Performance Analysis of Non-uniform Filter Bank with Equi ripple Band pass Filter

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Abstract : *In this paper we present a non-uniform filter bank (NUFB) matched to a given signal. To obtain matched M-channel NUFB, first, we choose the decimation set having M-down sampling/decimation factors for which perfect reconstruction NUFB exist and then using novel approach proposed in this paper, M-channel signal matched analysis bank is estimated. The outputs of all filters at the analysis side of proposed filter bank are mutually as well as across various channels are uncorrelated. By using well established theory of multirate filter bank, M-channel NUFB matched to signal is obtained. The equiripple band pass filter will provide better tradeoff compared to previous non uniform Filter bank obtained in this fashion will be useful to compress code or represent the signal or image in the best possible manner.*

Keywords : filter bank ,NUFB ,PRE

1. Introduction

Many advancements in the area of multirate filter banks in conjunction with the ever increasing numerous applications have made multirate filter banks design an increasingly important field of research. The research effort was first focused on design of a two channel quadrature mirror filter (QMF) bank [1,2], which was later extended to design of M-channel filter banks [2–5]. Since then, several techniques [6–11] were developed to enhance the performance of filter banks in different engineering fields. Among different types of the filter banks, non-uniform filter banks have been elicited immense interest in the researchers in recent years due to their ability to differentiate information into different frequency bands based on energy distribution of signal which is required for several specific applications. These specific applications include sub-band coding like audio coding, speech coding, data and image compression [1,2]. Uniform filter banks have many constraints like integer and uniform decimation in each sub-band, and limited time frequency resolution. These constraints catalyze the importance of non-uniform filter banks (NUFBs).

In addition, NUFBs are able to provide any sort of rational decimation in each channel, any extent of time–frequency resolution as per requirement of the application, less quantization error, and low computational complexity. Over the past few years, a number of design methods [12–15] have been proposed by different authors for the design of multi-channel filter banks. However, design of a linear phase multi-channel filter bank with linear optimization has been still an issue since a very few references [16–20] provide linear phase as well as zero aliasing error, which is very much essential in several applications such as videos and communication systems. Such an application oriented technique was presented in [21]. In this technique, the authors have been used the evolutionary programming algorithms to design the optimized prototype filter for designing modulated filter banks. Advances in filter banks have provided a new generation of sub band coders for audio, image and video signals, analog to digital converters, signal compression systems, design of wavelet bases, antenna systems, digital audio industry and biomedical signal processing [1,2].

Recently, several design methods [22–27] have been proposed and evaluated for designing the non-uniform filter bank based on optimization and non-optimizations. But still, there is no such iterative technique reported in the literature which can reduce the computation time, converse in low number of iteration and also reduces the peak reconstruction error which can be used for filter banks with larger taps. Therefore, the authors in [28] have proposed an optimized algorithm for designing NUFB with Blackman Window family based on the algorithm given in [16]. Similar to the cut-off frequency, a suitable value of pass band edge frequency (ω_p) can reduce the amplitude distortion. There are very few references [16] available in which ω_p has been optimized for designing a prototype filter

for two-channel QMF banks and M-channel CM filter banks. Literature available so far on non-uniform filter banks reveals that there is still need for a computationally efficient technique ,which shall use linear optimization technique for designing non-uniform linear-phase filter bank. Apart from multirate filter banks designing techniques, the authors in [29] have recently proposed efficient optimization techniques to design two-dimensional IIR filters. This technique is based on the particle swarm intelligence approach [30], which was initially introduced for simulating human social behaviors. Later on, this particle swarm optimization (PSO) approach was improved by authors in [31] to give a new optimization algorithm called fitness-adaptive differential evolution algorithm to design QMF banks. A comparative study of modern search techniques is presented in [32] for designing two dimensional IIR filters.

