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Energy-based Controller with Optimization Tuning by Using Nelder-Mead Algorithm for Overhead Cranes

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Abstract. This paper presents a combination of a nonlinear PD controller and Nelder-Mead algorithm to design an optimal controller for a nonlinear overhead crane system. The nonlinear PD controller is derived based on the passivity of the system and Nelder-Mead algorithm is exploited to find optimal parameters for the controller. The system dynamic model is derived by using Lagrangian equation. Simulations are conducted within Matlab environment to determine the optimal control parameters and to verify the performance of the controller. The simulations demonstrates that the controller is effective to move the trolley as fast as possible to the desired position while the oscillation of the payload is suppressed at the end of the operation. The robustness of the controller against uncertainties in cable length and payload is also indicated by the simulations.

Keywords: underactuated nonlinear systems, overhead crane, energy-based control, passivity, optimal control.

1. Introduction

Overhead cranes are widely used in a large number of fields, such as heavy industries, seaports, automotive factories, and construction facilities. The productivity and efficiency of an overhead crane depend not only on payload weight and velocity but also on the capability of the crane to reduce the swing angle of the payload quickly at the end of each operation. Theory and practice have shown that faster acceleration and deceleration correspond to larger swing angles. This condition leads to a dangerous situation and may cause severe accidents if the cargo swing angle becomes too large. A large cargo swing angle could break the crane, damage other equipment and infrastructures, or even hurt people nearby.

The task of control the trolley and suppression the swing angle has attracted the interest of numerous researchers. A number of control algorithms have been developed for overhead cranes. One commonly used approach is the linear and gain scheduling methodology. This method linearizes the complex nonlinear crane model around the target position [14]. To increase performance, several authors exploited the nonlinearity of the system in designing the controller and investigated nonlinear, advanced, and intelligent control strategies. Several controllers have been investigated such as partial linearization [15;17;18], sliding mode [1;10], adaptive controller [2;12], energy-based controllers and gain scheduling [14; 5; 3].

In recent years, researchers have examined a design method based on the energy and passivity of the system. This approach was successfully applied in the control of underactuated systems, such as overhead cranes [4;16], ball-and-beam systems [11] and underactuated manipulators [6]. The main advantage of these methods is the simplicity in deriving the controller from the energy storage function, which adopts the mechanical energy of the system as well as the "artificial" kinetic and potential energy. The artificial energy affects control performance. In the traditional approach of energy-based methods, authors directly use the mechanical energy of the system or its quadratic form in the Lyapunov functions to derive proper control laws [4]. Hence, the obtained control laws include system parameter-related terms. Another limitation of traditional energy-based control methods is that they only exploit the passivity of the control input with actuated velocity and completely exclude the passive payload swing. [16] aimed to derive a controller that includes the passivity of the payload swing by utilizing the payload as an end effector of a manipulator. However, how to choose the controller parameters is not considered in the study. In the current paper, by introducing a performance index the optimal parameters of the controller then are tuned with Nelder Mead algorithm.

The remainder of this paper is organized as follows. Section 2 introduces the nonlinear dynamics of an overhead crane with two degrees of freedom as well as several useful properties of a dynamic system. In Sections 3, the nonlinear PD controller is given based on passive property of the system and the controller parameters is tuned by using Nelder-Mead algorithm. Section 4 presents the numerical verifications of the controller. Finally, Section 5 concludes.

2. Dynamic model of a 2-D overhead crane

2.1 Dynamic model

The control problem of a crane during the horizontal transportation phase is addressed in this paper. The rope has constant length, and the system has two degrees of freedom.

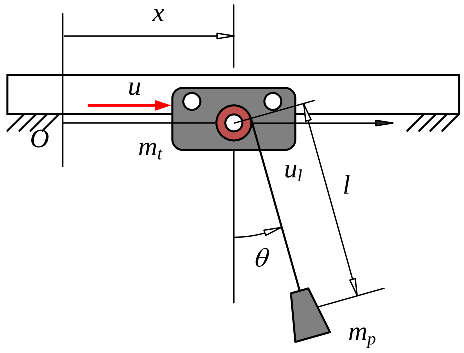

Fig. 1 The crane model

To obtain the dynamic model of the system, the following assumptions are established: i) the payload is considered as a point mass; ii) the mass and stiffness of the hoisting rope are neglected; iii) the effects of wind disturbances are not considered. Based on the Lagrangian formulation (Spong, Hutchinson, & Vidyasagar, 2005), the dynamic model of a 2-D overhead crane system is represented by the following:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{B}u \qquad (1)$$

