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Availability and Reliability analysis of a system having main unit and its sub unit & two associate units

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ABSTRACT

Reliability is one of the most vital elements in Information industry. An understanding of failure process is central to any effort to model development and availability is closely related to reliability and can be calculated from reliability. Reliability measurements such as Transition Probability, Mean time to system failure, Availability and Busy period of repairman in repairing the failed units can be used to guide managerial and engineering decisions on various projects. This basic model represents a system having one main unit along with its sub unit and two of its associate units. Functioning of main unit is essential for the functioning of the system. Associate units depend on the main unit and sub unit. The system fails as soon as the main unit fails. There is a single repairman who is always available and repairs the failed unit. After a random period of time, the whole system goes for preventive maintenance. In this paper the failure and repair time distributions are taken to be exponential. Using Markov regenerative process, several system characteristics using chapman-kolmogorov equations are evaluated and profit analysis is done. This kind of analysis is of immense help to the owners of small scale industry. Also the involvement of preventive maintenance in the model increases the reliability of the functioning units.

Keywords: Regenerative Point, MTSF, Availability, Busy Period

Academic Discipline: Mathematics

Subject Classification: 53A04, 53A05

1. Introduction: In the past, Arora et-al[2] has done reliability analysis of two unit standby redundant system with constrained repair time. Gupta et-al [6] has worked on a compound redundant system involving human failure. Rander et-al [2] has evaluated the cost analysis of two dissimilar cold standby systems with preventive maintenance and replacement of standby units. A pioneer work in this field was done by Gopalan [1] and Osaki [3] by performing analysis of warm standby system and parallel system with bivariate exponential life respectively. Earlier Pathak et al [7&8] studied reliability parameters of a main unit with its supporting units and also compared the results with two different distributions. This particular system consists of four units namely one main unit M, one sub unit A and two associate units B & C. Here the associate units B and C dependents upon main units M and sub unit A . The system is operable when the main unit, sub unit and at least one associate unit is in operation. Main unit and sub units can also function with either of B or C. As soon as a job arrives, all the units work with load. It is assumed that only one job is taken for processing at a time. There is a single repairman who repairs the failed units on priority basis. Using regenerative point technique several system characteristics such as transition probabilities, mean sojourn times, availability and busy period of the repairman are evaluated. In the end the expected profit is also calculated.

3. Assumptions used in the model:

a. The system consists of one main unit, one sub unit and two associate units.

b. The associate unit A and B works with the help of main unit and sub unit.

c. There is a single repairman which repairs the failed units on priority basis.

d. After random period of time the whole system goes to preventive maintenance.

e. All units work as new after repair.

f. The failure rates of all the units are taken to be exponential whereas the repair time distributions are arbitrary.

g. Switching devices are perfect and instantaneous.

4. Symbols and Notations:

p_{ij} = Transition probabilities from S_i to S_j

μ_i = Mean sojourn time at time t

E_0 =State of the system at epoch t=0

E=set of regenerative states $S_0 - S_6$

$q_{ij}(t)$ = Probability density function of transition time from S_i to S_j

$Q_{ij}(t)$ = Cumulative distribution function of transition time from S_i to S_j

$\pi_i(t)$ = Cdf of time to system failure when starting from state $E_0 = S_i \in E$

$\mu_i(t)$ = Mean Sojourn time in the state $E_0 = S_i \in E$

$B_i(t)$ = Repairman is busy in the repair at time t / $E_0 = S_i \in E$

$r_1 / r_2 / r_3 / r_4$ =repair rate of Main unit M /Sub Unit A / Unit B/ Unit C

$\alpha / \beta / \gamma / \delta$ =Failure rate of Main unit M /Sub Unit A / Unit B/ Unit C

$g_1 / g_2 / g_3 / g_4$ =Probability density function of repair time of Main unit M /Sub Unit A / Unit B/ Unit C

$\overline{G}_1 / \overline{G}_2 / \overline{G}_3 / \overline{G}_4$ =Cumulative distribution function of repair time of Main unit M /Sub Unit A / Unit B/ Unit C

a(t) = Probability density function of preventive maintenance .

b(t) = Probability density function of preventive maintenance completion time.

$\overline{A}(t)$ = Cumulative distribution functions of preventive maintenance.

$\overline{B}(t)$ = Cumulative distribution functions of preventive maintenance completion time.

$\boxed{s}$ = Symbol for Laplace -stieltjes transforms**.**

$\boxed{c}$ = Symbol for Laplace-convolution.