In this paper, a new improved iterative methodology is presented for the design of non-uniform filter bank. Organization of the paper is as follows: a brief introduction has been provided in this section on design techniques of NUFBs. Section 2 gives an overview of NUFBs. Section 3 presents the proposed methodology for NUFB. In Section 4, design results and application of the proposed method to sub band coding is carried out, followed by the concluding remarks in Section 5.
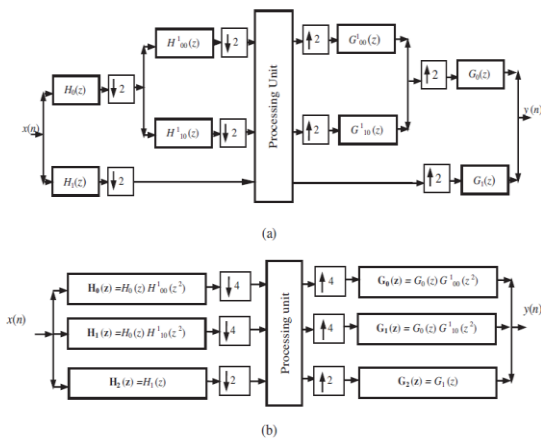

Fig. 1. A block diagram of multichannel non-uniform filter bank. (a) A block diagram of a tree structured filter bank and

 (b) its equivalent parallel structure.

2. Overview of non-uniform filter bank

The non-uniform filter bank with integer decimation and linear phase is realized with the help of tree structured techniques, which is based on building the filter bank using a two-channel filter bank as basic building blocks [1,2,]. The generalized structure of M channel filter banks based on tree structure approach is depicted in Fig. 1. For M-channel NUFB having decimation M0, M1, M2, . . ., M_M-1 ,for each band, then decimation factors are such that [1,2]

$$\sum_{k=0}^{M-1} \frac{1}{M_k} = 1 \qquad (1)$$

and the reconstructed signal $\hat{X}(z)$ is

$$\hat{X}(z) = \sum_{k=0}^{M-1} G_k(z) \frac{1}{M_k} \sum_{l=0}^{M_k-1} X\left(zW_{M_k}^l\right) H_k\left(zW_{M_k}^l\right) \quad (2)$$

where $H_k(z)$ and $G_k(z)$ are the analysis and synthesis filters respectively,

while $\qquad W^L = e^{-j2\pi/L}$.

The terms $H_k\left(zW_{M_k}^l\right)$ and $X\left(zW_{M_k}^l\right)$ aliasing terms introduced due to decimation/interpolation. For the perfect reconstruction (PR), these must be minimized by any strategic approach. In a tree-structured approach, a two-channel QMF bank is used as building blocks; hence the aliasing error is completely eliminated by proper selection of synthesis filters in terms of analysis filters. In a two-channel QMF bank, the aliasing error is eliminated with

$$G_1(z) = -2H_0(-z) \quad and \quad G_0(z) = 2H_1(-z) \quad (3)$$

As depicted in Fig. 1(a), the input signal x(n) was decomposed into two sub bands in which the sampling rate is reduced to 1/2 of the original sampling rate. These sub bands can be further extended to more sub bands by applying the same decomposition. In general, 2P sub bands can be obtained by repeating the same decomposition process P times and at each of the P stages of decomposition process, the number of two-channel QMF bank structure required, is 2P−1.

Total number of two-channel systems required is 2P−1. On the other side, the process of reconstructing the original input signal can be seen as mirror image of the decomposition process at analysis side as illustrated in Fig. 1(a) and its equivalent parallel structure shown in Fig. 1(b). After resolving the tree structured non uniform filter bank into its parallel forms, the following relations can be deduced [1,2,28]:

$$H_0(z) = H_L(z), \ H_L(z^2), \ G_0(z) = G_L(z), \ G_L(z^2) \quad (4)$$

$$H_1(z) = H_L(z), \ H_H(z^2), \ G_1(z) = G_L(z), \ G_H(z^2) \quad (5)$$

$$H_2(z) = H_H(z), \qquad G_2(z) = G_H(z) \qquad (6)$$

Performance of tree structured approach depends on the efficient design of QMF bank. The computation load is reduced due to some similarities between the coefficients of high-pass and low pass filters of a two-channel filter bank. However, the frequency characteristics of the filters might differ from stage to stage, but they must be the same within a particular stage. Depending on which of the filter bank is

used in the design, there would be perfect reconstruction (PR) or near perfect reconstruction (NPR). But the realization of perfect reconstruction filter bank requires very tedious and complex approach, which makes it practically unrealizable.