where $\mathbf{q} = [x, \theta]^T$ denotes the system state vector with $x(t)$ as the trolley displacement and $\theta(t)$ as the payload swing angle (Fig. 1), and u is the force acting on the trolley. The variables $\mathbf{M}(\mathbf{q}), \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$, $\mathbf{g}(\mathbf{q})$, and $\mathbf{B}$ represent the inertia matrix, centripetal-Coriolis matrix, and gravitational forces which are derived from kinetic and potential energy, and the input control matrix, respectively. These variables are explicitly defined as follows:

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} m_t + m_p & m_p l\cos\theta \\ m_p l\cos\theta & m_p l^2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix} 0 & -m_p l\dot{\theta}\sin\theta \\ 0 & 0 \end{bmatrix}, \quad \mathbf{g}(\mathbf{q}) = \begin{bmatrix} 0 \\ m_p g l\sin\theta \end{bmatrix}.$$

In this equation of motion, m_t and m_p represent the trolley mass and the payload mass, respectively, l denotes the length of the rope. Equation (1) can be rewritten as follows:

$$(m_t + m_p)\ddot{x} + m_p l\cos\theta\,\ddot{\theta} + m_p l\dot{\theta}^2\sin\theta = u, \qquad (2)$$

$$m_p l\cos\theta\,\ddot{x} + m_p l^2\ddot{\theta} + m_p g l\sin\theta = 0. \qquad (3)$$

Equation (1) has several important properties: (i) the inertia matrix is positive definite and symmetric, $\mathbf{M} = \mathbf{M}^T > 0$; (ii) the matrix $\mathbf{N} = (\dot{\mathbf{M}} - 2\mathbf{C})$ is skew-symmetric, $\mathbf{s}^T(\dot{\mathbf{M}} - 2\mathbf{C})\mathbf{s} = 0$ for $\forall \mathbf{s} \in \mathbb{R}^2$.

2.2 Passivity of the open-loop system

Given the storage energy function E comprising the kinetic and potential energy of the system, we obtain the following:

$$E(\mathbf{q},\dot{\mathbf{q}}) = \tfrac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} + P(\mathbf{q}) \geq 0, \qquad (4)$$

with $P(\mathbf{q}) = m_p g l(1 - \cos\theta) \geq 0$.

The derivative of the storage function with regard to time is calculated as follows:

$$\dot{E} = \dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \tfrac{1}{2}\dot{\mathbf{q}}^T\dot{\mathbf{M}}(\mathbf{q})\dot{\mathbf{q}} + \dot{\mathbf{q}}^T\mathbf{g}(\mathbf{q}). \qquad (5)$$

Substituting the term $\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}$ from Equation (1) and using the skew-symmetric property of $\tfrac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) - \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$, we obtain the following:

$$\dot{E} = u\dot{x} \qquad (6)$$

The term $u\dot{x}$ denotes the power supplied by the actuator for the trolley. The inequality in Equation (6) shows that the system is passive. Integrating both sides from zero to t, we obtain the following:

$$E(t) - E(0) = \int_0^t \dot{E}\,ds = \int_0^t u(t)\dot{x}(t)\,dt$$

or

$$E(t) - E(0) = \int_0^t u(t)\dot{x}(t)\,dt\ . \qquad (7)$$

When the forces $u = 0$ and $|\theta| < \tfrac{1}{2}\pi$ for the zero input, the system has a stable equilibrium $(x,\theta,\dot{x},\dot{\theta}) = (*,0,0,0)$, where the total energy is minimized after taking the zero values.

3. Energy-based controller design

The control objective is to bring the trolley from an initial condition to a desired position and for the payload swing angles to vanish completely at the load destination. This objective indicates that the state variables $\mathbf{q} = [x, \ \theta]^T$ should reach their desired values $\mathbf{q}_d = [x_d, \ 0]^T$ after a short time.

3.1 Controller design directly from system energy

In this section, the controller is derived by considering the energy storage function:

$$V = \frac{1}{p}k_E E^p(\mathbf{q},\dot{\mathbf{q}}) + \tfrac{1}{2}k\dot{x}^2 + \tfrac{1}{2}k_p(x - x_d)^2, \qquad (8)$$

with $p = 1$ or 2, and $k_E, k_p > 0, k \geq 0$.

