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Performance Measures for a three-unit compact circuit

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Abstract

This paper analyses a three component system with single repair facility. Denoting the failure times of the components as T_1 and T_2 and the repair time as R, the joint survival function of (T_1, T_2, R) is assumed to be that of trivariate distribution of Marshall and Olkin. Here, R is an exponential variable with parameter α and T_1 and T_2 are independent of each other. In this paper use of Laplace-Transform is taken for finding Mean Time Between Failure, Availability and Mean Down Time and table for Reliability measure is shown in the end.

Key Words : MTBF, Availability, MDT, Reliability.

[1] Introduction

Reliability measures for a two-component standby system with repair facility were obtained by several authors under different assumptions in the past. Lie et al. (1977) and Yearout et al. (1986) have done extensive reviews for the failure times and repair times assuming that these are statistically independent. Joshi and Dharmadhikari (1989) considered the bivariate exponential distribution to derive the performance measures associated with a two-component standby system. Goel and Srivastava (1991) considered a correlated structure for the failure and repair times and obtained various reliability measures.

In many situations, a unit or system can be repaired immediately after breakdown. In such cases, the mean time between failures refers to the average time of breakdown until the device is beyond repair. When a system is often unavailable due to breakdowns and is put back into operation after each breakdown with proper repairs, the mean time between breakdowns is often defines as the mean time between failures. If we consider only active repair time i.e. the time spent for actual repair, then the mean time to repair (MTTR) is the statistical mean time for active repair. It is the total active repair time during a given period divided by the number of during the same interval. Frequently, a system may be unavailable on account of periodic inspections and not because of breakdowns. By the systematic inspection or preventive maintenance for the detection of defects and prevention of failures, the system is kept in a satisfactory operational condition. The time spent for this is termed as the preventive maintenance downtime. There is difference between mean time between maintenance (MTBM) and mean time between failures (MTBF). When preventive maintenance downtime is zero or is not considered, MTBM is same as MTBF.

Here in this paper, we perform the analysis of a three-component standby system with single repair facility. Here T_1 is the exponential variable with parameter k_1 and T_2 is another random variable with parameter k_2. R is an exponential variable with parameter α . T_1 and T_2 are independent of each other. It is further assumed that these components are identical in nature and each unit works as new after the repair and switching devices are perfect and instantaneous.

The following are the assumptions for the model:

(i) The system is composed of three components linked in parallel-series configuration (Fig. 1).

(ii) The components are non-identical in nature.

(iii) At time t=0, all the components are in operable mode.

(iv) After repair each unit works as new.

(v) Switching devices are perfect and instantaneous.

Define $p_i(t) = \Pr\{X(t) = i : X(0) = 0\}, i = 0, 1, 2$

Here in this model, we consider the following trivariate exponential distribution for (T_1, T_2, R) with survival function of (Marshall and Olkin, 1967) of the form :

$$\overline{F}(t_1, t_2, t_3) = \exp[-k_1(t_1 + t_2) - k_2 t_3 - k_3\{\max(t_1, t_2) + \max(t_2 + t_3)\}]$$
$$t_1, t_2, t_3 \geq 0; k_1, k_2 > 0; k_3 \geq 0 \qquad [1.1]$$

It may be observed that

i) T_1 and T_2 are independent and identically distributed exponential random variables with the parameter (k_1+k_2),

ii) R is exponential with the parameter (k_2+2k_3), not necessarily independent of (T_1,T_2) and

iii) (T_1,R) and (T_2,R) are identically distributed as bivariate exponential with the parameter (k_1, k_2+k_3, k_3).

By considering (1) as the survival function of (T_1,T_2,R) , we obtain expressions for reliability, MTBF and the gain due to repair facility. The state transition diagram for the system, in the interval $(t, t + \Delta t)$ is given below:

