

Prediction of post overload fatigue crack growth life of HSLA steel under mixed-mode (I and II) spike overload by using genetic programming

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Abstract

In the present investigation, fatigue crack growth tests under mixed-mode (I and II) overload have been conducted on HSLA steel and subsequently genetic programming has been applied to predict post overload fatigue life. It is observed that the proposed model predicts fatigue life of HSLA steel with reasonable accuracy.

Keywords: Genetic programming; Delay cycle; Mixed-mode; HSLA steel; Fatigue life;

1. INTRODUCTION

High strength low alloy (HSLA) steels are designed and developed to provide better mechanical properties and greater resistance to atmospheric corrosion than conventional carbon steels. They have high strength, toughness, weldability, formability and corrosion resistance which render them suitable for use in a wide variety of structural applications. These structural components are frequently subjected to cyclic loading during their service lives which causes fatigue failure. As the failure due to fatigue is one of the prime concerns in structural design, its evaluation and prediction of fatigue life is thus very important to avoid catastrophic failure. For this purpose, the principles of fracture mechanics are used to determine whether the cracks will grow large enough to cause catastrophic failure before they can be detected during a periodic inspection. To predict fatigue crack growth life a large data base has to be created which requires large number of fatigue tests on varieties of materials. However, these tests are costly and also time consuming. With the advent of sophisticated computational facilities, now a day, alternative methods have been designed and developed to predict fatigue life based on experimental base line data in order to avoid costly fatigue tests.

Fatigue crack propagation is a path dependent process and is strongly affected by the load sequence [1]. The load sequence may consist of simple constant amplitude load, superimposed overloads and under loads, variable amplitude loads, block loads etc. In real situations, components and structures are frequently exposed

to complex stress field because of the inconsistent variation of applied fatigue loads and the mode-mixity. Over many years, there have been many studies on crack behavior under complex stress fields [2–5]. However, studies on load interactions (particularly superimposed overloads) under mixed-mode (I and II) loading conditions [6–9] on fatigue behavior are still limited. As far as prediction of fatigue life under interspersed mixed-mode (I and II) overload is concerned, it is a complex phenomena due to the interaction effects that exist during fatigue crack growth under this loading situation. Therefore, evolutionary computational methods such as artificial neural network (ANN), genetic algorithm (GA), fuzzy-logic, adaptive neuro-fuzzy inference system (ANFIS) etc. have emerged as alternative modeling tools in the field of fatigue. Genel [10] has applied ANN for predicting the strain-life fatigue properties using tensile material data of steels. Fotovati and Goswami [11] have used ANN approach to predict fatigue crack growth rate in Ti-6Al-4V alloy at elevated temperature. Jarrah et al. [12] has applied ANFIS to model the fatigue behavior of unidirectional glass fiber / epoxy composites under tension-tension and tension-compression loading. Genetic programming (GP) has been applied by Vassilopoulos and Georgopoulos [13] in modeling fatigue life of FRP composite materials. As far as prediction of fatigue crack growth life under mixed-mode (I and II) by GP is concerned, almost no work has been reported till date. Thus, the present investigation aims at developing GP model to predict post overload fatigue crack growth life of HSLA (ASTM A633 Gr. A) steel under the above loading condition.

2. EXPERIMENTATION

The material used in this study was HSLA (ASTM A633 Gr. A) steel. The chemical composition and the mechanical properties of the alloy are summarized in Table 1 and 2 respectively. Single edge notched tension (SENT) specimens having thickness of 6.47mm were used for conducting the fatigue crack growth tests. The specimens were made in the LT plane, with the loading aligned in the longitudinal direction. The detail geometry of the specimens is given in Fig. 1.

Table 1 – Chemical compositions of HSLA steel (wt%)

Material	Fe	C	Mn	P	S	Si	Nb
ASTM A633 Gr. A	Main constituent	0.16	1.35	0.015	0.006	0.30	0.038

Table 2 – Mechanical properties of HSLA steel

Material	Tensile strength (σ_{ut}) MPa	Yield strength (σ_{ys}) MPa	Young's modulus (E) MPa	Poisson's ratio (ν)	Plane Strain Fracture toughness (K_{IC}) MPa \sqrt{m}	Plane Stress Fracture toughness (K_{IC}) MPa \sqrt{m}	Elongation in 40 mm
ASTM A633 Gr. A	532.00	493.00	205,000	0.28	91.00	571.68	23 %

