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Stochastic Analysis Of The LMS And NLMS Algorithms For Cyclostationary White Gaussian Inputs

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Abstract

This paper studies the stochastic behavior of the LMS and NLMS algorithms for a system identification framework when the input signal is a Cyclostationary white Gaussian process. The input Cyclostationary signal is modeled by a white Gaussian random process with periodically time-varying power. Mathematical models are derived for the mean and mean-square-deviation (MSD) behavior of the adaptive weights with the input Cyclostationary. These models are also applied to the non-stationary system with a random walk variation of the optimal weights. Finally, the performance of the two algorithms is compared for a variety of scenarios.

KEYWORDS: Adaptive Filters, Analysis, LMS Algorithm, NLMS Algorithm, Stochastic Algorithms

1. INTRODUCTION

An important aspect of adaptive filter performance is the ability to track time variations of the underlying signal statistics. The standard analytical model assumes the input signal is stationary.

However, a non-stationary signal model can be provided by a random walk model for the optimum weights. The form of the mean-square error performance surface remains unaltered while the surface moves in the weight space over time. This model provides the conditions for the adaptive algorithm to track the optimum solution.

Alternatively, the input signal can be modeled as a Cyclostationary process in many practical applications. In these cases, the form of the performance surface is periodic with the same period as the input autocorrelation matrix. This performance surface deformation affects the adaptive filter convergence and is independent of changes in the optimum weights. This transient performance surface deformation can be modeled by standard analytical models. However, it is still desirable to understand the adaptive performance with non-stationary inputs.

Analysis of the LMS behavior for Cyclostationary inputs studied only its convergence in the mean. The special case of a pulsed variation of the input power and a linear combiner structure has recently been studied for both LMS and NLMS algorithms. An analysis of the Least Mean Fourth (LMF) algorithm behavior for non stationary inputs has been recently presented. The analytical model derived for the LMF behavior was valid only for a specific form of the input autocorrelation matrix, and cannot be easily extended to a general time-varying input statistics. Also, as the LMF weight update equation is a function of a higher power of the estimation error, the statistical assumptions used are necessarily different from those required for the analysis of the LMS and NLMS algorithms.

Hence, the study of the behaviors of the LMS and NLMS algorithms under Cyclostationary inputs cannot be inferred from the analysis and new models must be derived. Adaptive solutions involving Cyclostationary signals have been sought for many application areas. In particular, communication, radar, and sonar systems frequently need such solutions, as several man-made signals encountered in these areas have parameters that vary periodically with time.

Thus, a statistical analysis of adaptive algorithms under Cyclostationary inputs could have a significant impact on a wide variety of problems involving Cyclostationary processes. The analysis of the adaptive filter behavior for Cyclostationary inputs is not easy because of the difficulty of modeling the input cyclostationarity in a mathematically treatable way. Thus, relatively simple models are needed from which to infer algorithm behavior for inputs with time-varying statistics.

This paper presents statistical analyses of the Least Mean Square (LMS) and the Normalized Least Mean Square (NLMS) algorithms with specific Cyclostationary input signals and an unknown system in a system identification framework. The input Cyclostationary signal is modeled by a white Gaussian random process with periodically time-varying power. These models are used to study the adaptive filter performance for input signals with sinusoidal and pulsed power variations and a transversal filter structure. The cases of fast, moderate and slow power variations are considered. Mathematical models are derived for the mean and mean-square-deviation (MSD) behavior of the adaptive weights with these input cyclostationarities. These models are derived via extension of well-known results for the LMS and NLMS algorithms to the Cyclostationary case. These models are also applied to the non stationary channel with a random walk variation of the optimal weights. Simulation results show excellent agreement with the theoretically predicted behaviors, confirming the usefulness of the analytical model to study the adaptive filter behavior.

2. Statistical Assumptions

A. System Identification and the Markov Channel Model

This paper studies the system identification model given in Fig. 1. The N dimensional input vector to the adaptive filter tap weights is given by

$X(n) = [x(n), x(n-1), \ldots, x(n-N+1)]^T$.

