}\log\left[1+\frac{(x_i-\mu_l)^2}{c_l}\right] p(l|x_i,\Theta^g)} - 1 \quad (16)$$

The update equations for Θ for TCNGM are

$$\mu_l = \frac{\sum_{i=1}^{N} x_i\, p(l|x_i,\Theta^g)}{\sum_{i=1}^{N} p(l|x_i,\Theta^g)} + \frac{2c_l^{1/2}\left[\left[\frac{c_l}{c_l+(a-\mu_l)^2}\right]^{\frac{v_l+1}{2}} + \left[\frac{c_l}{c_l+(b-\mu_l)^2}\right]^{\frac{v_l+1}{2}}\right]}{(v_l+1)B\left(\frac{v_l}{2},\frac{1}{2}\right)\left[2-\left[I_{a_1}\left(\frac{v_l}{2},\frac{1}{2}\right)+I_{b_1}\left(\frac{v_l}{2},\frac{1}{2}\right)\right]\right]} \quad (17)$$

$$c_l = \frac{(v_l+1)\sum_{i=1}^{N}(x_i-\mu_l)^2 p(l|x_i,\Theta^g)}{\sum_{i=1}^{N} p(l|x_i,\Theta^g)} + \frac{2c_l^{1/2}\left[(a-\mu_l)\left[\frac{c_l}{c_l+(a-\mu_l)^2}\right]^{\frac{v_l+1}{2}} + (b-\mu_l)\left[\frac{c_l}{c_l+(b-\mu_l)^2}\right]^{\frac{v_l+1}{2}}\right]}{B\left(\frac{v_l}{2},\frac{1}{2}\right)\left[2-\left[I_{a_1}\left(\frac{v_l}{2},\frac{1}{2}\right)+I_{b_1}\left(\frac{v_l}{2},\frac{1}{2}\right)\right]\right]} \quad (18)$$

$$v_l = \frac{\left[2-\left[I_{a_1}\left(\frac{v_l}{2},\frac{3}{2}\right)+I_{b_1}\left(\frac{v_l}{2},\frac{3}{2}\right)\right]\right]}{\left[2-\left[I_{a_1}\left(\frac{v_l}{2},\frac{1}{2}\right)+I_{b_1}\left(\frac{v_l}{2},\frac{1}{2}\right)\right]\right]}\frac{\sum_{i=1}^{N} p(l|x_i,\Theta^g)}{\sum_{i=1}^{N}\log\left[1+\frac{(x_i-\mu_l)^2}{c_l}\right] p(l|x_i,\Theta^g)} - 1 \quad (19)$$

and α_l is same as that in Equation(13).

3.1 EM Algorithm For The Proposed Model

In this section, we describe how image segmentation is performed using EM algorithm for the mixture model defined by truncated compound normal with gamma distribution i.e., $Normal(\mu, \sigma^2) \underset{\sigma^{-2}}{\Lambda} Gamma(c\chi_v^2)$. The basic steps here are

Step1: Decide M, the number of segments based on the number of components of the mixture i.e., fix $\Theta = (\alpha_1, \dots, \alpha_M, \theta_1, \dots, \theta_M)$.

Step2: Initialize Θ.
Step3: Invoke EM algorithm.
EM Algorithm: /*Repeat E-step and M-step until convergence is reached*/
E-step: Compute the expectation as

$$p^{(q)}(l|x_i, \Theta^g) = \frac{\alpha_l^{(q)} p_l\left(x_i|\theta_l^{(q)}\right)}{\sum_{l=1}^{M}\alpha_l^{(q)} p_l\left(x_i|\theta_l^{(q)}\right)} \quad (q = 0,1,2, \dots)$$

where
$p_l(x_i|\theta_l)$ is defined as in Equation(12)

M-step: Compute update equations for
$\Theta = (\alpha_1, \dots, \alpha_M, \theta_1, \dots, \theta_M)$ using Equations (13),(17),(18),(19)
$(q = 0,1,2, \dots)$

The stopping criterion is
$\left|\log\mathcal{L}^{(q+1)} - \log\mathcal{L}^{(q)}\right| < \epsilon$

where $\mathcal{L}$ is the likelihood of the parameter estimates, ϵ is error tolerance. In the above algorithm a and b, respectively, are set to 0 and 255, since these values are considered as left and right truncation points for our TCNGM **[9],[10]**.