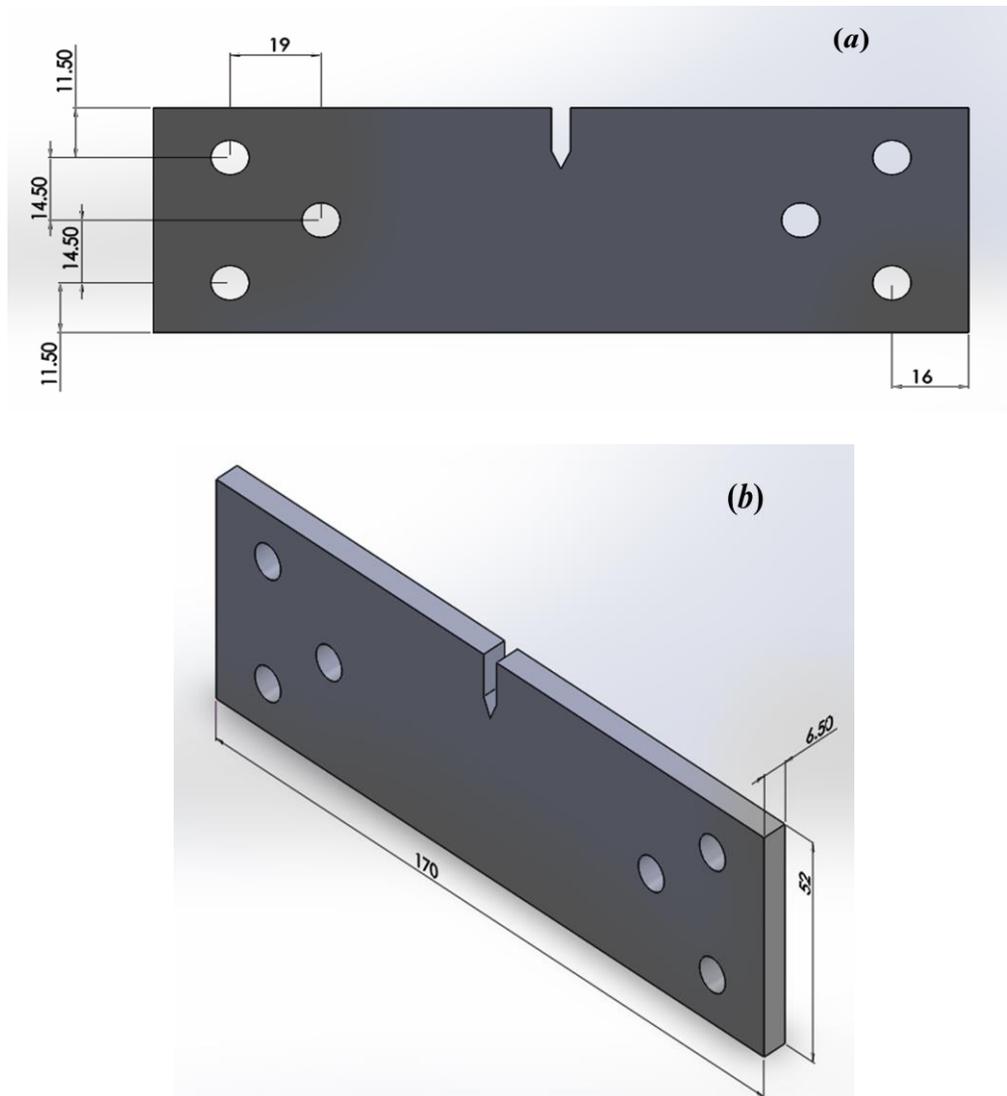


Fig. 1 – Single Edge Notch Tension (SENT) Specimen geometry

All the tests were performed on a servo-hydraulic *Instron-8502* machine having a load capacity of 250 kN, interfaced to a computer for machine control and data acquisition. The test specimens were fatigue pre-cracked under mode-I loading to an a/w ratio of 0.3 and were subjected to constant load test (i.e. progressive increase in ΔK with crack extension) maintaining a load ratio of 0.1. The sinusoidal loads were applied at a frequency of 6 Hz. The crack growth was monitored with the help of a COD gauge mounted on the face of the machined notch. The fatigue crack was allowed to grow up to an a/w ratio of 0.4 and subsequently subjected to single overload spike at a loading rate of 8 kN/min. The overloading was done by using a mixed-mode loading device similar to the one used in the authors' earlier work [14]. The overloading angle is defined as the angle between the loading axis and the normal to the plane of crack propagation. The arrangement of the experimental set-up in mode-I ($\beta = 0^\circ$) is shown in Fig. 2.