5. Symbols used for states of the system:

$M_0 / M_g / M_r$ -- Main unit ‘M’ under operation/good and non-operative mode/ repair mode

$A_0 / A_g / A_r$ -- Sub unit ‘A’ under operation/good and non-operative mode/ repair mode

$B_0 / B_r / B_g$ -- Associate Unit ‘B’ under operation/repair/ good and non-operative mode

$C_0 / C_r / C_g$ -- Associate Unit ‘C’ under operation/repair/good and non-operative mode

P.M. -- System under preventive maintenance.

Up states: $S_0 = (M_0, A_0, B_0, C_0)$; $S_3 = (M_0, A_0, B_r, C_0)$; $S_4 = (M_0, A_0, B_0, C_r)$

Down States: $S_1 = (M_r, A_g, B_g, C_g)$; $S_2 = (M_g, A_r, B_g, C_g)$; $S_6 = (S.D.)$; $S_5 = (P.M.)$

6. Transition Probabilities:

Simple probabilistic considerations yield the following non-zero transition probabilities:

1. $Q_{01}(t) = \int_0^t \alpha\, e^{-(\alpha+\beta+\gamma+\delta)t}\, \overline{A}(t)\, dt$

2. $Q_{02}(t) = \int_0^t \beta\, e^{-(\alpha+\beta+\gamma+\delta)t}\, \overline{A}(t)\, dt$

3. $Q_{03}(t) = \int_0^t \gamma\, e^{-(\alpha+\beta+\gamma+\delta)t}\, \overline{A}(t)\, dt$

4. $Q_{04}(t) = \int_0^t \delta\, e^{-(\alpha+\beta+\gamma+\delta)t}\, \overline{A}(t)\, dt$

5. $Q_{05}(t) = \int_0^t a(t) e^{-(\alpha+\beta+\gamma+\delta)t}\, dt$

6. $Q_{10}(t) = \int_0^t g_1(t)\, dt$

7. $Q_{20}(t) = \int_0^t g_2(t)\, dt$

8. $Q_{30}(t) = \int_0^t e^{-(\alpha+\beta+\delta)t} g_3(t)\, dt$

9. $Q_{36}(t) = \int_0^t (\alpha + \beta + \delta) e^{-(\alpha+\beta+\delta)t}\, \overline{G}_3(t)\, dt$

10. $Q_{40}(t) = \int_0^t e^{-(\alpha+\beta+\gamma)t} g_4(t)\, dt$

11. $Q_{46}(t) = \int_0^t (\alpha + \beta + \gamma) e^{-(\alpha+\beta+\gamma)t}\, \overline{G}_4(t)\, dt$

12. $Q_{60}(t) = \int_0^t g_5(t)\, dt$

13. $Q_{50}(t) = \int_0^t b(t)\, dt$

Where $x_1 = \alpha + \beta + \gamma + \delta$, Now letting $t \to \infty$, we get $\lim_{t\to\infty} Q_{ij}(t) = p_{ij}$

14. $p_{01} = \int_0^\infty \alpha\, e^{-(\alpha+\beta+\gamma+\delta)t}\, \overline{A}(t)\, dt = \frac{\alpha}{x_1}[1 - a^*(x_1)]$,

15. $p_{02} = \int_0^\infty \beta\, e^{-(\alpha+\beta+\gamma+\delta)t}\, \overline{A}(t)\, dt = \frac{\beta}{x_1}[1 - a^*(x_1)]$,

16. $p_{03} = \int_0^\infty \gamma\, e^{-(\alpha+\beta+\gamma+\delta)t}\, \overline{A}(t)\, dt = \frac{\gamma}{x_1}[1 - a^*(x_1)]$,

17. $p_{04} = \int_0^\infty \delta\, e^{-(\alpha+\beta+\gamma+\delta)t}\, \overline{A}(t)\, dt = \frac{\delta}{x_1}[1 - a^*(x_1)]$,

18. $p_{05} = \int_0^\infty a(t)\, e^{-(\alpha+\beta+\gamma+\delta)t} dt = a^*(x_1)$, 19. $p_{10} = \int_0^\infty g_1(t) dt = 1$,

20. $p_{20} = \int_0^\infty g_2(t) dt = 1$, 21. $p_{30}(t) = \int_0^\infty e^{-(\alpha+\beta+\delta)t} g_3(t) dt = g_3^*(\alpha+\beta+\delta)$,

22. $p_{36} = \int_0^\infty (\alpha+\beta+\delta) e^{-(\alpha+\beta+\delta)t} \overline{G_3}(t) dt = 1 - g_3^*(\alpha+\beta+\delta)$,

