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A Generic Computer Tool for the Solution of Fluid Flow Problems: Case Study of a Thermal Radiative Flow of an Optically Thick Gray Gas in the Presence of Indirect Natural Convection

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Abstract : In this paper, we present a generic computer tool based on the Nakamura finite difference scheme in order to solve laminar fluid flow problems. The present study is restricted to the category of one-dimensional, two-dimensional and three-dimensional fluid flows expressed in one spatial coordinate. All problems are assumed to be time dependant. The equations describing the flow and other relevant parameters are defined in a generic file which is used as input to the system. A generic interpreter is used to generate postfix codes that it will interpret in the process of calculations. For the purpose of application, we consider a two-dimensional unsteady flow of an incompressible electrically conducting viscous fluid along an infinite flat plate. The effects of the various parameters entering into the problem are discussed extensively and shown graphically.

Keywords: Generic Software, Computational Solution, Interpreter, Nakamura Scheme, Unsteady Flow.

1. Introduction

The study of fluid flows has important applications in engineering. That importance has made it necessary for researchers to try and know more about the motion of fluids. Some of the authors who made important contributions in that area are Soundalgekar and Takhar [10], Bestman and Adiepong [1], Nakamura [6], Brewster [2], Raptis and Massalas [8], Raptis and Perdikis [9], Naroua et al [7], Ghosh and Pop [3]. Yamauchi et al. [11] presented modified finite-difference formulas for a general proposition of an interface that they applied to the propagating beam analysis of z-variant rib waveguides. They found the modified formula based on the semivectorial H-field to be more insensitive to variation in an interface position than that on the E-field. They also observed that a discretization error is satisfactorily reduced in tilted and tapered rib waveguides. Zhu et al. [12] analysed explicit/implicit schemes for parabolic equations with discontinuous coefficients. Numerical experiments, which were given for both linear and nonlinear problems, showed that their theoretical estimates are optimal in some sense. Khader and Ahmed [4] introduced a numerical simulation using finite difference method with the theoretical study for the problem of the flow and heat transfer over an unsteady stretching sheet embedded in a porous medium in the presence of a thermal radiation. Matsuoka and Nakamura [5] proposed a stable numerical scheme for a Cahn-Hilliard type equation with long-range interaction describing the micro-phase separation of diblock copolymer melts. They designed their scheme by using the discrete variational derivative method which is one of the structure preserving numerical methods. They observed that their proposed scheme has the same characteristic properties, mass conservation and energy dissipation, as the original equation does. They also discussed the stability and unique solvability of their proposed scheme.

Although many improvements have been made in the use of numerical methods, we are proposing a generic computer tool that can be used to solve laminar fluid flow problems. It is generic in the sense that it is a common solution to the category of time dependant problems expressed in one spatial coordinate. For the purpose of application, we consider a two-dimensional unsteady flow of an incompressible electrically conducting viscous fluid along an infinite flat plate.

2. Generic Simulator Using Finite Differences

In the process of solving fluid flow problems, various mathematical tools have been developed and applied, among which is the finite difference method. Different schemes have been developed with different stability issues. One of the finite difference schemes used in numerical simulation of fluid flow problems is that of Nakamura [6] due to its stability. In this paper, we present a generic software tool based on the Nakamura model as shown in Figure 1.

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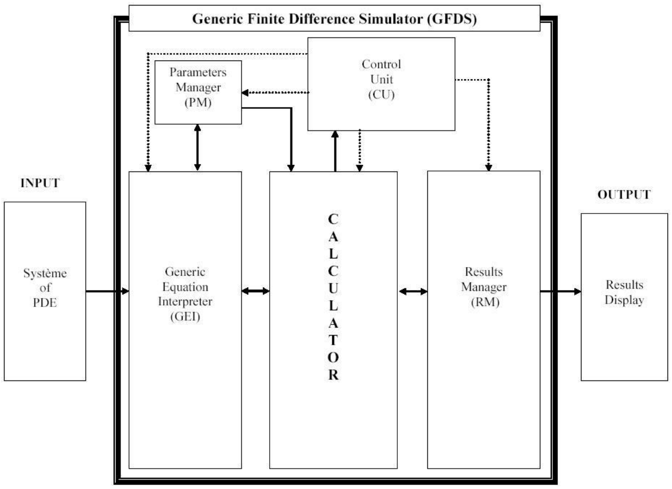