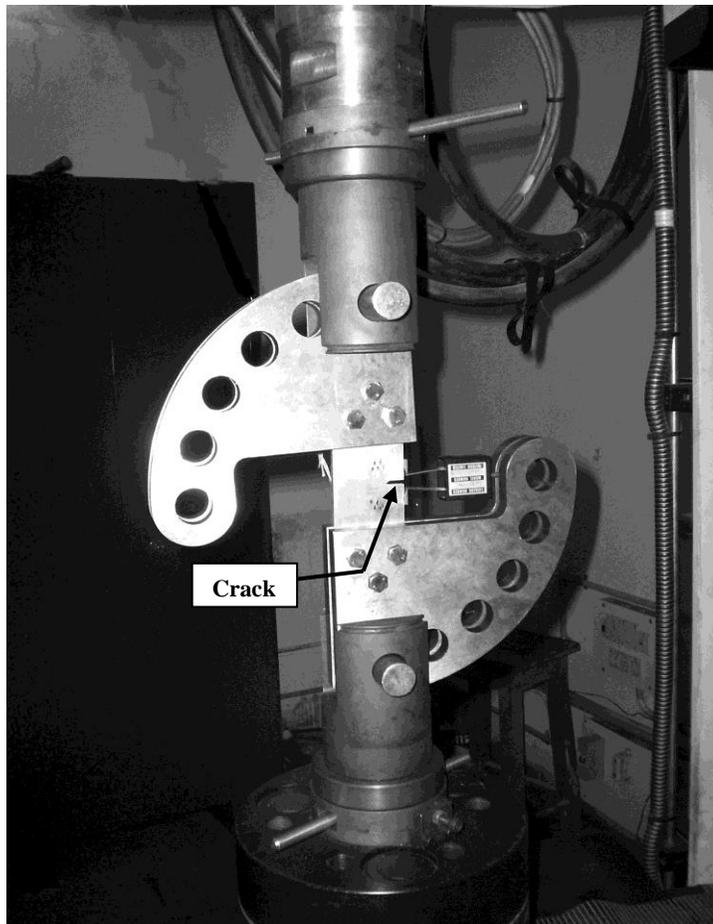


Fig. 2 – Experimental set-up showing mixed mode fixture in mode-I position

The following equations are used to determine stress intensity factors in mode-I and II (K_I and K_{II}) for different angles of overload application.

$$K_I = f(g) \cdot \frac{F \cos \beta \cdot \sqrt{\pi a}}{wB} \quad (1)$$

$$K_{II} = f(g) \cdot \frac{F \sin \beta \cdot \sqrt{\pi a}}{wB} \quad (2) \quad \text{where,}$$

$$f(g) = 1.12 - 0.231(a/w) + 10.55(a/w)^2 - 21.72(a/w)^3 + 30.39(a/w)^4$$

The specimens were subjected to mode I, mode II, and mixed-mode overloads at different loading angles, β ($= 18^\circ, 36^\circ, 54^\circ$ and 72°) at an overloading ratio of 2.5. Overloading ratio is defined as

$$R^{ol} = \frac{K_{eq}^{ol}}{K_{max}^B} \quad (3)$$

where K_{max}^B is the maximum stress intensity factor for base line test. The equivalent stress intensity factors (K_{eq}^{ol}) are calculated according to the following equation:

$$K_{eq}^{ol} = 0.5K_I^{ol} + 0.5\sqrt{(K_I^{ol})^2 + 4(\alpha_1 K_{II}^{ol})^2} \quad (4)$$

where $\alpha_1 = (K_{IC}/K_{IIC}) = 0.95$ according to strain energy density theory and K_I^{ol} and K_{II}^{ol} are the of stress intensity factors of modes I and II during the overload respectively. Then the fatigue test was continued in mode I. Since, plotting of all the overloading angle data points in a single graph is difficult to differentiate due to large amount of scatter, only three overloading angles (i.e. $\beta = 18^\circ, 36^\circ$, and 72°) along with base line data of $a - N$ curve has been plotted in Fig. 3.