23. $p_{40}(t) = \int_0^\infty e^{-(\alpha+\beta+\gamma)t} g_4(t) dt = g_4^*(\alpha+\beta+\gamma)$,

24. $p_{46} = \int_0^\infty (\alpha+\beta+\gamma) e^{-(\alpha+\beta+\gamma)t} \overline{G_4}(t) dt = 1 - g_4^*(\alpha+\beta+\gamma)$,

25. $p_{50}(t) = \int_0^\infty b(t) dt = 1$, 26. $p_{60} = \int_0^\infty g_5(t) dt = 1$,

27. $p_{10} = p_{20} = p_{50} = p_{60} = 1$ [6.1-6.27]

It is easy to see that

$p_{01} + p_{02} + p_{03} + p_{04} + p_{05} = 1$, $p_{30} + p_{36} = 1$, $p_{40} + p_{46} = 1$ [6.28-6.30]

And mean sojourn time are given by

31. $\mu_0 = \frac{1}{x_1}[1 - a^*(x_1)]$, 32. $\mu_1 = \int_0^\infty \overline{G_1}(t) dt$,

33. $\mu_2 = \int_0^\infty \overline{G_2}(t) dt$, 34. $\mu_3 = \frac{1}{\alpha+\beta+\delta}[1 - g_3^*(\alpha+\beta+\delta)]$,

35. $\mu_4 = \frac{1}{\alpha+\beta+\gamma}[1 - g_4^*(\alpha+\beta+\gamma)]$, 36. $\mu_5 = \int_0^\infty \overline{B}(t) dt$,

37. $\mu_6 = \int_0^\infty \overline{G_5}(t) dt$

[6.31-6.37]

We note that the Laplace-stieltjes transform of $Q_{ij}(t)$ is equal to Laplace transform of $q_{ij}(t)$ i.e. $\tilde{Q}_{ij}(s) = \int_0^\infty e^{-st} Q_{ij}(t) dt = L\{Q_{ij}(t)\} = q_{ij}^*(s)$ [6.38]

39. $\tilde{Q}_{01}(s) = \int_0^\infty \alpha\, e^{-(s+x_1)t} \overline{A}(t) dt = \frac{\alpha}{s+x_1}[1 - a^*(s+x_1)]$,

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40. $\tilde{Q}_{02}(s) = \int_0^\infty \beta\, e^{-(s+x_1)t}\,\overline{A}(t)dt = \dfrac{\beta}{s+x_1}[1-a^*(s+x_1)]$,

41. $\tilde{Q}_{03}(s) = \int_0^\infty \gamma\, e^{-(s+x_1)t}\,\overline{A}(t)dt = \dfrac{\gamma}{s+x_1}[1-a^*(s+x_1)]$,

42. $\tilde{Q}_{04}(s) = \int_0^\infty \delta\, e^{-(s+x_1)t}\,\overline{A}(t)dt = \dfrac{\delta}{s+x_1}[1-a^*(s+x_1)]$,

43. $\tilde{Q}_{05}(s) = \int_0^\infty e^{-(s+x_1)t} a(t)dt = a^*(s+x_1)$,

44. $\tilde{Q}_{10}(s) = \int_0^\infty e^{-st} g_1(t)dt = g_1^*(s)$,

45. $\tilde{Q}_{20}(s) = \int_0^\infty e^{-st} g_2(t)dt = g_2^*(s)$,

46. $\tilde{Q}_{30}(s) = \int_0^\infty e^{-(s+\alpha+\beta+\delta)t} g_3(t)dt = g_3^*(s+\alpha+\beta+\delta)$,

47. $\tilde{Q}_{36}(s) = \int_0^\infty (\alpha+\beta+\delta)\, e^{-(s+\alpha+\beta+\delta)t}\,\overline{G}_3(t)dt = \dfrac{(\alpha+\beta+\delta)}{s+\alpha+\beta+\delta}[1-g_3^*(s+\alpha+\beta+\delta)]$,

48. $\tilde{Q}_{40}(s) = \int_0^\infty e^{-(s+\alpha+\beta+\gamma)t} g_4(t)dt = g_4^*(s+\alpha+\beta+\gamma)$,

49. $\tilde{Q}_{46}(s) = \int_0^\infty (\alpha+\beta+\gamma)\, e^{-(s+\alpha+\beta+\gamma)t}\,\overline{G}_4(t)dt = \dfrac{(\alpha+\beta+\gamma)}{s+\alpha+\beta+\gamma}[1-g_4^*(s+\alpha+\beta+\gamma)]$,

