

Laws of Duality of soft mathematical morphology

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ABSTRACT: This paper will give a new type of laws, which will belong to soft mathematical morphology. These laws correspond to a property called duality in soft mathematical morphology, which is discussed in a new way.. It means this property is thoroughly discussed by the author. It can be said that the laws are also new laws. So it is named by the name of the author.

KEY WORDS: : Mathematical morphology, Mathematical soft morphology, Soft morphology, Erosion, Dilation, Soft erosion, Soft dilation, Primitive morphological operation, equality,duality, threshold , Iterative morphology, Multi scale morphology, soft open, soft close.

1. INTRODUCTION:

The images attained interest of human beings from the beginning. An image is equal to even more than thousand words. So it has attained the attention from the evolution of the people. It has started initially from drawing of images in caves and went to drawing of images with ink and taking photo graphs with camera and now processing with computers. Now so many developments have come from morphing to animation, satellite data processing to medical equipment design, geological applications to printing technology. Image processing has entered into all the areas from office work to movie industry, soldering to machine assembling. Now a days the image processing has entered in aadhar card generation to ration card generation ,criminal identification, construction of faces of suspected people , blood analysis , finger print analysis in forensic science. It has attracted the attention of researchers who are interested in research , to the business people who are interested in earning money.

No doubt it will satisfy the desires all the wings of the people.

In getting images.....inks and paints contributed .

Kodak company..... contributed a lot.

NASA..... contribution is un measurable.

In the development of image processing NASA contributed a lot.

2 MATHEMATICAL MORPHOLOGY:

Mathematical Morphology is a peculiar area generated peculiarly and developed peculiarly and entered in to Image Processing in a peculiar way.

MATHERON and SERRA started mathematical morphology for a peculiar purpose basically for ore mining as well as geology purpose.(12,13)

But it was found peculiarly that these concepts have applications in Image Processing and it was found surprisingly that the design of algorithms are also very simple.

In parallel FRANCE also contributed in this mathematical morphology field, by a group of mining scientists.

To the surprise of the scientists that these mathematical morphology algorithms are found suitable in smoothening of images , edges, identification of images etc.. what not, every area of image processing is influenced by mathematical morphology , substituting its simple algorithms of image processing. Even complex frequency domain operations are also, substituted by simple Mathematical Morphology algorithms. Surprisingly simple erosion and dilation are extended to design composite operations like open, close, skeletonization, thinning, thickening etc. Even complex algorithms like water shed algorithms, segmentation etc. also developed simply by mathematical morphology algorithms

- Morphological operations provide for the systematic alteration of the geometric content of an image, while maintaining the stability of important geometric characteristics.

- There exists a well developed morphological algebra that can be employed for representation and optimization.

- It is possible to express digital algorithms in terms of a very small class of primitive morphological operations.

• There exists rigorous representation theorems by means of which one can obtain the expressions of morphological filters in terms of the primitive morphological operations.

3 SOFT MORPHOLOGY:

The Soft Morphology is extension to Mathematical Morphology. Even though mathematical morphological operators are efficient, they suffer with a few drawbacks. In primitive morphological operations, erosion, one or two mismatched pixels of image prevent the structuring element from fitting perfectly. It is the basic morphological operation, quantifies the way in which, the structuring element fits into the image. Erosion is an "All or nothing" transformation, implemented using bitwise "and". So, erosion will be sensitive to noise.

In primitive morphological operation, dilation, isolated pixels, even though, they are irrelevant to the image's content, significantly affect the output of the transformation. The net effect is an increased number of large spurious particles, increasing the confusion in the dilated image. So, noise will be added, which may be named as additive noise.

But, many applications require more tolerance to noise than is provided by erosion and dilation. Soft morphological operators possess many of the characteristics, which are desirable, perform better in noisy environments. The soft morphological filters are better compared to mathematical morphology in small detail preservation and impulse noise. In soft morphology, it preserves details, by adjusting its parameters. With simple calculations or simple algorithms its performance is equal to filters designed in frequency domain, which involves complex computations.

The idea of soft morphological operations(14) is to relax, the standard morphological definition, a little, in such a way that, a degree of Robustness is achieved, While, most of the desirable properties of standard morphological operations are maintained.

4 ITERATIVE SOFT MORPHOLOGY:

Iterative morphology is a very important concept, which is having wide range of applications. Its applications are in skeletonization, thinning, thickening, structuring element Decomposition, segmentation, etc. Iterative morphology may be extended to iterative soft morphological environment also.

5 MULTI SCALE SOFT MORPHOLOGY:

The structuring element size may be 3/3,5/5,7/7,9/9,11/11,13/13 Applying the structuring elements at various sizes on the images we get different out puts. This type of process is called multiscale environment and using morphological operations, performing this type of processing is called multi scale morphology Multi Scale Mathematical Morphology is having applications in Image Smoothing, Edge Enhancement, Segmentation, Remote Sensing, Radar image analysis, Medical area etc. It is having special applications, like enhancing weak Edges, Decay analysis of wood, critical analysis of Cardio imagery.

Extending this concept to soft morphology is called multi scale soft morphology.

6 MULTI SCALE ITERATIVE SOFT MORPHOLOGY

If Multi Scale Soft Morphology and iterative soft morphology are combined, a new area Multi Scale iterative Soft Morphology Emerges. It is a new area, having wide applications and require a lot of attention..

6.1 DUALITY:

The duality may be defined in the following way, in a very simple manner.

Two operations $*$, $.$ are duals to each other if $(A*B)=(A.C.B)^C$ or $(A.B)=(A^C*B)^C$

In M.M. \oplus and \ominus are duals. i.e. $(I\oplus S) = (I^C\ominus S)^C$ or $(I\ominus S) = (I^C\oplus S)^C$

In these discussions the following convention is also followed where ever necessary. If dual of "X" is "Y" then it is represented as $X^d = Y$ or $(X)^d = Y$

In Soft morphology, the duality will exist in a different way because depending upon threshold values, many soft morphological operations will exist. . It is explained in the following sections. (In depth study of duality is not done, till now, in soft morphology. It is done in this study.)

In soft morphology self duality also will exist. Soft erosion at one threshold is dual to same soft erosion at different threshold. Also another observation is soft erosion at one threshold is equal to soft dilation at a different threshold. So soft erosion at one threshold will have multiple duals.

The same situation is for soft dilation also.

Because open ,close are composite operations , and multiple duals exists for primitive soft morphological operations, duals of these

composite operations are also multiple in number ,even more than two.

In iterative soft morphology also duality will exist, and it is multiple in number.

More and more number of duals exist in iterative environment.

These duals are very important and they have applications in noise smoothening as well as edge enhancement. So plenty of medical applications are there for duality operation.

All these are discussed in reference 11.

Some of the applications and importance is discussed in 15, in the author's paper.

All these duals are discussed in papers 1 to 10.

Now in this paper the essence is discussed in brief.

The laws of duality are given in detail.

7 KOMPPELLA V R laws of DUALITY of soft mathematical morphology:

The soft morphology has to be studied in multi scale as well as iterative environment with respect to properties. The reason is that the properties play major role in applying these operations. In this paper an important property called duality is discussed .This duality is also there in mathematical morphology and present in mathematical soft morphology also. It is discussed in multi scale as well as iterative environment ,in detail.

In soft mathematical morphology due to thresholds one soft morphological operation at one threshold may be equal to another soft morphological operation at a different threshold. So duality may be established among them and may be discussed in an elaborative way .The duality and dual operators are discussed in the author's Ph.D thesis and in his papers which are published in various international journals.(given in reference).

The duality may be discussed in multi scale soft morphological environment as well as iterative multi scale soft morphological environment. Here the soft morphological operations are soft erosion, soft dilation, soft open and soft close.

In multi scale environment the duality of soft erosion , soft dilation, soft open and soft close may be discussed. Due to equality multiple duals also exists. So multiple duals of soft erosion ,soft dilation, soft open, soft close also may be discussed.

In iterative environment also duality may be discussed. So duality of soft erosion in iterative

environment, duality of soft dilation in iterative environment, duality of soft open in iterative environment, duality of soft close in iterative environment may be discussed in detail and the corresponding formulae also may be given.

The equality in between soft erosion and soft dilation in multi scale environment is discussed in the author' s paper(10) in detail. The equality is also proved and established. The necessary charts ,tables, diagrams are also given. It is discussed in multi scale environment also, It means the equality is discussed with 3/3 structuring element size, 5/5 structuring element size,7/7 structuring element size,9/9 structuring element size ,11/11 structuring element size, and so on.

This paper is applied in all duality papers. The understanding is required in multiple duals, especially.

In ref 3,duality of soft erosion is discussed in detail. Definition is given. Duality is also proved.

The dualities are also given.

In the same way in ref 4,duality of soft dilation is also explained. The dualities are also given.

In ref 5 duality of soft open and soft close are also explained.