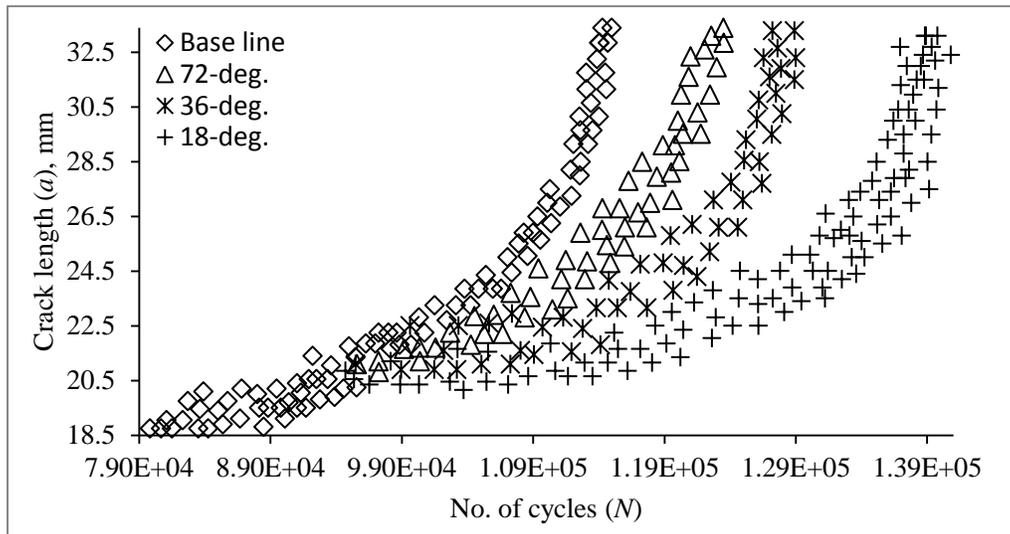


Fig. 3 – Comparison of experimental $a - N$ curves for different overloading angles ($\beta = 18^\circ, 36^\circ$, and 72°)

3. CRACK GROWTH RATE DETERMINATION

The determination of fatigue crack growth rate (da/dN) from raw laboratory data (shown in Fig. 3 as a sample copy) is of course a tedious task because of large amount of scatter. Out of various techniques proposed till date, the exponential equation method [15] has been proved to be better as it is possible to fit the entire $a - N$ data in a single equation. The same method has been adopted in this work to determine the crack growth rate which is described below.

It has been already established [15] that the experimental $a - N$ data can be well fitted by an exponential equation of the form:

$$a_j = a_i e^{m_{ij}(N_j - N_i)} \quad (5)$$

where, a_i and a_j = crack length in i^{th} step and j^{th} step in 'mm' respectively,

N_i and N_j = No. of cycles in i^{th} step and j^{th} step respectively,

m_{ij} = specific growth rate in the interval $i-j$,

i = No. of experimental steps,

and $j = i+1$

In the above equation the exponent ' m_{ij} ' (i.e. specific growth rate) is an important parameter which can be obtained by taking logarithm of equation (5) as follows:

$$m_{ij} = \frac{\ln\left(\frac{a_j}{a_i}\right)}{(N_j - N_i)} \quad (6)$$

The raw values of specific growth rate (m_{ij}) from experimental $a-N$ data are calculated using the above equation. These are then fitted with corresponding crack lengths by a polynomial curve-fit which gives a 3rd order polynomial equation of ' m ' vs. ' a '. To get a better result, crack lengths (modified) at small increments (0.005 mm) are obtained in excel sheet keeping the initial and final values (recorded from fatigue test) intact. Using the above polynomial equation the new (smoothened values) of m_{ij} are obtained which can be subsequently used to get the smoothened values of the number of cycles as per the following equation:

$$N_j = \frac{\ln\left(\frac{a_j}{a_i}\right)}{m_{ij}} + N_i \quad (7)$$

Finally, the crack growth rates (da/dN) are calculated directly by using the above calculated ' N ' values and modified ' a ' values as follows:

$$\frac{da}{dN} = \frac{(a_j - a_i)}{(N_j - N_i)} \quad (8)$$

The smoothed values of $a - N$ and $da/dN - \Delta K$ have been presented in Figs. 4 and 5 respectively which illustrate the respective superimposed experimental data sets for different overloading angles (i.e. $\beta = 0^\circ, 18^\circ, 36^\circ, 54^\circ, 72^\circ$ and 90°) including base line data of HSLA steel.