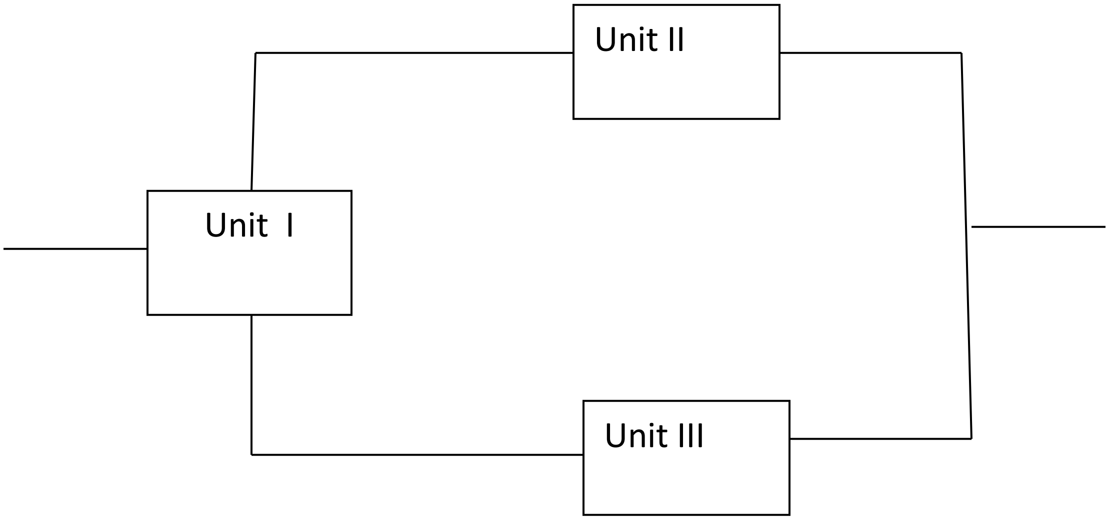

Figure 1 State Transition Diagram of three-unit standby System

[2] MTBF Calculation

Since reliability of the system is given by R(t)= P_0(t) + P_1(t) + P_2(t), we want to find the expressions for P_0(t), P_1(t) and P_2(t). We know that MTBF = $\int_0^\infty R(t)\,dt$

We have, $p_i(t) = p_r\{X(t) = i : X(0) = 0\}\ , i = 0,1,2$

So, $p_i(t+\Delta t) = p_r\{X(t+\Delta t) = i : X(0) = 0\}$

$$= \sum_{j=0}^{2} p_r\{X(t+\Delta t) = 0\ , X(t) = j : X(0) = 0\}$$

Using $P(AB) = P(A/B)\,P(B)$, we have

$$= \sum_{j=0}^{2} p_r\{X(t+\Delta t) = 0\ , X(t) = j\}\{X(t) = j :\ X(0) = 0\}$$

$$= \sum_{j=0}^{2} p_r\{X(t+\Delta t) = 0\ , X(t) = j\}\ p_j(t)$$

$$= p_r\{X(t+\Delta t) = 0 : X(t) = 0\}\ p_0(t) + p_r\{X(t+\Delta t) = 0 : X(t) = 1\}\ p_1(t)$$
$$+ p_r\{X(t+\Delta t) = 0 : X(t) = 2\}\ p_2(t)$$

$$p_0(t+\Delta t) = (1-\alpha\,\Delta t)\,p_0(t) + \gamma\,\Delta t\;p_1(t) + 0(\Delta t)$$

$$p_0(t+\Delta t) - p_0(t) = -\alpha\,\Delta t\;p_0(t) + \gamma\,\Delta t\;p_1(t) + 0(\Delta t)$$

Dividing by Δt and taking limit $\Delta t \to 0$ we have

$$\underset{\Delta t\to 0}{Lim}\frac{p_0(t+\Delta t)-p_0(t)}{\Delta t} = -\alpha\;p_0(t) + \gamma\;p_1(t) + \underset{\Delta t\to 0}{Lim}\frac{0(\Delta t)}{\Delta t}$$

$$p_0^{/}(t) = -\alpha\;p_0(t) + \gamma\;p_1(t) \qquad [2.1]$$

Similarly, we can write $p_1(t+\Delta t) = pr(X(t+\Delta t)=1 : X(0)=0\}$

$$p_1(t+\Delta t) = \sum_{j=0}^{2} pr(X(t+\Delta t)=1\;,\;X(t)=j : X(0)=0\}$$

$$= \sum_{j=0}^{2} pr(X(t+\Delta t)=1\;,\;X(t)=j\,\}\;p_j(t)$$

$$= pr\{X(t+\Delta t)=1 : X(t)=0\}\;p_0(t) + pr\{X(t+\Delta t)=1 : X(t)=1\}\;p_1(t)$$
$$+\; pr\{X(t+\Delta t)=1 : X(t)=2\}\;p_2(t)$$