Due to equality of soft erosion and soft dilation multiple duals will exist.

So multiple duals of soft erosion and soft dilation are explained in ref 1 and 2.

multiple duals of soft open and soft close are also given in ref 5.

Duality of iterative soft erosion is discussed in paper 7. The corresponding dualities are also derived .This concept is given in detail in this author ' s paper.

Duality of iterative soft dilation is discussed in paper 9. The corresponding dualities are also derived .This concept is given in detail in this author ' s paper.

Duality of iterative soft open is discussed in paper 6. The corresponding dualities are also derived .This concept is given in detail in this author ' s paper.

Duality of iterative soft close is discussed in paper 8. The corresponding dualities are also derived .This concept is given in detail in this author ' s paper.

In this paper the essence of the above papers is given. The formulae are given and these are named after the author's name as KOMPPELLA V R laws of DUALITY of soft mathematical morphology.

8 DUALITY IN SOFT DILATION IN MULTI SCALE SOFT MORPHOLOGICAL ENVIRONMENT

The concepts are given in detail in paper 4. The duality is discussed in detail in this environment. The proofs etc. are also given in this paper. So essence is given in this section.

8.1. $\frac{3}{3}$ Structuring Element

$D(1), D(9)$ are dual operations.

$D(2)$ and $D(8)$ are dual operations.

$D(3), D(7)$ are duals.

$D(4), D(6)$ are duals.

$D(5)$ may be defined as self dual

In general $(D(m))^d = D(10 - m)$ where $m = 1$ to 9

$m = 5$... self dual.

8.2. $\frac{5}{5}$ Structuring Element

$D(1) \& D(25)$ are duals.

$D(2) \& D(24)$ are duals.

$D(3), D(23)$ are duals.

$D(4), D(22)$ are duals. $D(5), D(21)$ are duals. $D(6), D(20)$ are duals.

$D(7), D(19)$ are duals. $D(8), D(18)$ are duals. $D(9), D(17)$ are duals.

$D(10), D(16)$ are duals. $D(11), D(15)$ are duals. $D(12), D(14)$ are duals.

$D(13)$ is self dual.

In general $(D(m))^d = D(26 - m)$ where $m = 1$ to 25. $m = 13$... self dual.

In the same way the duals may be listed in 7/7 structuring element environment,

9/9 structuring element environment, 11/11

structuring element environment, 13/13

structuring element environment, 15/15

structuring element environment, 17/17

structuring element environment..... w/w

structuring element environment.

8.3. $\frac{w}{w}$ Structuring Element

Now for general case w/w , where w is sub image size, it can be shown that

$D(1), D(w^2)$ are duals. $D(2), D(w^2 - 1)$

are duals. $D(3), D(w^2 - 2)$ are duals.

$D(4), D(w^2 - 3)$ are duals. $D(5), D(w^2 - 4)$

are duals. $D(6), D(w^2 - 5)$ are duals.

$D(7), D(w^2 - 6)$ are duals. $D(8), D(w^2 - 7)$

are duals. $D(9), D(w^2 - 8)$ are duals.

$D(10), D(w^2 - 9)$ are duals. $D(11), D(w^2 - 10)$

are duals. $D(12), D(w^2 - 11)$ are duals.

$D\left(\frac{w^2+1}{2} - 5\right), D\left(\frac{w^2+1}{2} + 5\right)$ are duals.

$D\left(\frac{w^2+1}{2} - 4\right), D\left(\frac{w^2+1}{2} + 4\right)$ are duals.

$D\left(\frac{w^2+1}{2} - 3\right), D\left(\frac{w^2+1}{2} + 3\right)$ are duals.

$D\left(\frac{w^2+1}{2} - 2\right), D\left(\frac{w^2+1}{2} + 2\right)$ are duals.

$D\left(\frac{w^2+1}{2} - 1\right), D\left(\frac{w^2+1}{2} + 1\right)$ are duals.

$D\left(\frac{w^2+1}{2}\right)$ is self dual.

In general

$(D(m))^d = D(w^2 + 1 - m)$ where $m = 1$ to w^2 .

$m = \frac{w^2+1}{2}$... self dual.

9. DUALITY IN SOFT EROSION IN MULTI SCALE MORPHOLOGICAL ENVIRONMENT :

This section is basing upon the author's paper 3. All the details are discussed in this paper. The essence is given in this section.

9.1. $\frac{3}{3}$ Structuring Element

$E(1), E(9)$ are dual operations.

$E(2)$ and $E(8)$ are dual operations.

$E(3), E(7)$ are duals.

$E(4), E(6)$ are duals.

$E(5)$ is self dual.

In general, $(E(m))^d = E(10 - m)$ where $m = 1$ to 9

$m = 5$... self dual.

9.2. $\frac{5}{5}$ Structuring Element

$E(1), E(25)$ are duals. $E(2), E(24)$ are duals. $E(3), E(23)$ are duals.

$E(4), E(22)$ are duals. $E(5), E(21)$ are duals. $E(6), E(20)$ are duals.

$E(7), E(19)$ are duals. $E(8), E(18)$ are duals. $E(9), E(17)$ are duals.

$E(10), E(16)$ are duals. $E(11), E(15)$ are duals. $E(12), E(14)$ are duals.

$E(13)$ is self dual.

In general, $(E(m))^d = E(26 - m)$ where $m = 1$ to 25

$m = 13$... self dual.

In the same way the duals may be listed in 7/7 structuring element environment,

9/9 structuring element environment, 11/11
 structuring element environment, 13/13
 structuring element environment, 15/15
 structuring element environment, 17/17
 structuring element environment..... w/w
 structuring element environment.

9.3. W/w Structuring Element

Now for general case W/w , where w is sub image size, we can show that

$E(1), E(w^2)$ are duals. $E(2), E(w^2 - 1)$
 are duals. $E(3), E(w^2 - 2)$ are duals.
 $E(4), E(w^2 - 3)$ are duals. $E(5), E(w^2 - 4)$
 are duals. $E(6), E(w^2 - 5)$ are duals.
 $E(7), E(w^2 - 6)$ are duals. $E(8), E(w^2 - 7)$
 are duals. $E(9), E(w^2 - 8)$ are duals.
 $E(10), E(w^2 - 9)$ are duals. $E(11), E(w^2 - 10)$
 are duals. $E(12), E(w^2 - 11)$ are duals.

.....

 $E\left(\frac{w^2+1}{2} - 5\right), E\left(\frac{w^2+1}{2} + 5\right)$ are duals.
 $E\left(\frac{w^2+1}{2} - 4\right), E\left(\frac{w^2+1}{2} + 4\right)$ are
 duals.
 $E\left(\frac{w^2+1}{2} - 3\right), E\left(\frac{w^2+1}{2} + 3\right)$ are duals.
 $E\left(\frac{w^2+1}{2} - 2\right), E\left(\frac{w^2+1}{2} + 2\right)$ are
 duals.
 $E\left(\frac{w^2+1}{2} - 1\right), E\left(\frac{w^2+1}{2} + 1\right)$ are duals.
 $E\left(\frac{w^2+1}{2}\right)$ self dual.

In general,
 $(E(m))^d = E(w^2 + 1 - m)$ where $m = 1$ to w^2 .

$m = \frac{w^2+1}{2}$ self dual.

10. MULTIPLE DUALS IN SOFT DILATION IN MULTI SCALE SOFT MORPHOLOGICAL ENVIRONMENT

This is basing upon the author's paper 1.

Considering the equivalencies of soft morphological operations which are discussed in paper 10, of the author, and paper 1 the essence is given in this section.

10.1. $3/3$ Structuring Element

The duals of soft dilation operations are
 $(D(1))^d = D(9), E(1)$ $(D(2))^d = D(8), E(2)$
 $(D(3))^d = D(7), E(3)$
 $(D(4))^d = D(6), E(4)$ $(D(5))^d = D(5), E(5)$
 $(D(6))^d = D(4), E(6)$

$(D(7))^d = D(3), E(7)$ $(D(8))^d = D(2), E(8)$ $(D(9))^d = D(1), E(9)$
 In general, $(D(m))^d = D(10 - m), E(m)$ where $m = 1$ to 9

10.2. $5/5$ Structuring Element

The duals of soft dilation operations are

$(D(1))^d = D(25), E(1)$	$(D(2))^d = D(24), E(2)$
$(D(3))^d = D(23), E(3)$	$(D(4))^d = D(22), E(4)$
$(D(5))^d = D(21), E(5)$	$(D(6))^d = D(20), E(6)$
$(D(7))^d = D(19), E(7)$	$(D(8))^d = D(18), E(8)$
$(D(9))^d = D(17), E(9)$	$(D(10))^d = D(16), E(10)$
$(D(11))^d = D(15), E(11)$	$(D(12))^d = D(14), E(12)$
$(D(13))^d = D(13), E(13)$	$(D(14))^d = D(12), E(14)$
$(D(15))^d = D(11), E(15)$	$(D(16))^d = D(10), E(16)$
$(D(17))^d = D(9), E(17)$	$(D(18))^d = D(8), E(18)$
$(D(19))^d = D(7), E(19)$	$(D(20))^d = D(6), E(20)$
$(D(21))^d = D(5), E(21)$	$(D(22))^d = D(4), E(22)$
$(D(23))^d = D(3), E(23)$	$(D(24))^d = D(2), E(24)$
$(D(25))^d = D(1), E(25)$	

In general, $(D(m))^d = D(26 - m), E(m)$ where $m = 1$ to 25

In the same way the duals may be listed in 7/7 structuring element environment,
 9/9 structuring element environment, 11/11
 structuring element environment, 13/13
 structuring element environment, 15/15
 structuring element environment, 17/17
 structuring element environment..... w/w
 structuring element environment.

