

# Performance Evaluation Of Ppwr Reduction In Ofdm System Using Non Linear Companding Transform

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**Abstract**— The main limitation in employing orthogonal frequency division multiplexing (OFDM) system is high peak-to-average power ratio (PAPR) of the transmitted signal. A new nonlinear companding algorithm that transforms the OFDM signals into the desirable statistics form defined by a linear piecewise function is proposed. The more adjustability in companding form and an effective trade-off between the PAPR and bit error rate (BER) performances can be obtained by introducing an inflexion point and the variable slopes in the target probability density function. Theoretical analyses for this algorithm is presented and expressions regarding the achievable signal attenuation factor and transform gain are produced. The selection criteria of transform parameters focusing on its robustness and performance aspects are also examined. The conferred theoretical analyses are well verified via computer simulations.

**Keywords**— Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), nonlinear companding transform (NCT), high power amplifier (HPA).

## I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) systems have been extensively applied in wireless communication systems, e.g. Worldwide Interoperability for Microwave Access (Wi MAX). It is widely known that OFDM is an attractive technique for achieving high data transmission rate in wireless communication systems and it is robust to the frequency selective fading channels [1] OFDM systems have one major disadvantage, i.e. a very high Peak-to-Average Power Ratio (PAPR) at the transmitter [2] which causes signal distortion such as in-band distortion and out-of-band radiation due to the nonlinearity of the high power amplifier (HPA) and a worse bit error rate (BER) [3]. To reduce the distortions caused by the nonlinearity of HPA it requires a large back off from the peak power which is a significant burden, especially in mobile terminals. The large PAPR increases the complexity of analog-to-digital converter (ADC) and digital-to-analog converter (DAC). Thus, PAPR

reduction is one of the major problem in OFDM systems. PAPR reduction schemes can be classified according to several criteria. First, with respect to the computational operation in the frequency domain the PAPR schemes can be categorized as multiplicative and additive schemes tone reservation (TR) [5], peak canceling, and clipping [6] are additive schemes, because peak reduction vectors are added to the input symbol vector. On the other hand, Selected mapping (SLM) and partial transmit sequences (PTS) are examples of the multiplicative scheme because the phase sequences are multiplied by the input symbol vectors in the frequency domain [4]. Second, the PAPR reduction schemes can be also categorized based on whether they are deterministic or probabilistic. Deterministic schemes, such as peak canceling, clipping Probabilistic schemes, however, statistically improve the characteristics of the PAPR distribution of the OFDM signals avoiding signal distortion. SLM and PTS are examples of the probabilistic scheme because several candidate signals are generated and that which has the minimum PAPR is selected for transmission. In this paper, we propose a new NCT algorithm which transforms the Gaussian distributed signal into a desirable distribution form defined by a linear piecewise function with an inflexion point. Compared to the previous methods, this algorithm will choose the proper transform parameters to reduce the impact of companding distortion on the BER performance. In addition, it also allows more flexibility and freedom in the companding form to satisfy various design requirements. The results regarding the achievable reduction in PAPR, signal attenuation factor, and the selection criteria of transform parameters are derived and verified through computer simulations..

## II. CHARACTERIZATION OF OFDM SIGNAL

### A. OFDM system model

Let  $X=[X_0 X_1 \dots X_{(N-1)}]^T$  denote an input symbol vector in the frequency domain, where  $X_k$  represents the complex data of the  $k$ th subcarrier and the number of subcarriers are represented by the  $N$ . The input symbol vector

is also called the input symbol sequence. Generally, an OFDM signal is the sum of independent data symbols modulated by phase-shift keying (PSK) or quadrature amplitude modulation (QAM) each of which is separated by  $1/Nt_s$  in the frequency domain, where  $t_s$  is the sampling period. Then, a continuous time baseband OFDM signal is defined as

$$a_t = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{k}{N} t}, \quad 0 \leq t \leq Nt_s \quad (1)$$

The discrete time base band OFDM signal  $a_n$  sampled at the nyquist rate  $t=nT_s$  can be given as

$$a_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{k}{N} n}, \quad n = 0, 1, \dots, N-1 \quad (2)$$