The last two terms in Equation (8) are called the "artificial" kinetic and potential energy related to actuated coordinates. The derivative of V with regard to time under equation (6) is as follows:

$$\begin{aligned}\dot{V} &= k_E E\dot{E}(\mathbf{q},\dot{\mathbf{q}}) + k\dot{x}\ddot{x} + \dot{x}k_p(x - x_d) \\ &= \dot{x}[uk_E E + k\ddot{x} + k_p(x - x_d)].\end{aligned} \qquad (9)$$

Taking $\ddot{x}$ from Equation (2)

$$\ddot{x} = f_1(\dot{x},\theta,\dot{\theta}) + g_1(\theta)u\ ,$$

with $g_1(\theta) = (m_t + m_p\sin^2\theta)^{-1}$, and

$$f_1(\dot{x},\theta,\dot{\theta}) = \frac{(m_p g\sin\theta\cos\theta + m_p l\dot{\theta}^2\sin\theta)}{(m_t + m_p\sin^2\theta)}$$

and substituting into Equation (9), we obtain the following:

$$\dot V = \dot x\left([k_E E + k g_1(\theta)]u + k f_1(\dot x, \theta, \dot\theta) + k_p(x - x_d)\right). \quad (10)$$

The controller is chosen as

$$u = \frac{-k_p(x - x_d) - k_d\dot x - k f_1(\dot x, \theta, \dot\theta)}{k_E E(\mathbf{q}, \dot{\mathbf{q}}) + k g_1(\theta)}, \quad (11)$$

and Equation (10) becomes

$$\dot V = -k_d \dot x^2 \le 0. \quad (12)$$

As $E(\mathbf{q}, \dot{\mathbf{q}}), \mathbf{M}(\mathbf{q})$ and x_d are bounded, $\mathbf{q}$ and $\dot{\mathbf{q}}$ are also bounded. Thus, $\ddot{\mathbf{q}}$ from Equation (1) and $\ddot V = -2(k_d + k_E f_{11})\dot x\ddot x$ are bounded. Consequently, $\dot V(\mathbf{q}, \dot{\mathbf{q}})$ is uniformly continuous. According to Barbalat's lemma, $\lim_{t\to\infty}\dot V(\mathbf{q}, \dot{\mathbf{q}}) = 0$; thus, $\lim_{t\to\infty}\dot x(t) = 0$. Based on the passivity properties of the system and Equations (8), (11) and (12), $\lim_{t\to\infty} E(\mathbf{q}, \dot{\mathbf{q}})$, $\lim_{t\to\infty} V(\mathbf{q}, \dot{\mathbf{q}})$, $\lim_{t\to\infty}(x - x_d)$, and $\lim_{t\to\infty} u$ are constant. Thereafter, these constants need to be proven as zero by contradiction. Assume that $[x, \theta] \ne [x_d, 0]$; that is, assume that these variables have other constant values from the equilibrium. From the control law in Equation (11), we obtain $u = u_o \ne 0$; that is, the input takes a constant value that leads to a contradiction because constant forces produce changes in x and θ. According to Equation (11), system dynamics will change until the minimum storage function in Equation (8) is achieved. At this position, $[x, \theta, \dot x, \dot\theta] = [x_d, 0, 0, 0]$, the system stabilizes at the desired position.

In the simplest case with $p = 1, k_E = 1, k = 0$ the nonlinear controller in Equation (11) becomes the following:

$$u = -k_p(x - x_d) - k_d\dot x \quad (13)$$

The control law in Equation (13) is a proportional-derivative (PD) controller that can be modified into a nonlinear PD controller by adding a coupling part [8]:

$$u = -k_p(x - x_d) - (k_d + k_{\dot\theta}\dot\theta^2)\dot x, \quad k_{\dot\theta} \ge 0. \quad (14)$$

The controller (11) is derived based on the passivity of the input u with respect to the output $\dot x$. The obtained controller in special case becomes the PD controller (13).

3.2 Control design based on passivity of input with respect to the combination of $\dot x$ and $\dot\theta$

An other alternative by presented in [16], in which the authors try to find a new energy storage function such that the system is passive with respect to both actuated and unactuated coordinates

$$\dot E_1 = u[\dot x + g(\theta)\dot\theta].$$

So we need find an additional engery storage function E_a that satisfy

$$E_1 = E + E_a, \qquad \dot E_a = u g(\theta)\dot\theta \quad (15)$$

where the function E is defined in (4).