10.3. W/w Structuring Element

$(D(1))^d = D(w^2), E(1)$
 $(D(2))^d = D(w^2 - 1), E(2)$
 $(D(3))^d = D(w^2 - 2), E(3)$
 $(D(4))^d = D(w^2 - 3), E(4)$
 $(D(5))^d = D(w^2 - 4), E(5)$
 $(D(6))^d = D(w^2 - 5), E(6)$
 $(D(7))^d = D(w^2 - 6), E(7)$

$$\begin{aligned}
 & \dots \dots \dots \\
 (D(w^2 - 6))^d &= D(7), E(w^2 - 6) \\
 (D(w^2 - 5))^d &= D(6), E(w^2 - 5) \\
 (D(w^2 - 4))^d &= D(5), E(w^2 - 4) \\
 (D(w^2 - 3))^d &= D(4), E(w^2 - 3) \\
 (D(w^2 - 2))^d &= D(3), E(w^2 - 2) \\
 (D(w^2 - 1))^d &= D(2), E(w^2 - 1) \\
 (D(w^2))^d &= D(1), E(w^2) \\
 \text{In general,} \\
 (D(m))^d &= D(w^2 + 1 - m), E(m) \text{ Where} \\
 m &= 1 \text{ to } w^2
 \end{aligned}$$

11 MULTIPLE DUALS IN SOFT EROSION IN MULTI SCALE SOFT MORPHOLOGICAL ENVIRONMENT

This section is given basing upon paper of the author which is given as ref 2.

In this section, paper 10, of the author, which discusses equalities of soft erosion and soft dilation, is also a base in deriving formulae and discussing concepts. The essence is only given in this section.

11.1. 3/3 Structuring Element

The duals of soft erosion operations are

$$\begin{aligned}
 (E(1))^d &= E(9), D(1) & (E(2))^d &= E(8), D(2) \\
 (E(3))^d &= E(7), D(3) & (E(4))^d &= E(6), D(4) \\
 (E(5))^d &= E(5), D(5) & (E(6))^d &= E(4), D(6) \\
 (E(7))^d &= E(3), D(7) & (E(8))^d &= E(2), D(8) \\
 (E(9))^d &= E(1), D(9) \\
 \text{In general, } (E(m))^d &= E(10 - m), D(m) \text{ where} \\
 m &= 1 \text{ to } 9
 \end{aligned}$$

11.2. 5/5 Structuring Element

The duals of soft erosion operations are

$$\begin{aligned}
 (E(1))^d &= E(25), D(1) & (E(2))^d &= E(24), D(2) \\
 (E(3))^d &= E(23), D(3) & (E(4))^d &= E(22), D(4) \\
 (E(5))^d &= E(21), D(5) & (E(6))^d &= E(20), D(6) \\
 (E(7))^d &= E(19), D(7) & (E(8))^d &= E(18), D(8) \\
 (E(9))^d &= E(17), D(9) & (E(10))^d &= E(16), D(10) \\
 (E(11))^d &= E(15), D(11) & (E(12))^d &= E(14), D(12)
 \end{aligned}$$

$$\begin{aligned}
 (E(13))^d &= E(13), D(13) & (E(14))^d &= E(12), D(14) \\
 (E(15))^d &= E(11), D(15) & (E(16))^d &= E(10), D(16) \\
 (E(17))^d &= E(9), D(17) & (E(18))^d &= E(8), D(18) \\
 (E(19))^d &= E(7), D(19) & (E(20))^d &= E(6), D(20) \\
 (E(21))^d &= E(5), D(21) & (E(22))^d &= E(4), D(22) \\
 (E(23))^d &= E(3), D(23) & (E(24))^d &= E(2), D(24) \\
 (E(25))^d &= E(1), D(25) \\
 \text{In general, } (E(m))^d &= E(26 - m), D(m) \text{ where} \\
 m &= 1 \text{ to } 25
 \end{aligned}$$

In the same way the duals may be listed in 7/7 structuring element environment, 9/9 structuring element environment, 11/11 structuring element environment, 13/13 structuring element environment, 15/15 structuring element environment, 17/17 structuring element environment..... w/w structuring element environment.

11.3. w/w Structuring Element

$$\begin{aligned}
 (E(1))^d &= E(w^2), D(1) & (E(2))^d &= E(w^2 - 1), D(2) \\
 (E(3))^d &= E(w^2 - 2), D(3) & (E(4))^d &= E(w^2 - 3), D(4) \\
 (E(5))^d &= E(w^2 - 4), D(5) & (E(6))^d &= E(w^2 - 5), D(6) \\
 (E(7))^d &= E(w^2 - 6), D(7) \\
 & \dots \dots \dots
 \end{aligned}$$

$$\begin{aligned}
 & \dots \dots \dots \\
 (E(w^2 - 6))^d &= E(7), D(w^2 - 6) \\
 (E(w^2 - 5))^d &= E(6), D(w^2 - 5) \\
 (E(w^2 - 4))^d &= E(5), D(w^2 - 4) \\
 (E(w^2 - 3))^d &= E(4), D(w^2 - 3) \\
 (E(w^2 - 2))^d &= E(3), D(w^2 - 2) \\
 (E(w^2 - 1))^d &= E(2), D(w^2 - 1) \\
 (E(w^2))^d &= E(1), D(w^2) \\
 \text{In general, } (E(m))^d &= E(w^2 + 1 - m), D(m) \text{ Where } m = 1 \text{ to } w^2
 \end{aligned}$$

12. DUALITY IN SOFT OPEN

It is basing upon author's paper 5.
 $(O(m, n))^d = O(w^2 + 1 - m, w^2 + 1 - n)$ or $C(m, n)$
 or $E(w^2 + 1 - m)E(n)$ or $D(m)D(w^2 + 1 - n)$
 Soft open will have four duals
 (1) soft open (with different thresholds)

(2) soft close (with different thresholds)

(3) Iterative soft erosion (two times and with different thresholds)

(4) Iterative soft dilation (two times and with different thresholds)

13. DUALITY IN SOFT CLOSE

It is also basing upon author's paper 5.

$$(C(m, n))^d = C(w^2 + 1 - m, w^2 + 1 - n) \quad \text{or} \quad O(m, n)$$

$$\text{or } D(w^2 + 1 - m)D(n) \text{ or } E(m)E(w^2 + 1 - n)$$

Soft close will have four duals

(1) soft open (with different thresholds)

(2) soft close (with different thresholds)

(3) Iterative soft erosion (two times and with different thresholds)

(4) Iterative soft dilation (two times and with different thresholds)

14 DUALITY OF ITERATIVE SOFT EROSION IN MULTI SCALE SOFT MORPHOLOGICAL ENVIRONMENT

In this section the duality of iterative soft erosion is discussed, with various cases like same thresholds, different thresholds, and with various duals. The detailed discussion is given in the author's paper 7, where discussions of concepts, discussions of derivations of formulae is done in detail. In this section the essence is only presented.