50. $\tilde{Q}_{50}(s) = \int_0^\infty e^{-st} b(t)dt = b^*(s)$,

51. $\tilde{Q}_{60}(s) = \int_0^\infty e^{-st} g_5(t)dt = g_5^*(s)$

[6.39-6.51]

We define m_{ij} as follows:-

$$m_{ij} = -[\frac{d}{ds}\tilde{Q}_{ij}(s)]_{s=0} = -Q_{ij}^{/}(0) \qquad [6.52]$$

It can to show that $m_{01}+m_{02}+m_{03}+m_{04}+m_{05}=\mu_0; m_{30}+m_{36}=\mu_3; m_{40}+m_{46}=\mu_4$

Where $\alpha+\beta+\gamma+\delta = x_1$ [6.53-6.55]

7. Mean time to System failure-

Time to system failure can be regarded as the first passage time to the failed state. To obtain it we regard the down state as absorbing. Using the argument as for the regenerative process, we obtain the following recursive relations.

$$\pi_0(t) = Q_{01}(t) + Q_{02}(t) + Q_{03}(t)\ \boxed{s}\ \pi_3(t) + Q_{04}(t)\ \boxed{s}\ \pi_4(t) + Q_{05}(t)$$

$$\pi_3(t) = Q_{30}(t)\ \boxed{s}\ \pi_0(t) + Q_{36}(t)$$

$$\pi_4(t) = Q_{40}(t)\ \boxed{s}\ \pi_0(t) + Q_{46}(t) \qquad [7.1\text{-}7.3]$$

Taking Laplace -stieltjes transform of above equations and writing in matrix form.

We get

$$\begin{bmatrix} 1 & -\tilde{Q}_{03} & -\tilde{Q}_{04} \\ -\tilde{Q}_{30} & 1 & 0 \\ -\tilde{Q}_{40} & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi_3 \\ \pi_4 \end{bmatrix} = \begin{bmatrix} \tilde{Q}_{01} + \tilde{Q}_{02} + \tilde{Q}_{05} \\ \tilde{Q}_{36} \\ \tilde{Q}_{46} \end{bmatrix}$$

$$D_1(s) = \begin{vmatrix} 1 & -\tilde{Q}_{03} & -\tilde{Q}_{04} \\ -\tilde{Q}_{30} & 1 & 0 \\ -\tilde{Q}_{40} & 0 & 1 \end{vmatrix} = 1 - \tilde{Q}_{30}\tilde{Q}_{03} - \tilde{Q}_{04}\tilde{Q}_{40} \qquad [7.4]$$

And $N_1(s) = \begin{vmatrix} \tilde{Q}_{01} + \tilde{Q}_{02} + \tilde{Q}_{05} & -\tilde{Q}_{03} & -\tilde{Q}_{04} \\ \tilde{Q}_{36} & 1 & 0 \\ \tilde{Q}_{46} & 0 & 1 \end{vmatrix} = (\tilde{Q}_{01} + \tilde{Q}_{02} + \tilde{Q}_{05} + \tilde{Q}_{03}\tilde{Q}_{36} + \tilde{Q}_{04}\tilde{Q}_{46})$

Now letting $s \to 0$ we get

[7.5]

$$D_1(0) = 1 - p_{03}p_{30} - p_{04}p_{40} \qquad [7.6]$$

The mean time to system failure when the system starts from the state S_0 is given by

$$\text{MTSF} = E(T) = -[\frac{d}{ds}\tilde{\pi}_0(s)]_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)} \qquad [7.7]$$

To obtain the numerator of the above equation, we collect the coefficients of relevant of m_{ij} in $D_1'(0) - N_1'(0)$.

Coeff . Of $(m_{01} = m_{02} = m_{03} = m_{04} = m_{05}) = 1$

Coeff .of $(m_{30} = m_{36}) = p_{03}$

Coeff .of $(m_{40} = m_{46}) = p_{04}$ [7.8-7.10]

From equation [7.7]

$$\text{MTSF} = E(T) = -[\frac{d}{ds}\tilde{\pi}_0(s)]_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)}$$

$$= \frac{\mu_0 + \mu_3 p_{03} + \mu_4 p_{04}}{1 - p_{03} p_{30} - p_{04} p_{40}} \qquad [7.11]$$