Therefore, a common interest of researchers is towards NPR, since it is a practically realizable approach. But during realization of this approach, there is always an introduction of three distortions: amplitude distortion, phase distortion and aliasing error. The aliasing error can be removed by using proper pairs of analysis and synthesis filters, phase error by using linear phase filters, and amplitude distortion can be minimized by using different optimization techniques.

The performance of this method is evaluated in terms of number of iterations (NOI), computation time (CPU time), and reconstruction error (PRE) given by

$$PRE = \max\left\{\sum_{k=0}^{M-1}\left|H_k\left(e^{j\omega}\right)\right|^2\right\} - \min\left\{\sum_{k=0}^{M-1}\left|H_k\left(e^{j\omega}\right)\right|^2\right\} \quad (7)$$

Example-I: A 3-channel linear-phase non-uniform filter bank with decimation factors 4, 4, and 2 is designed by the proposed method. Design specifications for the prototype filter are N + 1 = 96, stop band attenuation (As) = 80 dB, pass band ripple (A_p) = 0.000521 dB, ω_s = 0.51_, and ω_p = 0.41_. The peak reconstruction error (PRE), computational time (CPU time) and number of iterations (NOI) obtained are 2.8×10^{-3}, 0.826 s, and 11, respectively.

Example-II: A 4-channel linear-phase non-uniform filter bank (8, 8, 4, 2) with the prototype filter design specifications: N + 1 = 48, As = 80 dB, A_p = 0.0005 dB, ω_s = 0.61_ and ω_p = 0.41_ is designed with this method. In this case, PRE and computational time are 3.11×10^{-3} and 0.733 s, respectively, while NOI = 15

2.1 Algorithm For the Non Uniform Filter bank:

Step 1: Specify design specifications stop band attenuation (A_s), pass band ripple (A_p), and normalized pass band (ω_p), stop band frequency (ω_s), number of band (M), and step size.

Step 2: Initialize counter and the magnitude response (MR) of the prototype filter given by Eq

$$X = \left|H_L\left(e^{j\pi/2}\right)\right| = 0.7071$$

and also specify tolerance (Tol).

Step 3: Design the prototype filter using constrained equi ripple finite impulse response (FIR) technique before the optimization start. Calculate the magnitude response of designed filter (MRD)

at $\omega = \pi/2$. Also calculate error = MR − MRD.

Step 4: (A) If error is not comparable to tolerance (Tol), the pass band edge frequency (ω_p) is varied using the step size. It is varied in two ways:

a. if MRD < MR, then increase ω_p by step
b. Otherwise, decrease ω_p by step

Step 4: (B) If error is not comparable to tolerance. Then, design the other filters composing the parallel equivalent NUFB using Eqs.(4)–(6) in case of 3-channel NUFB, Eq. is

$$\left|H_L\left(e^{j\omega}\right)\right|^6 + \left|H_L\left(e^{j\omega}\right)\right|^4\left|H_L\left(e^{j(\omega-\pi)}\right)\right|^2 + \left|H_L\left(e^{j\omega}\right)\right|^2\left|H_L\left(e^{j(\omega-\pi)}\right)\right|^2 + \left|H_L\left(e^{j(\omega-\pi)}\right)\right|^2 = 1$$

in 4-channel NUFB Eq.is

$$H_0(z) = H_L(z)H_L(z^2)H_L(z^4)H_L(z^8)$$

$$H_1(z) = H_L(z)H_L(z^2)H_L(z^4)H_H(z^8), \quad H_2(z) = H_L(z)H_L(z^2)H_H(z^4)$$

$$H_3(z) = H_L(z)H_H(z^2) \quad and \quad H_4(z) = H_H(z)$$

Step 5: Redesign the prototype filter using new ωp and same order. Calculate MRD and also error.