3.2 Implementation And Results

We have implemented the EM algorithm **[13],[15]** for the CNGM and its truncated version(TCNGM) in MATLAB and obtained fruitful segmentation results for those images as detailed in **[9],[10]**.In our experiment, the initialization of the parameters for the specified number of clusters is done using K-means clustering.

For those images considered for examination, we have obtained fruitful segmentation results as shown in Fig. 2 and Table 1.

The main objective behind the proposed CNGM model described in**[9]**and the extended TCNGM model**[10]** is to study their feasibility to solve mixture density estimation problem in the context of leptokurtic deviations of normal mixture distributions. We understand that image data distributions, where coefficient of kurtosis is more than 3, are well modeled as compound normal with gamma mixture model.

The additional objective behind the proposed TCNGM model **[10]** is to study its feasibility to solve mixture density estimation problem in the context of truncated data distributions. We understand that image data distributions are well modeled as truncated distributions since the pixel values fall within a finite range.

Fig. 2 Original Images(Leftmost) and their Segments

Table 1: Maximum Likelihood Estimates For TCNGM

Image	α_l	θ_l			$\log \mathcal{L}$
		μ_l	c_l	v_l	
Snake	0.9381	130.6596	3673.7409	72.2379	-35023.6902 (K=2)
	0.0619	77.7117	9605.9975	62.1075	
Sunset	0.6692	78.6252	400.4247	23.1710	-37102.2624 (K=3)
	0.3242	35.5839	2410.8514	13.7319	
	0.0066	165.9272	43587.8925	5.5880	
Fox	0.0886	116.0769	33501.1298	6.0447	-36632.3612 (K=3)
	0.3303	253.0665	204.1824	17.7048	
	0.5811	13.9604	517.3162	20.7158	
Eagle	0.1089	160.9267	3922.3254	9.2782	-44456.4675 (K=4)
	0.1203	111.4112	6938.3657	8.9274	
	0.6446	195.2787	603.0092	10.6518	
	0.1089	160.9267	3922.3254	9.2782	
Lady	0.2372	130.9775	36563.1525	23.8566	-50330.2265 (K=4)
	0.2180	14.5879	662.7173	12.8592	
	0.2341	99.4459	25535.1006	12.6374	
	0.3107	196.0037	3827.2277	28.6799	
Church	0.0242	40.8355	560.9751	10.1860	-46073.7706 (K=5)
	0.4780	92.6533	851.9156	8.8252	
	0.3174	246.0361	692.6319	7.6565	
	0.0682	185.3611	5247.4521	6.6640	
	0.1122	135.8699	4479.3548	7.4084	
Man	0.1568	51.1117	4511.8358	36.5965	-46812.1734 (K=6)
	0.2128	104.0818	5213.8420	40.1993	
	0.0994	67.0034	10259.8758	35.7816	
	0.3328	107.1731	4078.2365	40.7203	
	0.0454	127.2467	6914.7080	37.2054	
	0.1528	175.8975	60375.5438	9.7511	
Crane	0.0729	119.5908	12838.0214	17.0312	-46840.0419 (K=7)
	0.2266	60.6184	2696.5310	16.9804	
	0.0929	174.5387	1195.2051	22.8335	
	0.0357	244.9591	4116.7951	12.4592	
	0.1596	92.8795	5036.9914	17.5185	
	0.1626	48.0420	1856.1957	17.4924	
	0.2497	75.5196	2790.3153	17.5613	
Horse	0.0699	113.5461	1604.9862	7.9121	-48815.3291 (K=7)
	0.2182	139.4819	804.7983	8.2259	
	0.2365	156.3763	893.9370	8.0697	
	0.0872	87.5816	1160.9240	8.0030	
	0.1062	208.1673	1637.6123	7.7005	
	0.1825	180.0689	1507.3609	7.8942	
	0.0995	57.9416	627.8301	8.1048	

4. SEGMENTATION QUALITY ANALYSIS OF THE MODEL

It is important to note that the success or failure of a proposal will be revealed when it undergoes a thorough examination of its end results. Since CNGM and TCNGM have effectively accomplished the fundamental task of their use for image segmentation, it now becomes necessary to examine the proposed segmentation algorithms for their performance vis-à-vis the other popular ones.

