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Optimal Design of Direct Adaptive Fuzzy Control Scheme for a Class of Uncertain Nonlinear Systems Using Firefly Algorithm

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Abstract: ***In this paper, an optimal direct adaptive fuzzy controller is designed for a class of uncertain nonlinear continuous systems through the following three steps: first, some fuzzy sets whose membership functions cover the state space are defined; then, firefly algorithm is used as a novel nature inspired optimization approach to construct an initial adaptive fuzzy controller in which some parameters are free to change. In other words, the control knowledge (fuzzy IF-THEN rules) is incorporated into the fuzzy controller through the setting of its initial parameters and simultaneously determining a suitable adaptation parameter by using the FA; finally, an adaptive law is developed to adjust the free parameters based on a Lyapunov synthesis method. It is confirmed that i) the closed-loop system using this optimal adaptive fuzzy controller is globally stable in the sense that all signals involved are bounded and ii) the tracking error converges to zero asymptotically. Finally, the proposed control scheme applies to the two well-known examples in the nonlinear control problems. Moreover, the two different methods that are non-optimal Direct Adaptive Fuzzy (DAF) control, and DAF based on particle swarm optimization method are also implemented for comparison. The results demonstrate the effectiveness of the proposed optimal direct adaptive fuzzy control methodology.***

Keywords: Direct adaptive fuzzy control, firefly algorithm, Lyapunov synthesis, particle swarm optimization.

1. Introduction

Uncertainties are inevitable in nonlinear systems, due to the idealization of the plant via a mathematical model and the experience of human operators who can adequately control the plant and present qualitative control rules in terms of vague and non-crisp sentences [1, 2]. Therefore, conventional nonlinear control approaches cannot address those uncertainties and will be failed in controlling uncertain nonlinear systems.

Fuzzy logic systems (FLS) are important tools for incorporating human expert knowledge in complement to mathematical knowledge. Based on this feature, utilization of fuzzy knowledge-based control to deal with nonlinear systems whose dynamics are not explicitly understood and whose models cannot be simply established has been increased [3, 4]. Adaptive fuzzy control is by far the most successful application of FLS to practical problems.

Generally, adaptive fuzzy control schemes are classified into two categories, *i.e.* direct and indirect, based on how they use FLS. Direct Adaptive Fuzzy (DAF) controllers use FLS as controllers. In the other words, they incorporate fuzzy IF-THEN rules directly into themselves. Adaptive fuzzy controllers which use FLS as models of the plants are Indirect Adaptive Fuzzy (IAF) controllers, *i.e*. they use fuzzy IF-THEN rules which are described the plant [4, 5, 6].

Universally in designing an adaptive controller the initial adaptive fuzzy controller is made based on the knowledge of the human experts or some random rules; an adaptive law is then derived from Lyapunov synthesis and used to adjust the parameters of the adaptive fuzzy controller during the adaptation procedure. For the rapid convergence of the adaptive process good fuzzy IF-THEN rules are needed to provide perfect control strategies. On the other hand, if there are no available linguistic rules from the human experts, then the adaptive fuzzy controller changed to a conventional nonlinear adaptive controller analogous to the radial basis function adaptive controller [7, 8], and also the neural network nonlinear adaptive controller [9, 10]. The initial adaptive controllers are constructed by some arbitrary values in the conventional nonlinear adaptive control schemes. Therefore, it is transparent that without sufficient and efficient fuzzy IF-THEN rules the convergence speed needs more time. Another challenging and yet rewarding problem in adaptive control scheme, DAF or IAF, is how properly determined the existing adaptation parameters in the adaptation law, which derived from the Lyapunov theory and designed by trial-and-error by the user.

Therefore, overcoming those restrictions and improving the tracking performance of DAF and IAF have gained a lot of attention these days. Many researchers have been focusing on using bio-inspired methods and evolutionary strategies to cope with those shortages, due to the lack of analytical approaches. Genetic algorithm (GA) has been widely applied to DAF and IAF. The GA is employed to optimize all the configuration parameters of the adaptive fuzzy such as the number of membership functions and rules [11, 12], the initial values of the consequent parameter vector [13], and the parameters in adaptive laws [14, 15]. Particle Swarm Optimization (PSO) has recently received much interest for achieving high efficiency and simpler implementation algorithm in comparison with GA. Commonly, PSO is utilized in adaptive fuzzy control such as optimizing both its structures and free parameters [16], simultaneously tune the shape of the fuzzy membership functions for all the consequences of rules in fuzzy rule-base [17]. Furthermore, PSO is used to update the premise part of the fuzzy system while the consequent part is updated by the other methods [18].

