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Comparative Study on Thresholding Techniques of Discrete Wavelet Transform (DWT) to De-noise Corrupted ECG Signals

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Abstract: ***The electrocardiogram (ECG) signals which are recording of bioelectric potential caused by rhythmical activities are usually corrupted with various sources of noise. In this paper, Discrete Wavelet Transform (DWT) based technique is used to de-noise ECG signals. A comparative study is done on the soft and hard thresholding DWT for several ECG signals for different levels of signal to noise ratio (SNR). The SNR improvement (SNR_{imp}) and Percent Root mean square Difference (PRD (%)) is analyzed. The results show that the soft-thresholding technique performs better than the hard-thresholding to de-noise ECG signal***

Keywords: ECG, Discrete Wavelet Transform (DWT), Soft thresholding, Hard thresholding.

1. Introduction

Biomedical signals reflect the nature and activities of different physiological processes. ECG reflects the state of heart and hence is like a pointer to the health conditions of a human being. The ECG is essential for diagnosis, patient monitoring and therefore management of abnormal cardiovascular activity. ECG signal is corrupted by noise or unwanted signal which makes it difficult to analyze. The typical sources of noise are, high frequency noise, motion artifacts in ECG, maternal interference in fetal ECG, EMG noise, instrumentation noise etc. Therefore de-noising the ECG signal is a pre-requisite to arrive at proper diagnosis by analyzing it.

A number of methods have been applied to de-noise ECG signals such as, digital filters, ICA, PCA, adaptive filtering etc. The existing de-noising techniques have certain limitations. The filter bank based de-noising process smoothes the P and R amplitude of the ECG signal, and it is more sensitive to different levels of noise [1]; The statistical model derived in PCA, ICA is not only fairly arbitrary but also extremely sensitive to small changes in either the signal or the noise unless the basis functions are trained on a global set of ECG beat types, moreover, the ICA doesn't allow the prior information about the signals for efficient filtering; adaptive filtering requires reference signal information for the effective filtering process, and the reference signal has to be additionally recorded together with ECG [2].

In this paper, technique based on discrete wavelet transform (DWT) is used to de-noise ECG signals. As ECG is a non-stationary signal and requires good time-frequency resolution to be de-noised wavelet transform technique overcomes the limitations of the previous de-noising methods.

2. Wavelet Theory

A wavelet is a wave-like oscillation with an amplitude that starts out at zero, increases, and then decreases back to zero [3]. So, it's a waveform of effectively limited duration that has an average value of zero. Compared with sine waves, which are the basis of Fourier analysis, they do not have limited duration- they extend from minus to plus infinity. Moreover, sinusoids are smooth and predictable, whereas wavelets tend to be irregular and asymmetric. Fourier analysis consists of breaking up a signal into sine waves of various frequencies; similarly, wavelet analysis is the breaking up of a signal into shifted and scaled versions of the original or mother wavelet.

2.1 From Fourier to Wavelet Transform

In 19th century, the French mathematician J.Fourier, showed that any periodic function can be expressed as an infinite sum of periodic complex exponential functions. Many years after he had discovered this remarkable property of functions, his ideas were generalized to first non-periodic functions, and then periodic or non-periodic discrete time signals. It is after this generalization that it became a very suitable tool for computer calculations. In 1965, a new algorithm called Fast Fourier Transform (FFT) was developed and Fourier Transform (FT) became even more popular [4].

The definition of FT is given by,

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt \tag{1}$$

$$f(t) = \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega \tag{2}$$

The information provided by the integral, corresponds to all time instances, since the integration is from minus infinity to plus infinity over time. This is why Fourier transform is not suitable if the signal has time varying frequency, i.e., the signal is non-stationary. This means that the FT tells whether a certain frequency component exists or not. This information is independent of where in time this component appears.

Therefore a linear time frequency representation called Short Fourier Transform (STFT) was introduced. In STFT, the signal is divided into small enough segments, where these segments (portions) of the signal can be assumed to be stationary. For this purpose, a window function is chosen. The width of this window must be equal to the segment of the signal where its stationarity is valid.

The following definition of the STFT summarizes all the above explanations in one line:

$$STFT(l, w) = \int_t \left[f(t) w^* (t-l)\right] e^{-j\omega t}\, dt \tag{3}$$

Where, w is a window function.

The important feature of STFT is the width of the window that is used. The width is also called the support of the window. The narrower we make the window, the better the time resolution, and better the assumption of stationarity, but poorer the frequency resolution and vice versa.

The problem with STFT is the fact whose roots go back to what is known as the *Heisenberg's Uncertainty Principle.* This principle originally applied to the momentum and location of moving particles can be applied to time-frequency information of a signal. Simply, this principle states that one cannot know the exact time-frequency representation of a signal, i.e., one cannot know what spectral components exist at what instances of times. What one can know are the time intervals in which certain bands of frequencies exist, which is a resolution problem.