8. Availability Analysis:

Let $M_i(t)(i = 0,3,4)$ denote the probability that system is initially in regenerative state $S_i \in E$ is up at time t without passing through any other regenerative state or returning to itself through one or more non regenerative states .i.e. either it continues to remain in regenerative S_i or a non regenerative state including itself . By probabilistic arguments, we have the following recursive relations

$$M_0(t) = e^{-(\alpha+\beta+\gamma+\delta)t}\overline{A}(t), \qquad M_3(t) = e^{-(\alpha+\beta+\delta)t}\overline{G_3}(t), \qquad M_4(t) = e^{-(\alpha+\beta+\gamma)t}\overline{G_4}(t), \qquad [8.1\text{-}8.3]$$

Recursive relations giving point wise availability $A_i(t)$ given as follows:

$$A_0(t) = M_0(t) + \sum_{i=1,2,3,4,5} q_{0i}(t) \boxed{c} A_i(t) \quad ; \quad A_1(t) = q_{10}(t) \boxed{c} A_0(t) \, ;$$

$$A_2(t) = q_{20}(t) \boxed{c} A_0(t) \quad ; \quad A_3(t) = M_3(t) + \sum_{i=0,6} q_{3i}(t) \boxed{c} A_i(t) \, ;$$

$$A_4(t) = M_4(t) + \sum_{i=0,6} q_{4i}(t) \boxed{c} A_i(t) \quad ; \quad A_5(t) = q_{50}(t) \boxed{c} A_0(t) \, ;$$

$$A_6(t) = q_{60}(t) \boxed{c} A_0(t) \qquad [8.4\text{-}8.10]$$

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

$$q_{7X7}[A_0^*, A_1^*, A_2^*, A_3^*, A_4^*, A_5^*, A_6^*]' = [M_0^*, 0, 0, M_3^*, M_4^*, 0, 0]' \qquad [8.11]$$

Where $q_{7X7} = \begin{bmatrix} 1 & -q_{01}^* & -q_{02} & -q_{03}^* & -q_{04}^* & -q_{05}^* & 0 \\ -q_{10}^* & 1 & 0 & 0 & 0 & 0 & 0 \\ -q_{20} & 0 & 1 & 0 & 0 & 0 & 0 \\ -q_{30}^* & 0 & 0 & 1 & 0 & 0 & -q_{36}^* \\ -q_{40} & 0 & 0 & 0 & 1 & 0 & -q_{46}^* \\ -q_{50}^* & 0 & 0 & 0 & 0 & 1 & 0 \\ -q_{60}^* & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{7X7}$

Therefore $D_2(s) = \begin{vmatrix} 1 & -q_{01}^* & -q_{02}^* & -q_{03}^* & -q_{04}^* & -q_{05}^* & 0 \\ -q_{10}^* & 1 & 0 & 0 & 0 & 0 & 0 \\ -q_{20}^* & 0 & 1 & 0 & 0 & 0 & 0 \\ -q_{30}^* & 0 & 0 & 1 & 0 & 0 & -q_{36}^* \\ -q_{40}^* & 0 & 0 & 0 & 1 & 0 & -q_{46}^* \\ -q_{50}^* & 0 & 0 & 0 & 0 & 1 & 0 \\ -q_{60}^* & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$

$$= 1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^* - q_{03}^* q_{30}^* - q_{04}^* q_{40}^* - q_{05}^* q_{50}^* - q_{03}^* q_{36}^* q_{60}^* - q_{04}^* q_{46}^* q_{60}^*$$

If $s \to 0$ we get $D_2(0) = 0$ which is true [8.12]

Now $N_2(s) = \begin{vmatrix} M_0^* & -q_{01}^* & -q_{02}^* & -q_{03}^* & -q_{04}^* & -q_{05}^* & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ M_3^* & 0 & 0 & 1 & 0 & 0 & -q_{36}^* \\ M_4^* & 0 & 0 & 0 & 1 & 0 & -q_{46}^* \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$

Solving this Determinant, we get

$$N_2(s) = M_0^* + M_3^* q_{03}^* + M_4^* q_{04}^* \qquad [8.13]$$

If $s \to 0$ we get

$$N_2(0) = \mu_0 + \mu_3 p_{03} + \mu_4 p_{04} \qquad [8.14]$$

To find the value of $D_2^{/}(0)$ we collect the coefficient m_{ij} in $D_2(s)$ we get

Coeff. of $(m_{01} = m_{02} = m_{03} = m_{04} = m_{05}) = 1 = L_0$

Coeff. of $(m_{10}) = p_{01} = L_1$

Coeff. of $(m_{20}) = p_{02} = L_2$

Coeff.of $(m_{30} = m_{36}) = p_{03} = L_3$

Coeff.f $(m_{40} = m_{46}) = p_{04} = L_4$

Coeff. of $m_{50} = p_{05} = L_5$

Coeff. of $m_{60} = p_{03} p_{36} + p_{04} p_{46} = L_6$

[8.15-8.21]