14.1. Duality of iterative soft erosion with same threshold and with one type of dual.

$$((E(m))^n)^d = (D(m))^n$$

14.2. Duality of iterative soft erosion with same threshold and with another type of dual.

$$((E(m))^n)^d = (E(w^2 + 1 - m))^n$$

14.3. Duality of iterative soft erosion with different thresholds and with one type of dual.

$$\left(\begin{matrix} E(m)E(n)E(o)E(p) & \dots & E(z) \\ 1 & 2 & 3 & 4 & \dots & n \end{matrix} \right)^d = \left(\begin{matrix} D(m)D(n)D(o)D(p) & \dots & D(z) \\ 1 & 2 & 3 & 4 & \dots & n \end{matrix} \right)$$

14.4. Duality of iterative soft erosion with different thresholds and with another type of dual.

$$\left(\begin{matrix} E(m)E(n)E(o)E(p) & \dots & E(z) \\ 1 & 2 & 3 & 4 & \dots & n \end{matrix} \right)^d =$$

$$E(w^2 + 1 - m)E(w^2 + 1 - n)E(w^2 + 1 - o)E(w^2 + 1 - p) \dots E(w^2 + 1 - z)$$

14.5. Duality of iterative soft erosion with same threshold and with two duals

$$(a) \quad ((E(m))^2)^d = (E(w^2 + 1 - m), D(m))(E(w^2 + 1 - m), D(m))$$

$$= (E(w^2 + 1 - m) E(w^2 + 1 - m)) \quad \text{or} \quad (E(w^2 + 1 - m)D(m))$$

$$\text{or } (D(m)E(w^2 + 1 - m)) \text{ or } D(m)D(m)$$

$$((E(m))^2)^d = (E(w^2 + 1 - m) E(w^2 + 1 - m)) \text{ or } O(w^2 + 1 - m, m)$$

$$\text{or } C(m, w^2 + 1 - m)$$

$$\text{or } (D(m)D(m))$$

= Double soft erosion or soft open or soft close or double soft dilation

$$(b) \quad ((E(m))^4)^d =$$

$$(E(w^2 + 1 - m), D(m)) (E(w^2 + 1 - m), D(m)) (E(w^2 + 1 - m), D(m)) (E(w^2 + 1 - m), D(m))$$

=

$$(E(k), D(m)) (E(k), D(m)) (E(k), D(m)) (E(k), D(m))$$

(by putting $w^2 + 1 - m = k$)

$$= (E(k)E(k)E(k)E(k)) \quad \text{or}$$

$$(E(k)E(k)E(k)D(m)) \text{ or } (E(k)E(k)D(m)E(k))$$

or

$$(E(k)E(k)D(m)D(m)) \text{ or } (E(k)D(m)E(k)E(k)) \text{ or}$$

$$(E(k)D(m)E(k)D(m)) \text{ or } (E(k)D(m)D(m)E(k)) \text{ or}$$

$$(E(k)D(m)D(m)D(m)) \text{ or}$$

$$(D(m)E(k)E(k)E(k)) \quad \text{or}$$

$$(D(m)E(k)E(k)D(m)) \text{ or } (D(m)E(k)D(m)E(k))$$

or

$$(D(m)E(k)D(m)D(m)) \text{ or } (D(m)D(m)E(k)E(k))$$

or

$$(D(m)D(m)E(k)D(m)) \text{ or } (D(m)D(m)D(m)E(k))$$

or

$$(D(m)D(m)D(m)D(m))$$

= Iterative soft erosion four times or iterative soft erosion three times followed by soft dilation or iterative soft erosion two times followed by soft close or iterative soft erosion two times followed by soft dilation two times or soft open followed by iterative soft erosion two times or soft open two times or soft open soft close or soft open followed soft dilation two times or soft close followed by soft erosion two times or soft close followed by soft open or soft close followed by soft close or soft close followed by soft dilation two times or soft dilation two time followed by soft erosion two times or soft dilation two times followed by soft open or soft dilation two times followed soft close or soft dilation four times.

In the same way formulae for

open once with different thresholds or soft dilation twice and soft close once with different thresholds or soft dilation four times with different thresholds.

15 DUALITY OF ITERATIVE SOFT DILATION IN MULTI SCALE SOFT MORPHOLOGICAL ENVIRONMENT

In this section the discussion is done basing upon author's paper 9.

15.1. Duality of iterative soft dilation with same threshold and with one type of dual.

$$((D(m))^n)^d = (E(m))^n$$

15.2. Duality of iterative soft dilation with same threshold and with another type of dual.

$$((D(m))^n)^d = (D(w^2 + 1 - m))^n$$

15.3. Duality of iterative soft dilation with different thresholds and with one type of dual.

$$\left(\begin{matrix} D(m)D(n)D(o)D(p) & & D(z) \\ 1 & 2 & 3 & 4 & \dots & n \end{matrix} \right)^d$$

$$= \begin{matrix} E(m)E(n)E(o)E(p) & & E(z) \\ 1 & 2 & 3 & 4 & \dots & n \end{matrix}$$

15.4. Duality of iterative soft dilation with different thresholds and with another type of dual.

$$\left(\begin{matrix} D(m)D(n)D(o)D(p) & & D(z) \\ 1 & 2 & 3 & 4 & \dots & n \end{matrix} \right)^d$$

$$= \begin{matrix} D(w^2 + 1 - m)D(w^2 + 1 - n)D(w^2 + 1 - o)D(w^2 + 1 - p) & & D(w^2 + 1 - z) \\ 1 & 2 & 3 & 4 & \dots & n \end{matrix}$$

15.5. Duality of iterative soft dilation with one threshold and with multiple duals

$$(D(n))^d = D(w^2 + 1 - n), E(n)$$

15.6. Duality of two iterative soft dilations with one threshold and with multiple duals

$$((D(n))^2)^d = (D(n)D(n))^d = (D(n))^d (D(n))^d$$

$$= \begin{matrix} (D(w^2 + 1 - n), E(n))(D(w^2 + 1 - n), E(n)) \\ (D(w^2 + 1 - n))^2, (D(w^2 + 1 - n)E(n)), (E(n)D(w^2 + 1 - n)), (E(n))^2 \end{matrix}$$

$$= \begin{matrix} (D(w^2 + 1 - n))^2, (C(w^2 + 1 - n, n)), (O(n, w^2 + 1 - n)), (E(n))^2 \end{matrix}$$

= soft dilation twice with one threshold, soft close with two different thresholds, soft open with two different thresholds, soft erosion twice with another threshold.

15.7. Duality of three iterative soft dilations with one threshold and with multiple duals

$$((D(n))^3)^d = (D(n)D(n)D(n))^d =$$

$$(D(n))^d (D(n))^d (D(n))^d$$

$$= (D(w^2 + 1 - n), E(n)) (D(w^2 + 1 - n), E(n)) (D(w^2 + 1 - n), E(n))$$

(by substituting x for $D(w^2 + 1 - n)$ and y for $E(n)$)

$$= \begin{matrix} ((x, y) (x, y) (x, y)) \\ ((xxx, xxy, xyx, xyy, yxx, yxy, yyx, yyy)) \\ \left(\begin{matrix} D(w^2 + 1 - n)D(w^2 + 1 - n)D(w^2 + 1 - n), \\ D(w^2 + 1 - n)D(w^2 + 1 - n)E(n), \\ D(w^2 + 1 - n)E(n)D(w^2 + 1 - n), \\ D(w^2 + 1 - n)E(n)E(n), \\ E(n)D(w^2 + 1 - n)D(w^2 + 1 - n), \\ E(n)D(w^2 + 1 - n)E(n), \\ E(n)E(n)D(w^2 + 1 - n), \\ E(n)E(n)E(n) \end{matrix} \right) \end{matrix}$$

= Soft dilation thrice, soft dilation once and soft close once, soft dilation once and soft open once, soft close once and soft erosion once, soft open once and soft dilation once, soft open once and soft erosion once, soft erosion once and soft open once, soft erosion thrice.

15.8. Duality of four iterative soft dilations with one threshold and with multiple duals

$$\begin{matrix} (D(n))^4 \\ ((D(n))^4)^d = (D(n)D(n)D(n)D(n))^d = \\ (D(n))^d (D(n))^d (D(n))^d (D(n))^d \end{matrix}$$

$$= \begin{matrix} (D(w^2 + 1 - n), E(n)) (D(w^2 + 1 - n), E(n)) \\ (D(w^2 + 1 - n), E(n)) (D(w^2 + 1 - n), E(n)) \\ \left(\begin{matrix} D(w^2 + 1 - n)D(w^2 + 1 - n)D(w^2 + 1 - n)D(w^2 + 1 - n), \\ D(w^2 + 1 - n)D(w^2 + 1 - n)D(w^2 + 1 - n)E(n), \\ D(w^2 + 1 - n)D(w^2 + 1 - n)E(n)D(w^2 + 1 - n), \\ D(w^2 + 1 - n)D(w^2 + 1 - n)E(n)E(n), \\ D(w^2 + 1 - n)E(n)D(w^2 + 1 - n)D(w^2 + 1 - n), \\ D(w^2 + 1 - n)E(n)D(w^2 + 1 - n)E(n), \\ D(w^2 + 1 - n)E(n)E(n)D(w^2 + 1 - n), \\ D(w^2 + 1 - n)E(n)E(n)E(n), \\ E(n)D(w^2 + 1 - n)D(w^2 + 1 - n)D(w^2 + 1 - n), \\ E(n)D(w^2 + 1 - n)D(w^2 + 1 - n)E(n), \\ E(n)D(w^2 + 1 - n)E(n)D(w^2 + 1 - n), \\ E(n)D(w^2 + 1 - n)E(n)E(n), \\ E(n)E(n)D(w^2 + 1 - n)D(w^2 + 1 - n), \\ E(n)E(n)D(w^2 + 1 - n)E(n), \\ E(n)E(n)E(n)D(w^2 + 1 - n), \\ E(n)E(n)E(n)E(n) \end{matrix} \right) \end{matrix}$$

$$= \left(\begin{array}{c} (D(w^2 + 1 - n))^4, \\ D(w^2 + 1 - n)D(w^2 + 1 - n)C(w^2 + 1 - n, n), \\ D(w^2 + 1 - n)C(w^2 + 1 - n, n), D(w^2 + 1 - n), \\ D(w^2 + 1 - n)C(w^2 + 1 - n, n)E(n), \\ C(w^2 + 1 - n, n)D(w^2 + 1 - n)D(w^2 + 1 - n), \\ C(w^2 + 1 - n, n)C(w^2 + 1 - n, n), \\ C(w^2 + 1 - n, n), \\ C(w^2 + 1 - n, n)E(n)E(n), \\ O(n, w^2 + 1 - n)D(w^2 + 1 - n)D(w^2 + 1 - n), \\ O(n, w^2 + 1 - n)C(w^2 + 1 - n, n), \\ O(n, w^2 + 1 - n)O(n, w^2 + 1 - n), \\ O(n, w^2 + 1 - n)E(n)E(n), \\ E(n)O(n, w^2 + 1 - n)D(w^2 + 1 - n), \\ E(n), E(n)C(w^2 + 1 - n, n), \\ E(n)E(n)O(n, w^2 + 1 - n), \\ (E(n))^4 \end{array} \right)$$