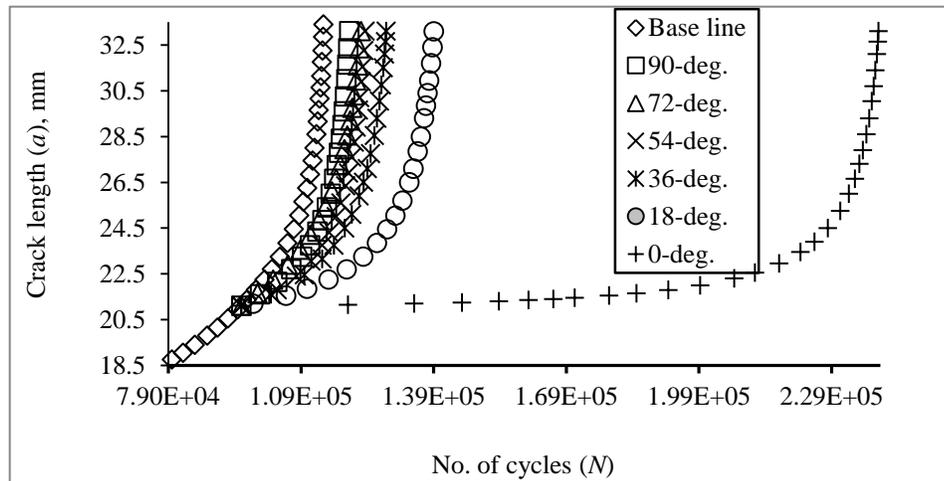


Fig. 4 – Smoothened values of $a - N$ curves for different overloading angles

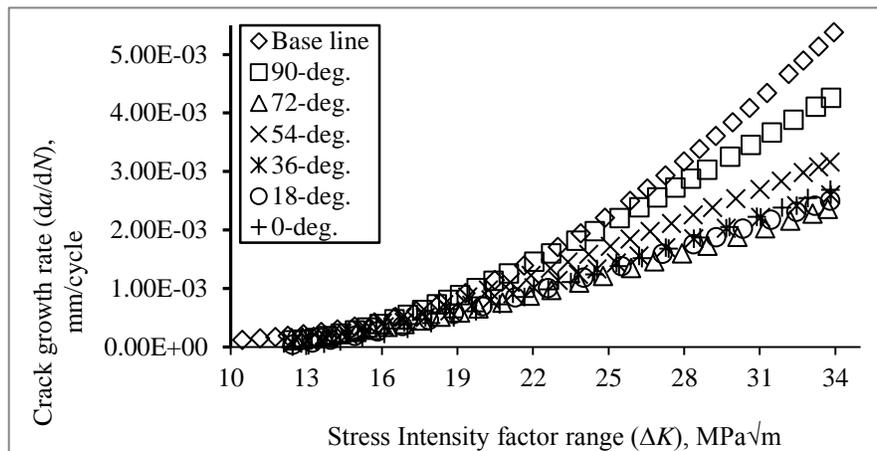


Fig. 5 – Smoothened values of $da/dN - \Delta K$ curves for different overloading angles

4. GENETIC PROGRAMMING APPROACH

Genetic programming (GP), an extension of genetic algorithms (GA), is an evolutionary algorithm-based methodology, which consists of evolving computer programs that perform a user-defined task. GP, first developed by Koza [16] in 1990, is based on the Darwinian principle of reproduction and survival of the fittest. It is similar to the biological genetic operations such as crossover and mutation. In GP, there is a population of computer programs (individuals) that reproduce with each other. Over time, the best individuals will survive

and eventually evolve to do well in the given environment (Fig. 6). A high-level description of GP algorithm can be divided into a number of sequential steps [17]:

1. An initial population (generation 0) of models is generated in random which is represented by tree-like structure comprising of functions and terminals. Each tree, having variable length, is constructed of nodes and represents one candidate model. The nodes can be terminal nodes (called also leafs) placed at the end of a branch signifying an input or a constant, or non-terminal nodes representing functions performing some action on their terminal nodes. A typical model representing the expression $x_0 + x_1 - x_3 - x_2 + \frac{x_0}{x_1}$ is shown in Fig. 7.
2. The performance of each model in the population is evaluated by simulating the corresponding model and calculating some fitness measure like Mean Square Error, Mean Relative Error and so on, that can measure the capability of the model to solve the problem with respect to the experimental data.
3. The reproduction operators are used to copy existing programs into the new generation.
4. A new population of models is created, using certain selection schemes (like proportional selection, tournament selection, rank based selection, e.t.c.) and evolutionary operators like crossover and mutation from randomly chosen set of parents. For the new population, step 2 onwards is repeated until a predefined termination criterion is satisfied, or a fixed number of generations are completed. After some number of generations the algorithm converges at a near-optimum for the problem model.

For the present work the main points of the implemented GP evolutionary algorithm in pseudo code has been shown in Fig. 8. First, the initial population $P(t)$ of random organism (i.e., models for prediction of crack growth rate da/dN) consisting of the available function and terminal genes is generated. The organisms are in fact computer programs of various shapes and sizes. The variable t represents the generation time.