A discrete time OFDM signal vector denoted by  $\mathbf{a} = [a_0, a_1, \dots, a_{N-1}]^T$ . Then,  $\mathbf{a}$  corresponds to the inverse fast Fourier transform (IFFT) of  $\mathbf{X}$ , that is,  $\mathbf{a} = \mathbf{Q}\mathbf{X}$ , where  $\mathbf{Q}$  is the IFFT matrix. An oversampled discrete time OFDM signal vector can be denoted by  $\mathbf{a}_1 = [a_{0,1}, a_{1,1}, \dots, a_{(N-1),1}]^T$  where  $a_{n,1}$  the oversampled discrete time OFDM signal sampled at  $t = \frac{nT_s}{l}$  written as

$$a_{n,1} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X'_k e^{j2\pi \frac{k}{lN} n}, \quad n = 0, 1, \dots, lN-1 \quad (3)$$

Where i.e  $X'_k$  is

$$X'_k = \begin{cases} X_k, & 0 \leq k \leq N-1 \\ 0, & N \leq k \leq lN-1 \end{cases} \quad (4)$$

Continuous time baseband OFDM signals can be approximately represented by  $l$  times oversampled discrete time baseband OFDM signals. It is shown in [10] that choosing  $l = 4$  is sufficient to approximate the peak value of the continuous time OFDM signals.

### III. PROPOSED ALGORITHM

The main aim of the proposed algorithm is to transform the statistics of the amplitude  $|y_n|$  into the desirable PDF defined by a piecewise function, which consists of two linear functions with an inflexion point. Assume the inflexion point and cutoff point of the PDF of  $|x_n|$  are a  $B(0 < a < 1)$  and  $B(B > 0)$ , respectively. Thus, the desirable target PDF can be expressed as

$$f_{|x_n|}(y) = \begin{cases} l_1 y, & 0 \leq y \leq aB \\ l_2 y + (l_1 - l_2)aB, & aB < y \leq B \end{cases} \quad (5)$$

Where two slopes  $l_1 > 0$  and  $l_2 < 0$  are variable parameters that determine the desired companding form i.e. the ultimate PAPR, while controlling the average output power in this transform. Based on the definition of PDF  $\int_{-\infty}^{+\infty} f_{|x_n|}(y) dy = 1$ , we have

$$l_1 = \frac{2 - B^2 l_2 (a-1)^2}{B^2 a (2-a)} \quad (6)$$

$$F_{|x_n|}(y) = \begin{cases} \frac{1}{2} y^2, & 0 \leq y \leq aB \\ \frac{1}{2} y^2 + (l_1 - l_2)aBy - \frac{l_1 - l_2}{2} (aB)^2, & aB < y \leq B \\ 1, & y > B \end{cases} \quad (7)$$

$$F^{-1}_{|x_n|}(x) = \begin{cases} \sqrt{\frac{2x}{l_1}} & y \leq \frac{1}{2} (aB)^2 \\ \frac{1}{l_2} ((l_2 - l_1)aB + \sqrt{l_1 - l_2 \sqrt{(l_1 - l_2)l_1 a^2 B^2 + 2l_2 x}}) & y > \frac{1}{2} (aB)^2 \end{cases} \quad (8)$$

Clearly, CDF is a strictly monotonic increasing function and has the corresponding inverse function as follows.

Given that  $h(x)$  is also a strictly monotonic increasing function, we can obtain the following relationship

$$F_{|y_n|}(y) = \text{prob}\{|y_n| \leq y\} = \text{prob}\{h(|y_n|) \leq h(y)\} = F_{|x_n|}(h(y)) \quad (9)$$

Thus, the proposed companding function is given by

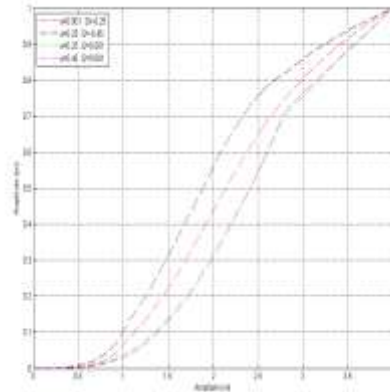


Figure 1 Transfer curves of the proposed companding function

$$h(y) = \text{sgn}(y) \cdot f^{-1}_{|y_n|} ( F^{-1}_{|y_n|}(y) )$$

From (6), the CDF of  $|x_n|$  can be represented as

$$\begin{cases} \text{sgn}(y) \sqrt{\frac{1}{l_1} (1 - e^{-\frac{y^2}{\beta^2}})} & |y| \leq \chi_0 \\ \text{sgn}(y) \frac{1}{l_2} ((l_2 - l_1)aB + \sqrt{(l_1 - l_2)l_1 a^2 B^2 + 2k_2 \left( (1 - e^{-\frac{y^2}{\beta^2}}) \right)}) & |y| > \chi_0 \end{cases} \quad (10)$$

$$B = \left( \frac{1}{2l_2} \left( \left( \frac{\beta^2}{2} - 4\frac{\beta^2}{l_2} \right)^{\frac{1}{2}} - \frac{\beta^2}{l_1} \right) \right)^{\frac{1}{2}} \quad (11)$$