15.9. Duality of iterative soft dilation with different thresholds and with two duals.

$$\begin{aligned} (D(n)D(m))^d &= (D(n))^d (D(m))^d \\ &= (D(w^2 + 1 - n), E(n))(D(w^2 + 1 - m), E(m)) \\ &= (D(w^2 + 1 - n)D(w^2 + 1 - m)) \text{ or } (D(w^2 + 1 - n)E(m)) \text{ or} \\ &\quad (E(n)D(w^2 + 1 - m)) \text{ or } (E(n)E(m)) \end{aligned}$$

$$\begin{aligned} &= (D(w^2 + 1 - n)D(w^2 + 1 - m)) \text{ or } C(w^2 + 1 - n, m) \text{ or} \\ &\quad O(n, w^2 + 1 - m) \text{ or } E(n)E(m) \end{aligned}$$

= Soft dilation twice with different thresholds or soft close or soft open or soft erosion with different thresholds

15.10. Duality of iterative soft dilation with three different thresholds and with two duals.

$$\begin{aligned} (D(n)D(m)D(p))^d &= (D(n))^d (D(m))^d (D(p))^d \\ &= (D(w^2 + 1 - n), E(n)) (D(w^2 + 1 - m), E(m)) (D(w^2 + 1 - p), E(p)) \\ &= (D(w^2 + 1 - n)D(w^2 + 1 - m)D(w^2 + 1 - p)) \end{aligned}$$

$$\begin{aligned} \text{or} \quad &(D(w^2 + 1 - n)E(m)D(w^2 + 1 - p)) \text{ or} \\ &(E(n)D(w^2 + 1 - m)D(w^2 + 1 - p)) \end{aligned} \quad \text{or}$$

$$\begin{aligned} &(E(n)E(m)D(w^2 + 1 - p)) \text{ or} \\ &(D(w^2 + 1 - n)D(w^2 + 1 - m)E(p)) \end{aligned} \quad \text{or}$$

$$\begin{aligned} &(D(w^2 + 1 - n)E(m)E(p)) \text{ or} \\ &(E(n)D(w^2 + 1 - m)E(p)) \text{ or } (E(n)E(m)E(p)) \end{aligned}$$

$$\begin{aligned} &= (D(w^2 + 1 - n)D(w^2 + 1 - m)D(w^2 + 1 - p)) \\ \text{or} \quad &(C(w^2 + 1 - n, m) D(w^2 + 1 - p)) \text{ or} \\ &(O(n, w^2 + 1 - m)D(w^2 + 1 - p)) \end{aligned} \quad \text{or}$$

$$\begin{aligned} &(E(n)O(m, w^2 + 1 - p)) \text{ or} \\ &(D(w^2 + 1 - n)C(w^2 + 1 - m, p)) \text{ or } (C(w^2 + 1 - n, m)E(p)) \text{ or} \\ &(O(n, w^2 + 1 - m)E(p)) \text{ or } (E(n)E(m)E(p)) \end{aligned}$$

= soft dilation thrice or soft close and soft dilation or soft open and soft dilation or soft erosion and soft open or soft dilation and soft close or soft close and soft erosion or soft open and soft erosion or soft erosion thrice.

15.11. Duality of iterative soft dilation with four different thresholds and with two duals.

$$(D(n)D(m)D(p)D(r))^d = (D(n))^d (D(m))^d (D(p))^d (D(r))^d$$

$$\begin{aligned} &= (D(w^2 + 1 - n), E(n)) (D(w^2 + 1 - m), E(m)) \\ &\quad (D(w^2 + 1 - p), E(p)) (D(w^2 + 1 - r), E(r)) \\ &= (D(w^2 + 1 - n)D(w^2 + 1 - m)D(w^2 + 1 - p)D(w^2 + 1 - r)) \text{ or} \end{aligned}$$

$$\begin{aligned} &\quad (D(w^2 + 1 - n)E(m)D(w^2 + 1 - p)D(w^2 + 1 - r)) \text{ or} \\ &\quad (E(n)D(w^2 + 1 - m)D(w^2 + 1 - p)D(w^2 + 1 - r)) \text{ or} \end{aligned}$$

$$\begin{aligned} &\quad (E(n)E(m)D(w^2 + 1 - p)D(w^2 + 1 - r)) \text{ or} \\ &\quad (D(w^2 + 1 - n)D(w^2 + 1 - m)E(p)D(w^2 + 1 - r)) \text{ or} \end{aligned}$$

$$\begin{aligned} &\quad (D(w^2 + 1 - n)E(m)E(p)D(w^2 + 1 - r)) \text{ or} \\ &\quad (E(n)D(w^2 + 1 - m)E(p)D(w^2 + 1 - r)) \text{ or} \end{aligned}$$

$$\begin{aligned} &\quad (E(n)E(m)E(p)D(w^2 + 1 - r)) \text{ or} \\ &\quad (D(w^2 + 1 - n)D(w^2 + 1 - m)D(w^2 + 1 - p)E(r)) \text{ or} \end{aligned}$$

$$\begin{aligned} &\quad (D(w^2 + 1 - n)E(m)D(w^2 + 1 - p)E(r)) \text{ or} \\ &\quad (E(n)D(w^2 + 1 - m)D(w^2 + 1 - p)E(r)) \text{ or} \end{aligned}$$

$$\begin{aligned} &\quad (E(n)E(m)D(w^2 + 1 - p)E(r)) \text{ or} \\ &\quad (D(w^2 + 1 - n)D(w^2 + 1 - m)E(p)E(r)) \text{ or} \end{aligned}$$

$$\begin{aligned} &\quad (D(w^2 + 1 - n)E(m)E(p)E(r)) \text{ or} \\ &\quad (E(n)D(w^2 + 1 - m)E(p)E(r)) \text{ or} \end{aligned}$$

$$\begin{aligned} &\quad (E(n)E(m)E(p)E(r)) \\ &= (D(w^2 + 1 - n)D(w^2 + 1 - m)D(w^2 + 1 - p)D(w^2 + 1 - r)) \text{ or} \end{aligned}$$

$$\begin{aligned} &\quad (C(w^2 + 1 - n, m) D(w^2 + 1 - p)D(w^2 + 1 - r)) \text{ or} \\ &\quad (O(n, w^2 + 1 - m)D(w^2 + 1 - p)D(w^2 + 1 - r)) \text{ or} \end{aligned}$$

$$\begin{aligned} &\quad (E(n)O(m, w^2 + 1 - p)D(w^2 + 1 - r)) \text{ or} \\ &\quad (D(w^2 + 1 - n)C(w^2 + 1 - m, p)D(w^2 + 1 - r)) \text{ or} \end{aligned}$$

$$\begin{aligned} &\quad (C(w^2 + 1 - n, m)O(p, w^2 + 1 - r)) \text{ or} \\ &\quad (O(n, w^2 + 1 - m)O(p, w^2 + 1 - r)) \text{ or} \end{aligned}$$

$$\begin{aligned} &\quad (E(n)E(m)O(p, w^2 + 1 - r)) \text{ or} \\ &\quad (D(w^2 + 1 - n)D(w^2 + 1 - m)C(w^2 + 1 - p, r)) \end{aligned}$$

$$\begin{aligned} \text{or} \quad &(C(w^2 + 1 - n, m)C(w^2 + 1 - p, r)) \text{ or} \\ &(O(n, w^2 + 1 - m)C(w^2 + 1 - p, r)) \end{aligned} \quad \text{or}$$

$$\begin{aligned} &(E(n)O(m, w^2 + 1 - p)E(r)) \text{ or} \\ &(D(w^2 + 1 - n)C(w^2 + 1 - m, p)E(r)) \end{aligned} \quad \text{or}$$

$(O(n, w^2 + 1 - m)E(p)E(r))$ or
 $(E(n)E(m)E(p)E(r))$
 = soft dilation four times or soft close once and soft dilation twice or soft open once and soft dilation twice or soft erosion once soft open once and soft dilation once or soft dilation once soft close once and soft dilation once or soft close once and soft open once or soft open twice or soft erosion twice and soft open once or soft dilation twice and soft close once or soft close twice or soft open once and soft close once or soft erosion once and soft open once and soft erosion once or soft dilation once soft close once and soft erosion once or soft close once and soft erosion twice or soft open once and soft erosion twice or soft erosion four times.

