

# A Wavelet Based Image Restoration for MR images

*Jyoti S. Gadakh<sup>1</sup>, Prof. P.R.Thorat<sup>2</sup>*

<sup>1</sup>Savitribai Phule Women's Engineering College, Dr. Babasaheb Ambedkar Marathwada University  
 Aurangabad, Maharashtra, India  
*jiyotigadakh8@gmail.com*

<sup>2</sup>Principal, Savitribai Phule Women's Engineering College, Dr. Babasaheb Ambedkar Marathwada University  
 Aurangabad Maharashtra, India  
*thorat.popat.r@gmail.com*

**Abstract:** In medical image processing, medical images are corrupted by different type of noises. It is very important to obtain precise images to facilitate accurate observations for the given application. Removing of noise from medical images is now a very challenging issue in the field of medical image processing. Most well known noise reduction methods, which are usually based on the local statistics of a medical image, are not efficient for medical image noise reduction. This paper presents a new and fast method for removal of noise and blur from Magnetic Resonance Imaging (MRI) using wavelet transform. In this work we utilize a fact that wavelets can represent magnetic resonance images well, with relatively few coefficients. We use this property to improve MRI restoration with arbitrary k-space trajectories. Image restoration is posed as an optimization problem that also could be solved with the Fast iterative shrinkage thresholding algorithm (FISTA). Using mathematical analysis we show that our non linear method is performing fast than other regularization algorithms.

**Keywords-** Fast iterative shrinkage thresholding algorithm (FISTA), Magnetic resonance imaging (MRI), wavelets.

## 1. Introduction

Magnetic Resonance Imaging (MRI) is a medical imaging technique that has proven to be particularly valuable for examination of the soft tissues in the body. MRI is an imaging examination of the soft tissues in the body. MRI is an imaging that makes use of the phenomenon of nuclear spin resonance. Since the discovery of MRI, this technology has been used for many medical applications. Because of the resolution of MRI and the technology being essentially harmless it has emerged as the most accurate and desirable imaging technology [1]. MRI is primarily used to demonstrate pathological or other physiological alterations of living tissues and is a commonly used form of medical imaging. Despite significant improvements in recent years, magnetic resonance (MR) images often suffer from low signal-to-noise ratio (SNR) or contrast-to-noise ratio (CNR), especially in cardiac and brain imaging. Therefore, noise reduction techniques are of great interest in MR imaging as well as in other imaging modalities.

Magnetic Resonance Imaging scanners provide data that are samples the special Fourier transform i.e. K-space of the object under investigation. The Shannon–Nyquist sampling theory in both spatial and k-space domains suggests that the sampling density should correspond to the field-of-view (FOV) and that the highest sampled frequency is related to the pixel width of the reconstructed images. However, constraints in the implementation of the k-space trajectory that controls the sampling pattern (e.g., acquisition duration,

scheme, smoothness of gradients) may impose locally reduced sampling densities. Insufficient sampling results in reconstructed images with increased noise and artifacts, particularly when applying gridding methods.

The common and generic approach to alleviate the reconstruction problem is to treat the task as an inverse problem. In this framework, ill-posedness due to a reduced sampling density is overcome by introducing proper regularization constraints. They assume and exploit additional knowledge about the object under investigation to robustify the reconstruction. Earlier techniques used a quadratic regularization term, leading to solutions that exhibit a linear dependence upon the measurements. Unfortunately, in the case of severe undersampling (i.e., locally low sampling density) and depending on the strength of regularization, the reconstructed images still suffer from noise propagation, blurring, ringing, or aliasing errors. It is well known in signal processing that the blurring of edges can be reduced via the use of nonquadratic regularization.

In particular, -wavelet regularization has been found to outperform classical linear algorithms such as Wiener filtering in the deconvolution task.