Where  $\chi_0 = \beta \left( -\ln \left( 1 - \left( \frac{l_1}{2} \right) a^2 B^2 \right) \right)^{\frac{1}{2}}$  Additionally, in

order to keep the input and output signal with a constant average power level, namely  $E\{|y_n|^2\} = \beta^2$

The transfer curves of this algorithm with various parameters are plotted in Fig. 1, from that we can see that the transform can achieve more reduction in the PAPR with

$l_2$  or increasing. Especially, it is noteworthy that the EC [9] and TC are two special cases of the proposed algorithm. More specifics about the selection criteria of parameters are shown in Section IV.

At the receiver side, the companded signal can be recovered by the corresponding de-companding function as seen in (11), shown at the bottom of the page. In practice, since actual signal processed at the transmitter and receiver are the quantized signal with finite set of values

IV. PERFORMANCE STUDY

The theoretical performance of the proposed algorithm is characterized in this section by using the two main evaluation criteria: the achievable reduction in PAPR and the impact of companding distortion on the BER performance at the receiver.

A. DESIRED REDUCTION IN PAPR

The new algorithm the ultimate PAPR of the companded signal is given by

$$PAPR_y = \frac{\max_{n \in [0, N-1]} \{|y_n|^2\}}{E\{|y_n|^2\}} = \frac{B^2}{\beta^2} = \frac{(\xi_1^2 - 4\xi_2\xi_0)^{\frac{1}{2}} - \xi_1}{2\xi_2\beta^2} \quad (12)$$

Furthermore, a transform gain G is defined as the ratio of the PAPR of original signal to that of the companded signal, i.e.

$$G(\text{dB}) = 10\log_{10} \frac{PAPR_x}{PAPR_y} = 10\log_{10} \frac{2\xi_2 B_{i \max}^4}{(\xi_1^2 - 4\xi_2\xi_0)^{\frac{1}{2}} - \xi_1} \quad (13)$$

Where  $B_{i \max} = \max_{0 \leq n \leq N-1} \{|y_n|\}$

The theoretical results of  $PAPR_y$  and G are depicted in Fig. 2(a) and (b), respectively.

As can be seen, by adjusting the values of  $l_2$  and a this algorithm offers an adequate flexibility in the PAPR reduction. Consequently, the ultimate PAPR of the companded signal can be effectively confined in the interval [4.1 dB, 5.7 dB], or in other words, the achievable transform gain G in the PAPR is from 6 dB to 7.7 dB. Moreover, substituting (16) into (7), the CCDF of the PAPR with the proposed algorithm can be written as follows.

$$CCDF_y(\gamma_0) = \text{prob}\{PAPR_y > \gamma_0\} = CCDF_x\left(\frac{2\xi_2 B_{i \max}^4}{(\xi_1^2 - 4\xi_2\xi_0)^{\frac{1}{2}} - \xi_1} \gamma_0\right) \quad (14)$$

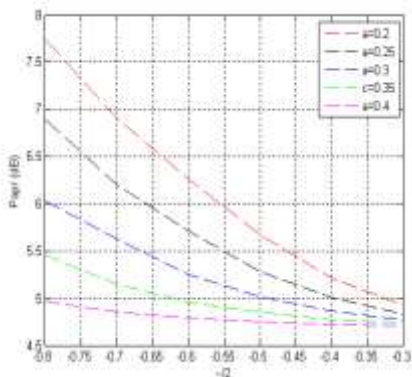


Figure 2 The ultimate PAPR of companded signals

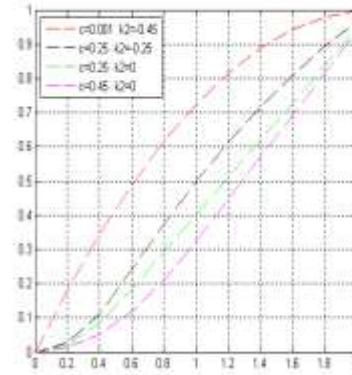


Figure 3 Theoretical results of PAPR and G versus of the proposed algorithm. The ultimate PAPR of companded signals. (b) Transform gain G.