16. DUALITY OF ITERATIVE SOFT OPEN IN MULTI SCALE SOFT MORPHOLOGICAL ENVIRONMENT

In this section the essence of author's paper 6 , is presented.

16.1. Duality of iterative soft open with same set of thresholds and with one type of dual.

$$((O(m, n))^k)^d = (C(m, n))^k$$

16.2. Duality of iterative soft open with same set of thresholds and with another type of dual.

$$((O(m, n))^k)^d = (O(w^2 + 1 - m, w^2 + 1 - n))^k$$

16.3. Duality of iterative soft open with different sets of thresholds and with one type of dual.

$$\begin{pmatrix} O(a, b)O(c, d)O(e, f)O(g, h) & \dots & O(x, y) \\ 1 & 2 & 3 & 4 & \dots & n \end{pmatrix}^d = \begin{pmatrix} C(a, b)C(c, d)C(e, f)C(g, h) & \dots & C(x, y) \\ 1 & 2 & 3 & 4 & \dots & n \end{pmatrix}$$

16.4. Duality of iterative soft open with different sets of thresholds and with another type of dual.

$$\begin{pmatrix} O(a, b)O(c, d)O(e, f)O(g, h) & \dots & O(x, y) \\ 1 & 2 & 3 & 4 & \dots & n \end{pmatrix}^d = O(w^2 + 1 - a, w^2 + 1 - b)O(w^2 + 1 - c, w^2 + 1 - d) \dots \dots O(w^2 + 1 - x, w^2 + 1 - y)$$

16.5. Duality of two iterative soft opens with one set of thresholds and with multiple duals.

$$(O(m, n)^2)^d$$

$$= \left(\begin{aligned} & (O(w^2 + 1 - m, w^2 + 1 - n)O(w^2 + 1 - m, w^2 + 1 - n)), \\ & (O(w^2 + 1 - m, w^2 + 1 - n)E(w^2 + 1 - m)E(n)), \\ & (O(w^2 + 1 - m, w^2 + 1 - n)D(m)D(w^2 + 1 - n)), \\ & (O(w^2 + 1 - m, w^2 + 1 - n)C(m, n)), \\ & (E(w^2 + 1 - m)E(n)O(w^2 + 1 - m, w^2 + 1 - n)), \\ & (E(w^2 + 1 - m)E(n)E(w^2 + 1 - m)E(n)), \\ & (E(w^2 + 1 - m)O(n, m)D(w^2 + 1 - n)), \\ & (E(w^2 + 1 - m)O(n, m)E(n)), \\ & (D(m)D(w^2 + 1 - n)O(w^2 + 1 - m, w^2 + 1 - n)), \\ & (D(m)C(w^2 + 1 - n, w^2 + 1 - m)E(n)), \\ & (D(m)D(w^2 + 1 - n)D(m)D(w^2 + 1 - n)), \\ & (D(m)D(w^2 + 1 - n)C(m, n)), \\ & (C(m, n)O(w^2 + 1 - m, w^2 + 1 - n)), \\ & (C(m, n)E(w^2 + 1 - m)E(n)), \\ & (C(m, n)D(m)D(w^2 + 1 - n)), (C(m, n)C(m, n)), \end{aligned} \right)$$

= soft open twice or soft open and soft erosion twice or soft open and soft dilation twice or soft open and soft close or soft erosion twice and soft open or soft erosion four times or soft erosion and soft open and soft dilation or soft erosion and soft open and soft erosion or soft dilation twice and soft open or soft dilation twice and soft close and soft erosion or soft dilation four times or soft dilation twice and soft close or soft close and soft open or soft close and soft erosion twice or soft close and soft dilation twice or soft close twice.

16.6. Duality of two iterative soft opens with two sets of thresholds and with multiple duals.

$$(O(m, n) O(u, v))^d = \left(\begin{aligned} & (O(w^2 + 1 - m, w^2 + 1 - n)O(w^2 + 1 - u, w^2 + 1 - v)), \\ & (O(w^2 + 1 - m, w^2 + 1 - n)E(w^2 + 1 - u)E(v)), \\ & (O(w^2 + 1 - m, w^2 + 1 - n)D(u)D(w^2 + 1 - v)), \\ & (O(w^2 + 1 - m, w^2 + 1 - n)C(u, v)), \\ & (E(w^2 + 1 - m)E(n)O(w^2 + 1 - u, w^2 + 1 - v)), \\ & (E(w^2 + 1 - m)E(n)E(w^2 + 1 - u)E(v)), \\ & (E(w^2 + 1 - m)O(n, u)D(w^2 + 1 - v)), \\ & (E(w^2 + 1 - m)O(n, u)E(v)), \\ & (D(m)D(w^2 + 1 - n)O(w^2 + 1 - u, w^2 + 1 - v)), \\ & (D(m)C(w^2 + 1 - n, w^2 + 1 - u)E(v)), \\ & (D(m)D(w^2 + 1 - n)D(u)D(w^2 + 1 - v)), \\ & (D(m)D(w^2 + 1 - n)C(u, v)), \\ & (C(m, n)O(w^2 + 1 - u, w^2 + 1 - v)), \\ & (C(m, n)E(w^2 + 1 - u)E(v)), \\ & (C(m, n)D(u)D(w^2 + 1 - v)), (C(m, n)C(u, v)), \end{aligned} \right)$$

= soft open twice or soft open and soft erosion twice or soft open and soft dilation twice or soft open and

soft close or soft erosion twice and soft open or soft erosion four times or soft erosion once and soft open once and soft dilation once or soft erosion once soft open once and soft erosion once or soft dilation twice and soft open once or soft dilation once and soft close once and soft erosion once or soft dilation four times or soft dilation twice and soft close once or soft close once and soft open once or soft close once and soft open once or soft close once and soft erosion twice or soft close once and soft dilation twice or soft close twice.

16.7. Duality of soft open with one set of thresholds and with multiple duals.

$$(O(m, n))^d = (O(w^2 + 1 - m, w^2 + 1 - n), D(m)D(w^2 + 1 - n), E(w^2 + 1 - m)E(n), C(m, n))$$

= Soft open or soft dilation twice or soft erosion twice or soft close.

17. DUALITY OF ITERATIVE SOFT CLOSE IN MULTI SCALE SOFT MORPHOLOGICAL ENVIRONMENT

In this section the essence of author's paper 8 , is presented.

17.1. Duality of iterative soft close with same set of thresholds and with one type of dual.

$$= \left(\begin{array}{l} (C(w^2 + 1 - m, w^2 + 1 - n)C(w^2 + 1 - m, w^2 + 1 - n)), \\ (C(w^2 + 1 - m, w^2 + 1 - n)D(w^2 + 1 - m)D(n)), \\ (C(w^2 + 1 - m, w^2 + 1 - n)E(m)E(w^2 + 1 - n)), \\ (C(w^2 + 1 - m, w^2 + 1 - n)O(m, n)), \\ (D(w^2 + 1 - m)D(n)C(w^2 + 1 - m, w^2 + 1 - n)), \\ (D(w^2 + 1 - m)D(n)D(w^2 + 1 - m)D(n)), \\ (D(w^2 + 1 - m)C(n, m)E(w^2 + 1 - n)), \\ (D(w^2 + 1 - m)C(n, m)D(n)), \\ (E(m)E(w^2 + 1 - n)C(w^2 + 1 - m, w^2 + 1 - n)), \\ (E(m)O(w^2 + 1 - n, w^2 + 1 - m)D(n)), \\ (E(m)E(w^2 + 1 - n)E(m)E(w^2 + 1 - n)), \\ (E(m)E(w^2 + 1 - n)O(m, n)), \\ (O(m, n)C(w^2 + 1 - m, w^2 + 1 - n)), \\ (O(m, n)D(w^2 + 1 - m)D(n)), \\ (O(m, n)E(m)E(w^2 + 1 - n)), (O(m, n)O(m, n)), \end{array} \right)$$

= soft close twice or soft close and soft dilation twice or soft close and soft erosion twice or soft close and soft open or soft dilation twice and soft close or soft dilation four times or soft dilation and soft close and soft erosion or soft dilation and soft close and soft dilation or soft erosion twice and soft close or soft erosion twice and soft open and soft dilation or soft erosion four times or soft erosion twice and soft open or soft open and soft close or soft open and soft dilation twice or soft open and soft erosion twice or soft open twice.