The Firefly Algorithm (FA) is a recent nature inspired method [19], which has been used for solving multimodal optimization problems. This algorithm is based on the behavior of social insects (fireflies) and the phenomenon of bioluminescent communication. Preliminary studies indicate that FA is superior over GA and PSO [20].

In this paper, an optimal direct adaptive fuzzy controller is designed for a class of uncertain nonlinear continuous system by using the firefly algorithm. The FA is utilized to construct an initial adaptive fuzzy controller with some adjustable parameters. In other words, the control knowledge of skilled human operators, fuzzy IF-THEN rules, is incorporated into the fuzzy controller through the setting of its initial parameters and simultaneously determining a suitable adaptation parameter by using the FA; finally, an adaptive law is developed to tune the free parameters based on a Lyapunov theory. Furthermore, the two different methods that are non-optimal DAF, and DAF PSO-based methods are also implemented for comparison.

This paper is organized as follows. In section 2 the problem formulation and its assumptions are defined. Section 3 represents a summary of the FLS and its universal approximation theorem. In section 4 after explaining the general method of the DAF designing, the proposed control scheme for the optimal designing the DAF is presented and its stability and error convergence of the overall closed-loop control scheme are proved based on Lyapunov synthesis. In section 5, the firefly algorithm and particle swarm optimization method are expressed. Three well-known examples are performed to support the effectiveness of the proposed control scheme in section 6. Finally, Conclusions are drawn in Section 7.

2. Problem Statement and Assumption

Consider a class of Single-Input Single-Output (SISO) n-th order nonlinear system in the following form:

$$x^{(n)} = f\left(x, \dot{x}, \dots, x^{(n-1)}\right) + bu(t) \quad (1)$$

$$y = x$$

where f and g are unknown bounded nonlinear functions in which no prior knowledge for bounds need to be known. Furthermore, $u \in R$and $y \in R$ are the input and the output of the system, respectively.

$X^T = \left[x, \dot{x}, \dots, x^{(n-1)}\right] = [x_1, x_2, \dots, x_n] \epsilon R^n$ is the state vector of the system assumed to be available for measurement. The following assumption is considered for the nonlinear system mentioned in Eq. (1) by the authors.

Assumption 1. In order to have a controllable system, it's required that $b \neq 0$ in Eq. (1). b is assumed to be positive, *i.e.* $b > 0$. By contrast, b can be negative and the control signal is derived in a similar way. The *control objective* is to design a controller u based on fuzzy system and an adaptation law for adjusting controller parameters, such that the X state vector of the system in Eq. (1), follows a given desired trajectory state $X_d^T = \left[x_d, \dot{x}_d, \dots, x_d^{(n-1)}\right]$ in the presence of system uncertainties. Therefore, by using the designed controller the tracking error in Eq. (2) should converge to zero.

$$E = x - x_d = [e, \dot{e}, \dots, e^{(n-1)}]^T \quad (2)$$

Assumption 2. The desired trajectory vector X_d is continuous, measurable, and bounded with a known positive constant Ψ.

$$\|x_d\| < \Psi \quad (3)$$

3. Fuzzy Logics and universal approximation theorem

The fuzzy logic system (FLS) that is detailed in [4] is used in this research and is briefly explained below for continuity of the discussion. FLS performs a mapping from a compact set $U_1 \times U_2 \times \dots \times U_n = U \subset R^n$ to a compact set $V \subset V$. Any fuzzy system consists of a fuzzifier, a fuzzy rule base, a fuzzy inference engine and a defuzzifier. The fuzzy rule base consists of a collection of fuzzy IF-THEN rules such as

$$R^{(l)}: \text{IF } \{x_1 \; is \; F_1^l \; and \dots and \; x_n \; is \; F_n^l\} \text{ THEN } y \; is \; G_i^l \quad l = 1, \dots, M \quad (4)$$

where $X = [x_1, x_2, \cdots, x_n]^T \in U$and $y \in V$are the input and output of the fuzzy system, respectively. $F_i^l$and G^l are fuzzy sets in U_i and V, respectively. The fuzzy inference engine performs a mapping from fuzzy sets in U to fuzzy sets in V, based on fuzzy rule base. Furthermore, the fuzzifier maps a crisp point $X = [x_1, x_2, \cdots, x_n]^T \in U$ to a fuzzy set in U and the defuzzifier maps fuzzy sets in V to a crisp point in V.