Figure 1: Generic simulator

It is intended to be generic in providing a solution to the category of systems of partial differential equations describing laminar fluid flow problems. To be more precise, we restricted our study to the types of problems expressed in time and one spatial coordinate. It may be one-dimensional, two-dimensional or three-dimensional fluid flows. In order to ensure that the system is generic, it is important to follow it with applications. The proposed generic system takes as input a generic file and contains a generic equation interpreter, a module responsible for parameters management, a results manager and a calculator. A special unit designated as "Control Unit" supervises the totality of the system and ensures the correctness of computations. In the generic file, the following items are defined:

- Active parameters;
- Dependent variables;
- Energy equation;
- Momentum equations;
- Boundary conditions;
- Initial conditions.

The generic equation interpreter reads the generic file and produces an interpretable code in postfix notation. All operations on the parameters and the generated results are under the responsibilities of the parameters manager and the results manager respectively. The calculator is responsible for all computations and interacts directly with the equation interpreter, the parameters manager and the results manager. The first step is to rewrite the equations using backward difference approximation (which is stable) in the time coordinate. For each dependent variable, the central difference scheme (which is unconditionally stable) is used to evaluate the derivatives with respect to the spatial coordinate. The resulting system of equations cannot be solved individually for each grid point. The equations for all the grid points must be solved simultaneously. The set of equations for all the grid points forms a tridiagonal system of equations as described by Nakamura [6] of the form:

$[A].\{u_{t+1}\} = \{p\}$

This system of equations for each time step requires an iterative procedure due to the presence of non-linear coefficients. Successive substitutions and iterations are continuously executed for each time step until convergence is reached. The mesh system is shown in Figure 2.

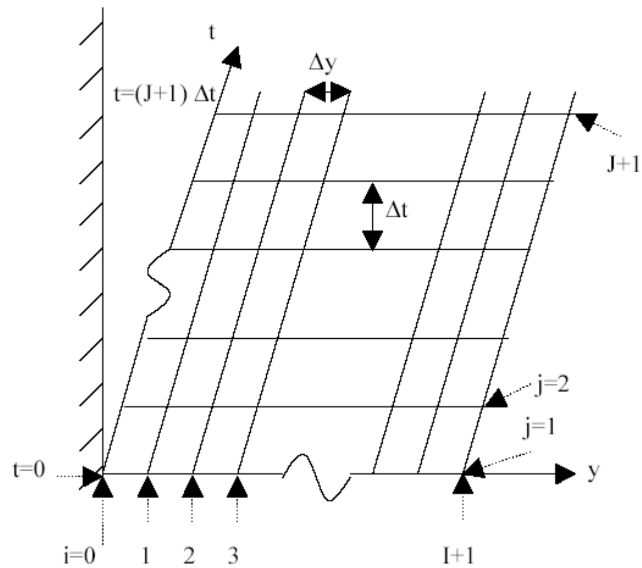

Figure 2: Mesh system

The operations of the simulator are expressed in the following algorithm:

```
Algorithm simulator
begin
  define region of the flow (Ω);
  define boundary conditions;
  define initial conditions;
  define number of spatial grid points;
  define number of  time grid points;
  set parameters;
  define equations;
  for each equation
     optimize equation;
     produce an interpretable code;
  for each iteration
     for each equation
        compute results;
     edit parameters;
end
```

3. Application

For the sake of application, we consider a two-dimensional unsteady flow of an incompressible electrically conducting viscous fluid along an infinite flat plate. The geometry and the unsteady flow fields for this problem are described by Raptis and Massalas [8]. The problem considered reduces to the following non-dimensional differential equations:

$$\begin{cases} \frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{1}{4}\frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial y^2} + M\frac{\partial H}{\partial y} & (1) \\ \frac{1}{4}\frac{\partial H}{\partial t} - \frac{\partial H}{\partial y} = M\frac{\partial u}{\partial y} + \frac{1}{Pm}\frac{\partial^2 H}{\partial y^2} & (2) \\ \frac{Pr}{4}\frac{\partial T}{\partial t} - Pr\frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + Pr.Ec\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Pr.Ec}{Pm}\left(\frac{\partial H}{\partial y}\right)^2 + \frac{4}{3R}\frac{\partial^2 T}{\partial y^2} & (3) \end{cases}$$