17.6. Duality of two iterative soft close operations with two sets of thresholds and with multiple duals.

$$(C(m, n) C(u, v))^d =$$

$$((C(m, n))^k)^d = (O(m, n))^k$$

17.2. Duality of iterative soft close with same set of thresholds and with another type of dual.

$$\therefore ((C(m, n))^k)^d = (C(w^2 + 1 - m, w^2 + 1 - n))^k$$

17.3. Duality of iterative soft close with different sets of thresholds and with one type of dual.

$$\begin{aligned} & (C(a, b)C(c, d)C(e, f)C(g, h) \dots C(x, y))^d \\ &= O(a, b)O(c, d)O(e, f)O(g, h) \dots O(x, y) \end{aligned}$$

17.4. Duality of iterative soft close with different sets of thresholds and with another type of dual.

$$\begin{aligned} & (C(a, b)C(c, d)C(e, f)C(g, h) \dots C(x, y))^d \\ &= C(w^2 + 1 - a, w^2 + 1 - b)C(w^2 + 1 - c, w^2 + 1 - d) \dots C \end{aligned}$$

17.5. Duality of two iterative soft close operations with one set of thresholds and with multiple duals.

$$(C(m, n)^2)^d =$$

$$\left(\begin{array}{l} (C(w^2 + 1 - m, w^2 + 1 - n)C(w^2 + 1 - u, w^2 + 1 - v)), \\ (C(w^2 + 1 - m, w^2 + 1 - n)D(w^2 + 1 - u)D(v)), \\ (C(w^2 + 1 - m, w^2 + 1 - n)E(u)E(w^2 + 1 - v)), \\ (C(w^2 + 1 - m, w^2 + 1 - n)O(u, v)), \\ (D(w^2 + 1 - m)D(n)C(w^2 + 1 - u, w^2 + 1 - v)), \\ (D(w^2 + 1 - m)D(n)D(w^2 + 1 - u)D(v)), \\ (D(w^2 + 1 - m)C(n, u)E(w^2 + 1 - v)), \\ (D(w^2 + 1 - m)C(n, u)E(v)), \\ (E(m)E(w^2 + 1 - n)C(w^2 + 1 - u, w^2 + 1 - v)), \\ (E(m)O(w^2 + 1 - n, w^2 + 1 - u)D(v)), \\ (E(m)E(w^2 + 1 - n)E(u)E(w^2 + 1 - v)), \\ (E(m)E(w^2 + 1 - n)O(u, v)), \\ (O(m, n)C(w^2 + 1 - u, w^2 + 1 - v)), \\ (O(m, n)D(w^2 + 1 - u)D(v)), \\ (O(m, n)E(u)E(w^2 + 1 - v)), (O(m, n)O(u, v)), \end{array} \right)$$

= soft close twice or soft close and soft dilation twice or soft close and soft erosion twice or soft close and soft open or soft dilation twice and soft close or soft dilation four times or soft dilation once and soft close once and soft erosion once or soft dilation once soft close once and soft dilation once or soft erosion twice and soft close once or soft erosion once and soft open once and soft dilation once or soft erosion four times or soft erosion twice and soft open once or soft open once and soft close once or soft open once and soft close once or soft open once and soft dilation twice or soft open once and soft erosion twice or soft open twice.

17.7. Duality of soft close with one set of thresholds and with multiple duals.

$$(C(m, n))^d = \left(\begin{array}{l} C(w^2 + 1 - m, w^2 + 1 - n), E(m)E(w^2 + 1 - n), \\ D(w^2 + 1 - m)D(n), O(m, n) \end{array} \right)$$

= Soft close or soft erosion twice or soft dilation twice or soft open

18 CONCLUSION: In this paper duality is discussed for various situations. The detailed discussions are given in references 1 to 10. The discussion in this paper is basing upon the papers and thesis 11. The essence of the concept is given here. The formulae is named after the author's name .

In this paper the essence of the concepts of duality of soft morphological operations (in multi scale as well as iterative environment) are presented..

The formulae is listed here.

➤ $(D(m))^d = D(w^2 + 1 - m)$ where $m = 1$ to w^2 .

$m = \frac{w^2+1}{2}$... self dual.

➤ $(E(m))^d = E(w^2 + 1 - m)$ where $m = 1$ to w^2 .

$m = \frac{w^2+1}{2}$... self dual.

➤ $(D(m))^d = D(w^2 + 1 - m), E(m)$ Where $m = 1$ to w^2

➤ $(E(m))^d = E(w^2 + 1 - m), D(m)$ Where $m = 1$ to w^2

- $(O(m, n))^d = O(w^2 + 1 - m, w^2 + 1 - n)$ or $C(m, n)$
- or $E(w^2 + 1 - m)E(n)$ or $D(m)D(w^2 + 1 - n)$
- $(C(m, n))^d = Close(w^2 + 1 - m, w^2 + 1 - n)$ or $Open(m, n)$
- or $D(w^2 + 1 - m)D(n)$ or $E(m)E(w^2 + 1 - n)$

➤ $((E(m))^n)^d = (D(m))^n$

➤ $((E(m))^n)^d = (E(w^2 + 1 - m))^n$

➤ $\left(\begin{array}{cccccccc} E(m) & E(n) & E(o) & E(p) & \dots & E(z) \\ 1 & 2 & 3 & 4 & \dots & n \end{array} \right)^d = \left(\begin{array}{cccccccc} D(m) & D(n) & D(o) & D(p) & \dots & D(z) \\ 1 & 2 & 3 & 4 & \dots & n \end{array} \right)^d$

➤ $\left(\begin{array}{cccccccc} E(m) & E(n) & E(o) & E(p) & \dots & E(z) \\ 1 & 2 & 3 & 4 & \dots & n \end{array} \right)^d = E(w^2 + 1 - m)E(w^2 + 1 - n)E(w^2 + 1 - o)E(w^2 + 1 - p) \dots E(w^2 + 1 - z)$

$$\begin{aligned} &\triangleright ((E(m))^2)^d = (E(w^2 + 1 - m) E(w^2 + 1 - m)) \text{ or } O(w^2 + 1 - m, m) \\ &\qquad\qquad\qquad \text{or } C(m, w^2 + 1 - m) \text{ or } (D(m)D(m)) \\ &\qquad\qquad\qquad = \text{Double soft erosion or soft open or soft close or double soft dilation} \end{aligned}$$

$$\triangleright (E(m)E(n))^d = (E(w^2 + 1 - m) E(w^2 + 1 - n)) \text{ or } (O(w^2 + 1 - m, n)) \text{ or } (C(m, w^2 + 1 - n)) \text{ or } D(m)D(n)$$

$$\begin{aligned} &\triangleright (E(m)E(n)E(p))^d \\ &= (E(w^2 + 1 - m)E(w^2 + 1 - n)E(w^2 + 1 - p)) \text{ or} \\ &\quad (E(w^2 + 1 - m)E(w^2 + 1 - n)D(p)) \text{ or} \\ &\quad (E(w^2 + 1 - m)D(n)E(w^2 + 1 - p)) \text{ or} \\ &\quad (E(w^2 + 1 - m)D(n)D(p)) \text{ or} \end{aligned}$$

$$\begin{aligned} &\quad (D(m)E(w^2 + 1 - n)E(w^2 + 1 - p)) \text{ or} \\ &\quad (D(m)E(w^2 + 1 - n)D(p)) \text{ or} \\ &\quad (D(m)D(n)E(w^2 + 1 - p)) \text{ or} \\ &\quad (D(m)D(n)D(p)) \end{aligned} \qquad \text{or}$$

$$\begin{aligned} &\triangleright ((D(m))^n)^d = (E(m))^n \\ &\triangleright ((D(m))^n)^d = (D(w^2 + 1 - m))^n \\ &\triangleright (D(n))^d = D(w^2 + 1 - n), E(n) \\ &\triangleright ((D(n))^2)^d = (D(w^2 + 1 - n))^2, (C(w^2 + 1 - n, n)), (O(n, w^2 + 1 - n)), (E(n))^2 \end{aligned}$$

$$\begin{aligned} &\triangleright ((D(n))^3)^d = \\ &\quad \left(\begin{array}{c} D(w^2 + 1 - n)D(w^2 + 1 - n)D(w^2 + 1 - n), \\ D(w^2 + 1 - n)D(w^2 + 1 - n)E(n), \\ D(w^2 + 1 - n)E(n)D(w^2 + 1 - n), \\ D(w^2 + 1 - n)E(n)E(n), \\ E(n)D(w^2 + 1 - n)D(w^2 + 1 - n), \\ E(n)D(w^2 + 1 - n)E(n), \\ E(n)E(n)D(w^2 + 1 - n), \\ E(n)E(n)E(n) \end{array} \right) \end{aligned}$$