so, we have $p_1(t+\Delta t) = \alpha\,\Delta t\;p_0(t) + \{1-(\beta+\gamma)\Delta t\}\;p_1(t)$ which on simplification gives

$$p_1^{/}(t) = \alpha\;p_0(t) - (\beta+\gamma)\,p_1(t)$$

Assuming, $\alpha = k_1 + k_3$, $\beta = k_1$ and $\gamma = k_2 + k_3$, we get the equations [6.2.3] and [6.2.4] in the following reduced form:

$$p_0^{/}(t) = -(k_1+k_3)\;p_0(t) + (k_2+k_3)\;p_1(t)$$

$$p_1^{/}(t) = (k_1+k_3)\;p_0(t) - (k_1+k_2+k_3)\;p_1(t)$$

$$p_2^{/}(t) = k_2\;p_0(t) - (k_1+k_2+k_3)\;p_1(t) + k_2\,p_2(t) \qquad [2.2\text{-}2.4]$$

Taking Laplace transform on both the sides of [6.1.5] and [6.1.6] and noting that $L\{p_i(t)\} = L_i(s)$

We get $(k_1+k_3+s)\,L_0(s) - (k_2+k_3)\,L_1(s) + (k_3+s)\,L_2(s) = 1$

$$(k_1 + k_3) L_0(s) - (k_1 + k_2 + k_3 + s) L_1(s) + k_2 L_2(s) = 0$$

$$and \quad (k_2 + s) L_0(s) - (k_1 + s) L_1(s) + (k_1 + k_3 + s) L_2(s) = 0$$

solving using Cramer rule, we get

$$L_0(s) = \frac{(k_1 + k_2 + k_3 + s)}{\{s^2 + (2k_1 + k_2 + 2k_3)s + k_1(k_1 + k_3)\}} \qquad L_1(s) = \frac{(k_1 + k_3)}{\{s^2 + (2k_1 + k_2 + 2k_3)s + k_1(k_1 + k_3)\}} \text{ and}$$

$$L_2(s) = \frac{(k_2 + k_3 + s)}{\{s^2 + (2k_1 + k_2 + 2k_3)s + k_1(k_1 + k_3)\}}$$

Let s_1 and s_2 be the roots of the equation $s^2 + (2k_1 + k_2 + 2k_3)s + k_1(k_1 + k_3) = 0$

$$let \quad s_1 = \frac{-(2k_1 + k_2 + 2k_3) + \sqrt{k_2^2 + 4(k_1 + k_3)(k_2 + k_3)}}{2}$$

$$and \quad s_2 = \frac{-(2k_1 + k_2 + 2k_3) - \sqrt{k_2^2 + 4(k_1 + k_3)(k_2 + k_3)}}{2} \qquad [2.5\text{-}2.6]$$

Since s_1 , s_2 <0 . Thus we have,

$$L_0(s) = \frac{(k_1 + k_2 + k_3 + s)}{\{(s - s_1)(s - s_2)\}} \; ; \quad L_1(s) = \frac{(k_1 + k_3)}{\{(s - s_1)(s - s_2)\}} \quad ; \quad L_2(s) = \frac{(k_2 + k_3 + s)}{\{(s - s_1)(s - s_2)\}}$$

Resolving into partial fractions, we have

$$L_0(s) = \frac{(k_1 + k_2 + k_3 + s_1)}{(s - s_1)(s_1 - s_2)} - \frac{(k_1 + k_2 + k_3 + s_2)}{(s - s_2)(s_1 - s_2)}$$

$$L_1(s) = \frac{(k_1 + k_3)}{\{(s - s_1)(s - s_2)\}}$$

$$L_2(s) = \frac{(k_2 + k_3)}{\{(s - s_1)(s - s_2)\}} \qquad [2.7\text{-}2.9]$$

Taking inverse Laplace transforms of the above equations , we get

$$p_0(t) = \frac{(k_1 + k_2 + k_3 + s_1)e^{s_1 t} - (k_1 + k_2 + k_3 + s_2)e^{s_2 t}}{s_1 - s_2} \text{ and}$$

$$p_1(t) = \frac{(k_1 + k_3 + s_1)(e^{s_1 t} - e^{s_2 t})}{s_1 - s_2}$$

$$p_2(t) = \frac{(k_2 + k_3)(e^{s_1 t} - e^{s_2 t})}{s_1 - s_2} \qquad [2.10\text{-}2.12]$$

Hence the reliability of the system is given by $R(t) = p_0(t) + p_1(t) + p_2(t) = \dfrac{s_2 e^{s_1 t} - s_1 e^{s_2 t}}{s_1 - s_2}$

MTBF = $\int_0^\infty R(t)\,dt = -\dfrac{(s_1 + s_2)}{s_1 s_2}$ where s_1 and s_2 are given by equation above.