B. COMPANDING DISTORTION IMPACT ON OFDM

NCT is an extra nonlinear operation applied to the transmitted signal. For this reason, choosing the optimal companding form and parameters is the key reason to minimize the impact of companding distortion on the BER performance. Based on the analysis results for the Gaussian signals in [15] and [16], two performance criteria: signal attenuation and companding noise  $a_n$  can be used to characterize this impact, i.e.

$$y_n = \alpha x_n + a_n \quad (15)$$

Where attenuation factor  $\alpha$ , which is given by

$$\alpha = \frac{1}{\beta^2} \int_0^{\infty} xh(x) f_{|x_n|}(x) dx \quad (16)$$

Smaller value obviously gives larger companding distortion and the reduced BER performance. It is shown that noise power of  $a_n$  is also increasing as decreases. The attenuation factor of the new algorithm can be calculated as

$$\alpha = \frac{2}{\beta^4} \int_0^{\xi_0} x^2 e^{-\frac{x^2}{\sigma^2}} \left( \frac{2}{l_1} \left( 1 - e^{-\frac{x^2}{\sigma^2}} \right) \right)^{\frac{1}{2}} dx + \frac{2}{l_2 \beta^4} \int_{\xi_0}^{\infty} x^2 e^{-\frac{x^2}{\sigma^2}} \times \left( (l_2 - l_1) a B + \sqrt{(l_2 - l_1) l_1 a^2 + 2l_2 \left( 1 - e^{-\frac{x^2}{\sigma^2}} \right)} \right) dx$$

The theoretical result of  $\alpha$  is depicted in Fig. 3, from that we can see that  $\alpha$  gradually tends to 1 as  $l_2$  and a decrease. As a result, Fig. 2 and Fig. 3 demonstrate that, to obtain an expected PAPR reduction, it may be preferable for this algorithm to make the undesired signal distortion as small as possible by choosing proper parameters. This conclusion is quite helpful to design the optimal companding form to offer an effective trade-off between the PAPR reduction and BER performance in practice.

## V. SIMULATION RESULTS

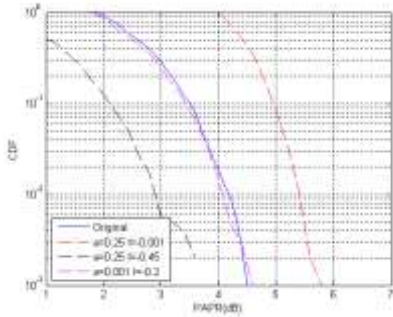


Figure 4 PAPR reduction performance of different transforms for, OFDM system With  $N=1024$ , QPSK modulation, and oversampling ratio  $J=4$ .

Fig. 4 plots the simulated PDFs of the companded signal with different transforms. As shown in this figure, by adjusting the variable parameters, this algorithm transforms the original Gaussian distribution into the desired PDF profile. In particular, unlike the EC and UC methods, this algorithm can effectively refrain from greatly increasing the probability of large signals.

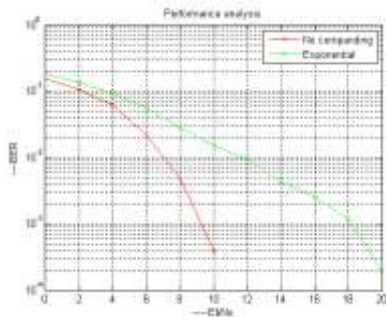


Figure 5 BR performance of different transforms under an AWGN channel for the OFDM system with  $N=1024$  and 16QAM modulation

Fig. 5 depicts the simulation results of CCDF of the PAPR with different transforms. The theoretical results are also presented. that the predicted theoretical and the experimental curves almost coincide. As a reference, the curve of 'original signal' is the ideal performance bound. As it is apparent, while NCT methods effectively reduce the PAPR,

## VI. CONCLUSION

In OFDM systems the high PAPR is considered to be one of the major drawback because the large signal fluctuation gives rise to the low power efficiency, NCT is an attractive solution to reduce the PAPR of OFDM signal due to its simplicity and effectiveness, In this paper we investigate a new NCT algorithm which changes the statistics of original signal from the complex Gaussian to a desirable PDF defined as a linear piecewise function. Thus, an effective and flexible trade-off between the PAPR and BER performance can be achieved to satisfy various system requirements. Theoretical performance of this algorithm is

characterized by means of the achievable reduction in PAPR and signal attenuation factor. Compared to the original signal the proposed algorithm can offer the transform gain in PAPR of 6.0 dB to 7.7 dB. In addition, the impact caused by companding distortion can be significantly reduced by choosing proper transform parameters; simulation results indicate that the new proposed algorithm substantially outperforms the existing NCT methods in the overall performance of OFDM system.

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