Thus the solution for the steady-state availability is given by

$$A_0^*(\infty) = \lim_{t\to\infty} A_0^*(t) = \lim_{S\to 0} sA_0^*(s) = \frac{N_2(0)}{D_2'(0)} = \frac{\mu_0 L_0 + \mu_3 L_3 + \mu_4 L_4}{\sum_{i=0,1,2,3,4,5,6} \mu_i L_i} \qquad [8.22]$$

9. BUSY PERIOD ANALYSIS:

(a) Let $W_i(t)$ $(i = 1,2,3,4)$ denote the probability that the repairman is busy initially with repair in regenerative state S_i and remain busy at epoch t without transiting to any other state or returning to itself through one or more regenerative states.

By probabilistic arguments we have

$$W_1(t) = \overline{G}_1(t), W_2(t) = \overline{G}_2(t), W_3(t) = \overline{G}_3(t), W_4(t) = \overline{G}_4(t) \qquad [9.1\text{-}9.4]$$

Developing similar recursive relations as in availability, we have

$$B_0(t) = \sum_{i=1,2,3,4,5} q_{0i}(t) \boxed{c} B_i(t) \quad ; \quad B_1(t) = W_1(t) + q_{10}(t) \boxed{c} B_0(t) \quad ;$$

$$B_2(t) = W_2(t) + q_{20}(t) \boxed{c} B_0(t) \quad ; \quad B_3(t) = W_3(t) + \sum_{i=0,6} q_{3i}(t) \boxed{c} B_i(t) \quad ;$$

$$B_4(t) = W_4(t) + \sum_{i=0,6} q_{4i}(t) \boxed{c} B_i(t) \quad ; \quad B_5(t) = q_{50}(t) \boxed{c} B_0(t) \quad ;$$

$$B_6(t) = q_{60}(t) \boxed{c} B_0(t)$$

$$[9.5\text{-}9.11]$$

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

$$q_{7X7}[B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*, B_6^*]' = [0, W_1^*, W_2^*, W_3^*, W_4^*, 0, 0]' \qquad [9.12]$$

Where q_{7x7} is denoted by [8.11] and therefore $D_2'(s)$ is obtained as in the expression of availability.

$$\text{Now } N_3(s) = \begin{vmatrix} 0 & -q_{01}^* & -q_{02}^* & -q_{03}^* & -q_{04}^* & -q_{05}^* & 0 \\ W_1^* & 1 & 0 & 0 & 0 & 0 & 0 \\ W_2^* & 0 & 1 & 0 & 0 & 0 & 0 \\ W_3^* & 0 & 0 & 1 & 0 & 0 & -q_{36}^* \\ W_4^* & 0 & 0 & 0 & 1 & 0 & -q_{46}^* \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

Solving this Determinant, In the long run, we get the value of this determinant after putting $s \to 0$ is

$$N_3(0) = (\mu_1 p_{01} + \mu_2 p_{02} + \mu_3 p_{03} + \mu_4 p_{04}) = \mu_1 L_1 + \mu_2 L_2 + \mu_3 L_3 + \mu_4 L_4 = \sum_{i=1,2,3,4} \mu_i L_i \qquad [9.13]$$

Thus the fraction of time for which the repairman is busy with repair of the failed unit is given by:

$$B_0^{1^*}(\infty) = \lim_{t\to\infty} B_0^{1^*}(t) = \lim_{s\to 0} sB_0^{1^*}(s) = \frac{N_3(0)}{D_2^{/}(0)} = \frac{\sum_{i=1,2,3,4} \mu_i L_i}{\sum_{i=0,1,2,3,4,5,6} \mu_i L_i} \qquad [9.14]$$

(b)Busy period of the Repairman in preventive maintenance in time (0, t], By probabilistic arguments we have

$$W_5(t) = \overline{B}(t) \qquad [9.15]$$

Similarly developing similar recursive relations as in 9(a), we have

$$B_0(t) = \sum_{i=1,2,3,4,5} q_{0i}(t) \boxed{c} B_i(t) \quad ; \qquad B_1(t) = q_{10}(t) \boxed{c} B_0(t) \quad ;$$

$$B_2(t) = q_{20}(t) \boxed{c} B_0(t) \quad ; \qquad B_3(t) = \sum_{i=0,6} q_{3i}(t) \boxed{c} B_i(t) \quad ;$$

$$B_4(t) = \sum_{i=0,6} q_{4i}(t) \boxed{c} B_i(t) \quad ; \qquad B_5(t) = W_5(t) + q_{50}(t) \boxed{c} B_0(t) \quad ;$$

$$B_6(t) = q_{60}(t) \boxed{c} B_0(t)$$

[9.16-9.22]