Step 6: Increment the counter by 1 and step = step/2. Go to step 4 till error is not comparable to tolerance.

3. Proposed methodology

We now present a method that approximates the desired frequency response by a linear-phase FIR amplitude function according to the following optimality criterion.

The integral of the weighted square frequency-domain error is given by

$$\varepsilon 2 = \int E2(\omega)d\omega$$

and we assume that the order and the type of the filter are known. Under this assumptions designing the FIR filter now reduces to determining the coefficients that would minimize $\varepsilon 2$.

3.1 Equiripple Design

The least-square criterion of minimizing is not entirely satisfactory. A better approach is to minimize the maximum error at each band

$\varepsilon 2 = \int E2(\omega)d\omega$

$\varepsilon = \max\omega |E(\omega)|$

The method is optimal in a sense of minimizing the maximum magnitude of the ripple in all bands of interest, the filter order is fixed.

It can be shown that this leads to an Equiripple filter – a filter which amplitude response oscillates uniformly between the tolerance bounds of each band.

3.2. Remez Method

There exists a computational procedure known as the Remez method to solve this mathematical optimization problem. There are also exist formulae for estimating the required filter length in the case of low-pass, band-pass and narrow transition bandwidths. However, these formulae are not always reliable so it might be necessary to iterate the procedure so as to satisfy the design constraints [5].

3.3 Realization of Optimized Equiripple Method using Matlab

We use remez function in the optimized Equiripple method.
Function c=remez(n,f,a,w,'ftype') where n shows filter order;

f shows a vector, and is a positive number between 0 and 1;

a shows a vector, and represents the amplitude in the specified frequency domain;

w shows the weighted value of each frequency band;

b shows a vector whose length is n+1.

3.4 Designing Equiripple band-pass filter using Remez function.

Requirements:

1. Sampling frequency is 2 kHz. Stop-band cutoff frequency is 0.2 and 0.7π and Pass-band cutoff frequency is 0.3 and 0.6π.

2. The stop-band attenuation is $\geq$40dB. Stop-band ripple is 0.01 and pass-band ripple is 0.17.

Algorithm Steps for the Proposed Method:

Step 1:	User Input: Filter Type (LP,HP,BP,BR)
Step 2 :	User Input: Frequency Edges (vector f, depending on the filter type)
Step 3:	User Input: Sampling Frequency (fs)
Step 4:	User Input: Attenuation on the passband (A_p)
Step 5:	User Input: Attenuation on the passband (A_s)
Step 6:	Calculate δ_p and δ_s using Equations 9.0 and 10.0 and populate vector **dev**.
step7:	If filter type is LP then a=[1 0]
step8:	If filter type is HP then a=[0 1]
step 9:	If filter type is BP then a=[0 1 0]
step 10:	If filter type is BR then a=[1 0 1]
step 11:	Use the **remezord** function: [n,f0,a0,w] = remezord(f,a,dev,fs)
step 12:	Use the **remez** function: b=remez(n,f0,a0,w)
step 13:	Use the freqz function to obtain the h[k] coefficients
step 14:	Plot the frequency response.

Example .