Solving for u from Eq. (2) and (3) ones obtains

$$u = -\frac{(m_t + m_p\sin^2\theta)l\ddot\theta}{\cos\theta} - (m_t + m_p)g\frac{\sin\theta}{\cos\theta} - m_p l\dot\theta^2\sin\theta. \quad (16)$$

Putting (16) into Eq. (15) one yields

$$\dot E_a = -\frac{g(\theta)}{\cos\theta}\dot\theta\begin{bmatrix}(m_t + m_p\sin^2\theta)l\ddot\theta + \\ m_p l\dot\theta^2\sin\theta\cos\theta + (m_t + m_p)g\sin\theta\end{bmatrix} = -\frac{g(\theta)}{\cos\theta}\frac{d}{dt}\begin{bmatrix}(m_t + m_p\sin^2\theta)l\dot\theta^2 + \\ (m_t + m_p)g(1 - \cos\theta)\end{bmatrix} \quad (17)$$

Assuming that $|\theta| < \pi/2$, so $\cos\theta > 0$, Eq. (17) suggests us to choose $g(\theta) = -\lambda l\cos\theta$, then we have

$$\dot E_a = \lambda l\frac{d}{dt}\begin{bmatrix}\frac{1}{2}(m_t + m_p\sin^2\theta)l\dot\theta^2] + \\ (m_t + m_p)g(1 - \cos\theta)\end{bmatrix} \quad (18)$$

and the additional energy function is given as

$$E_a = \tfrac{1}{2}\lambda(m_t + m_p\sin^2\theta)l^2\dot\theta^2 + \lambda(m_t + m_p)gl(1 - \cos\theta). \quad (19)$$

In order to guarantee $E_a \ge 0$, we choose the parameter $\lambda > 0$. So from Eq. (4) and Eq. (19) we have a new energy storage function as

$$\begin{aligned}E_1(\mathbf{q}, \dot{\mathbf{q}}) &= E + E_a \\ &= \tfrac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} + m_p gl(1 - \cos\theta) \\ &\quad + \tfrac{1}{2}\lambda(m_t + m_p\sin^2\theta)l^2\dot\theta^2 \\ &\quad + \lambda(m_t + m_p)gl(1 - \cos\theta)\end{aligned} \quad (20)$$

and its derivative as

$$\dot E_1 = u_x(\dot x - \lambda l\cos\theta\dot\theta) = u_x\tfrac{d}{dt}[x - \lambda l\sin\theta]. \quad (21)$$

In order to derive a controller based on new energy storage function, the generalized error of the payload is defined as the following

$$e_p = x - x_d - \lambda l\sin\theta = e - \lambda l\sin\theta,$$
$$\dot e_p = \dot x - \lambda l\dot\theta\cos\theta = \dot e - \lambda l\dot\theta\cos\theta. \quad (22)$$

This definition guarantees that if

$e_p \to 0, \theta \to 0$ then $x \to x_d$ or $x_p \to x_d$;

$\dot e_p \to 0, \dot\theta \to 0$ then $\dot e \to 0$.

The control law is deriven by choosing a Lyapunov function as

$$V = E_1 + \tfrac{1}{2}k_p e_p^2 \quad (23)$$

The derivative of V w.r.t. time under consideration of Eq. (21) is

$$\begin{aligned}\dot V &= \dot E_1 + k_p e_p\dot e_p \\ &= u(\dot x - \lambda l\cos\theta\dot\theta) + k_p e_p(\dot x - \lambda l\cos\theta\dot\theta) \\ &= (u + k_p e_p)(\dot x - \lambda l\cos\theta\dot\theta) = (u + k_p e_p)\dot e_p\end{aligned}$$

This derivative $\dot V$ suggest us to choose control law as

$$\begin{aligned}u &= -k_p e_p - k_d\dot e_p \\ &= -k_p(x - x_d - \lambda l\sin\theta) - k_d(\dot e - \lambda l\cos\theta\dot\theta),\end{aligned} \quad (24)$$

with $k_d > 0$, so we obtain

$$\dot V = -k_d\dot e_p^2 = -k_d(\dot x - \lambda l\dot\theta\cos\theta)^2 \le 0. \quad (25)$$

The detail of stability analysis can be found in ref. [Sun N., & Fang, Y. (2012)].