The observation noise $\eta_0(n)$ is assumed to be a zero-mean i.i.d. random sequence, with variance σ_0^2 and statistically independent of any other signal.

The standard random-walk model is used for the unknown channel

$$H(n+1) = H(n) + Q(n) \qquad (2.1)$$

Where Q(n) is a white Gaussian vector with zero mean and covariance matrix $E[Q(n)Q^T(n)]=\sigma_q^2(n)I.$ (2.2)

The Vector sequence $Q(n)$ is assumed independent of both $X(n)$ and $n_0(n)$.

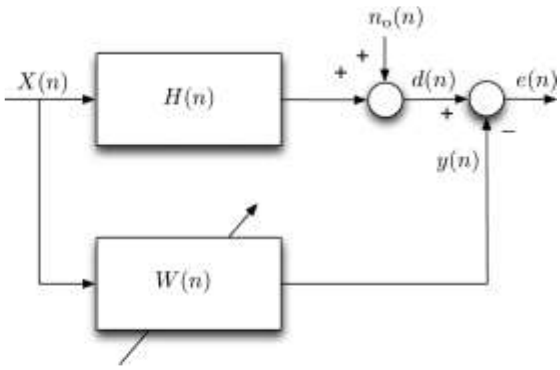

Fig.1. System Identification Model

B. Independence Theory and the Performance Measure

The Independence Theory (IT) of adaptive filtering assumes that the weights at time n are statistically independent of the input vector at time n. The use of this assumption considerably simplifies the stochastic analysis of the adaptive filter.

MSD is given by

$$MSD(n)=E[(W(n)-H(n))^T(W(n)H(n))] \quad (2.3)$$

C. Cyclostationary Input Signal Model

A wide sense Cyclostationary random process y(t) is defined as

$$E[y(t_1+T)]=E[y(t_1)] \qquad (2.4)$$

$$E[y(t_1+T)y(t_2+T)]=E[y(t_1)y(t_2)] \quad (2.5)$$

For all t_1 and t_2 and where T is the period. simple model is considered sinusoidal power time variation.

$$\sigma_x^2(n)=\beta(1+sin(\omega_0 n)) \text{ for } \beta>0, \omega_0>0 \quad (2.6)$$

The time variations can be classified as slow, moderate or fast as compared to the length of the filter. Hence, the variations are slow if $\omega_0 N \ll 2\pi$ and if N<<T . The variations are fast for if $\omega_0 N \gg 2\pi$ and if N>>T . The variations are moderate for if $\omega_0 N = 2\pi$ and if N=T.

3. LMS Algorithm

The LMS weight update recursion is

$$W(n+1)=W(n)+\mu e(n)X(n) \qquad (3.1)$$

where

$$e(n)=H^T(n)X(n)+n_0(n) - W^T(n)X(n) \quad (3.2)$$

and μ is the step-size. Defining the weight error vector $V(n)=W(n)-H(n)$

$$V(n+1)=\{I-\mu X(n)X^T(n)\}V(n)+\mu n_0(n)X(n)-Q(n)$$

(3.3)

A. LMS Mean Behavior

$$E(V(n+1)]= \{I-\mu R_X(n)\}E[v(n)] \qquad (3.4)$$

B. LMS MSD Behavior

$$K_{vv}(n+1)= K_{vv}(n)- \mu[R_X(n) K_{vv}(n)+ \quad K_{vv}(n)R_X(n)]+ \mu^2\{2R_X(n) K_{vv}(n)R_X(n)+Tr[R_X(n) K_{vv}(n)R_X(n)]\}+\mu^2 \sigma_0^2(n)R_X(n)+ \sigma_q^2(n)I \qquad (3.5)$$

4. NLMS Algorithm

The NLMS weight update recursion is

$$w(n+1)=w(n)+\mu\frac{e(n)X(n)}{X(n)X^T(n)} \qquad (4.1)$$

where

$$e(n)=H^T(n)X(n)+n_0(n)-W^T(n)X(n)$$

(4.5)

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and μ is the step-size.