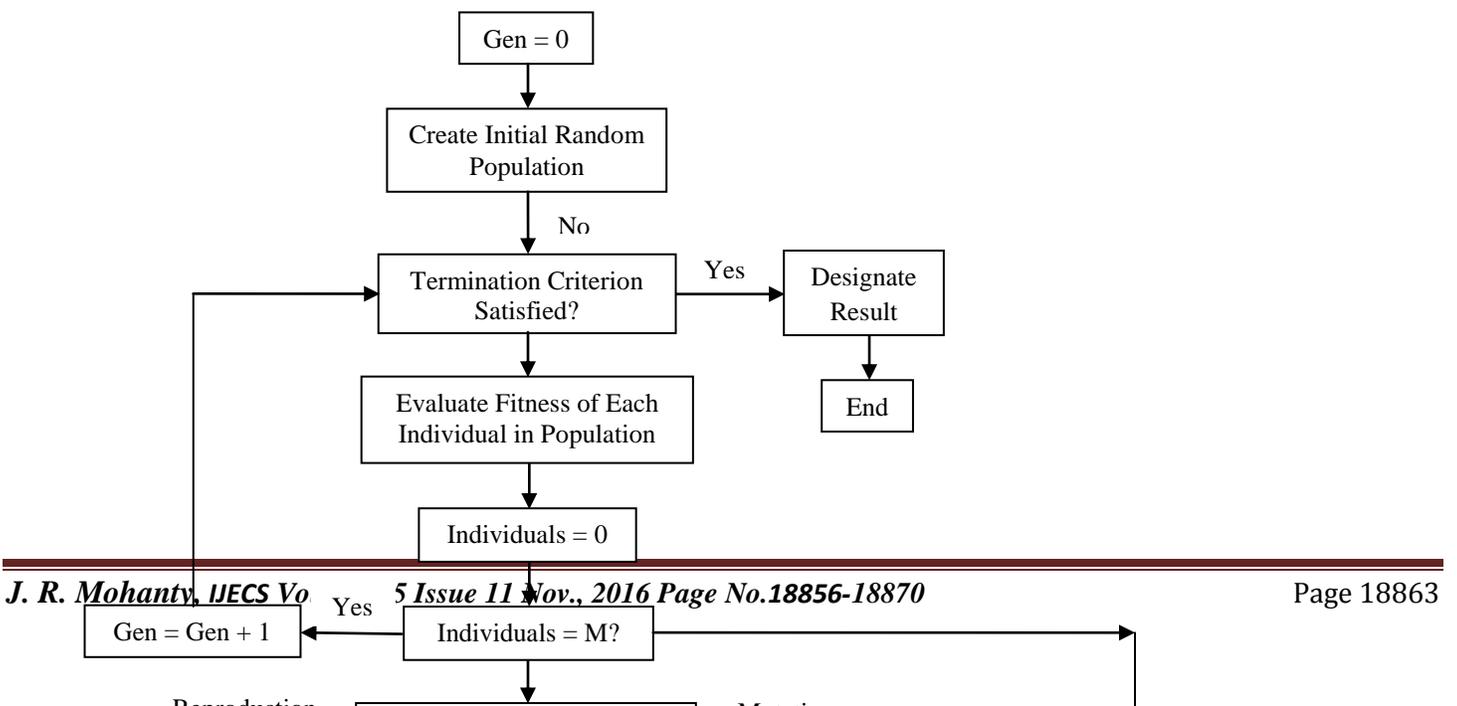


Figure 6 – Genetic Programming Flow Chart [18]