So, MTBF = $\dfrac{(k_1 + k_2 + 2k_3)}{k_1(k_1 + k_3)}$ [2.11]

it may be noted that MTBF when there is no repair facility is given by

MTBF (no repair facility) = E (T_1+T_2) = E (T_1) + E (T_2) = $\dfrac{1}{k_1 + k_3} + \dfrac{1}{k_1 + k_3} = \dfrac{2}{k_1 + k_3}$

[2.12]

[3] Availability analysis of the system

In this section, we consider the transient solution of the system and the availability measures such as the point wise availability and the steady-state availability by considering the above model. By considering the equation [1.1] as the survival function of (T_1, T_2, R), we obtain the expressions for point wise availability and the steady-state availability. Using similar arguments as in the case of MTBF, we obtain the following differential equations:

$$p_0^{/}(t) = -(k_1 + k_3)p_0(t) + (k_2 + k_3)p_1(t)$$

$$p_1^{/}(t) = (k_1 + k_3)p_0(t) - (k_1 + k_2 + k_3)p_1(t) + (k_2 + 2k_3)p_2(t)$$

$$p_2^{/}(t) = k_1 p_1(t) - (k_2 + 2k_3)p_2(t)$$

[3.1-3.3]

Taking Laplace transforms of above three equations and applying the Cramer’s rule, we get

$$L_0(s)=\frac{s^2+(k_1+2k_2+2k_3)s+(k_2+k_3)(k_2+2k_3)}{s^3+2(k_1+k_2+2k_3)s^2+\{(k_1+k_2+2k_3)^2-k_1(k_2+k_3)\}s}$$

$$L_1(s)=\frac{(k_1+k_3)(k_2+2k_3+s)}{s^3+2(k_1+k_2+2k_3)s^2+\{(k_1+k_2+2k_3)^2-k_1(k_2+k_3)\}s}$$ and

$$L_2(s)=\frac{k_1(k_1+k_2)}{s^3+2(k_1+k_2+2k_3)s^2+\{(k_1+k_2+2k_3)^2-k_1(k_2+k_3)\}s}$$

[3.4-3.6]

Let s_1 and s_2 be the roots of the equation

$$s^2+2(k_1+k_2+2k_3)s\ \ +(k_1+k_2+2k_3)^2-k_1(k_2+k_3)=0$$

Resolving into partial fractions, we have

$$L_0(s)=\frac{s_2\{s_1^2+(k_1+2k_2+2k_3)s_1+(k_2+k_3)(k_2+2k_3)\}}{(s_1-s_2)(s-s_1)s_1s_2}+\frac{s_1\{s_2^2+(k_1+2k_2+2k_3)s_2+(k_2+k_3)(k_2+2k_3)\}}{(s_2-s_1)(s-s_2)s_1s_2}$$
$$+\frac{(k_2+k_3)(k_2+2k_3)}{s\,s_1s_2}$$

Similarly, we can write

$$L_1(s)=\frac{(k_1+k_3)[s_2(k_2+2k_3+s_1)]}{(s_1-s_2)(s-s_1)s_1s_2}+\frac{s_1(k_2+2k_3+s_2)}{(s_2-s_1)(s-s_2)s_1s_2}+\frac{(k_2+2k_3)}{s\,s_1s_2}$$

$$L_2(s)=\frac{s_2k_1(k_1+k_3)}{(s_1-s_2)(s-s_1)s_1s_2}+\frac{s_1}{(s_2-s_1)(s-s_2)s_1s_2}+\frac{1}{s\,s_1s_2}$$

[3.7- 3.8]