Using Singleton fuzzifier, product inference engine and center average defuzzifier, the output of fuzzy system can be expressed as

$$y = \frac{\sum_{l=1}^{M} \bar{y}^l \left(\prod_{i=1}^{n} \mu_{f_i^l}(x_i)\right)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{f_i^l}(x_i)\right)} = \theta^T \xi(x) \quad (5)$$

where M is the total number of rules, $\theta = [\bar{y}^1, \bar{y}^2, \dots, \bar{y}^M]^T$ is the center of output fuzzy membership functions and is the adjustable parameter vector, and $\xi(X) = [\xi_1(X), \xi_2(X), \dots, \xi_M(X)]^T$is the fuzzy basis function defined as follows:

$$\xi_j(x) = \frac{\prod_{i=1}^{n} \mu_{f_i^l}(x_i)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{f_i^l}(x_i)\right)} \quad j = 1, \dots, M \quad (6)$$

Now, the following theorem could be expressed.

Theorem 1: For any given real continuous function g on the compact set $U \subset R^n$ and $\varepsilon > 0$ arbitrary, there exists a fuzzy system $f^*(x) = \theta^{*T}\xi(x)$ in the form of (5) such that

$$sup_{x\epsilon U}|f(x) - g(x)| < \varepsilon \quad (7)$$

The above theorem represents that the fuzzy systems in the form of (5) can approximate any real continuous function to any degree of accuracy. This means the fuzzy systems in the form of (5) have universal approximation property as also reported earlier in [4].

4. Optimal Direct Fuzzy Adaptive

In this section, first the direct fuzzy adaptive strategy is expressed and then the optimal direct fuzzy adaptive control scheme is presented.

4.1 Direct Fuzzy Adaptive

If the nonlinear function in Eq. (1), $f(x)$ is known, then the control u can be chosen in such that cancel the nonlinearity and design the controller based on linear control theory, *e.g.*, pole placement. Let $e = y_m - y$, $e = (e, \dot{e}, \cdots, e^{(n-1)})^T$ and $K = (k_n, \cdots, k_1)^T$ be such that all roots of the polynomial $s^n + k_1 s^{n-1} + \cdots + k_n$ are in the open left-half complex plane, *i.e.* Hurwitz, and choose the control law as follows:

$$u_D^* = \frac{1}{b}[-f(x,t) + y_d^{(n)} - K^T e] \quad (8)$$

Substituting (8) into (1), we obtain the closed-loop system governed by

$$e^n + k_1 e^{n-1} + \cdots + k_n e = 0 \quad (9)$$

Because of the choice of K, we have $e(t) \longrightarrow 0$ as $t \longrightarrow \infty$ that is, the plant output y converges to the ideal output y_m asymptotically. Since $f(x)$ is unknown, the ideal controller (8) cannot be implemented. The control objective in direct adaptive control is not insisted that the plant output y should converge to the ideal output y_m, asymptotically; it is only required that y follows, y_m as close as possible. Hence for coping with the uncertainties and achieving to the control objective the following control law is proposed.

$$u_D(X,\theta) = \theta^T\xi(X) \quad (10)$$

where $u_D(X,\theta)$ is the fuzzy system in the form of (5) with free parameters θ to approximate the unknown function. By substituting (10) into (1) and rearrangement it the following closed-loop dynamics is obtained.

$$e^{(n)} = -K^T e + b[u^* - u_D(X|\theta) \quad (11)$$

Let Λ and b are defined as follow:

$$\Lambda = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \dots & \dots & \cdots & \dots & 0 \end{bmatrix}_{n\times n} \quad (12)$$

and

$$B = [0 \cdots 0 \ b]_{1\times n} \quad (13)$$

the closed-loop dynamics can be written into the following vector form

$$\dot{e} = \Lambda e + B[u^* - u_D(X|\theta)] \quad (14)$$

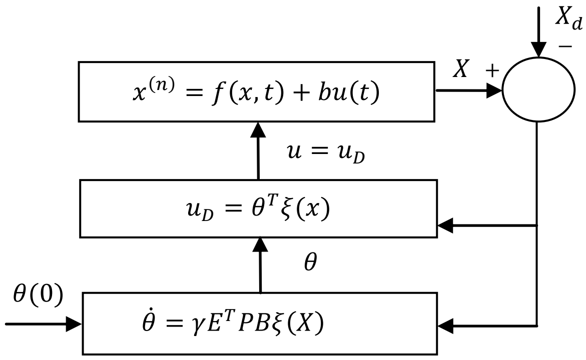

Figure 1: The direct adaptive fuzzy control scheme

Based on the universal approximation ability of fuzzy system and its *Theorem 1* in section 2, there exist optimal θ^* such that

$$\theta^* = \arg\ min_{\theta\in R^M}\ [sup_{x\in R^n}|u_D(X|\theta) - u^*|] \quad (15)$$