Towards the aforementioned goal, the normal mixture model based segmentation method has been considered first for comparison with our methods since the genesis for our models is normal distribution. The primary objective of this choice is to critically examine the segmentation deviations of CNGM and TCNGM from normal mixture model and possibly understand the inherent model driven characteristics of the proposed two models in terms of the leptokurtic and platykurtic deviations of the more general mesokurtic normal mixture model.

Further, the K-means clustering based segmentation is also used for segmentation performance since K-means was used to in our work for parameter initialization.

4.1 Segmentation Quality Metrics

In this section, we consider three performance metrics, viz., Global Consistency Error(GCE), Probabilistic Rand Index(PRI), and Variation of Information(VoI) which are widely used in the current literature**[8],[16]**. All the three metrics compare two segmentation techniques by finding classification errors in the proposed method with respect to pixel labeling in the bench mark classifier.

In particular, the GCE is a measure of overlap between the segments produced by two segmentation methods. It is the average of minimum of the sums of the local refinement error(LRE) in both directions of the two segmentation methods for all pixels, where LRE is the degree of overlap between two sets; it is 0 if there is complete overlap in the given direction, and is 1 if there is no overlap at all. Thus GCE ranges between 0 and 1 and lower GCE values for test segmentation vis-à-vis the benchmark segmentation suggest their closeness with each other. It is formally defined in **[8]** as stated hereunder.

Let S and S' be two segmentations of an image $X = (x_1, x_2, \dots, x_n)$consisting of N pixels. For a given Pixel x_i, consider the classes (segments) that contain x_i in S and S'. We denote these sets of pixels by $C(S, x_i)$and $C(S', x_i)$, respectively. The local refinement error (LRE) is then defined at point x_i as:

$$LRE(S, S', x_i) = \frac{|C(S,x_i)\backslash C(S',x_i)|}{C(S,x_i)} \qquad (20)$$

where \ denotes the set differencing operator. Since this error measure is not symmetric, we use Global Consistency Error (GCE) defined as:

$$GCE(S, S^i) = \frac{1}{N} \min\{\sum_i LRE(S, S', x_i), \sum_i LRE(S', S, x_i)\} \qquad (21)$$

The PRI is defined based on the assumption that the test segmentation follows Bernoulli distribution given benchmark segmentation yielding values between 0 and 1, higher values being better. PRI is an extension of the definition of Rand index and Rand index is formally defined as given below.

Rand index, as defined in **[17]**, is a measure of the similarity between two clusterings of the same data, Y and Y', can be defined as c(Y, Y') equal to the number of similar assignments of point-pairs normalized by the total number of point- pairs.

More precisely, given N points, $X_1, X_2, \dots, X_N$, and two clusterings of them Y= $\{Y_1, Y_2, \dots, Y_{k_1}\}$and $Y' = \{Y'_1, Y'_2, \dots, Y'_{k_2}\}$, we define

$$c(Y, Y') = \sum_{i<j}^{N} \gamma_{ij} / \binom{N}{2} \qquad (22)$$

where

$$\gamma_{ij} = \begin{cases} 1 \text{ if there exist } k \text{ and } k' \text{such that both } X_i \text{ and } X_j \\ \text{are in both } Y_k \text{ and } Y'_{k'} \\ 1 \text{ if there exist } k \text{ and } k' \text{such that } X_i \text{ is in both } Y_k \text{ and } Y'_{k'} \\ \text{while } X_j \text{ is in neither } Y_k \text{ or } Y'_{k'} \\ 0 \text{ otherwise} \end{cases}$$

The VoI is based on information theory and is a function of individual entropies of both the segmentations and mutual information between them **[16]** yielding values between (0 ..VoI_max] , lower values being better. It is defined as given below.