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

$$q_{7X7}[B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*, B_6^*]^{/} = [0,0,0,0,0,W_5^*,0]^{/}$$

Where q_{7x7} is denoted by [8.11] and therefore $D_2^{/}(s)$ is obtained as in the expression of availability.

$$\text{Now } N_4(s) = \begin{vmatrix} 0 & -q_{01}^* & -q_{02}^* & -q_{03}^* & -q_{04}^* & -q_{05}^* & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -q_{36}^* \\ 0 & 0 & 0 & 0 & 1 & 0 & -q_{46}^* \\ W_5^* & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

Solving this Determinant, In the long run, we get the value of this determinant after putting $s \to 0$ is

$$N_4(0) = \mu_5 p_{05} = \mu_5 L_5 \qquad [9.23]$$

Thus the fraction of time for which the system is under preventive maintenance is given by:

$$B_0^{2^*}(\infty) = \lim_{t\to\infty} B_0^{2^*}(t) = \lim_{s\to 0} sB_0^{2^*}(s) = \frac{N_4(0)}{D_2^{/}(0)} = \frac{\mu_5 L_5}{\sum_{i=0,1,2,3,4,5,6} \mu_i L_i} \qquad [9.24]$$

(c)Busy period of the Repairman in Shut Down repair in time (0, t], By probabilistic arguments we have

$$W_6(t) = \overline{G}_6(t) \qquad [9.25]$$

Similarly developing similar recursive relations as in 9(b), we have

$$B_0(t) = \sum_{i=1,2,3,4,5} q_{0i}(t) \boxed{c} B_i(t) \quad ; \quad B_1(t) = q_{10}(t) \boxed{c} B_0(t) \quad ;$$

$$B_2(t) = q_{20}(t) \boxed{c} B_0(t) \quad ; \quad B_3(t) = \sum_{i=0,6} q_{3i}(t) \boxed{c} B_i(t) \quad ;$$

$$B_4(t) = \sum_{i=0,6} q_{4i}(t) \boxed{c} B_i(t) \quad ; \quad B_5(t) = q_{50}(t) \boxed{c} B_0(t) \quad ;$$

$$B_6(t) = W_6(t) + q_{60}(t) \boxed{c} B_0(t) \qquad [9.26\text{-}9.32]$$

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

$$q_{7X7}[B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*, B_6^*]' = [0,0,0,0,0,0,W_6^*]'$$

Where q_{7x7} is denoted by [8.11] and therefore $D_2'(s)$ is obtained as in the expression of availability.

$$\text{Now } N_5(s) = \begin{vmatrix} 0 & -q_{01}^* & -q_{02}^* & -q_{03}^* & -q_{04}^* & -q_{05}^* & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -q_{36}^* \\ 0 & 0 & 0 & 0 & 1 & 0 & -q_{46}^* \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ W_6^* & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

In the long run, we get the value of this determinant after putting $s \to 0$ is

$$N_5(0) = \mu_6(p_{03}p_{36} + p_{04}p_{46}) = \mu_6 L_6 \qquad [9.33]$$

Thus the fraction of time for which the system is under shut down is given by:

$$B_0^{3^*}(\infty) = \lim_{t\to\infty} B_0^{3^*}(t) = \lim_{s\to 0} sB_0^{3^*}(s) = \frac{N_5(0)}{D_2'(0)} = \frac{\mu_6 L_6}{\sum_{i=0,1,2,3,4,5,6} \mu_i L_i} \qquad [9.34]$$

10. Particular cases: When all repair time distributions are n-phase Erlangian distributions i.e.

Density function $g_i(t) = \dfrac{nr_i(nr_it)^{n-1}e^{-nr_it}}{n-1!}$ And Survival function $\overline{G}_j(t) = \sum_{j=0}^{n-1} \dfrac{(nr_it)^j e^{-nr_it}}{j!}$

[10.1-10.2]

And other distributions are negative exponential

$$a(t) = \theta e^{-\theta t}, b(t) = \eta e^{-\eta t}, \overline{A}(t) = e^{-\theta t}, \overline{B}(t) = e^{-\eta t} \qquad [10.3\text{-}10.6]$$