And let ω be the *minimum approximation error* defined by

$$\omega = u_D(X|\theta) - u^* \quad (16)$$

Using (16) and (10), the error equation (14) can be rewritten as follows:

$$\dot{e} = \Lambda e + B(\theta^* - \theta)^T\xi(X) - B\omega \quad (17)$$

Consider the Lyapunov function candidate as follows:

$$V = \frac{1}{2}e^T Pe + \frac{b}{2\gamma}(\theta^* - \theta)^T(\theta^* - \theta) \quad (18)$$

where P is a positive definite matrix satisfying the following Lyapunov equation:

$$\Lambda^T P + P\Lambda = -Q \quad (19)$$

where Q is an arbitrary $n \times n$ positive definite matrix and γ is a positive constant (recall that $b > 0$ by assumption, so V is positive). The time derivative of V along the closed-loop system trajectory (17) is

$$\dot{V} = -\frac{1}{2}e^T Qe + e^T PB[(\theta^* - \theta)^T\xi(X) - \omega] - \frac{b}{\gamma}(\theta^* - \theta)^T\dot{\theta} \quad (20)$$

Considering (20) and a few manipulations yields

$$\dot{V} = -\frac{1}{2}e^T Qe + \frac{b}{\gamma}(\theta^* - \theta)^T[\gamma e^T P\xi(X) - \dot{\theta}] - e^T PB\omega \quad (21)$$

To minimize the tracking error e and the parameter errors $\theta^* - \theta$, or equivalently, to minimize V, the adaptation law should be chosen such that $\dot{V}$ is negative. Since $-\frac{1}{2}e^T Qe$ is negative and we can choose the fuzzy systems such that the minimum approximation error ω is small, a common strategy is to choose the adaptation law such that the middle term in Eq. (21) be zero. The adaptation law is as follows:

$$\dot{\theta} = \gamma e^T P\xi(X) \quad (22)$$

then

$$\dot{V} = -\frac{1}{2}e^T Qe - e^T PB\omega \quad (23)$$

Since $Q > 0$, Eq. (19), and ω is the minimum approximation

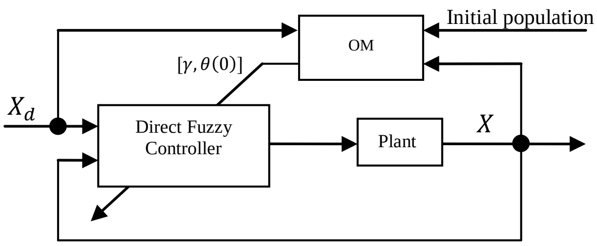

Figure 2: The overall scheme of the proposed method based on an OM

error, by designing the fuzzy system $u_D(X|\theta)$ with a sufficient number of rules, the ω will be small enough such that $|e^TPB\omega| < \frac{1}{2}e^TQe$, which results in $\dot{V} < 0$. The direct adaptive control scheme is depicted in Fig. 1.

4.2 Optimal Direct Fuzzy Adaptive

Direct Adaptive Fuzzy (DAF) controllers use fuzzy systems as controllers. In the other words, they incorporate fuzzy IF-THEN rules directly into themselves. It should be emphasize that the human expert knowledge (fuzzy IF-THEN rules (4)) is incorporated through the initial parameters. Therefore, the major advantage of the FLS is emerged while the initial parameters are chosen accurately. Another significant parameter is γ, which is known as adaptation parameter. There is no specific approach to choose a suitable adaptation parameter. Therefore, an Optimization Method (OM) should be employed to determine the initial parameters and the adaptation parameter simultaneously. Moreover, the control objective in optimal adaptive control scheme is not only the X state vector of the system in Eq. (1), follows a given desired trajectory state but also the tracking error converges to zero asymptotically. The Mean Square Error (MSE) is defined in Eq. (24) to use as an objective function for evaluating the performance index in the optimal designing of direct fuzzy system, *i.e.* assigning the initial parameters and the adaptation parameter. The MSE formulates as follows:

$$MSE = \frac{1}{K}\sum (X - X_d)^2 \qquad (24)$$

where, X is actual state of fuzzy system, X_d is desired state. K is the total number of data.
The proposed control scheme based on an optimization method is shown in Fig. 2.