The corresponding boundary conditions are:

$$\begin{cases} y = 0: \quad u = 0, \qquad T = 1, \quad H = 0 \\ y \to \infty: \quad u \to U(t), \qquad T \to 0, \quad H \to 0 \end{cases} \qquad (4)$$

where

- u is the dimensionless velocity;
- H is the dimensionless induced magnetic field;
- T is the dimensionless temperature;
- Pr is the Prandtl number;
- Pm is the magnetic Prandtl number;
- Ec is the Eckert number;
- M is the magnetic field;
- R is the Radiation parameter;
- $U(t) = 1+\varepsilon e^{i\omega t}$ is the stream velocity.

The above system of equations (1 - 3) with boundary conditions (4) has been solved numerically by the proposed generic tool. The mesh system used is as shown in Figure 2. The system of equations is first transformed using backward difference approximation in time as shown below:

$$\begin{cases} u^{//} + u^{/} - \frac{1}{4\Delta t}u = -\frac{1}{4\Delta t}u_{i,j-1} - MH^{/} - \frac{1}{4}U_t & (5) \\ \frac{1}{Pm}H^{//} + H^{/} - \frac{1}{4\Delta t}H = -\frac{1}{4\Delta t}H_{i,j-1} - Mu^{/} & (6) \\ \left(1+\frac{4}{3R}\right)T^{//} + Pr.T^{/} - \frac{Pr}{4\Delta t}T = -\frac{Pr}{4\Delta t}T_{i,j-1} - Pr.Ec(u^{/})^2 - \frac{Pr.Ec}{Pm}(H^{/})^2 & (7) \end{cases}$$

where $u^{/}, u^{//}, H^{/}, H^{//}, T^{/}, T^{//}$ are derivatives with respect to y.

For the sake of simplicity, we write:

$C_2^1 = 1$; $C_1^1 = 1$; $C_0^1 = -\frac{1}{4\Delta t}$;

$P_{i,j-1}^1 = -\frac{1}{4\Delta t}u_{i,j-1} - MH^{/} - \frac{1}{4}U_t$;

$C_2^2 = \frac{1}{Pm}$; $C_1^2 = 1$; $C_0^2 = -\frac{1}{4\Delta t}$;

$P_{i,j-1}^2 = -\frac{1}{4\Delta t}H_{i,j-1} - Mu^{/}$;

$C_2^3 = 1 + \frac{4}{3R}$; $C_1^3 = \mathrm{Pr}$; $C_0^3 = -\frac{\mathrm{Pr}}{4\Delta t}$;

$P_{i,j-1}^3 = -\frac{Pr}{4\Delta t}T_{i,j-1} - Pr.Ec\left(u^{/}\right)^2 - \frac{Pr.Ec}{Pm}\left(H^{/}\right)^2$;

The C_m^n coefficients and the $P_{i,j-1}^n$ terms are transformed into an interpretable code which is in this case in postfix notion. For any computation involved in the process, the postfix code is interpreted in order to return a real value.

Using the above formulation, Equations (5-7) take the form:

$$\begin{cases} C_2^1\, u_{i,j}^{//} + C_1^1 u_{i,j}^{/} + C_0^1 u_{i,j} = P_{i,j-1}^1 & (8) \\ C_2^2 H_{i,j}^{//} + C_1^2 H_{i,j}^{/} + C_0^2 H_{i,j} = P_{i,j-1}^2 & (9) \\ C_2^3 T_{i,j}^{//} + C_1^3 T_{i,j}^{/} + C_0^3 T_{i,j} = P_{i,j-1}^3 & (10) \end{cases}$$