Therefore the problem is to choose a window function, once and for all, and use that window in the entire analysis. The answer, of course, is application dependent. If the frequency components are well separated from each other in the original signal, then some frequency resolution can be sacrificed and may go for good time resolution, since the spectral components are already well separated from each other. However, if this is not the case, then it is a very difficult to find a good window function.

Although the time and frequency resolution problems are results of a physical phenomenon (the Heisenberg's uncertainty principle) and exist regardless of the transform used, it is possible to analyze any signal by using the alternative approach of wavelet transform (WT). WT analyses the signal at different frequencies with different resolutions. Every spectral component is not resolved equally as was the case in the STFT.

WT is designed to give good time resolution and poor frequency resolution at high frequencies and good frequency resolution and poor time resolution at low frequencies. This approach makes sense especially when the signal at hand has high frequency components for short durations and low frequency components for long durations, which is the case in most biological signals, mainly EEG, EMG and ECG.

2.2 Signal De-noising by Discrete Wavelet Transform

In wavelet transform, a signal is analyzed and expressed as a linear combination of the sum of the product of the wavelet coefficients and mother wavelet, as discussed in the previous sections. *Donoho* and *Johnstone* [5,6] proposed wavelet thresholding de-noising method based on discrete wavelet transform (DWT) with universal threshold which is suitable for non-stationary signals such as ECG.

The time-frequency representation of DWT is performed by repeated filtering of the input signal with a pair of low pass filter (LPF) and high pass filter (HPF) [7]. The coefficient corresponding to the low pass filter is called as Approximation Coefficients (CA) and similarly, high pass filtered coefficients are called as Detailed Coefficients (CD). Furthermore, the CA is consequently divided into new approximation and detailed coefficients. This decomposition process is carried out until the required frequency response is achieved from the given input signal.

3. De-noising Algorithm

In practice, the raw signal acquired using data acquisition system is expressed by *X(n)*,

$$X(n) = s(n) + u(n) \tag{4}$$

In assumption, the raw signals are usually contaminated with noise as shown in equation (4), where *s(n)* is the useful signal and *u(n)* is the noise information, which includes all (power line interference, baseline wandering, high frequency noises, etc) sources of noises. In order to separate noises, the de-noising algorithm is given below [7,8].

Step 1: Decompose the input signal using DWT: Choose a wavelet and determine the decomposition level of a wavelet transform *N*, then implement *N* layers wavelet decomposition of signal *S*.

Step 2: Select the thresholding method and thresholding rule for quantization of wavelet coefficients. Normally those wavelet coefficients with smaller magnitudes than the preset threshold are caused by the noise and are replaced by zero. And, the others with larger magnitudes than the preset threshold are caused by original signal mainly and kept (hard-thresholding case) or shrunk (soft-thresholding).

Step 3: Finally, the de-noised signal is reconstructed without affecting any features of signal of interest. The reconstruction is done by Inverse Discrete Wavelet Transform (IDWT) of various wavelet coefficients for each decomposition level.

3.1 Wavelet Thresholding

Wavelet thresholding is the signal estimation technique that exploits the capabilities of signal de-noising. Thresholding method is categorized into two types such as hard thresholding and soft thresholding. Before definition, the thresholding rule is discussed.

3.2 Thresholding Rules

Donoho has initially proposed the fixed thresholding based denoising of signals and images [9].For each level a threshold value is found, and it is applied for the detailed coefficient *CD*. Now, σ is estimated by the wavelet coefficients as,

$$\sigma = \frac{(median\ (|CD|))}{0.6745} \tag{5}$$

Where, median ($|CD|$) denotes the median value of wavelet coefficients *CD*.

Now, the preset threshold is,

$$T_j = \sigma \sqrt{2 \log \|CD\|} \tag{6}$$

The hard-thresholding and soft-thresholding method are as following.

3.3 Hard-thresholding Method

According to the hard-thresholding method, hard-thresholding function w_{ht} is defined as,

$$w_{ht} = \begin{cases} CD, & |CD| \geq T_j \\ 0, & |CD| \leq T_j \end{cases} \tag{7}$$

3.4 Soft-thresholding Method

According to the soft-thresholding method, soft-thresholding function w_{st} is defined as,

$$w_{st} = \begin{cases} sgn(CD)\left(\left|CD - T_j\right|\right), & |CD| \geq T_j \\ 0, & |CD| \leq T_j \end{cases} \tag{8}$$

4. Implementation of the Algorithm

The de-noising algorithm is applied on ECG signals, which were taken from the Department of Biomedical Physics and Technology, University of Dhaka. To observe the versatility of the technique the algorithm is implemented on ECG signals of all 12 leads, and for different levels of input SNR. Fig. 1 and 2 show an uncorrupted signal and corrupted noisy signal.