5. Optimization Method

In this section, first the PSO method is reviewed, due to its popularity in the optimization approaches. Then the FA is introduced and its performance in designing the optimal direct fuzzy adaptive compared with the PSO.

5.1 PSO Algorithm

Particle Swarm Optimization (PSO) is a population-based optimization technique. It uses swarm of particles to find a global optimum solution in search space [21]. Each particle represents a candidate solution to the cost function and it has its own position and velocity. Assume particle swarms are in D-dimensional search space. Let the *i*th particle in a D-dimensional space be represented as$x_i = (x_{i1}, \dots, x_{id}, \dots, x_{iD})$. The best previous position of the *i*th particle is recorded and represented as $p_{bi} = (p_{bi1}, \dots, p_{bid}, \dots, p_{biD})$, which gives the best value in the cost function and also called *pbest*. General best position, *gbest*, denoted by p_{gb} is the best value of the *Pbest* among all the particles in the cost function. The velocity for the *i*th particle is represented as $v_i = (v_{i1}, \dots, v_{id}, \dots, v_{iD})$. In each of the iteration, the velocity and the position of each particle are updated according to Eq. (25) and Eq. (26), respectively.

$$v_{id} = wv_{id} + c_1r_1(p_{bid} - x_{id}) + c_2r_2(p_{gb} - x_{id}) \qquad (25)$$

$$x_{id} = x_{id} + v_{id} \qquad (26)$$

where w is called the inertia coefficient and it is in the interval [0,1]. We reduce the inertia weight, *w*, linearly during the iterations to enhance the search ability of the PSO as follows.

$$w(iter) = w_{max} - \frac{w_{max} - w_{min}}{Iter_{max}} \times iter \qquad (27)$$

where $iter_{max}$ is the maximum number of algorithm's iteration [22, 23], $C_1$and C_2 are non-negative constants of acceleration; $r_1$and r_2 are generated randomly in the interval [0,1], $v_{id} \in [-v_{max}, v_{max}]$ and v_{max} is the maximum velocity. The termination criterion of the PSO is determined by reaching the maximum iteration number. After the maximum number of iteration reached, global best particle represent an optimal solution consist of the best human experts knowledge (initial fuzzy IF-THEN rules) and the best adaptation parameter (γ) for the DAF. All parameters required to implement the PSO algorithm are presented in Appendix A (Table A.1).

5.2 Firefly Algorithm

The Firefly Algorithm is a population-based algorithm which was proposed by Yang [19, 20], and it was based on the three following idealized rules:

- All fireflies are unisex so that one firefly is attracted to other fireflies regardless of their sex;
- Attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less bright one will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If no one is brighter than a particular firefly, it moves randomly;
- The brightness or light intensity of a firefly is affected or determined by the landscape of the objective function to be optimized.

Initially, all the fireflies are randomly dispersed across the search-space. The firefly algorithms simply can be summarized as the two following stages:
I. *Variation of light intensity*: Light intensity is depended on the value of the objective function.
Therefore, it can be explained that in maximization /minimization problem a firefly with high/low intensity will attract another firefly with high/low intensity.
For all the fireflies, x_i, its brightness I_i is corresponding to its value of the objective function and it is represented as follows [24]:

$$I_i = f(x_i), \qquad i = 1, 2, \dots, n \qquad (28)$$

II. *Movement toward brighter firefly*: The movement of a firefly x_i is absorbed to another brighter (more attractive) firefly x_j is formulated by

$$x_i^{t+1} = x_i^t + \beta(r) \times \left(x_i^t - x_j^t\right) + \alpha\varepsilon_i^t \qquad (29)$$

where $\beta(r)$ is attractiveness function of the firefly and determined by

$$\beta(r) = \beta_0 e^{-\gamma r_{ij}^2} \qquad (30)$$

Table 1: A comparison among non-optimal DAF [2], the DAF PSO-based, and the DAF FA-based via different error measurement indices

Method	First-order system			Duffing forced system		
	SSE	MAE	MSE	SSE	MAE	MSE
Non-optimal DAF	343.93	0.0693	0.0114	3778.04	0.1874	0.1259
PSO DAF	108.64	0.0295	0.0036	2397.47	0.0837	0.0799
FA DAF	53.38	0.0163	0.0017	2158.17	0.0743	0.0719

where β_0 is the attractiveness at $r = 0$, and γ is the light absorption coefficient. r represents the distance between any two fireflies i and j at x_i and x_j can be the formulated by any norm such as, the l_2-norm as follows:

$$r_{ij} = \|x_i - x_j\|_2 \tag{31}$$

Finally, the third term is randomization with the vector of random variables ε_i being drawn from a Gaussian distribution. In essence, the parameter γ characterizes the variation of the attractiveness, and partly controls how the algorithm behaves. It is also possible to adjust γ so that multiple optima can be found at the same during iterations [25]. A meticulous detailed description of the FA is given in [20]. All parameters required to implement the FA are presented in Appendix A (Table A.2). A pseudo-code of this algorithm is given below.