Using the central difference scheme which is unconditionally stable, Equations (8-10) reduce to:

$$\begin{cases} C_2^1\left(\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{(\Delta y)^2}\right) + C_1^1\left(\frac{u_{i+1,j} - u_{i-1,j}}{2\Delta y}\right) + C_0^1 u_{i,j} = P_{i,j-1}^1 & (11) \\ C_2^2\left(\frac{H_{i+1,j} - 2H_{i,j} + H_{i-1,j}}{(\Delta y)^2}\right) + C_1^2\left(\frac{H_{i+1,j} - H_{i-1,j}}{2\Delta y}\right) + C_0^2 H_{i,j} = P_{i;j-1}^2 & (12) \\ C_2^3\left(\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta y)^2}\right) + C_1^3\left(\frac{T_{i+1,j} - T_{i-1,j}}{2\Delta y}\right) + C_0^3 T_{i,j} = P_{i,j-1}^3 & (13) \end{cases}$$

At time step j+1, Equations (11-13) reduce to:

$$\begin{cases} \left(\frac{C_2^1}{(\Delta y)^2} - \frac{C_1^1}{2\Delta y}\right).u_{i-1,j+1} + \left(C_0^1 - \frac{2C_2^1}{(\Delta y)^2}\right).u_{i,j+1} + \left(\frac{C_2^1}{(\Delta y)^2} + \frac{C_1^1}{2\Delta y}\right).u_{i+1,j+1} = P_{i,j}^1 & (14) \\ \left(\frac{C_2^2}{(\Delta y)^2} - \frac{C_1^2}{2\Delta y}\right).H_{i-1,j+1} + \left(C_0^2 - \frac{2C_2^2}{(\Delta y)^2}\right).H_{i,j+1} + \left(\frac{C_2^2}{(\Delta y)^2} + \frac{C_1^2}{2\Delta y}\right).H_{i+1,j+1} = P_{i,j}^2 & (15) \\ \left(\frac{C_2^3}{(\Delta y)^2} - \frac{C_1^3}{2\Delta y}\right)T_{i-1,j+1} + \left(C_0^3 - \frac{2C_2^3}{(\Delta y)^2}\right).T_{i,j+1} + \left(\frac{C_2^3}{(\Delta y)^2} + \frac{C_1^3}{2\Delta y}\right).T_{i+1,j+1} = P_{i,j}^3 & (16) \end{cases}$$

Equations (14 - 16) cannot be solved individually for each grid point i. The equations for all the grid points must be solved simultaneously. The set of equations for i = 1,2,...........,I forms a tridiagonal system of equations as described by Nakamura [6] and shown in Equations (17-19).0

$$\begin{bmatrix} \left(C_0^1 - \frac{2C_2^1}{(\Delta y)^2}\right)\left(\frac{C_2^1}{(\Delta y)^2} + \frac{C_1^1}{2\Delta y}\right) & & & \\ \left(\frac{C_2^1}{(\Delta y)^2} - \frac{C_1^1}{2\Delta y}\right)\left(C_0^1 - \frac{2C_2^1}{(\Delta y)^2}\right)\left(\frac{C_2^1}{(\Delta y)^2} + \frac{C_1^1}{2\Delta y}\right) & & & \\ & \left(\frac{C_2^1}{(\Delta y)^2} - \frac{C_1^1}{2\Delta y}\right)\left(C_0^1 - \frac{2C_2^1}{(\Delta y)^2}\right)\left(\frac{C_2^1}{(\Delta y)^2} + \frac{C_1^1}{2\Delta y}\right) & & \\ & \cdots\cdots\cdots\cdots\cdots\cdots & & \\ & & & \left(\frac{C_2^1}{(\Delta y)^2} - \frac{C_1^1}{2\Delta y}\right)\left(C_0^1 - \frac{2C_2^1}{(\Delta y)^2}\right) \end{bmatrix} .q_1 = p_1 \quad (17)$$

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$$\text{where } q_1 = \begin{bmatrix} u_{1,j+1} \\ u_{2,j+1} \\ u_{3,j+1} \\ \cdots\cdots\cdots \\ u_{i,j+1} \\ \cdots\cdots\cdots \\ u_{I,j+1} \end{bmatrix} \text{ and } p_1 = \begin{bmatrix} P^1_{1,j} \\ P^1_{2,j} \\ P^1_{3,j} \\ \cdots\cdots \\ P^1_{i,j} \\ \cdots\cdots \\ P^1_{I,j} \end{bmatrix}$$