Given two sets $X = (x_1, x_2, \dots, x_k)$ and $Y = (y_1, y_2, \dots, y_l)$,

$$VoI(X, Y) = H(X) + H(Y) - 2I(X, Y) \qquad (23)$$

where $H(X) and\ H(Y)$ are entropies of X and Y , and $I(X, Y)$ is the mutual information between X and Y. Further, entropy of X is defined as $H(X) = -\sum_i p(x_i) \log p(x_i)$ and mutual information *I(X,Y)* is defined as $I(X, Y) = \sum_{y \in Y} \sum_{x \in X} p(x, y) \log \frac{p(x,y)}{p(x)p(y)}$ where $p(x, y)$ is the joint probability of x and y, and $p(x)$ and $p(y)$are probabilities of x and y respectively.

4.2 Cluster Labeling for Segmentation data

The image segmentation data produced by the EM algorithm is subject to a labeling procedure, in which each pixel in image data is labeled with such component *l* in the mixture whose likelihood is maximum for the given pixel. This results in the image data being divided into clusters of labels. This labeled data is then used as input to the procedures defined for GCE, PRI, and VoI that have been introduced in the previous section.

4.3 Segmentation Performance of The Model

The quantitative measures obtained by applying the GCE, PRI, and VoI quality metrics for pair wise comparison of TCNGM, CNGM, Normal Mixture(NM), and K-means are presented in Tables 2, 3, and 4 and the corresponding bar charts are shown in Figures 3, 4, and 5.

The TCNGM in general is found to be closer to the other mixture models in terms of the said metrics for the images considered for examination. It is expected to perform even better in case the pixel distribution in images is highly influenced by scale and shape parameters.

For some of the images like Church, Lady, Fox, and Snake, the CNGM and TCNGM are observed to be more closer than Gaussian mixture model to K-means clustering in terms of all the three performance metrics. Furthermore, it has been observed that CNGM and TCNGM are closer to K-means for majority of the images in terms of one or the other performance metrics. This may probably be due to the reason that local distributions are more correctly modeled by our model than Gaussian mixture model.

For the images we have considered for experimentation, majority of them, viz., Church, Sunset, Crane, Lady, Snake, and Fox have component distributions which are more leptokurtic. For these figures, we have observed that our models(CNGM and TCNGM) are more closer than Gaussian mixture(NM) to K-means clustering. For the other images, e.g., Horse and Eagle, NM is more closer to K-means. This may be due to the reason that CNGM and TCNGM are more correctly modeling leptokurtic distributions than NM.

A general observation of the bar charts for PRI, GCE, and VoI suggests that the truncated version is proved to be a competing model for its use in solving mixture density estimation problem, since the results produced for the corresponding image segmentation problem show, more or less, its closeness to the other mixture models and K-means clustering.

Another important observation is that the classification error appears to increase with the number of segments K increasing across all images, irrespective of the mixture models or K-means clustering that are subjected to comparison between one another. This phenomenon might suggest for its validity and application in place of the model optimization criterion like minimum description length while deciding the optimal number of components in a mixture. For example, a given image may be subjected to segmentation by incrementing K value by one, from one run to the next, until classification error becomes stable.

5. CONCLUSIONS

In this paper, the EM algorithm for the model studied in [**10]** is presented which has been implemented using MATLAB. And it is applied to solve image segmentation problem as a mixture density estimation problem. Here, we have rerun the implementation for normal and compound normal with gamma mixture models to check for consistency in the results between runs and used these results to compare with those obtained for the truncated version of the compound normal with gamma mixture model. The results obtained have been thoroughly examined in respect of segmentation performance of the TCNGM vis-à-vis the other mixture models(CNGM and NM) and K-means clustering. The segmentation quality is quantitatively measured using Probabilistic Rand Index (PRI), Global Consistency Error (GCE), and Variation of Information (VoI).