Many recent works in MRI have focused on nonlinear reconstruction via total variation (TV) regularization, choosing finite differences as a sparsifying transform. Nonquadratic wavelet regularization has also received some attention, but we are not aware of a study that compares

the performance of TV against  $\ell_1$  wavelet regularization. Various algorithms have been recently proposed for solving general linear inverse problems subject to  $\ell_1$ -regularization. Some of them deal with an approximate reformulation of the  $\ell_1$  regularization term. This approximation facilitates reconstruction sacrificing some accuracy and introducing extra degrees of freedom that make the tuning task laborious. Instead, the iterative shrinkage/thresholding algorithm (ISTA) is an elegant and nonparametric method that is mathematically proven to converge. A potential difficulty that needs to be

overcome is the slow convergence of the method when the forward model is poorly conditioned (e.g., low sampling density in MRI). This has prompted research in large-scale convex optimization on ways to accelerate ISTA.

We propose a fast algorithm for solving the non linear reconstruction problem and presents arguments to explain its superior speed of convergence.

## 2. Model of Data Formation

We consider MRI in two dimensions, in which case a 2D plane is excited. The time-varying magnetic gradient fields that are imposed define a trajectory in the (spatial) Fourier domain that is often referred to as k-space. We denote by the coordinates in that domain. The excited spins, which behave as radio-frequency emitters, have their precessing frequency and phase modified depending on their positions. The modulated part of the signal received by a homogeneous coil is given by,

$$m(\mathbf{k}) = \hat{\rho}(k) = \int \rho(r) e^{-j(k,r)} dr \quad (1)$$

It corresponds to the Fourier transform of the spin density  $\rho$  that we refer to as *object*. The  $N$  measurements, concatenated in the vector  $m = (m_1, m_2 \dots m_N)$  correspond to sampled values of this Fourier transform at the frequency locations  $k_N$  along the k-space trajectory.

## 3. Model for the Original Data

We consider that the Fourier domain and, in particular, the sampling points  $k_N$ , are scaled to make the Nyquist sampling interval unity. This can be done without any loss of generality if the space domain is scaled accordingly. Therefore, we model the object as a linear combination of pixel-domain basis functions  $\varphi_p$  that are shifted replicates of some generating function  $\varphi$ , so that,

$$\rho = \sum_{p \in \mathbb{Z}^2} c[p] \varphi_p \quad (2)$$

With

$$\varphi_p(r) = \varphi_p(r-p). \quad (3)$$

In MRI, the implicit choice for  $\varphi$  is often Dirac's delta. The image to be reconstructed i.e. sampled version of object  $\rho(p)$  is obtained by filtering the coefficients  $c[p]$  with discrete filter.

$$P(e^{jw}) = \sum_{h \in \mathbb{Z}^2} \hat{\varphi}(w + 2\pi h) \quad (4)$$

Where  $\hat{\varphi}$  denotes the fourier transform of  $\varphi$ .

In the Wavelet formalism some constraints apply on  $\varphi$ . It must be a scaling function that satisfies the properties for a multiresolution. In that case, the wavelets can be defined as linear combinations of the  $\varphi_p$  and the object is equivalently characterized by its coefficients in the orthonormal wavelet basis. There exists a discrete wavelet

transform (DWT) that bijectively maps the coefficients to the wavelet coefficients that represent the same object in a continuous wavelet basis. In the rest of the paper, we represent this DWT by the synthesis matrix  $W$ .

## 4. Matrix Representation of a model

Since a FOV determines a finite number  $M$  of coefficients  $c[P]$ , we handle them as a vector  $c$ , keeping the discrete coordinates  $p$  as implicit indexing. By simulating the imaging of the object (2), and by evaluating (1) for  $k=k_n$ , we find that the noise-free measurements are given by,