3.3 Parameter tunning by Nelder-Mead algorithm

It is clear that, when we apply the controllers (11) or (24) the response of the system depends on the chosen parameters such as k_E, k, k_p, k_d or k_p, k_d, λ , respectively. In order to find reasonable parameters for the controller (24) a performance index is defined as a quantitative measure to depict the system performance. Using this technique an 'optimum system' can often be designed and a set of nonlinear PD parameters in the system can be adjusted to meet the required specification. For the proposed control structure, an integral squared error can be used as the performance index of the system. It is defined as follows:

$$J = f(k_p, k_d, \lambda) = \int_0^{t_f} [d_1 e_x^2 + d_2 \dot{e}_x^2 + d_3 \theta^2 + d_4 \dot{\theta}^2] dt, \quad (26)$$

where $e_x = (x - x_d)$ and $\dot{e}_x = (\dot{x} - \dot{x}_d)$ are tracking errors of the trolley position, and d_1, d_2, d_3, d_4 are weighting factors. In this paper, the Nelder-Mead iterative algorithm [13;7] is exploited to find the optimum parameters k_p, k_d, λ of the controller (24).

The Nelder-Mead simplex algorithm is the most widely used direct search method for solving the optimization problem

$$\min f(\mathbf{x}), \quad \text{in our case } \mathbf{x} = [k_p, k_d, \lambda]^T, \quad (27)$$

where $f: \mathbb{R}^n \to \mathbb{R}$ is called the objective function and n the dimension. A simplex is a geometric figure in n dimensions that is the convex hull of $n+1$ vertices. We denote a simplex with vertices $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{n+1}$ by Δ.

The Nelder-Mead method iteratively generates a sequence of simplices to approximate an optimal point of (27). At each iteration, the vertices $\{\mathbf{x}_j\}, j = 1, \ldots, n+1$ of the simplex are ordered according to the objective function values

$$f(\mathbf{x}_1) \le f(\mathbf{x}_2) \le \ldots \le f(\mathbf{x}_{n+1}). \quad (28)$$

We refer to $\mathbf{x}_1$ as the *best* vertex, and to $\mathbf{x}_{n+1}$ as the *worst* vertex. The algorithm uses four possible operations: *reflection*, *expansion*, *contraction*, and *shrink*, each being associated with a scalar parameter: α (reflection), β (expansion), γ (contraction), and δ (shrink). The values of these parameters satisfy $\alpha > 0$, $\beta > 1$, $0 < \gamma < 1$, and $0 < \delta < 1$. In the standard implementation of the Nelder-Mead method the parameters are chosen to be

$$\{\alpha, \beta, \gamma, \delta\} = \{1, 2, 1/2, 1/2\}. \quad (29)$$

Let $\mathbf{x}_0$ be the centroid of the n best vertices. Then

$$\mathbf{x}_0 = \frac{1}{n}\sum_{j=1}^{n} \mathbf{x}_j. \quad (30)$$

We now outline the Nelder-Mead algorithm given in [9]:

1. Sort. Evaluate f at the n+1 vertices of Δ and sort the vertices so that (28) holds

$$f(\mathbf{x}_1) \le f(\mathbf{x}_2) \le \ldots \le f(\mathbf{x}_{n+1}).$$

Calculate $\mathbf{x}_0$, the center of gravity of all points except $\mathbf{x}_{n+1}$, by (30).

2. Reflection. Compute the reflection point $\mathbf{x}_r$ from

$$\mathbf{x}_r = \mathbf{x}_0 + \alpha(\mathbf{x}_0 - \mathbf{x}_{n+1}).$$

Evaluate $f_r = f(\mathbf{x}_r)$. If the reflected point is better than the second worst, but not better than the best, i.e.: $f(\mathbf{x}_1) \le f(\mathbf{x}_r) < f(\mathbf{x}_n)$, then we obtain a new simplex by replacing the worst point $\mathbf{x}_{n+1}$ with the reflected point $\mathbf{x}_r$, and go to step 1.