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Table 2 Probabilistic Rand Index: TCNGM/CNGM/NM/K-means

Image #	Image	PRI						K
		CNGM/NM	CNGM/K-means	NM/ K-means	TCNGM/CNGM	TCNGM/ NM	TCNGM/ K-means	
1	Snake	1	0.9983	0.9983	1	1	0.9983	2
2	Sunset	0.8418	0.9186	0.8941	0.8826	0.8647	0.9333	3
3	Fox	0.9846	0.9434	0.9295	0.9887	0.9735	0.9540	3
4	Eagle	0.9629	0.9724	0.9761	0.9981	0.9611	0.9723	4
5	Lady	0.9659	0.9181	0.8886	0.9775	0.9446	0.9368	4
6	Church	0.9689	0.8397	0.8222	0.9341	0.9232	0.8397	5
7	Man	0.9374	0.9258	0.9423	0.8312	0.8257	0.8056	6
8	Crane	0.9464	0.8934	0.9267	0.9299	0.8919	0.8534	7
9	Horse	0.8750	0.9668	0.8798	0.8609	0.9501	0.8672	7

Table 3 Global Consistency Error: TCNGM/CNGM/NM/K-means

Image #	Image	GCE						K
		CNGM/ NM	CNGM/ K-means	NM/ K-means	TCNGM/ CNGM	TCNGM/ NM	TCNGM/ K-means	
1	Snake	0	0.0016	0.0016	0	0	0.0016	2
2	Sunset	0.1015	0.1067	0.0837	0.0736	0.1128	0.0325	3
3	Fox	0.0221	0.0596	0.0676	0.0218	0.0405	0.0482	3
4	Eagle	0.0464	0.0516	0.0547	0.0081	0.0541	0.0519	4
5	Lady	0.0603	0.1613	0.2028	0.0468	0.1020	0.1285	4
6	Church	0.0930	0.2021	0.2372	0.1288	0.1358	0.1378	5
7	Man	0.1300	0.1412	0.1379	0.1906	0.1649	0.2041	6
8	Crane	0.1402	0.2559	0.1875	0.1769	0.2493	0.3355	7
9	Horse	0.3441	0.0784	0.3409	0.3560	0.1392	0.3463	7

Table 4 Variation of Information: TCNGM/CNGM/NM/K-means

Image #	Image	VoI						K
		CNGM/ NM	CNGM/ K-means	NM/ K-means	TCNGM/ CNGM	TCNGM/ NM	TCNGM/ K-means	
1	Snake	0	0.0160	0.0160	0	0	0.0160	2
2	Sunset	0.7783	0.6230	0.5760	0.7203	0.7373	0.4609	3
3	Fox	0.1546	0.4377	0.5050	0.1482	0.2692	0.3566	3
4	Eagle	0.2866	0.3228	0.3210	0.0601	0.3352	0.3217	4
5	Lady	0.3692	0.8170	1.0005	0.3057	0.5758	0.6773	4
6	Church	0.5034	1.1326	1.3479	0.7575	0.7783	1.0346	5
7	Man	0.7207	0.7654	0.7250	1.2193	1.1455	1.3270	6
8	Crane	0.7495	1.1852	0.9461	0.9124	1.1710	1.5035	7
9	Horse	1.4790	0.4499	1.4920	1.5278	0.7489	1.5332	7