$$M_o = Ec \quad (5)$$

Where  $E$ , the MRI matrix, is decomposed as,

$$E = \text{diag}(\hat{\varphi}(k_n)[s_1, \dots, s_n]) \quad (6)$$

There,  $S_n$  a space-domain vector such that

$$S_n[P] = e^{-j(W_n, p)}. \quad (7)$$

A more realistic data-formation model is,

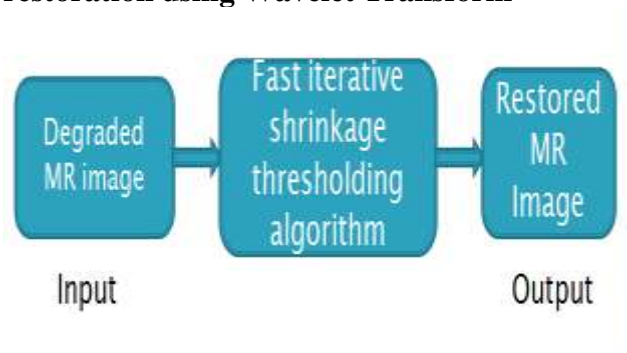
$$m = Ec + b \quad (8)$$

or

$$m = M\omega + b \quad (9)$$

with  $M = EW$  and the residual vector  $b$  representing the effect of measurement of noise and scanner imprecision. The inverse problem of MRI is then to recover the  $M$  coefficients  $\omega$  (or  $c$ ) from the  $N$  corrupted measurements  $m$ . Its degree of difficulty depends on the magnitude of the noise  $b$  and the conditioning of the matrix  $M$  (or  $E$ ).

## 5. Basic Block Diagram For MR image restoration using Wavelet Transform



## 5. Algorithm

- Step 1 : Read Image.
- Step 2: Apply wavelet transform to calculate wavelet coefficients.
- Step 3 : Apply inverse wavelet transform ;calculate Gaussian value and centre of mask.

- Step 4 :Call the function for deblurring.  
 Step 5 :Assign default values to parameters.  
 Step 6 :Calculate the Lipschitz constant of Gradient of  $\|A(x) - \text{image}\|^2$   
 Step 7 :Parameter initialization.  
 Step 8 : Store the current values of iterate and constant in some variables.  
 Step 9 : Calculate Gradient step.  
 Step 10 : Apply Wavelet transform.  
 Step 11 : Soft thresholding using Lipschitz constant.  
 Step 12 : Apply Inverse wavelet transform to calculate new iterate.  
 Step 13 : Update t and Y parameters.  
 Step 14 : Compute  $l_1$  -normalization of the wavelet transform and the function value . Store the values.

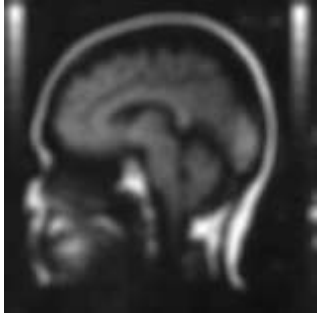
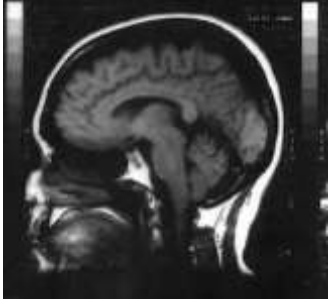
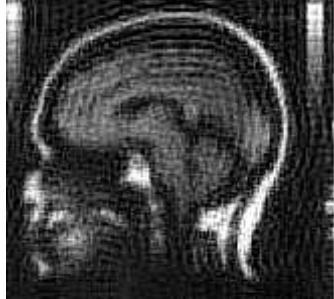
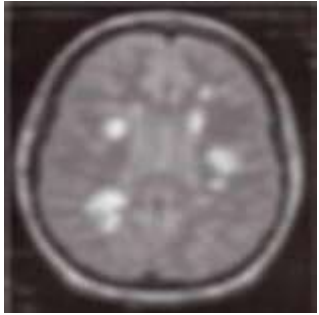