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evolutionary algorithm
begin
   $t \leftarrow 0$ 
  initialize  $P(t)$ 
  evaluate  $P(t)$ 
  while (not termination_condition) do
    begin
       $t \rightarrow t+1$ 
      alter  $P(t)$  by applying genetic operators
      evaluate  $P(t)$ 
    end
  end
end
```

Figure 7 – Evaluatory algorithm in pseudo code

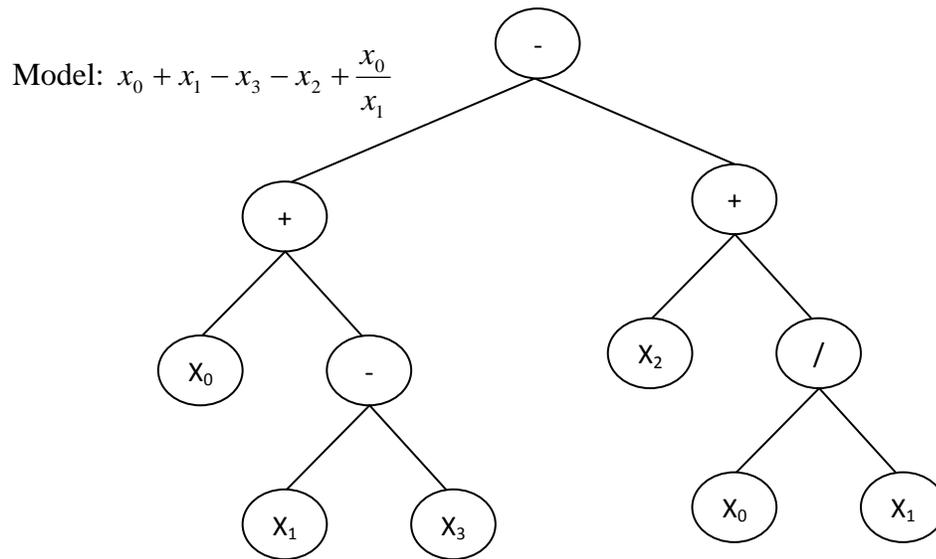


Figure 8 – A tree representation of a simple model

5. APPLICATION OF GP FOR CRACK GROWTH RATE DETERMINATION

In the present study genetic programming is used as a stochastic non-linear regression tool since one output i.e. crack growth rate (da/dN) is assigned to three input variables i.e. overloading angle (β), maximum stress intensity factor (K_{max}) and stress intensity factor range (ΔK). During the process, computer programs are evolved to describe the relation between these two parameters, i.e. output = f(input), or $da/dN = f(\beta, K_{max}, \Delta K)$. The best fitted program, according to the criterion of minimizing the error between targeted output and selected program outputs, is used to predict outputs for unknown input variables.

During the design of GP model the experimental data of the aforementioned data base has been used to predict the fatigue crack growth rate (da/dN) of HSLA steel. For the application of GP method the GP software tool of RML Technologies, Inc, DiscipulusTM [18] was used. DiscipulusTM genetic programming software is a powerful regression and classification tool.

In the context of the present work, the fatigue crack growth rate (FCGR) data of HSLA steel under mixed-mode (I and II) overloading conditions were treated as follows: All FCGR data, except the set $\beta = 36^\circ$, were used for the training of the model; a total of 5 sets (i.e. $\beta = 0^\circ, 18^\circ, 54^\circ, 72^\circ$ and 90°) with one set consisting of 300 input and output parameters respectively. Given the number of input and output parameters in

the training set, the process is characterized as a non-linear stochastic regression analysis. During the training phase the genetic programming tool established several relations (by regression analysis) in the form of computer programs between the input and output variables. It is worthwhile to mention here that the proposed GP formulation is valid for the ranges of training set as given in Table 3. Parameters of the GP models are presented in Table 4. Using an iterative process the parameters of the established relations were adjusted in order to minimize the error between targeted output and selected program outputs. The same model (the selected evolved program) can be stored and potentially be used to predict other output values for a new applied input data set (i.e. $\beta = 36^\circ$).

Table 3 – Variables used in model construction

Code	Input variable	Range	Code	Output variable	Range (Al-7020)
x_1	Overload angle (β)	$0^\circ - 90^\circ$ (with a diff. of 18°)		Crack growth rate (da/dN)	$6.37 \times 10^{-5} - 2.58 \times 10^{-3}$
x_2	Maximum stress intensity factor (K_{max})	13.35 – 37.22			
x_3	Stress intensity factor range (ΔK)	12.46 – 33.97			

Table 4 – Parameters of GP model for the alloy

P_1	Population size	1000
P_2	Number of generations	Between 100 to 7000
P_3	Function set	'-', '*', 'power'
P_4	Probability of reproduction	0.1
P_5	Probability of crossover	0.9
P_6	Maximum depth of initial random organisms	4
P_7	Maximum permissible depth organisms after crossover	10

6. RESULT AND DISCUSSION