A. NLMS Mean Behavior

$$E[V(n+1)]=\left\{I-\mu E\left[\frac{X^T(n)X(n)}{X(n)X^T(n)}\right]\right\}E[V(n)] \quad (4.6)$$

B. NLMS MSD Behavior

MSD Behavior of NLMS is given by

$$K_{vv}(n+1)=K_{vv}(n)-\mu E\left[\frac{X^T(n)X(n)}{X(n)X^T(n)}\right]K_{vv}(n)-\mu K_{vv}(n)E\left[\frac{X^T(n)X(n)}{X(n)X^T(n)}\right]\left[\frac{X(n)X^T(n)K_{vv}(n)X(n)X^T(n)}{[X^T(n)X(n)]^2}\right]+\mu^2E[n_0^2(n)]E\left[\frac{X(n)X^T(n)}{[X^T(n)X(n)]^2}\right]+\sigma_q^2(n)I \quad (4.7)$$

5. Simulation results of LMS and NLMS algorithms

This section provides simulation results of LMS and NLMS algorithms. The simulations are done for the case of Cyclostationary random processes with sinusoidal power variations and pulse time power variations. The analysis is done for a system identification framework. The unknown system is modeled by the standard random-walk model. For periodic variation in the input power, the steady-state mean-square deviation (MSD) of the NLMS algorithm is shown to be periodic with the same period as that of the variation in the input power.

The transient and the steady-state MSD performances are not affected by rapid variation in the input power.