Firefly Algorithm

Objective function $f(x)$, $x = (x_1, \dots, x_d)^T$
Initialize a population of fireflies $x_i (i = 1, 2, \dots, n)$ Define light absorption coefficient γ
while (t < MaxGeneration)
for I = 1: n all n fireflies
  for j = 1: i all n fireflies
    Light intensity I_i at x_i is determined by $f(x_i)$
      if ($I_j > I_i$)
        Move firefly i towards j in all d dimensions
      end if
    Attractiveness varies with distance r via $\exp[-\gamma r^2]$
    Evaluate new solutions and update light intensity
  end for j
end for i
Rank the fireflies and find the current best
end while
Post-process results and visualization

6. Simulation

In this section, the proposed control scheme is applied to the two examples and the effectiveness of the suggested method is discussed in comparison with non-optimal DAF proposed by Wang [2] and the DAF based on PSO. The first system is a first order uncertain nonlinear system and the other example is Duffing forced-oscillation system.

6.1 Simulation Conditions

In both examples, the goal of the control design is the output of the systems tracks the reference input $x_d = \sin(t)$ and the tracking error converges to zero asymptotically. In both examples the following six fuzzy input membership functions in the universe [-2, 2] are assumed:

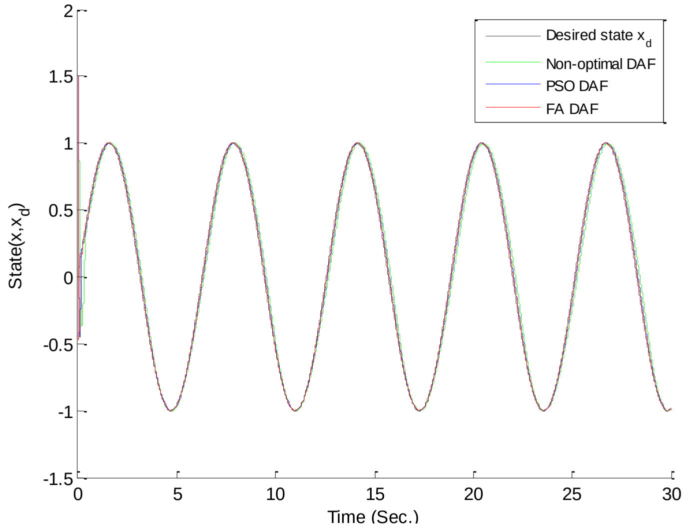

Figure 3: The tracking performance for first-order system: the desired signal x_d (dotted black line), Non-optimal DAF Wang, 1993) (solid green line), DAF PSO-based (solid blue line), and DAF FA-based (solid red line).

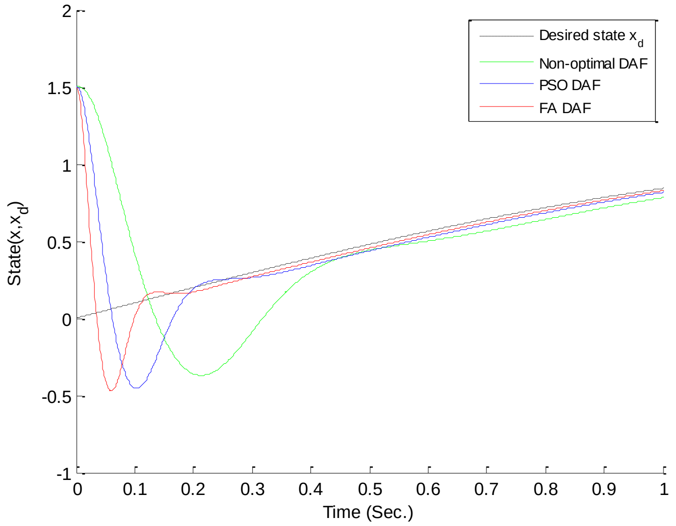

Figure 4: Portion of the tracking performance in Fig. 3 with an expanded time axis view.