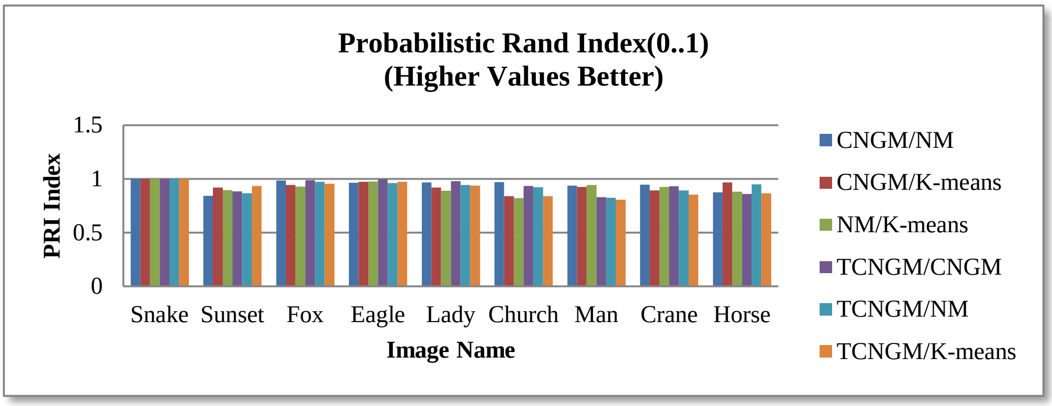

Fig. 3 Probabilistic Rand Index Plot: TCNGM/CNGM/NM/K-means

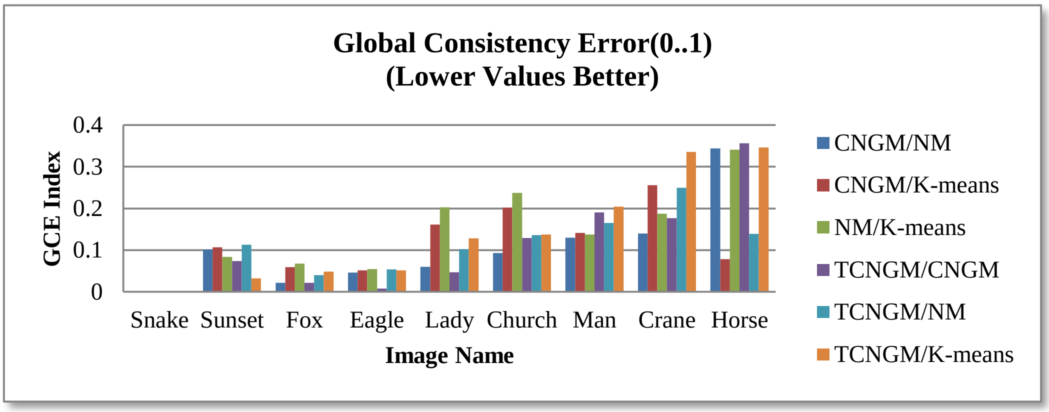

Fig. 4 Global Consistency Error Plot: TCNGM/CNGM/NM/K-means

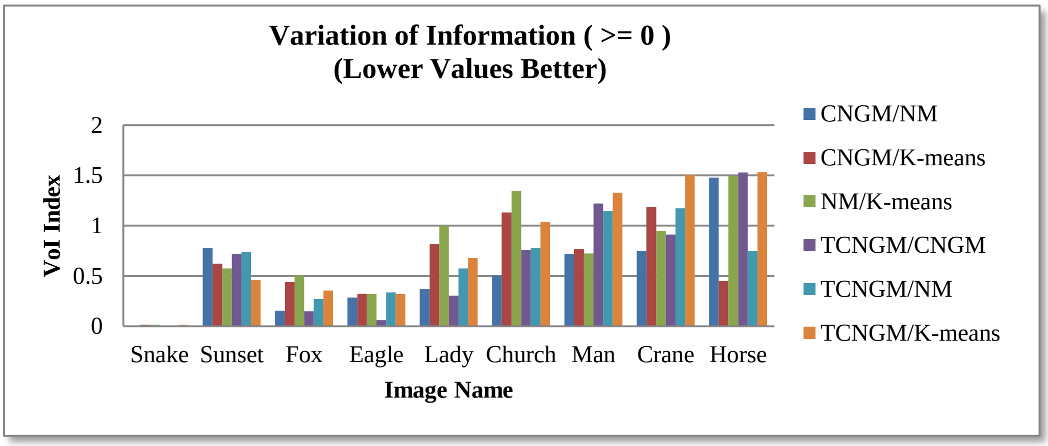

Fig. 5 Variation of Information Plot: TCNGM/CNGM/NM/K-means

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