$$\begin{aligned} &\triangleright (D(n)D(m))^d = (D(w^2 + 1 - n)D(w^2 + 1 - m)) \\ &\text{or } C(w^2 + 1 - n, m) \text{ or } O(n, w^2 + 1 - m) \text{ or } E(n)E(m) \\ &\quad ((O(m, n))^k)^d = (C(m, n))^k \end{aligned}$$

$$\begin{aligned} &\triangleright (O(m, n))^d \\ &= (O(w^2 + 1 - m, w^2 + 1 - n), D(m)D(w^2 + 1 - n), E(w^2 + 1 - m)E(n), C(m, n)) \\ &\triangleright ((O(m, n))^k)^d = (C(m, n))^k \\ &\triangleright ((O(m, n))^k)^d = (O(w^2 + 1 - m, w^2 + 1 - n))^k \\ &\triangleright (O(a, b)O(c, d)O(e, f)O(g, h) \dots O(x, y))^d \\ &= C(a, b)C(c, d)C(e, f)C(g, h) \dots C(x, y) \\ &= O(w^2 + 1 - a, w^2 + 1 - b)O(w^2 + 1 - c, w^2 + 1 - d) \dots \\ &\triangleright (O(m, n)^2)^d \\ &= \left(\begin{array}{l} (O(w^2 + 1 - m, w^2 + 1 - n)O(w^2 + 1 - m, w^2 + 1 - n)), \\ (O(w^2 + 1 - m, w^2 + 1 - n)E(w^2 + 1 - m)E(n)), \\ (O(w^2 + 1 - m, w^2 + 1 - n)D(m)D(w^2 + 1 - n)), \\ (O(w^2 + 1 - m, w^2 + 1 - n)C(m, n)), \\ (E(w^2 + 1 - m)E(n)O(w^2 + 1 - m, w^2 + 1 - n)), \\ (E(w^2 + 1 - m)E(n)E(w^2 + 1 - m)E(n)), \\ (E(w^2 + 1 - m)O(n, m)D(w^2 + 1 - n)), \\ (E(w^2 + 1 - m)O(n, m)E(n)), \\ (D(m)D(w^2 + 1 - n)O(w^2 + 1 - m, w^2 + 1 - n)), \\ (D(m)C(w^2 + 1 - n, w^2 + 1 - m)E(n)), \\ (D(m)D(w^2 + 1 - n)D(m)D(w^2 + 1 - n)), \\ (D(m)D(w^2 + 1 - n)C(m, n)), \\ (C(m, n)O(w^2 + 1 - m, w^2 + 1 - n)), \\ (C(m, n)E(w^2 + 1 - m)E(n)), \\ (C(m, n)D(m)D(w^2 + 1 - n)), (C(m, n)C(m, n)), \end{array} \right) \end{aligned}$$

$$\begin{aligned} & \triangleright (O(m, n) O(u, v))^d = \\ & \left(\begin{aligned} & (O(w^2 + 1 - m, w^2 + 1 - n)O(w^2 + 1 - u, w^2 + 1 - v)), \\ & (O(w^2 + 1 - m, w^2 + 1 - n)E(w^2 + 1 - u)E(v)), \\ & (O(w^2 + 1 - m, w^2 + 1 - n)D(u)D(w^2 + 1 - v)), \\ & (O(w^2 + 1 - m, w^2 + 1 - n)C(u, v)), \\ & (E(w^2 + 1 - m)E(n)O(w^2 + 1 - u, w^2 + 1 - v)), \\ & (E(w^2 + 1 - m)E(n)E(w^2 + 1 - u)E(v)), \\ & (E(w^2 + 1 - m)O(n, u)D(w^2 + 1 - v)), \\ & (E(w^2 + 1 - m)O(n, u)E(v)), \\ & (D(m)D(w^2 + 1 - n)O(w^2 + 1 - u, w^2 + 1 - v)), \\ & (D(m)C(w^2 + 1 - n, w^2 + 1 - u)E(v)), \\ & (D(m)D(w^2 + 1 - n)D(u)D(w^2 + 1 - v)), \\ & (D(m)D(w^2 + 1 - n)C(u, v)), \\ & (C(m, n)O(w^2 + 1 - u, w^2 + 1 - v)), \\ & (C(m, n)E(w^2 + 1 - u)E(v)), \\ & (C(m, n)D(u)D(w^2 + 1 - v)), (C(m, n)C(u, v)), \end{aligned} \right) \end{aligned}$$

$$\begin{aligned} & \triangleright ((C(m, n))^k)^d = (O(m, n))^k \\ & ((C(m, n))^k)^d \\ & = (C(w^2 + 1 - m, w^2 + 1 - n))^k \end{aligned}$$

$$(C(m, n))^d = \left(\begin{array}{c} C(w^2 + 1 - m, w^2 + 1 - n), E(m)E(w^2 + 1 - n), \\ D(w^2 + 1 - m)D(n), O(m, n) \end{array} \right)$$

$$\begin{aligned} & \triangleright \left(\begin{array}{c} C(a, b)C(c, d)C(e, f)C(g, h) \dots \dots C(x, y) \\ 1 \quad 2 \quad 3 \quad 4 \quad \dots \dots n \end{array} \right)^d \\ & = \begin{array}{c} O(a, b)O(c, d)O(e, f)O(g, h) \dots \dots O(x, y) \\ 1 \quad 2 \quad 3 \quad 4 \quad \dots \dots n \end{array} \\ & = C(w^2 + 1 - a, w^2 + 1 - b)C(w^2 + 1 - c, w^2 + 1 - d) \dots \dots C(w^2 + 1 - x, w^2 + 1 - y) \\ & \quad 1 \quad 2 \quad \dots \dots n \end{aligned}$$

$$\triangleright (C(m, n)^2)^d =$$

$$= \left(\begin{array}{l} (C(w^2 + 1 - m, w^2 + 1 - n)C(w^2 + 1 - m, w^2 + 1 - n)), \\ (C(w^2 + 1 - m, w^2 + 1 - n)D(w^2 + 1 - m)D(n)), \\ (C(w^2 + 1 - m, w^2 + 1 - n)E(m)E(w^2 + 1 - n)), \\ (C(w^2 + 1 - m, w^2 + 1 - n)O(m, n)), \\ (D(w^2 + 1 - m)D(n)C(w^2 + 1 - m, w^2 + 1 - n)), \\ (D(w^2 + 1 - m)D(n)D(w^2 + 1 - m)D(n)), \\ (D(w^2 + 1 - m)C(n, m)E(w^2 + 1 - n)), \\ (D(w^2 + 1 - m)C(n, m)D(n)), \\ (E(m)E(w^2 + 1 - n)C(w^2 + 1 - m, w^2 + 1 - n)), \\ (E(m)O(w^2 + 1 - n, w^2 + 1 - m)D(n)), \\ (E(m)E(w^2 + 1 - n)E(m)E(w^2 + 1 - n)), \\ (E(m)E(w^2 + 1 - n)O(m, n)), \\ (O(m, n)C(w^2 + 1 - m, w^2 + 1 - n)), \\ (O(m, n)D(w^2 + 1 - m)D(n)), \\ (O(m, n)E(m)E(w^2 + 1 - n)), (O(m, n)O(m, n)), \end{array} \right)$$

$$\triangleright (C(m, n) C(u, v))^d =$$

$$\left(\begin{array}{l} (C(w^2 + 1 - m, w^2 + 1 - n)C(w^2 + 1 - u, w^2 + 1 - v)), \\ (C(w^2 + 1 - m, w^2 + 1 - n)D(w^2 + 1 - u)D(v)), \\ (C(w^2 + 1 - m, w^2 + 1 - n)E(u)E(w^2 + 1 - v)), \\ (C(w^2 + 1 - m, w^2 + 1 - n)O(u, v)), \\ (D(w^2 + 1 - m)D(n)C(w^2 + 1 - u, w^2 + 1 - v)), \\ (D(w^2 + 1 - m)D(n)D(w^2 + 1 - u)D(v)), \\ (D(w^2 + 1 - m)C(n, u)E(w^2 + 1 - v)), \\ (D(w^2 + 1 - m)C(n, u)E(v)), \\ (E(m)E(w^2 + 1 - n)C(w^2 + 1 - u, w^2 + 1 - v)), \\ (E(m)O(w^2 + 1 - n, w^2 + 1 - u)D(v)), \\ (E(m)E(w^2 + 1 - n)E(u)E(w^2 + 1 - v)), \\ (E(m)E(w^2 + 1 - n)O(u, v)), \\ (O(m, n)C(w^2 + 1 - u, w^2 + 1 - v)), \\ (O(m, n)D(w^2 + 1 - u)D(v)), \\ (O(m, n)E(u)E(w^2 + 1 - v)), (O(m, n)O(u, v)), \end{array} \right)$$

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