$$\begin{aligned}
\mu_{N3} &= \frac{1}{1 + \exp(-(x+2))} \\
\mu_{N2} &= \exp(-(x+1.5)^2) \\
\mu_{N1} &= \exp((x+0.5)^2) \\
\mu_{p1} &= \exp((x-0.5)^2) \\
\mu_{p2} &= \exp((x-1.5)^2) \\
\mu_{p3} &= \frac{1}{1 + \exp(-5(x-2))}
\end{aligned} \tag{32}$$

Three error measurement indices such as Mean Absolute Error (MAE), Sum of the Squared Errors (SSE), and MSE are used to verify the validation of the proposed method in comparison with the non-optimal DAF, which was proposed by Wang in [2], PSO-DAF, which is used the PSO algorithm, and the proposed control scheme based on FA. The MAE is a quantity used to measure how close outcome state is to the desired state. The MAE formulates as follows:

$$MAE = \frac{1}{K}\sum |X - X_d| = \frac{1}{K}\sum |e_i| \qquad (33)$$

where, X is actual state of fuzzy system, X_d is the desired state. K is the total number of data. SSE is used to measure how differences between each error and its group's mean.

$$SSE = \sum (e_i - \bar{e})^2 \qquad (34)$$

6.2 Simulation Results and discussion

Example1: First order nonlinear system

Consider the first order nonlinear system in the following form:

$$\dot{x}(t) = \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} + b \times u(t) \qquad (35)$$

where $f(x)$ and b are considered $(1 - x_1^2)x_2 - x_1$ and 1, respectively. $f(x)$ is assumed to be unknown. The initial value of the system states is $x(0) = 1.5$. The results are simulated for time period [0, 60].

Fig. 3 shows the tracking performance for the non-optimal DAF method proposed by Wang [2], the DAF based on PSO algorithm, and the proposed approach. Fig. (4) shows a portion of the tracking performance of Fig. (3) with an expanded time axis to illustrate the fast convergence of the output to desired state. Fig. (5) shows the boundness of the control signal in the DAF based on FA. The results demonstrate that the proposed control scheme has a less tracking error and better tracking performance among the other methods.

Example2: Duffing forced-oscillation system

Consider the Duffing forced-oscillation system in the following form:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -0.1x_2 - x_1^3 + 12\cos(t) + b \times u(t) \qquad (36)$$

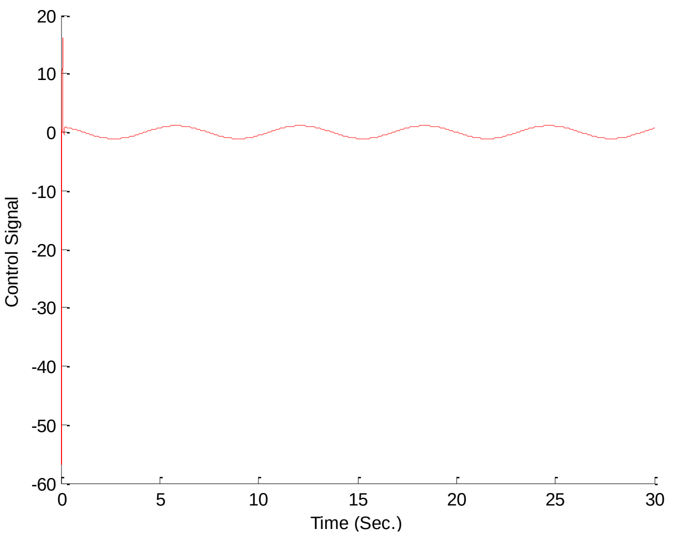

Figure 5: The Boundedness of the control signal (u) for DAF FA-based.

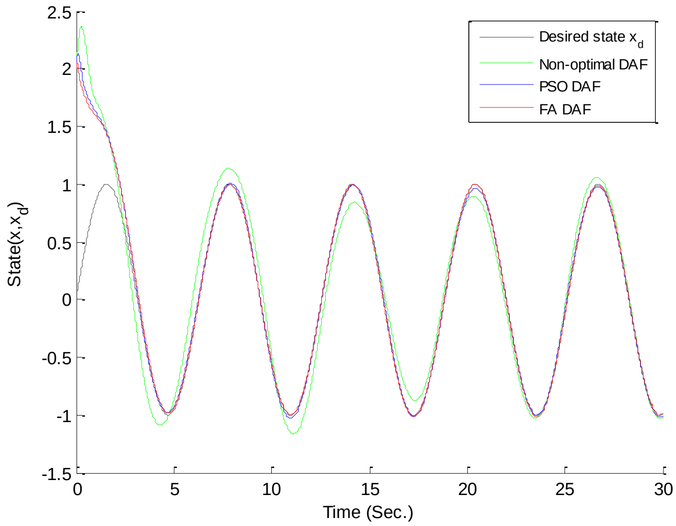

Figure 6: The tracking performance for Duffing forced-oscillation system: the desired signal x_d (dotted black line), Non-optimal DAF (Wang, 1993) (solid green line), DAF PSO-based (solid blue line), and DAF FA-based (solid red line).