The work by *Donoho* and *Johnstone* gives a better understanding of how wavelet transforms work. This new understanding combined with nonlinear processing solves currently problems and gives the potential of formulating and solving completely new problems [10]. The method is based on taking the discrete wavelet transform (DWT) of a signal, passing this transform through a threshold; those wavelet coefficients with smaller magnitudes than the preset threshold are caused by the noise and replaced by zero, and the others with larger magnitude than the preset threshold are caused by the signal mainly and kept (hard-thresholding case) or shrunk (soft-thresholding) case; then taking the inverse DWT (IDWT).

These results were carried out for the same signal. This able to remove noise and achieve high compression ratio because of the concentrating ability of the wavelet transform. If a signal has its energy concentrating in a small number of wavelet dimensions, its coefficients will be relatively large compared to any other signal or noise that has its energy spread over a large number of coefficient. This means that thresholding or shrinking the wavelet transform will remove the low amplitude noise or undesired signals and any noise overlap as little as possible in the frequency domain and linear time-invariant filtering will approximately spare them. It is the localizing or concentrating properties of the wavelet transform that make it particularly effective when used with this nonlinear method.

The working methodology of the de-noising process is as following.

4.1 Decomposition of the Noisy Signal

The noisy signal is decomposed into five levels of WT by using by using Daubechies wavelet (db4). The process of decomposition by WT is actually repeatedly filtering of the input signal with a pair of low pass filter (LPF) and high pass filter (HPF). The coefficient corresponding to the low pass filter is called as Approximation Coefficients (CA) and similarly, high pass filtered coefficients are called as Detailed Coefficients (CD). Furthermore, the CA is consequently divided into new approximation and detailed coefficients. So, decomposition level of 5 yields the approximation coefficients CA1, CA2, CA3, CA4 and CA5; and detailed coefficients CD1, CD2, CD3, CD4 and CD5. The coefficients of noisy ECG signal with 10 db SNR after WT is shown in Fig. 3

4.2 Obtaining Estimated Threshold Function

To obtain the estimated threshold *Donoho*'s global thresholding rule is used. The hard and soft thresholding rules to obtain estimated threshold functioin w_{ht} and w_{st} respectively are discussed in section 3.3 and 3.4. The estimated threshold function at each level is obtained from the corresponding detailed coefficient obtained from that very level.

4.3 Reconstruction of the Signal

After obtaining the estimated threshold functions w_{ht1}- w_{ht5} and w_{st1}- w_{st5} from hard and soft-thresholding, the signal is reconstructing by applying inverse discrete wavelet transform (IDWT) on these estimated threshold functions and the approximate coefficient obtained at the last level of decomposition *CA5*.

5. Result and Discussion

The performance measures are as following,

(a) Improvement in Signal to Noise Ratio ,

$$SNR_{imp} = 10 \log_{10} \frac{\sum_{n=1}^{N} |y[n] - x[n]|^2}{\sum_{n=1}^{N} |\hat{x}[n] - x[n]|^2}$$

Where, *x[n]* denotes the original ECG signal,

$y[n]$ denotes the noisy ECG signal, and
$\hat{x}[n]$ denotes the reconstructed de-noised ECG signal
(b) Percent Root Mean Square Difference,

$$PRD = \sqrt{\frac{\sum_{n=1}^{N}(\hat{x}[n]- x[n])^2}{\sum_{n=1}^{N} x^2[n]}} \times 100$$

The mean values of the measures for different values of input SNR, for both soft and hard-thresholding definition for all 12 leads are shown in the table.

The results show that the output SNR of the ECG signals is reasonably improved by de-noising by implementing the DWT-based de-noising technique.. The PRD is also decreased to a satisfactory value, which reveals the applicability of the algorithm in real-world environment effectively to de-noise non-stationary signals like ECG.

6. Conclusion

DWT is a very effective method to decompose non-stationary signals and it overcomes the limitations of de-noising method previously used to some extent. . However, when it comes to de-noise ECG signal, direct de-noising can't be done because it degrades the quality of the signal. In this paper, a comparative study is done on the thresholding definitions of DWT to de-noise ECG signal. The techniques have been proved to be quite good-performing. However the soft-thresholding method has better performance than the hard-thresholding because hard-thresholding leads to oscillation of reconstructed ECG signals. It is required to implement the algorithms for signals in noisy environment of the real world. If the developed algorithm can de-noise the noisy signals successfully enough, then the algorithm can be used in micro-controller chips to use the technique in ECG recorder machines through integrated circuits (ICs).