Design a Band-Pass Equiripple FIR Filter with the following specifications:

1) A passband attenuation of 0.1dB in the range 1000-1200Hz
2) A stopband attenuation of 40dB for frequencies <= 800 Hz
3) A stopband attenuation of 40dB for frequencies >= 1400 Hz
4) Sampling Frequency = 4000 Hz

MATLAB program, **Equiripple band pass filter** was run with the specifications parameters. And shows the amplitude response of a filter of order N = 100.
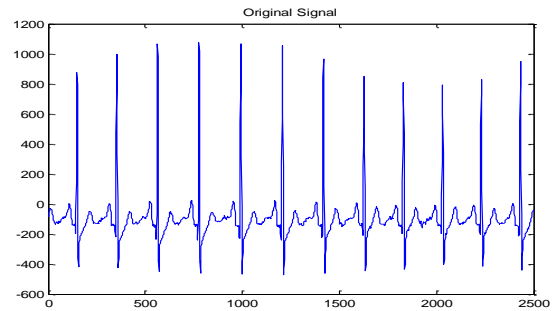
4. Results and Conclusions:

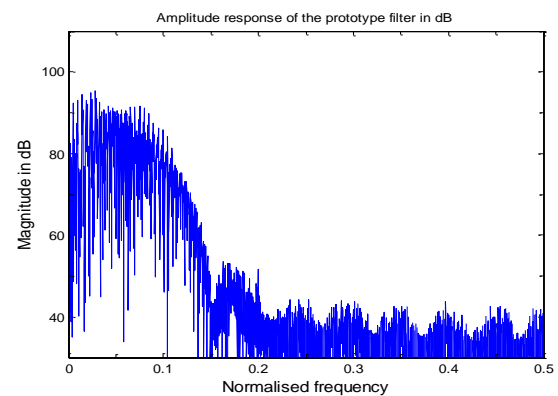Fig 2:Input signal response to an NUFB

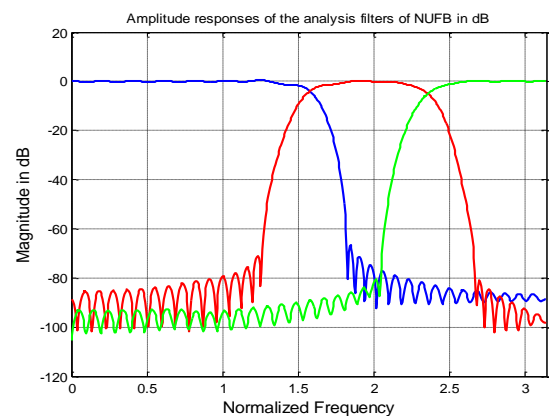
Fig 3: Amplitude response of NUFB
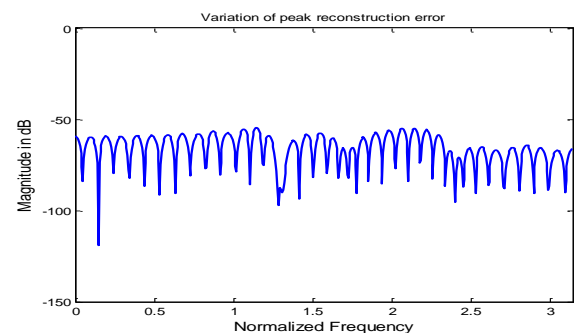

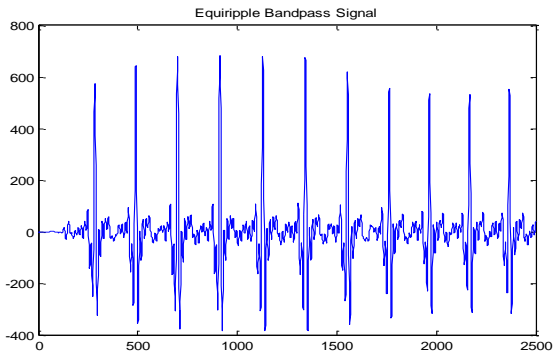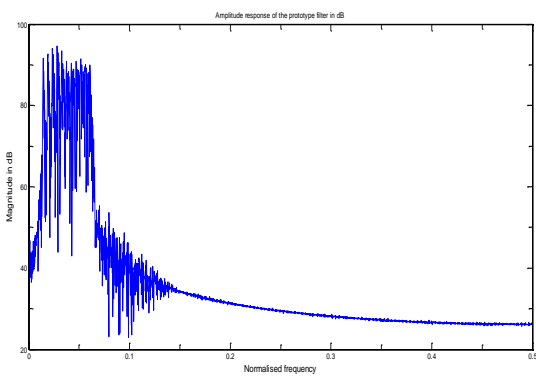Fig 4: Amplitude response of the analysis filter of NUFB