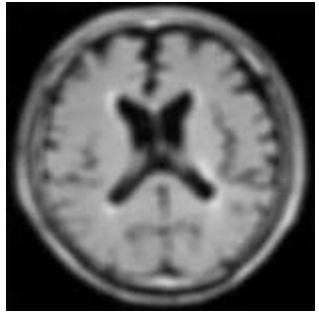
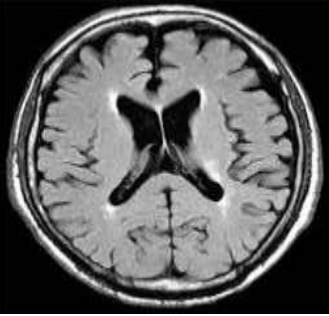



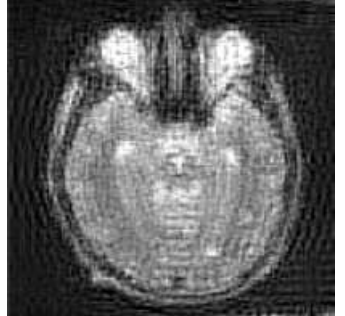


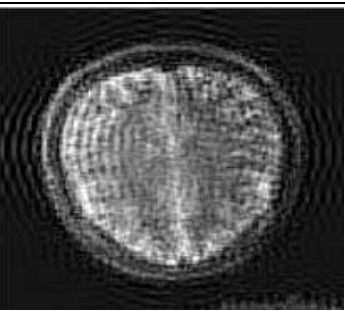
## 6. Result

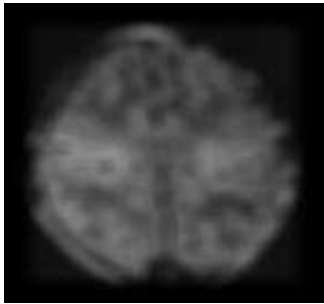
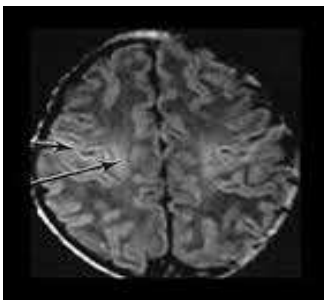
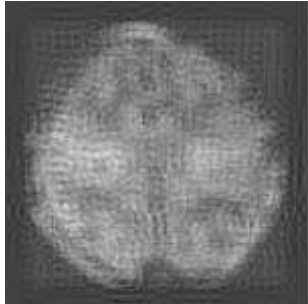
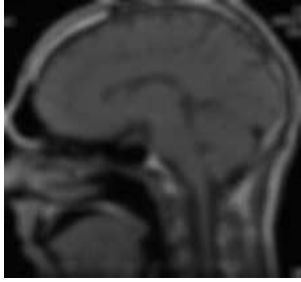


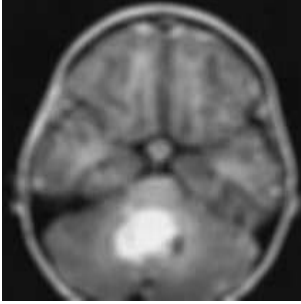
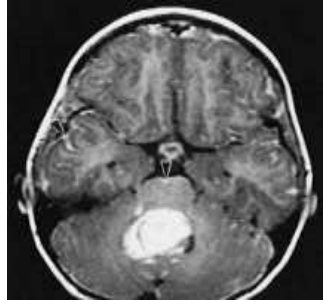
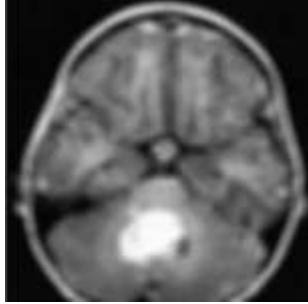
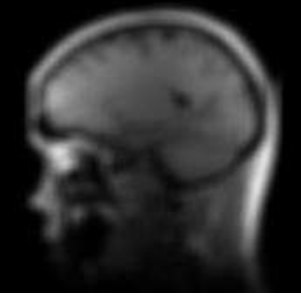