7. Figures and Tables

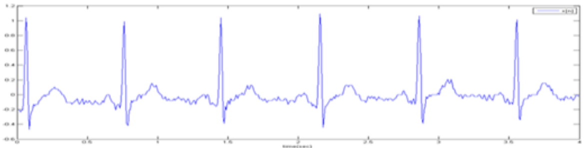


Figure 1: Original Uncorrupted Signal

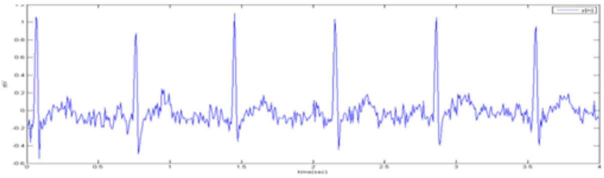
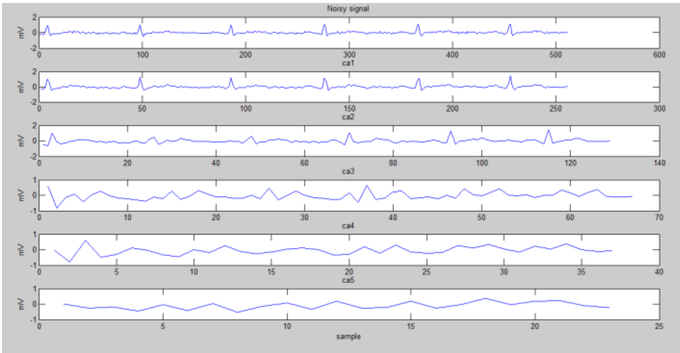
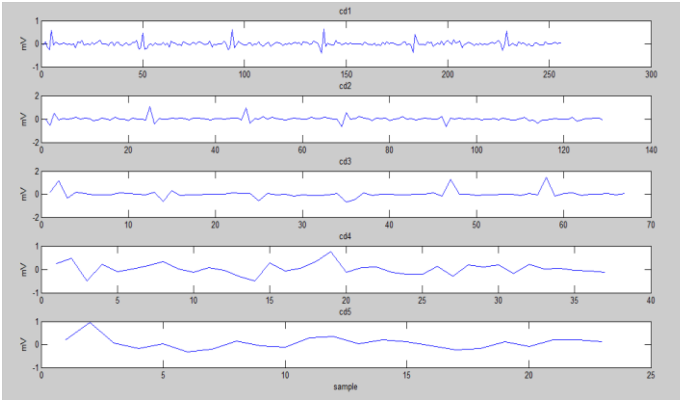


Figure 2: Signal Corrupted by Noise with SNR 10 db



(a)



(b)

Figure 3: (a) Approximate and (b) Detailed Coefficients of Noisy ECG Signal

Table: Results

Input SNR	*SNR_{imp} for Soft*	*SNR_{imp} for Hard*	*PRD for Soft*	*PRD for Hard*
5 db	1.4894	1.4348	6.4776	6.2685
10 db	2.1146	1.4047	5.777	5.3957
15 db	5.5958	4.1343	5.8158	5.4026
20 db	9.9297	8.6546	5.6548	5.4260

References

[1] L. Sharma, S. Dandapat, A. Mahanta, "Multiscale Wavelet Energies and Relative Energy Based De-noising of ECG Signal," In Communication Control and Computing Technologies, IEEE International Conference, pp. 491 –495, 2010.
[2] T. He, G. Clifford and L. Tarassenko, "Application of ICA in Removing Artifacts from the ECG," Neural Processing Letters, pp. 105–116, 2006.
[3] http://en.wikipedia.org/wiki/Wavelet
[4] I. Daubechies, "Ten Lectures on Wavelets" CBMS-NSF Regional Series in Applied Mathematics, SIAM, Philadelphia, 1992.

[5] D. L. Donoho, L. M. Johnstone, “Ideal Spatial Adaptation via Wavelet Shrinkage,” Biometrica, Vol. 81, pp. 425-455, 1994.
[6] D. L. Donoho, “De-noising by Soft-thresholding,” IEEE Trans. Inform Theory, Vol. 41, pp. 613-627, 1995.
[7] P. Karthikeyan, M. Murugappan, and S.Yaacob, “ECG Signal De-noising Using Wavelet Thresholding Techniques in Human Stress Assessment,” International Journal on Electrical Engineering and Informatics - Volume 4, Number 2, July 2012.
[8] A. R. Al-qawasmi, K. Daqrouq, “ECG Signal Enhancement Using Wavelet Transform,” WSEAS TRANSACTIONS on BIOLOGY and BIOMEDICINE.
[9] S. Poornachandra, N. Kumaravel, “A novel method for the elimination of power line frequency in ECG signal using hyper shrinkage function,” Digit. Signal Process, Vol. 18, pp. 116-126, 2008.
[10] M. Merri, M. Alberto M., and A. J. Moss, “Dynamic analysis of ventricular representation Duration from 24-hour recording,” IEEE Transactions on biomedical engineering, Vol. 40, No. 12.