For n=1 $\quad g_i(t) = r_i e^{-r_i t}$, $\overline{G}_i(t) = e^{-r_i t}$ $\quad$ If i=1, 2, 3, 4

$$g_1(t) = r_1 e^{-r_1 t}, g_2(t) = r_2 e^{-r_2 t}, g_3(t) = r_3 e^{-r_3 t}, g_4(t) = r_4 e^{-r_4 t}, g_5(t) = r_5 e^{-r_5 t}$$

$$\overline{G}_1(t)=e^{-r_1 t},\ \overline{G}_2(t)=e^{-r_2 t},\ \overline{G}_3(t)=e^{-r_3 t},\ \overline{G}_4(t)=e^{-r_4 t},\ \overline{G}_5(t)=e^{-r_5 t} \qquad [10.7\text{-}10.16]$$

Also

$$p_{01}=\frac{\alpha}{x_1+\theta},\ p_{02}=\frac{\beta}{x_1+\theta},\ p_{03}=\frac{\gamma}{x_1+\theta},\ p_{04}=\frac{\delta}{x_1+\theta},\ p_{05}=\frac{\theta}{x_1+\theta}$$

$$p_{10}=1,\ p_{20}=1,\ p_{30}=\frac{r_3}{\alpha+\beta+\delta+r_3},\ p_{36}=\frac{\alpha+\beta+\delta}{\alpha+\beta+\delta+r_3},$$

$$p_{40}=\frac{r_4}{\alpha+\beta+\gamma+r_4},\ p_{46}=\frac{\alpha+\beta+\gamma}{\alpha+\beta+\gamma+r_4},\ \mu_0=\frac{1}{x_1+\theta},\ \mu_1=\frac{1}{r_1},$$

$$\mu_2=\frac{1}{r_2},\ \mu_3=\frac{r_3}{\alpha+\beta+\delta+r_3},\ \mu_4=\frac{r_4}{\alpha+\beta+\gamma+r_4},\ \mu_5=\frac{1}{\eta},\ \mu_6=\frac{1}{r_5}$$

[10.17-10.34]

$$\text{MTSF}=\frac{\mu_0+\mu_3 p_{03}+\mu_4 p_{04}}{1-p_{03}p_{30}-p_{04}p_{40}},\ A_0(\infty)=\frac{\mu_0 L_0+\mu_3 L_3+\mu_4 L_4}{\sum\limits_{i=0,1,2,3,4,5,6}\mu_i L_i},\ B_0^{1^*}(\infty)=\frac{\sum\limits_{i=1,2,3,4}\mu_i L_i}{\sum\limits_{i=0,1,2,3,4,5,6}\mu_i L_i},$$

$$B_0^{2^*}(\infty)=\frac{\mu_5 L_5}{\sum\limits_{i=0,1,2,3,4,5,6}\mu_i L_i},\ B_0^{3^*}(\infty)=\frac{\mu_6 L_6}{\sum\limits_{i=0,1,2,3,4,5,6}\mu_i L_i} \qquad [10.35\text{-}10.39]$$

Where $L_0=1;\ L_1=p_{01};\ L_2=p_{02};\ L_3=p_{03};\ L_4=p_{04};\ L_5=p_{05};\ L_6=p_{03}p_{36}+p_{04}p_{46}$

[10.40-10.46]

11. Profit Analysis:-

The profit analysis of the system can be carried out by considering the expected busy period of the repairman in repair of the unit in (0,t].

Therefore, G(t) = Expected total revenue earned by the system in (0,t] -Expected repair cost of the failed units

-Expected repair cost of the repairman in preventive maintenance -Expected repair cost of the

Repairman in shut down

$$= C_1\mu_{up}(t)-C_2\mu_{b1}(t)-C_3\mu_{b2}(t)-C_4\mu_{b3}(t)$$

$$=C_1A_0-C_2B^1{}_0-C_3B^2{}_0-C_4B^3{}_0 \qquad [11.1]$$

Where $\mu_{up}(t)=\int_0^t A_0(t)\,dt;\ \mu_{b1}(t)=\int_0^t B_0^1(t)\,dt;\ \mu_{b2}(t)=\int_0^t B_0^2(t)\,dt;\ \mu_{b3}(t)=\int_0^t B_0^3(t)\,dt$

[11.2-11.5]

C_1 is the revenue per unit time and C_2, C_3, C_4 are the cost per unit time for which the system is under simple repair, preventive maintenance and shut down repair respectively.

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Figure: state transition diagram