Fig 5: PRE of NUFB

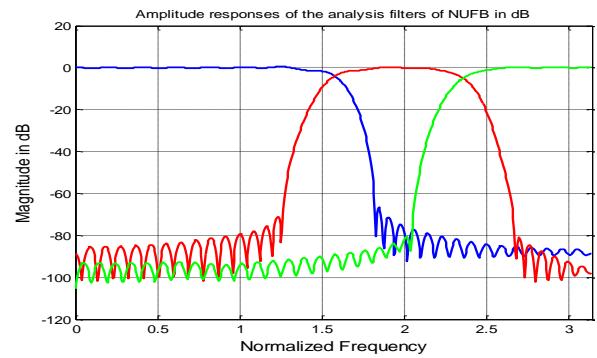
Fig 6 :Input signal to Proposed Method

Fig 8: Magnitude response of the analysis filter of NUFB using proposed method
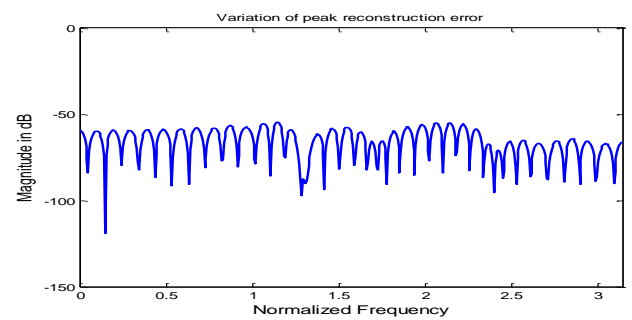

Fig 9: PRE of Proposed Method

Fig 7: Amplitude response of Proposed Method

Comparison of the proposed methodology with earlier published results.

Table 1: Comparison of the obtained results with earlier design methods

Algorithms	Band (M)	Filter taps (N)	As (dB)	PRE	NOI	CPU time (s)
Algorithm in [6]	Three band (4, 4, 2)	63	110	7.80×10^{-3}	-	-
Algorithm in [16]	Three band (4, 4, 2)	64	60	7.80×10^{-3}	-	-
Algorithm in [15]	Three band (4, 4, 2)	63	75	8.60×10^{-3}	153	165
Proposed method	Three band (4, 4, 2)	66	80	2.51×10-3	17	.782

V. CONCLUSIONS

The design of linear phase optimal FIR filters, with very flat pass-bands, can be done by solving the frequency response equations for different frequency components. To solve the filter parameters, one can use an iterative algorithm like Remez exchange algorithm [3]. The response of Remez algorithm. We can also do this, by sampling the desired frequency response with non-uniform frequency spacing as

shown the response in Fig. 1. Here we see that the degree of flatness in pass-bands and ripples in stop-bands depend on the transition bandwidth. If we set this bandwidth low then ripples increases and vice versa. We can get better stop-band attenuation using Blackman window method. In summary, the optimal solution is not always a good solution to the filter design problem. The major disadvantage of the window design method is the lack of precise control of the critical frequencies, such as and, in the design of a low pass FIR filter and its value depends on the type of window and the filter length N [3]. The frequency sampling method provides an improvement over the window design method, since $Hr(\omega)$ is specified at the frequencies and transition band is a multiple of [3]. This filter design method is particularly attractive when the FIR filter is realized either in the frequency domain by means of the DFT or any of the frequency sampling realizations.

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