3. Expansion. If the reflected point is the best point so far, $f(\mathbf{x}_r) < f(\mathbf{x}_1)$, then compute the expansion point $\mathbf{x}_e$ from

$$\mathbf{x}_e = \mathbf{x}_0 + \beta(\mathbf{x}_r - \mathbf{x}_0),$$

and evaluate $f_e = f(\mathbf{x}_e)$. If the expanded point is better than the reflected point, $f(\mathbf{x}_e) < f(\mathbf{x}_r)$, then we obtain a new simplex by replacing the worst point $\mathbf{x}_{n+1}$ with the expanded point $\mathbf{x}_e$, and go to step 1.

Else obtain a new simplex by replacing the worst point $\mathbf{x}_{n+1}$ with the reflected point $\mathbf{x}_r$, and go to step 1.

Else (i.e. reflected point is not better than second worst) continue at step 4.

4. Outside Contraction. If $f_n \le f_r < f_{n+1}$, compute the outside contraction point

$$\mathbf{x}_{oc} = \mathbf{x}_0 + \gamma(\mathbf{x}_r - \mathbf{x}_0)$$

and evaluate $f_{oc} = f(\mathbf{x}_{oc})$. If $f_{oc} \le f_r$, replace $\mathbf{x}_{n+1}$ with $\mathbf{x}_{oc}$; otherwise go to step 6.

5. Inside Contraction. If $f_r \ge f_{n+1}$, compute the inside contraction point $\mathbf{x}_{ic}$ from

$$\mathbf{x}_{ic} = \mathbf{x}_0 - \gamma(\mathbf{x}_r - \mathbf{x}_0)$$

and evaluate $f_{ic} = f(\mathbf{x}_{ic})$. If $f_{ic} < f_{n+1}$, replace $\mathbf{x}_{n+1}$ with $\mathbf{x}_{ic}$; otherwise, go to step 6.

6. Shrink. For $2 \le i \le n+1$, define

$$\mathbf{x}_i = \mathbf{x}_1 + \delta(\mathbf{x}_i - \mathbf{x}_1)$$

go to step 1.

The initial simplex is important, indeed, a too small initial simplex can lead to a local search, consequently the Nelder-Mead algorithm can get more easily stuck. So this simplex should depend on the nature of the problem.

4. Numerical simulation

In this paper, numerical simulations are conducted by using Matlab software to verify efficiency of the controller design approach. The control objective of the overhead crane is to move the trolley to its destination while complementing the anti-swing of the load. In the simulation, the system parameters are set as follows: $m_t = 2.0$ kg, $m_p = 0.85$ kg, $l = 0.7$ m, and $g = 9.81$ m/s². The fourth-order Runge–Kutta method with a time step of 0.01 s is applied. The target position of the trolley is set as $x_d = 1$ m. The controllers in Equation (24) is implemented in the simulation. The lower and upper boundaries of the controller parameters and the weighting factors are set by

$0 < k_p, k_d \le 250, \qquad 0 < l \le 10,$

$d_1 = d_2 = d_3 = d_4 = 1.$

For initializing the Nelder-Mead algorithm, the starting values of $\mathbf{x} = [k_p, k_d, l]^T$ are chosen as [15, 40, 1]. By applying Nelder-Mead algorithm the optimal parameters are obtained after 40 iterations. The optimal parameters and the value of the fitness function are $\mathbf{x}_{opt} = [10.5102 \quad 16.2436 \quad 2.9564]^T$ and J = 1.4868. The fitness values changing with the iterations are given in Fig. 2.