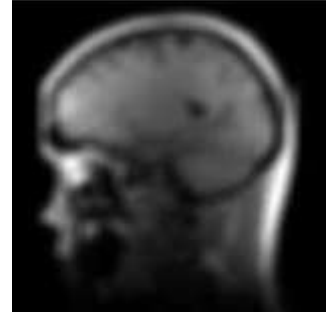
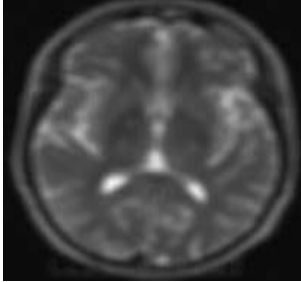
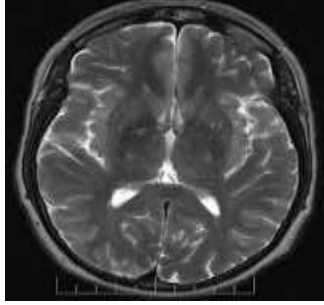
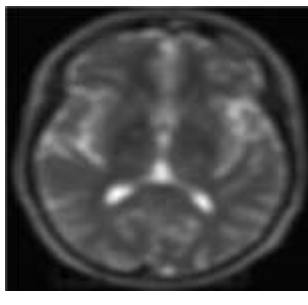
In this section we present different experiments we conducted on degraded MRI images of brain collected from different resources. The two main reconstruction performance measures that we considered are as follows.

- Signal to noise ratio with respect to a reference. Practically, the references are either the ground-truth images or the minimizer of the cost functional. It is known that SNR is not a foolproof measure of visual improvement but large SER values are encouraging and generally correlate with good image quality.
- Root Mean Square Error.

### 6.1 Result Table for Haar periodic Method

Figure No.	PSNR	RMSE
Fig. 1	10.019	80.4619
Fig. 2	6.6706	118.307
Fig. 3	6.6845	118.1174
Fig. 4	7.3171	109.8214
Fig. 5	9.5057	85.3597
Fig. 6	11.6033	67.046
Fig. 7	11.2754	69.6257
Fig. 8	7.6	106.3022
Fig. 9	12.4477	60.8353
Fig. 10	12.3896	61.2433
Fig. 11	11.905	64.7578
Fig. 12	11.0373	71.5609
Fig. 13	7.3146	109.8523
Fig. 14	11.4506	68.2356
Fig. 15	10.201	78.7931
Fig. 16	10.201	78.7931
Fig. 17	9.555	84.8766
Fig. 18	9.3537	86.8666

FigureNo.	Noisy Image	Reference Image	Restored Image
Fig 1			
Fig 2			
Fig 3			
Fig 4			
Fig 5			

FigureNo.	Noisy Image	Reference Image	Restored Image
Fig 11			
Fig 12			
Fig 13			
Fig 14			
Fig 15			

## 6.2 Result of restoration algorithm for different MRI Images

## Conclusion

Implementation of wavelet restoration technique has been proposed to restore the MR images. Theoretical evidence provided says that this algorithm leads to faster convergence with good image quality. Number of arithmetic computations required are less. Hence it is a best method to restore the Brain MR image.

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### Author Profile



**Jyoti S. Gadakh** received the B.E. degree in Electronics and telecommunication from Pune university, and worked as an assistant Professor at Mumbai Educational Trust's Institute of Engineering Bhujbal knowledge city,Nashik during 2008-20014.She is currently pursuing M.E. in Electronics Engineering from Savitribai Phule Women's Engineering College, Dr.Babasaheb Ambedkar Marathawada University,Aurangabad.

### Author Profile



**Prof.P.R.Thorat** received the B.E.& M.E. degree in Electronics Engineering from Dr.Babasaheb Ambedkar Marathawada University,Aurangabad.,and worked as a Lecturer in SSKS polytechnic at Sillod Dist. Aurangabad, worked as a Production engineer in AKAR group during 1995-2003 worked as an assistant Professor at Hi-techInstitute of technology Aurangabad during 2003-20010.He is currentiy working as a I/C Principal Savitribai Phule Women's Engineering College, Dr.Babasaheb Ambedkar Marathawada University,Aurangabad.