Simulation Results for Sinusoidal Input Power Variations

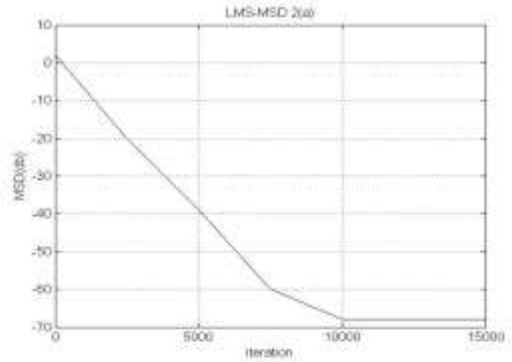

Fig.2. LMS Fast Variation

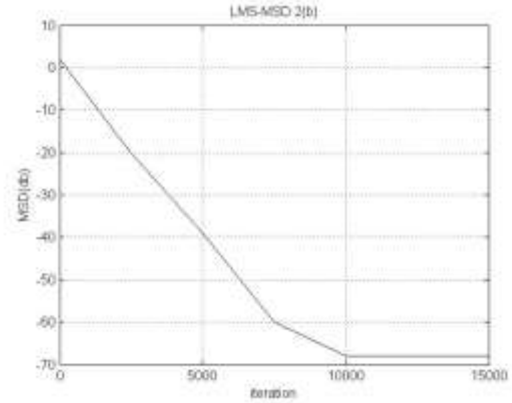

Fig.3. LMS Moderate Variation

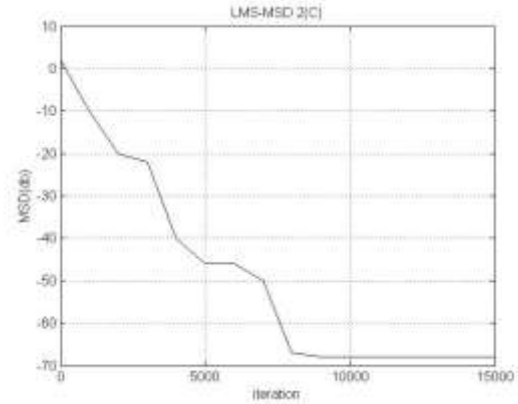

Fig.4. LMS Slow Variation

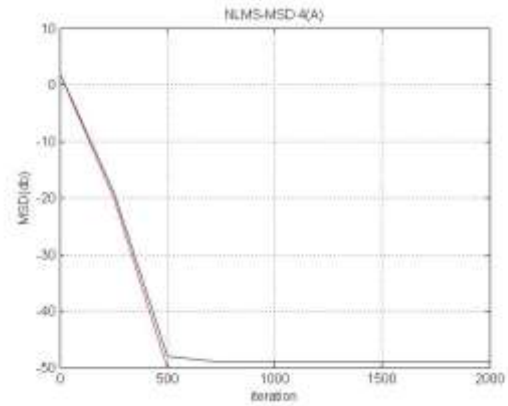

Fig.5. NLMS Fast Variation

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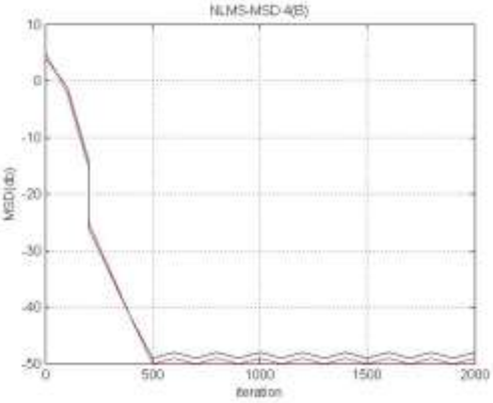