Taking Inverse Laplace transform on both the sides of equations [6.2.7-6.2.8], we get

$$p_0(t)=\frac{s_2e^{s_1t}\{s_1(s_1+k_1+2k_2+3k_3)+(k_2+k_3)(k_2+2k_3)\}}{s_1s_2(s_1-s_2)}$$
$$+\frac{s_1e^{s_2t}\{s_2(s_2+k_1+2k_2+3k_3)(k_2+k_3)(k_2+2k_3)\}}{s_1s_2(s_2-s_1)}+\frac{(k_2+k_3)(k_2+2k_3)}{s_1s_2}$$

$$p_1(t)=\frac{s_2e^{s_1t}(s_1+k_2+2k_3)(k_1+k_3)}{s_1s_2(s_1-s_2)}+\frac{s_1e^{s_2t}(s_2+k_2+2k_3)(k_1+k_3)}{s_1s_2(s_2-s_1)}+\frac{(k_2+2k_3)}{s_1s_2}$$

$$p_2(t)=\frac{s_2e^{s_1t}k_1(k_1+k_3)}{s_1s_2(s_1-s_2)}+\frac{s_1e^{s_2t}k_1(k_1+k_3)}{s_1s_2(s_2-s_1)}+\frac{k_1(k_1+k_3)}{s_1s_2}$$ [3.9-3.11]

The point availability of the system is given as A (t) $=p_0(t)+p_1(t)= 1-p_2(t)$ i.e.

$$A(t)=1-\frac{s_2e^{s_1t}k_1(k_1+k_3)}{s_1s_2(s_1-s_2)}+\frac{s_1e^{s_2t}k_1(k_1+k_3)}{s_1s_2(s_2-s_1)}+\frac{k_1(k_1+k_3)}{s_1s_2} \quad [3.12]$$

Thus the steady state availability of the system is given as

$$A_\infty=\lim_{t\to\infty}A(t)=1-\frac{k_1(k_1+k_3)}{s_1s_2}=\frac{(k_2+2k_3)(k_1+k_2+2k_3)}{(k_2+2k_3)^2+k_1(k_1+k_2+3k_3)} \quad [3.13]$$

[4] Mean Down Time Calculation: The system mean down time is an important aspect of Availability analysis and is evaluated by the formula $MDT=MTBF(\frac{1-A_\infty}{A})$. So, using the results from [3.10] and [3.13], we get

$$MDT=\frac{2k_1+k_2+2k_3}{(k_1+2k_3)(2k_1+k_2+2k_3)} \quad [4.1]$$

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[6] Observation:

We provide the data in the following table. The table gives the values for MDT for various values for k_1, k_2 and k_3.

Table 6.1

k_1	k_2	k_3	S_1	S_2	R(t)
.01	0.1	.01	-5.984	-6.293	$12.992e^{-2t}-10.526e^{-2.5t}-.02e^{-7.876t}$
			-3.123	-5.603	$10.965e^{-4t}-0.785e^{-6.5t}-.802e^{-4.976t}$
			-2.136	-1.610	$9.095e^{-6t}-0.584e^{-7.5t}-.04e^{-9.872t}$
.02	0.2	.02	-7.000	-6.177	$3.09e^{-2.07t}-5.506e^{-2.5t}-.076e^{-6.874t}$
			-6.667	-5.986	$2.654e^{-4.24t}-1.085e^{-6.86t}-2.202e^{-0.906t}$
			-3.125	-2.955	$4.889e^{-6.76t}-0.004e^{-2.56t}-3.045e^{-6.952t}$
.03	0.3	.03	-6.875	-6.172	$4.225e^{-2.0t}-3.793e^{-7.5t}-.068e^{-6.07t}$
			-4.414	-4.133	$6.259e^{-7.25t}-3.005e^{-2.85t}-2.762e^{-0.006t}$
			-3.250	-3.074	$7.809e^{-3.75t}-0.974e^{-2.06t}-0.049e^{-0.952t}$
.04	0.4	.04	-7.224	-6.179	$10.09e^{-5.07t}-5.506e^{-2.5t}-.084e^{-0.874t}$
			-4.454	-3.264	$12.690e^{-9.24t}-7.025e^{-9.82t}-9.002e^{-0.006t}$
			-1.667	-1.633	$14.249e^{-0.76t}-0.904e^{-8.56t}-3.045e^{-6.952t}$