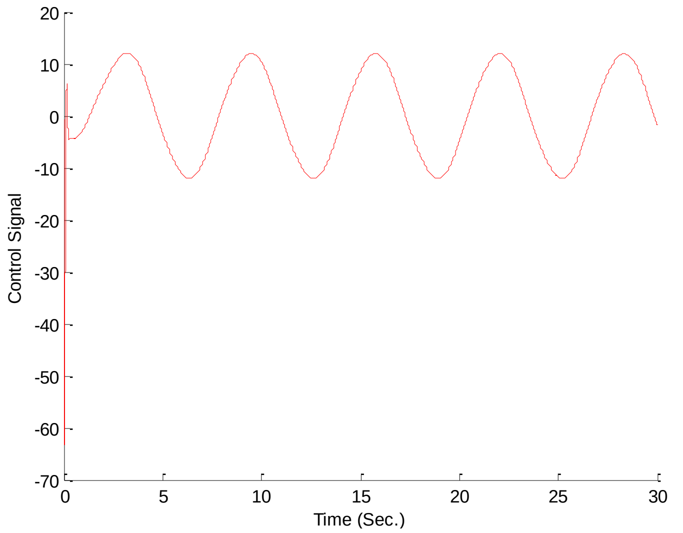

Figure 7: The Boundedness of the control signal (u) for DAF FA-based .

where $f(x)$ and b are considered $= -0.1x_2 - x_1^3 + 12\cos(t)$ and 1, respectively. $f(x)$ is supposed to be unknown. The results are simulated for time period [0, 60]. The initial values of the system are chosen to be $x(0) = (2,2)^T$. In Fig. 6, the tracking performance for the proposed method, the DAF proposed by Wang [2], and the DAF approach based on PSO algorithm are compared. Figs. 7 shows the boundness of the control signal in the DAF based on FA for the Duffing forced-oscillation system. The simulation results show the effectiveness of the proposed control scheme among the other methods to cope with uncertainty and asymptotical tracking is achieved.

Table 1 shows the comparison trackning error among the non-optimal DAF of Wang [2], the DAF based on the PSO algorithm, and the proposed method which is employed the FA for its optimization part based on the three error measurement indices. As a consequence, the proposed control scheme guarantees that the output tracking error and the systems states are uniformly ultimately bounded. This control strategy properly provides the requirement initial knowledge and also the adaptation parameter is determined appropriately.

7. Conclusion

The main contribution of this paper is to propose the direct adaptive fuzzy controller based on the FA for controlling uncertain and nonlinear systems. The FA is used to determine the initial knowledge and the adaptation parameter simultaneously. In the other words, the human experts knowledge, fuzzy IF-THEN rules, are incorporated through the initial parameters into the direct fuzzy control scheme via the FA. The smooth control signal and asymptotic convergence for error tracking are achieved in the proposed control scheme. All adaptive laws are derived from Lyapunov synthesis method, thereby guaranteeing the closed-loop stability and proving the asymptotical tracking for the desired output. Finally, twos well-known examples in control theory are discussed by providing numerical simulations. The results demonstrate the effectiveness of the optimal direct adaptive fuzzy control methodology based on FA in efficient control of complex and nonlinear systems. In future work, the authors aim to extend this methodology to more general forms of nonlinear systems.

Appendix A

All the parameters required to implement the PSO algorithm and the FA are presented in Tables A.1, Tables A.2, respectively.

Table A.1: The PSO algorithm parameters

Swarm Size	25
Max Iteration	40
Inertia weight (w)	Based on Eq.(27)
w_{min}	0.4
w_{max}	0.9
C_1	2.05
C_2	2.05
r_1	U~[0,1]
r_2	U~[0,1]

Table A.2: The FA algorithm parameters

Swarm Size	25
Max Iteration	40
β_0	1
γ	1
α	0.2
ε_i	N~[−1, 1]

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