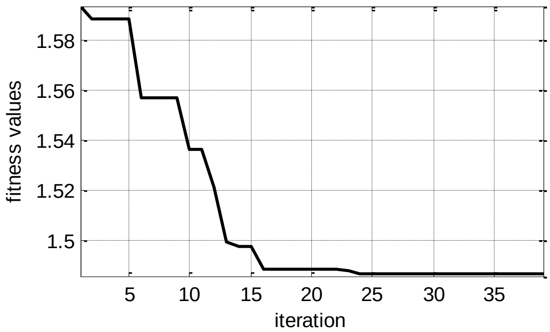

Fig.2 The fitness values with respect to iterations

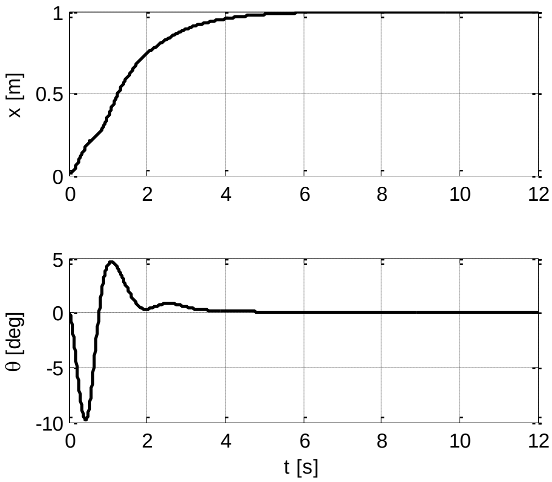

Fig.3 System response with m_p = 0.85 kg, L = 0.7 m

The simulation results for the displacement of the trolley and swing angle of the load are shown in Fig. 3. The simulation results show that the desired position of the trolley was reached after about 6s. During this time, the swing angle of the payload increased from zero at the staring time and decreased to zero when the trolley reached its destination. The maximum swing angle is about 10°. There is no overshoot in trolley motion and residual vibration in payload, that is very important in the operation of cranes.

Moreover, changes in system parameters including payload mass or cable length are also considered in the simulation. The system responses coresponding to changing in cable length and in mass of the payload are given in Figs. 4 and 5, respectively. Fig. 4 shows that in three cases the trolley reaches the desired position in about 6s. The maximum of swing angles corresponding to the case of shortest cable is the largest and that corresponding to the case of longest cable is smallest. Despite the swing angle is sensitive to the changing of cable length, but the trolley reaches the desired position. Moreover, the swing angle is suppressed decreasing to zero when the trolley gets its desired position (Fig. 4). The response of the system to changing of payload is shown in Fig. 5. This figure indicates that the controller is robust against payload change.

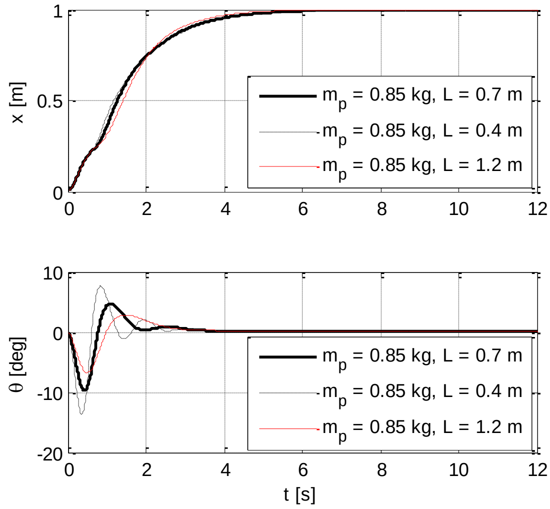

Fig.4 System response by changing length of the cable

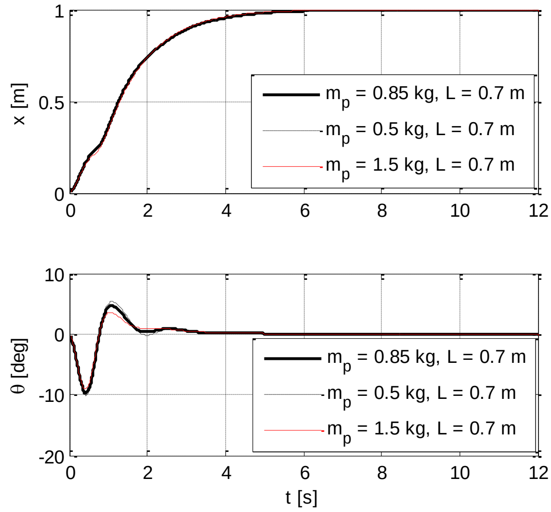

Fig.5 System response by changing weight of the payload

5. Conclusion

This paper presents a combination of a nonlinear PD controller and Nelder-Mead algorithm for underactuated overhead cranes. The main issue solved in this paper is to find a suitable parameters of controllers for the considered system. Nonlinear differential equations of the system including the motion of trolley displacement and payload oscillation has been derived and used for verification of control algorithm. Three parameters of nonlinear PD controller for the system are obtained by using Nelder-Mead algorithm. Simulation results showed that the controller is effective to move the trolley as fast as possible to the desired position, meanwhile the oscillation of the payload is suppressed at the end of the operation. In addition, the results also showed the robustness of the controller against uncertainties in payload and cable length.

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