$$\begin{bmatrix} \left(C_0^2 - \frac{2C_2^2}{(\Delta y)^2}\right) & \left(\frac{C_2^2}{(\Delta y)^2} + \frac{C_1^2}{2\Delta y}\right) & & & \\ \left(\frac{C_2^2}{(\Delta y)^2} - \frac{C_1^2}{2\Delta y}\right) & \left(C_0^2 - \frac{2C_2^2}{(\Delta y)^2}\right) & \left(\frac{C_2^2}{(\Delta y)^2} + \frac{C_1^2}{2\Delta y}\right) & & \\ & \left(\frac{C_2^2}{(\Delta y)^2} - \frac{C_1^2}{2\Delta y}\right) & \left(C_0^2 - \frac{2C_2^2}{(\Delta y)^2}\right) & \left(\frac{C_2^2}{(\Delta y)^2} + \frac{C_1^2}{2\Delta y}\right) & \\ & \cdots\cdots & \cdots\cdots & \cdots\cdots & \\ & & & \left(\frac{C_2^2}{(\Delta y)^2} - \frac{C_1^2}{2\Delta y}\right) & \left(C_0^1 - \frac{2C_2^2}{(\Delta y)^2}\right) \end{bmatrix} . q_2 = p_2 \quad (18)$$

$$\text{where } q_2 = \begin{bmatrix} H_{1,j+1} \\ H_{2,j+1} \\ H_{3,j+1} \\ \cdots\cdots\cdots \\ H_{i,j+1} \\ \cdots\cdots\cdots \\ H_{I,j+1} \end{bmatrix} \text{ and } p_2 = \begin{bmatrix} P^2_{1,j} \\ P^2_{2,j} \\ P^2_{3,j} \\ \cdots\cdots \\ P^2_{i,j} \\ \cdots\cdots \\ P^2_{I,j} \end{bmatrix}$$

$$\begin{bmatrix} \left(C_0^3 - \frac{2C_2^3}{(\Delta y)^2}\right) & \left(\frac{C_2^3}{(\Delta y)^2} + \frac{C_1^3}{2\Delta y}\right) & & & \\ \left(\frac{C_2^3}{(\Delta y)^2} - \frac{C_1^3}{2\Delta y}\right) & \left(C_0^3 - \frac{2C_2^3}{(\Delta y)^2}\right) & \left(\frac{C_2^3}{(\Delta y)^2} + \frac{C_1^3}{2\Delta y}\right) & & \\ & \left(\frac{C_2^3}{(\Delta y)^2} - \frac{C_1^3}{2\Delta y}\right) & \left(C_0^3 - \frac{2C_2^3}{(\Delta y)^2}\right) & \left(\frac{C_2^3}{(\Delta y)^2} + \frac{C_1^3}{2\Delta y}\right) & \\ & \cdots\cdots & \cdots\cdots & \cdots\cdots & \\ & & & \left(\frac{C_2^3}{(\Delta y)^2} - \frac{C_1^3}{2\Delta y}\right) & \left(C_0^3 - \frac{2C_2^3}{(\Delta y)^2}\right) \end{bmatrix} . q_3 = p_3 \quad (19)$$

$$\text{where } q_3 = \begin{bmatrix} T_{1,j+1} \\ T_{2,j+1} \\ T_{3,j+1} \\ \cdots\cdots\cdots \\ T_{i,j+1} \\ \cdots\cdots\cdots \\ T_{I,j+1} \end{bmatrix} \text{ and } p_3 = \begin{bmatrix} P^3_{1,j} \\ P^3_{2,j} \\ P^3_{3,j} \\ \cdots\cdots \\ P^3_{i,j} \\ \cdots\cdots \\ P^3_{I,j} \end{bmatrix}$$

For each time step, the system of equations (17-19) requires an iterative procedure due to the presence of non-linear coefficients. The designed tool continuously executes successive substitutions and iterations for each time step until convergence is reached.

4. Discussion of Results

To study the behavior of the velocity and temperature profiles, curves are drawn for various values of the parameters that describe the flow and are shown in Figures 3-6. The value of the magnetic field is kept constant (M = 0.4).

From Figure 3, we observe that:

i) the velocity profile (u) decreases due to an increase in time (t);

ii) there is an insignificant change in the velocity profile (u) due to an increase in the magnetic Prandtl number (Pm).

From Figure 4, we observe that the H profile decreases with time (t) whereas it increases due to an increase in the magnetic Prandtl number (Pm).