In the present study, genetic programming was applied on the training data sets for modeling post overload fatigue crack growth rates as described in the previous section. The data containing in the training file were used for learning by applying the fitness function. Subsequently, the new inputs of the test data set (i.e. $\beta =$

36°) were fed to the trained GP model to predict the corresponding predicted outputs. The overall performances of both sets were evaluated by the correlation coefficient (R) and mean squared error (MSE) given by:

$$R = \frac{\sum_{i=1}^m \left(\left(\frac{da}{dN} \right)_{\text{experimental}} - \left(\frac{da}{dN} \right)'_{\text{experimental}} \right) \left(\left(\frac{da}{dN} \right)_{\text{predicted}} - \left(\frac{da}{dN} \right)'_{\text{predicted}} \right)}{\sqrt{\sum_{i=1}^m \left(\left(\frac{da}{dN} \right)_{\text{experimental}} - \left(\frac{da}{dN} \right)'_{\text{experimental}} \right)^2 \left(\left(\frac{da}{dN} \right)_{\text{predicted}} - \left(\frac{da}{dN} \right)'_{\text{predicted}} \right)^2}} \quad (9)$$

$$MSE = \frac{\sum_{i=1}^m \left(\left(\frac{da}{dN} \right)_{\text{experimental}} - \left(\frac{da}{dN} \right)_{\text{predicted}} \right)^2}{n} \quad (10)$$

Where, $\left(\frac{da}{dN} \right)_{\text{experimental}}$ and $\left(\frac{da}{dN} \right)_{\text{predicted}}$ are the experimental and predicted crack growth rates,

$\left(\frac{da}{dN} \right)'_{\text{experimental}}$ and $\left(\frac{da}{dN} \right)'_{\text{predicted}}$ are their corresponding mean values and 'n' is the number of observations.

The GP estimates are compared to the experimental data for training and testing sets. The statistical performance of the GP model has been presented in Table 5.

Table 5 – Statistical results of GP for training and testing

Set	MSE	Corr. Coff. (R)
Train	2.4637	0.9856
Test	3.1587	0.9789

The training results proved that the proposed GP models have efficiently learned well the nonlinear relationship between the input and output variables with high correlation ($R = 0.9856$) and relatively low error (MSE = 2.4637) values. Comparing the GP predictions with the experimental data for the test stage (Fig. 9) demonstrates a high generalization capacity of the proposed model ($R = 0.9789$) and relatively low error (MSE = 3.1587) values. All these findings show a successful performance of the GP model for estimating fatigue crack growth rates in training and testing stages. The testing results (da/dN vs. ΔK) have been illustrated in Fig. 10 for HSLA steel. The numbers of cycles (i.e. post overload fatigue lives) were calculated from predicted and experimental results in the excel sheet (Fig. 11) as per the following equation:

$$N_{i+1} = \frac{a_{i+1} - a_i}{da/dN} + N_i \quad (11)$$

From the $a - N$ plot it is observed that the post overload fatigue life (at $\beta = 36^\circ$) of HSLA steel from GP model is 127300 cycles with an error of -0.787% in comparison to its experimental value which is 128310 cycles.

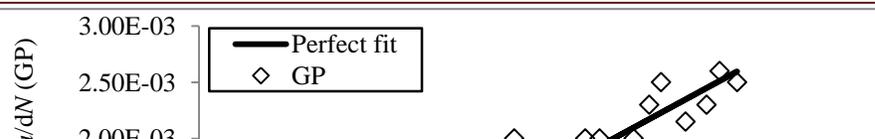


Fig. 9 – Modeling ability of genetic programming for the test set

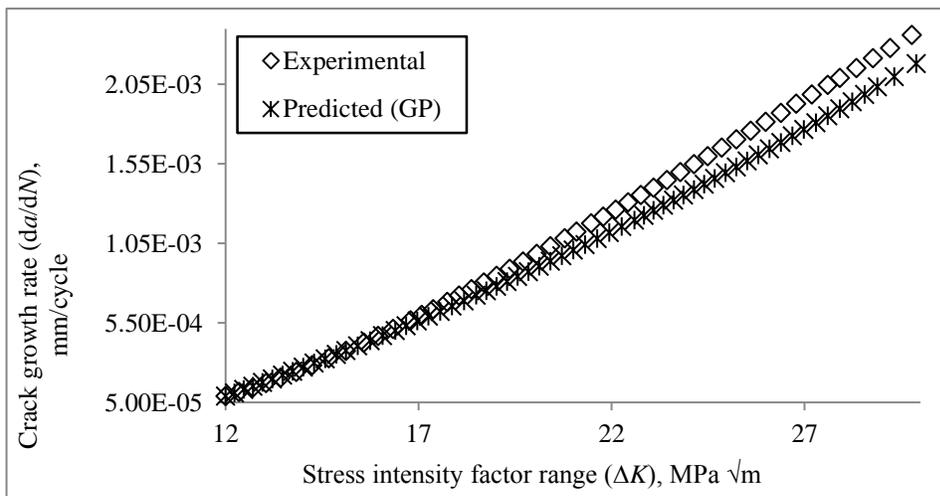
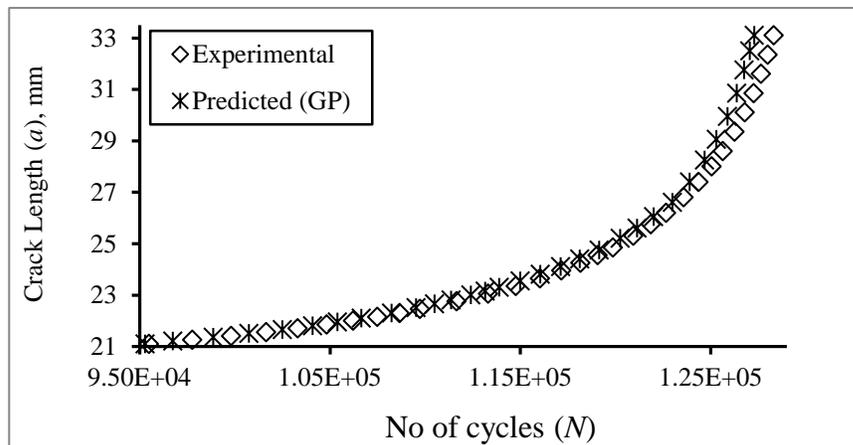
Fig.10 – Comparison of predicted (GP) and experimental $da/dN - \Delta K$ curves

Fig. 11 – Comparison of predicted (GP) and experimental $a - N$ curves

7. CONCLUSION

This work proved the ability of novel computational tools to model and predicts the post overload fatigue crack propagation life HSLA steel under interspersed mixed-mode overload conditions. Experimental results are used to build and validate the model. The proposed GP formulations show very good agreement with the experimental findings with quite satisfactory performance of accuracies ($R = 0.9789$ and $MSE = 3.1587$).

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