Fig.6. NLMS Moderate Variation

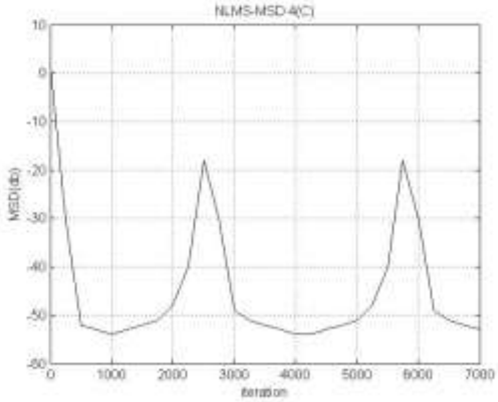

Fig.7. NLMS Slow Variation

Simulation Results for Pulse Input Power Variations

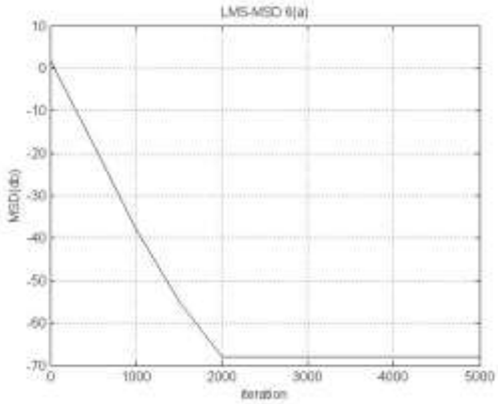

Fig.8. LMS Fast Variation

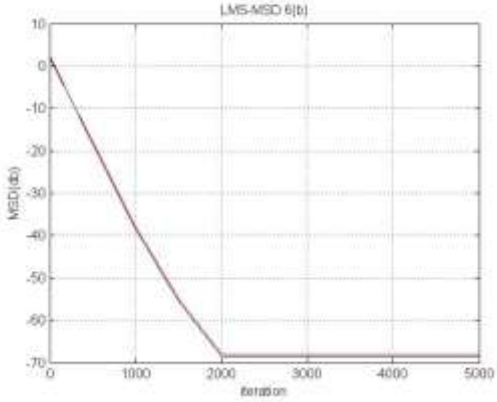

Fig.9. LMS Moderate Variations

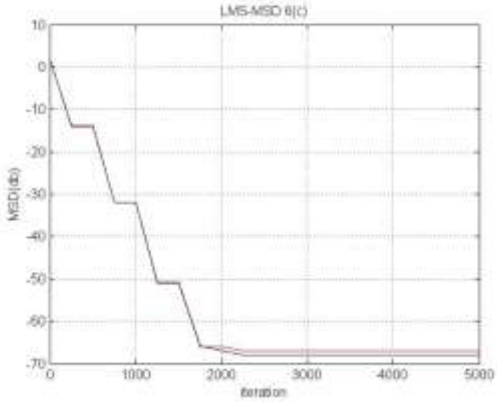

Fig.10. LMS Slow Variations

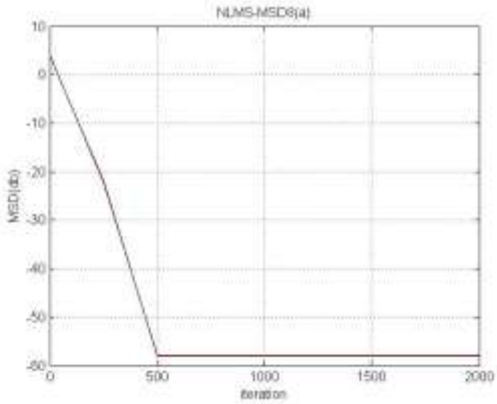

Fig.11. NLMS Fast Variation

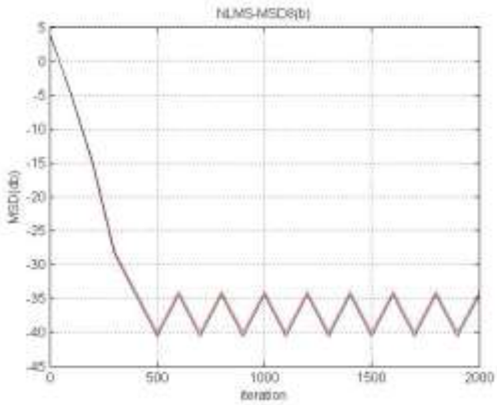

Fig.12. NLMS Moderate Variation

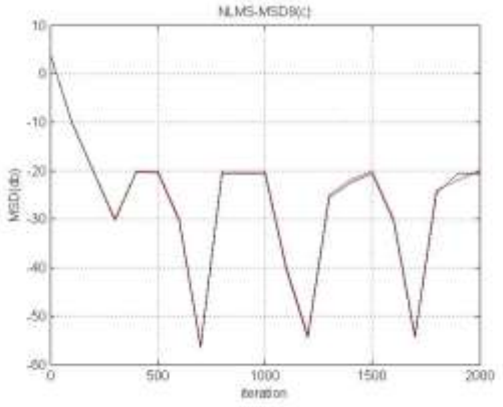

Fig.13. NLMS Slow Variation

6. COMPARISION OF LMS AND NLMS ALGORITHMS

The choice of algorithm depends upon the stability and tracking properties of each algorithm in the different environments studied in this paper. However, the NLMS algorithm is to be recommended based on stability, transient response and steady-state behavior as discussed above. Furthermore, the two algorithms share the following properties:

1) For periodic input power variations, the mean-square deviation (MSD) converges to a periodic sequence with the same period as the input power variations.

2) Neither the transient nor the steady-state performance is affected by rapid input power variations (period of the variation of the input power<<N).

They differ in the following properties:

1) For slow input power variations, the transient NLMS MSD behavior does not depend on the rate of variation of the input power, while the LMS MSD behavior does.

2) For a fixed plant with slow input power variations, the steady-state LMS MSD has negligible time-variations, while the NLMS MSD has significant time-variations.

7. CONCLUSION

This paper studies the stochastic behavior of the LMS and NLMS algorithms for a system identification framework when the input signal is a Cyclostationary white Gaussian process. It was found that the MSD converges to a periodic sequence with the same period as that of the periodic input power variation. Neither the transient nor the steady-state performance is affected by rapid input power variation. The performances of the two algorithms are compared and the NLMS algorithm is chosen on the basis of stability, transient response and steady-state behavior. The results of this paper suggest that the NLMS algorithm can be used effectively with Cyclostationary inputs such as voice data.

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Author Biographies

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