From Figures 5 and 6, we observe that:

i) the temperature profile (T) increases due to an increase in time (t);

ii) There is a fall in the temperature profile (T) due to an increase in the Prandtl number (Pr) and the radiation parameter (R), which is in agreement

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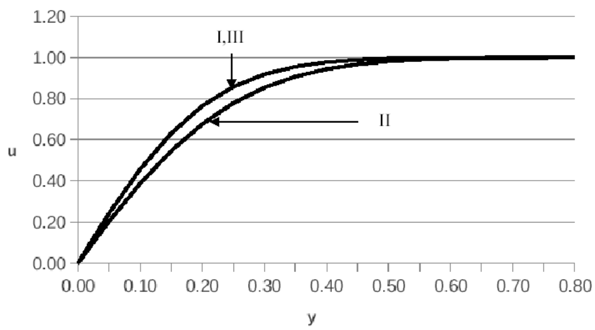
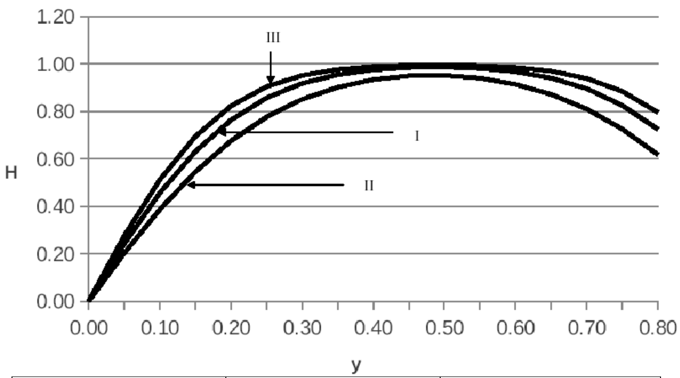
with Raptis and Massalas [8];

iii) there is an insignificant change in the temperature profile (T) due to an increase in the magnetic Prandtl number (Pm) and the Eckert number (Ec).



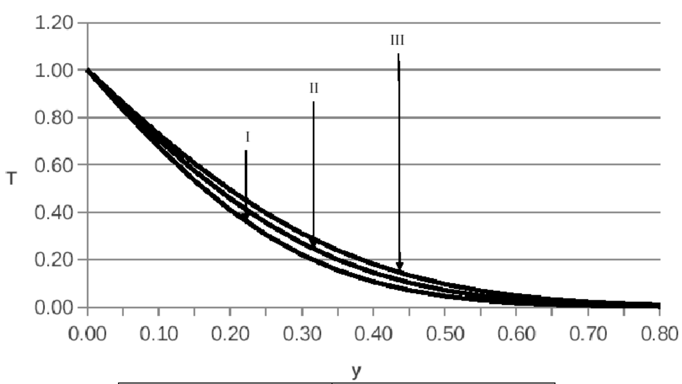
Series	**t**	**Pm**
I	0.004	1
II	0.005	1
III	0.004	5

Figure 3: Velocity profiles (u)



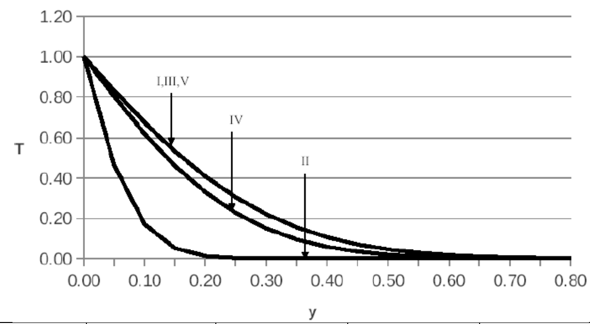
Series	**t**	**Pm**
I	0.004	1
II	0.005	1
III	0.004	5

Figure 4: Induced magnetic field profiles (H)



Series	**t**
I	0.004
II	0.005
III	0.006

Figure 5: Temperature profiles (T)



Series	**Pr**	**Pm**	**R**	**Ec**
I	0.71	1	3	0.001
II	7	1	3	0.001
III	0.71	5	3	0.001
IV	0.71	1	30	0.001
V	0.71	1	3	0.005

Figure 6: Temperature profiles (T)

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